

LOGARITHMS

①

(Formulas, Characteristic and Mantissa)

Class: VII, Mathematics

(F⁺)

TEACHING TASK

JEE MAINS LEVEL

01. $\log_{10} 0.001 = x$ | $x = -3$

$$\Rightarrow 0.001 = 10^x$$
$$\Rightarrow 10^{-3} = 10^x$$

Ans: A

02. $\log_{\sqrt{a^2-b^2}}(a+b)(a-b) = \log_{\sqrt{a^2-b^2}} a^2 - b^2 = 1$

Ans: A

03. $0.0001 = 10^x$

$$\Rightarrow 10^{-4} = 10^x$$

Ans: A

04. $\log_3 [\log_3 (\log_5 b)] = 0$

$$\Rightarrow \log_3 (\log_5 b) = 3^0 = 1$$
$$\Rightarrow \log_5 b = 3^1$$
$$\Rightarrow b = 5^3 = 125$$

Ans: B

05. $\log_2 (x^2 - 3x) = 10^0 = 1$

$$\Rightarrow x^2 - 3x = 2$$

$$\Rightarrow x^2 - 3x - 2 = 0$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9+8}}{2} = \frac{3 \pm \sqrt{17}}{2}$$

Ans: A



②

$$\begin{aligned}
 06. \log_{10} x &= -3.5196 \\
 &= -3 - 0.5196 \\
 &= -3 - 1 + 1 - 0.5196 \\
 &= -4 + 4804 \\
 &= \overline{4} - 4804
 \end{aligned}$$

Ans: C

$$\begin{aligned}
 07. & 2 \log 5\sqrt{3} + 3 \log 2 - \log 2 - \log 50 - \log 3 \\
 &= \log (5\sqrt{3})^2 + \log 2^3 - \log 2 - \log 50 - \log 3 \\
 &= (\log 75 + \log 8) - (\log 2 + \log 50 + \log 3) \\
 &= \log \frac{75 \times 8}{2 \times 50 \times 3} = \log 2
 \end{aligned}$$

Ans: D

$$\begin{aligned}
 08. \log_{\frac{1}{3}} 27 &= a \Rightarrow 27 = a^{\frac{1}{3}} \Rightarrow a = 3 \\
 \log_2 16 &= \log_2 2^4 = 4. \\
 \text{Now, opt. A: } 6 \times \left(\frac{3+a}{3a} \right) &= 6 \times \frac{6}{3 \times 3} = 4
 \end{aligned}$$

Ans: A

$$\begin{aligned}
 09. \text{Let } N &= 3^{40} \\
 \Rightarrow \log N &= 40 \times \log 3 \\
 &= 40 \times 0.477 \\
 &= 19.08
 \end{aligned}$$

∴ Characteristic of N is 19.
Hence N contains 20 digits

Ans: A

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$$10. \quad a^x = b^y = c^z = d^w$$

$$\Rightarrow a^x = b^y \quad \text{Similarly}$$

$$\Rightarrow x \log a = y \log b$$

$$\log c = \frac{x}{x}$$

$$\Rightarrow \frac{x}{y} = \frac{\log b}{\log a}$$

$$\log d = \frac{x}{w}$$

$$\therefore \log b = \frac{x}{y}$$

$$\therefore \log bcd = \log a^x + \log c + \log d$$

$$= x \left(\frac{1}{y} + \frac{1}{x} + \frac{1}{w} \right)$$

Ans: C

$$11) \quad \log_5 x^2 + 9 = 2$$

$$\Rightarrow x^2 + 9 = 5^2 = 25$$

$$\Rightarrow x = \pm 4$$

Ans: A, C

$$12. \quad \log_5 \frac{15}{\sqrt{2}} = 2$$

$$2 \log_2 15 = 2$$

$$\log_2 15^2 = 2$$

$$= 225$$

$$= \sqrt[3]{225^3}$$

Ans: A, B

$$13 \text{ Statement I: } \sqrt[5]{a^x} = y$$

$$\Rightarrow a^{\frac{x}{5}} = y$$

$$\Rightarrow \frac{x}{5} \log a = \log y$$

$$\Rightarrow x \log a = 5 \log y \quad (\text{True})$$

Statement II: Conceptual (True)

Ans: A

14. Statement I: Conceptual (~~True~~) (False) (4)
 Statement II: Conceptual (True) Ans: D

$$\begin{aligned}
 15. \quad & \frac{1}{\log_{a^2} abc} + \frac{1}{\log_{b^2} abc} + \frac{1}{\log_{c^2} abc} \\
 &= \log_{abc} a^2 + \log_{abc} b^2 + \log_{abc} c^2 \\
 &= \log_{abc} (abc)^2 = 2 \times \log_{abc} abc \\
 &= 2 \times 1 = 2 \quad \text{Ans: A}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \log_b^a \times \log_c^b \times \log_d^c \times \log_{a-1}^d \\
 &= -1 \cdot \log_b^a \times \log_c^b \times \log_d^c \times \log_{a-1}^d = -1 \times 1 \\
 &= -1 \quad \text{Ans: C}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \log_2^{\frac{-1}{3}} = \frac{-1}{3} \\
 & \Rightarrow \log_2^2 = 1 \\
 & \Rightarrow x = 2 \quad \text{Ans: 2}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \log_2 \{ \log_3 (\log_2 x) \} = 1 \\
 & \Rightarrow \log_3 (\log_2 x) = 2 \\
 & \Rightarrow \log_2 x = 9 \\
 & \Rightarrow x = 2^9 = 512 \quad \text{Ans: 512}
 \end{aligned}$$

(5)

$$19 \quad a) \log \sqrt[3]{x} = a \quad \Rightarrow \frac{1}{3} \cdot \log x = a$$

$$\Rightarrow \log x^{\frac{1}{3}} = a \quad \Rightarrow a = \frac{1}{3} : \sqrt{a} = \sqrt{\frac{1}{3}}$$

$$d) a = \frac{1}{10}$$

$$b) \log \sqrt[5]{a} = x$$

$$\Rightarrow x = \frac{1}{4}$$

$$\Rightarrow \sqrt[3]{x} = \sqrt[3]{\frac{1}{4}}$$

$$c) \log \sqrt[3]{a} = m$$

$$\Rightarrow m = \frac{1}{3}$$

$$\Rightarrow m^2 = \frac{1}{9}$$

$$a^2 = \frac{1}{100}$$

Ans: x, t, s, p

$$20 \quad a) \log (0.2)^3 = 3 \quad c) \frac{\log 10 \times \log 2^9}{\log 2} = 4 \log 10$$

$$b) \log_2^2 = \frac{1}{3}$$

$$d) \log_{6^{\frac{1}{2}}} 6^{\frac{3}{2}} = \frac{\frac{3}{2}}{\left(\frac{1}{2}\right)} = 6$$

Ans: x, p, -, s

LEARNERS TASK

Ques

$$01. \log_2 a = 2$$

$$\Rightarrow a = 2^2$$

$$\Rightarrow a = 4$$

$$\therefore \log_4 = \log_4^4 = 1$$

Ans: B

6

Ans: D

02.

$$3^{-2} = \frac{1}{9}$$

$$\Rightarrow -2 = \log_3\left(\frac{1}{9}\right)$$

03.

$$\log\left(\frac{1}{x}\right) = \frac{1}{2}$$

$$\Rightarrow \frac{1}{x} = \left(\frac{1}{y}\right)^{\frac{1}{2}}$$

$$\Rightarrow x^{-1} = \left(\frac{1}{y}\right)^{\frac{1}{2}}$$

$$\Rightarrow x^{-1} = \left(y^{-1}\right)^{\frac{1}{2}}$$

Ans: A

04.

$$\log_{25} = \log_{5^2} = 2 \cdot \log_5 = 2 \times 1 = 2$$

Ans: B

05.

$$\log_7 6^{-1} = \frac{-1}{6} = -\frac{1}{6}$$

Ans: B

06.

Conceptual

Ans: C

07.

Conceptual

Ans: B

08.

$$6 \cdot 325 = -6 + 0.325$$

Ans: A

09.

Conceptual

Ans: D

10.

$$\log \sqrt[3]{6} = \log 6^{\frac{1}{3}} = \frac{1}{3} \log 6$$

Ans: D

JEE MAINS LEVEL

01.

$$6^x = 7^{x+4}$$

$$\Rightarrow x \log 6 = (x+4) \log 7$$



⑦

$$\Rightarrow x \log 6 = x \log 7 + 4 \log 7$$

$$\Rightarrow x = \frac{4 \log 7}{\log 6 - \log 7} = \frac{4 \log 7}{\log 2 + \log 3 - \log 7} = \frac{4C}{a+b-c}$$

Ans: B

$$Q2. \log_3 9 = \log_3 3^2 = 2 \cdot \log_3 3 = 2 \times 1 = 2 \quad \text{Ans: A}$$

$$Q3. \text{Let } N = x^y \cdot y^z \cdot z^x$$

$$\log N = x \log x + y \log y + z \log z$$

$$= x \cdot K(y-z) + y \cdot K(z-x) + z \cdot K(x-y)$$

$$= 0$$

$$\therefore N = 1$$

Ans: C

$$Q4. a = \log_{12} 18 \quad \left| \quad b = \frac{\log_2 54}{\log_2 24} = \frac{1 + 3 \log_2 3}{3 + \log_2 3} \right.$$

$$= \frac{\log_2 18}{\log_2 12} = \frac{1 + 2 \log_2 3}{2 + \log_2 3}$$

$$\text{Let } \log_2 3 = t,$$

$$\therefore a = \frac{1 + 2t}{2 + t} \quad \left| \quad b = \frac{1 + 3t}{3 + t} \right. \quad \left. t = \frac{3b-1}{3-b} \right.$$

$$\Rightarrow t = \frac{2a-1}{2-a}$$

$$\Rightarrow \frac{2a-1}{2-a} = \frac{3b-1}{3-b}$$

$$\Rightarrow ab + 5(a-b) = 1$$

Ans: C

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05. $a^2 + b^2 = 7ab$

$\Rightarrow a^2 + b^2 + 2ab = 9ab$

$\Rightarrow (a+b)^2 = 9ab$

$\Rightarrow 2 \log(a+b) = \log 9 + \log ab$

$\Rightarrow 2 \log(a+b) = 2 \log 3 + \log ab$

$\Rightarrow 2 \left[\log \left(\frac{a+b}{3} \right) \right] = \log ab$

$\Rightarrow \log \left(\frac{a+b}{3} \right) = \frac{1}{2} \log ab$

Ans: B

06. $a = b^x, b = c^y, c = a^z$

$\Rightarrow \log a = x \log b, \log b = y \log c, \log c = z \log a$

Ans: A

$\Rightarrow xyz = 1$

07. $\log_{49} \sqrt{7} = p$

$p = \frac{1}{14}$

$\Rightarrow \log_{7^2} 7^{1/2} = p$

$\Rightarrow \frac{\log 7}{2} = p$

Ans: B

08. $x = 10^a$

$= 10^{3a-2}$

$= \frac{(10^a)^3}{10^2} = \frac{x^3}{100}$

Ans: B

09.	$m = 100^y \quad n = 100^m$ $m+n = 100^y + 100^m$ $m+n = \log_{100} 100^m + \log_{100} 100^n$ $= \log_{100} mn$	Ans: D
10.	$\left(\frac{x^2}{y^2}\right)^{\log x^2} = \log x^2$ $\Rightarrow \log \frac{x^2}{y^2} = \log x^2$ $\Rightarrow y = x$	Ans: B
11.	$\log_{10} (a+b)^2 = \log_{10} (a+b)$ $\Rightarrow (a+b)^2 = a+b$ $\Rightarrow a+b = 1$ $\Rightarrow a = \frac{1}{2}, b = \frac{1}{2}$	$a^3 + b^3 = \frac{1}{8} + \frac{1}{8}$ $= \frac{1}{4}$ $= \sqrt[3]{\frac{1}{64}}$
12.	Statement I: $a \cdot \log_m = \log_{m^2} = \log_m$ $\Rightarrow a = \log_m \times \log_m$ Statement II: Conceptual (True)	Ans: B, D
13.	Conceptual	Ans: C
14.	$\log_{2\sqrt{3}} 1728 = \log_{2\sqrt{3}} 2^6 \times 3^3 = \log_{(2\sqrt{3})^6} (2\sqrt{3})^6 = 6$	Ans: 6

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(a)

$$x \log a = y \log b$$

$$\frac{y \log a}{a} = \frac{x}{b} \Leftrightarrow$$

$$y \log a = x \log b \quad (a)$$

$$\frac{x \log a}{a} = \frac{y}{b} \Leftrightarrow$$

$$c) a \log x = y \log b$$

$$\frac{x \log a}{a} = \log \frac{b}{y} \Rightarrow$$

$$d) \frac{y \log a}{a} = \log \frac{b}{x} \Rightarrow$$

$$\frac{y \log a}{a} = \frac{x}{b} \Rightarrow$$

$\Rightarrow$ THE END $\in$

Ans: t, a, s, x

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