

BASIC DIFFERENTIATION

Class: IX, Mathematics ①

SOLUTIONS (A)

TEACHING TASK

$$01. \frac{d}{dx} \left(3\sqrt[3]{x} - \frac{4}{\sqrt{x}} \right)$$

$$= 3 \cdot \frac{d}{dx} \left(x^{\frac{1}{3}} \right) - 4 \frac{d}{dx} \left(x^{-\frac{1}{4}} \right)$$

$$= 3 \cdot \frac{1}{3} \cdot x^{\frac{1}{3}-1} - 4 \cdot \left(-\frac{1}{4} \right) \cdot x^{-\frac{1}{4}-1}$$

$$= x^{-2/3} + x^{-5/4}$$

Ans. B

$$02. f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$$

$$\Rightarrow f(x) = \sqrt{a} \cdot \sqrt{x} + \frac{a^2}{\sqrt{a} \cdot \sqrt{x}}$$

$$\Rightarrow f'(x) = \sqrt{a} \cdot \frac{1}{2\sqrt{x}} + \frac{a^2}{\sqrt{a}} \cdot \left(\frac{-1}{2x^{3/2}} \right)$$

$$\therefore f'(a) = \sqrt{a} \cdot \frac{1}{2\sqrt{a}} - \frac{a^2}{2\sqrt{a} \cdot a\sqrt{a}} = 0 \quad \text{Ans. B}$$

$$03. f(x) = x - x^2 + x^3 - x^4 + \dots \infty$$

$$f'(x) = 1 - 2x + 3x^2 - 4x^3 + \dots$$

$$f'(x) = (1+x)^{-2} \quad \therefore f'(x) = \frac{1}{(1+x)^2} \quad \text{Ans. B}$$

04. $x^4 + 7x^2y^2 + 9y^4 = 24xy^3$ (2)

$$\Rightarrow 4x^3 + 7 \left[x^2 \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot 2x \right] + 9 \cdot 4y^3 \frac{dy}{dx}$$

$$= 24 \left[x \cdot 3y^2 \cdot \frac{dy}{dx} + y^3 \cdot 1 \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

Ans. B

05. $f(x) = \frac{1}{1 + \frac{1}{x}} \quad \left| \quad g(x) = \frac{1}{1 + \left(\frac{x+1}{x}\right)} = \frac{x}{2x+1} \right.$

$$f(x) = \frac{x}{x+1}$$

$$\therefore g'(x) = \frac{(2x+1) \cdot 1 - \cancel{x} \cdot 2}{(2x+1)^2}$$

$$g'(2) = \frac{(2 \cdot 2 + 1) - 2 \cdot 2}{(2 \cdot 2 + 1)^2} = \frac{1}{25} \quad \text{Ans. B}$$

06. $y = 2^x \cdot 3^{2x} \cdot 5^{-x} \cdot 7^{-x}$

$$y = \frac{2^x \cdot (3^2)^x}{5^x \cdot 7^x} \Rightarrow y = \left(\frac{2 \cdot 3^2}{5 \cdot 7} \right)^x$$

$$\therefore \frac{dy}{dx} = \left(\frac{2 \cdot 3^2}{5 \cdot 7} \right)^x \cdot \log \left(\frac{2 \cdot 3^2}{5 \cdot 7} \right)$$

$$\frac{dy}{dx} = \frac{2^x \cdot 3^{2x}}{5^x \cdot 7^x} \cdot \log \left(\frac{18}{35} \right)$$

Ans. A

(3)

07.
$$\frac{d}{dx} \left(\frac{1+x^2+x^4}{1+x+x^2} \right)$$

$$= \frac{d}{dx} \left(\frac{(x^2+1)^2 - x^2}{1+x+x^2} \right)$$

$$= \frac{d}{dx} \left(\frac{(x^2+1+x)(x^2+1-x)}{1+x+x^2} \right)$$

$$= 2x-1 = ax+b$$

$$\therefore (a,b) = (2,-1)$$

Ans. C

08. $y = x \cdot \tan \frac{x}{2}$

$$\Rightarrow \frac{dy}{dx} = x \cdot \sec^2 \left(\frac{x}{2} \right) \cdot \frac{1}{2} + \tan \frac{x}{2} \cdot 1$$

$$\Rightarrow = \frac{x}{2 \cos^2 \left(\frac{x}{2} \right)} + \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}$$

$$= \frac{x + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}}$$

$$= \frac{x + \sin x}{1 + \cos x}$$

$$\therefore (+\cos x) \frac{dy}{dx} - \sin x = x$$

Ans. D

09.
$$(x+1)(x^2+1)(x^4+1)(x^8+1)$$

$$= \frac{(x-1)(x+1)(x^2+1)(x^4+1)(x^8+1)}{x-1} = \frac{x^{16}-1}{x-1}$$



$$\frac{d}{dx} \left(\frac{x^{16}-1}{x-1} \right) = \frac{(x-1)(16 \cdot x^{15}) - (x^{16}-1)(1)}{(x-1)^2} \quad (4)$$

$$\Rightarrow (16 \cdot x^{16} - 16x^{15} + x^{16} + 1)(x-1)^{-2}$$

$$= (15x^{16} - 16x^{15} + 1)(x-1)^{-2}$$

$$= (15x^{16} - 16x^{15} + 1)(x-1)^{-2}$$

$$\Rightarrow (p, q) = (16, 15)$$

Ans: D

$$10. \frac{d}{dx} \left[\log \left(\frac{x}{e^{\tan x}} \right) \right]$$

$$= \frac{d}{dx} \left[\log (x \cdot e^{-\tan x}) \right]$$

$$= \frac{1}{x \cdot e^{-\tan x}} \left(x \cdot e^{-\tan x} (-\sec^2 x) + e^{-\tan x} \cdot 1 \right)$$

$$= -\sec^2 x + \frac{1}{x}$$

Ans: B

$$11. f(x) = \frac{a - b \cos x}{a + b \cos x}$$

$$f'(x) = \frac{(a + b \cos x)(b \sin x) - (a - b \cos x)(-b \sin x)}{(a + b \cos x)^2}$$

$$f'(x) = \frac{2ab \sin x}{(a + b \cos x)^2} \quad \left| \begin{array}{l} f'(0) = 0 \\ f'(\pi) = 0 \end{array} \right.$$

$$f'\left(\frac{\pi}{2}\right) = \frac{2b}{a}$$

Ans: A, B, C

12. $f(x) = x \cdot \sin x$

(5)

$$f'(x) = x \cos x + \sin x$$

$$f'(0) = 0, \quad f'\left(\frac{\pi}{2}\right) = 1, \quad f'(\pi) = -\pi$$

Ans: A, B, C

13. Statement I: $y = e^x + 3 \log x$

$$y = e^x \cdot e^{3 \log x}$$

$$= e^x \cdot e^{\log x^3}$$

$$= x^3 \cdot e^x$$

$$\therefore \frac{dy}{dx} = x^3 \cdot e^x + e^x \cdot 3x^2$$

$$= e^x \cdot x^2 (x + 3) \quad (\text{True})$$

Statement II: Conceptual (True)

Ans: A

14. Statement I: $\frac{d}{dx} (1 - 2x + 3x^2 - 4x^3 \dots)$

$$= \frac{d}{dx} ((1+x)^{-2}) = (-2)(1+x)^{-3}$$

$$= \frac{-2}{(1+x)^3} \quad (\text{True})$$

Statement II: Conceptual (True)

Ans: A

15. $y = \exp \{ \cos^2 x + \cos^4 x + \cos^6 x + \dots \}$

$$y = \exp \left\{ \frac{\cos^2 x}{1 - \cos^2 x} \right\}$$

$$y = \exp\{\tan^{-1} x\} \quad \left| \quad \frac{dy}{dx} = \cancel{e^{\tan^{-1} x}} \cdot 2 \tan x \cdot \sec^2 x \right.$$

$$y = e^{\tan^{-1} x} \quad \left| \quad \frac{dy}{dx} = e^{\cot^{-1} x} \cdot 2 \cot x \cdot (-\operatorname{cosec}^2 x) \right.$$

(6)

$$y = \exp(\cot^{-1} x) \quad \left| \quad \frac{dy}{dx} = e^{\cot^{-1} x} \cdot 2 \cot x \cdot (-\operatorname{cosec}^2 x) \right.$$

$$y = e^{\cot^{-1} x} \quad \left| \quad \frac{dy}{dx} = -2 \cdot e^{\cot^{-1} x} \cdot \cot x \cdot \operatorname{cosec}^2 x \right.$$

Ans: C

16. $y = \exp\{\sin^2 x + \sin^4 x + \sin^6 x + \dots \infty\}$

$$y = \exp\left\{\frac{\sin^2 x}{1 - \sin^2 x}\right\}$$

$$y = \exp\{\tan^2 x\}$$

$$y = e^{\tan^2 x}$$

$$y = e^{\tan^2 x} \cdot 2 \tan x \cdot \sec^2 x$$

Ans: —

17. $\frac{d}{dx} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right]$

$$= \frac{x}{2} \cdot \frac{1}{2 \sqrt{a^2 - x^2}} \cdot (-2x) + \sqrt{a^2 - x^2} \cdot \frac{1}{2} +$$

$$\frac{a^2}{2} \cdot \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} \cdot \frac{1}{a}$$

$$\therefore \frac{dy}{dx} = \sqrt{a^2 - x^2}$$

Ans: A

$$18. f(x) = \frac{\cos x + \sin x}{\cos x - \sin x}$$

(7)

$$f(x) = \frac{1 + \tan x}{1 - \tan x}$$

$$= \tan\left(\frac{\pi}{4} + x\right)$$

$$f'(x) = \sec^2\left(\frac{\pi}{4} + x\right)$$

Ans. A

$$19. 5^x + 5^y = 5^{x+y}$$

$$\Rightarrow 5^x \cdot \log 5 + 5^y \cdot \log 5 \cdot \frac{dy}{dx} = 5^{x+y} \cdot \log 5 \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow 5^x + 5^y \frac{dy}{dx} = 5^{x+y} \left(1 + \frac{dy}{dx}\right)$$

$$\text{put } x=y=1$$

$$\Rightarrow 5 + 5 \frac{dy}{dx} = 25 \left(1 + \frac{dy}{dx}\right)$$

$$= 1 + \frac{dy}{dx} = 5 + 5 \frac{dy}{dx}$$

$$\Rightarrow 4 \frac{dy}{dx} = -4$$

$$\Rightarrow \frac{dy}{dx} = -1$$

Ans: -1

20.

$$xy = (x+y)^n$$

$$\Rightarrow x \cdot \frac{dy}{dx} + y \cdot 1 = n(x+y)^{n-1} \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow x \left(\frac{y}{x}\right) + y = n(x+y)^{n-1} \left(1 + \frac{y}{x}\right)$$



$$\Rightarrow 2y = n(x+y) \frac{(x+y)}{x} \quad (8)$$

*
20.
*

Given $x^3 y^4 = (x+y)^n$

$$x^3 \cdot 4y^3 \frac{dy}{dx} + y^4 \cdot 3x^2 = n(x+y)^{n-1} \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow 4x^3 y^3 \left(\frac{y}{x}\right) + 3x^2 y^4 = n(x+y)^{n-1} \left(1 + \frac{y}{x}\right)$$

$$\Rightarrow 4x^2 y^4 + 3x^2 y^4 = \frac{n}{x} (x+y)^n$$

$$\Rightarrow 7x^2 y^4 = \frac{n}{x} (x+y)^n$$

$$\Rightarrow 7x^3 y^4 = n(x+y)^n$$

$$\Rightarrow \boxed{n=7}$$

2) (i) $\frac{d}{da}(a^a + a^x) = a \cdot a^{a-1} + a^x \cdot \log a.$

(ii) $\frac{d}{dx}(\sin x^\circ) = \frac{d}{dx}\left(\sin \frac{\pi x}{180}\right) = -\sin \frac{\pi x}{180} \times \frac{\pi}{180}.$
 $= -\frac{\pi}{180} \sin x^\circ$

(iii) $\frac{d}{dx}(\log_a x) = \frac{d}{dx}(\log_e x \times \log_e a)$
 $= \log_e a \times \frac{1}{x} = \frac{1}{x \log_e a}$

$$(iv) \quad 3f(x) - 2f\left(\frac{1}{x}\right) = x \rightarrow (1)$$

(9)

$$x \leftrightarrow \frac{1}{x}$$

$$\Rightarrow 3f\left(\frac{1}{x}\right) - 2f(x) = \frac{1}{x} \rightarrow (2)$$

$$(1) \times 3 \Rightarrow 9f(x) - 6f\left(\frac{1}{x}\right) = 3x \rightarrow (3)$$

$$(2) \times 2 \Rightarrow 6f\left(\frac{1}{x}\right) - 4f(x) = \frac{2}{x} \rightarrow (4)$$

$$(3) + (4) \Rightarrow 5f(x) = 3x + \frac{2}{x}$$

$$\Rightarrow 5f'(x) = 3 - \frac{2}{x^2}$$

$$\therefore f'(2) = \frac{1}{2}$$

Ans t, p, x, s

22

$$(i) \quad \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 6$$

$$\Rightarrow \text{S.O.B.S}$$

$$\Rightarrow \frac{x}{y} + \frac{y}{x} + 2 = 6$$

$$\Rightarrow \frac{x}{y} + \frac{y}{x} = 4$$

$$\Rightarrow \frac{y \cdot 1 - x \cdot \frac{dy}{dx}}{y^2} + \frac{x \cdot \frac{dy}{dx} + y \cdot 1}{x^2} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

$$(ii) \quad y = \tan^{-1}\left(\cot\left(\frac{\pi}{2} - x\right)\right)$$

$$y = \tan^{-1}(\tan x)$$

$$y = x$$

$$\therefore \frac{dy}{dx} = 1.$$

$$(iii) \quad y = \tan^{-1}(\sec x + \tan x)$$

(10)

$$y = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right)$$

$$y = \tan^{-1}\left(\frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)$$

$$\therefore y = \frac{\pi}{4} + \frac{x}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}$$

$$(iv) \quad y = (\log x)^3$$

$$\frac{dy}{dx} = 3(\log x)^2 \cdot \frac{1}{x}$$

Ans: s, p, t, q

LEARNER'S TASK

Ques

01. Conceptual

Ans: B

02. Conceptual

Ans: C

03. Conceptual

Ans: D



04.	Conceptual	(11)	Ans: D
05.	Conceptual		Ans: D
06.	Conceptual		Ans: C
07.	Conceptual.		Ans: D
08.	Conceptual.		Ans: D
09.	Conceptual.		Ans: C
10.	Conceptual		Ans: D

JEE MAINS LEVEL

01.	$\frac{d}{dx} \left(\frac{5x-3}{2x+7} \right) = \frac{ad-bc}{(c^2+d^2)^2} = \frac{5 \cdot 7 - (-3) \cdot 2}{(2x+7)^2}$ $= \frac{41}{(2x+7)^2}$	Ans: C
02.	$y = 10^{-x}$ $\Rightarrow \frac{dy}{dx} = 10^{-x} \cdot \log_{10}(-1) = 10^{-x} \left(\log\left(\frac{1}{10}\right) \right)$	Ans: B
03.	$x\sqrt{1+y} + y\sqrt{1+x} = 0$ $\Rightarrow \cancel{x} \cdot \frac{1}{\cancel{2\sqrt{1+y}}} \cdot \left(\frac{dy}{dx} \right) + \sqrt{1+y} + \cancel{S.O.B.S}$ $\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ $S.O.B.S$ $\Rightarrow x^2(1+y) = y^2(1+x)$ $\Rightarrow x^2 + x^2y = y^2 + xy^2$	

(12)

$$\Rightarrow x^2 - y^2 = xy^2 - x^2y$$

$$\Rightarrow (x+y)(x-y) = xy(y-x)$$

$$\Rightarrow x+y+xy = 0$$

$$\Rightarrow (1+x)y = -x$$

$$\Rightarrow y = \frac{-x}{1+x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

Ans: B

04. $\frac{d}{dx}(\cos \sqrt{x}) = -\sin \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$ Ans: D

05. $\frac{d}{dx}(\cos 3x) = -\sin 3x \cdot 3$ Ans: C

06. $f(x) = \frac{x}{1+x}$
 $f(f(x)) = f\left(\frac{x}{1+x}\right) = \frac{\left(\frac{x}{1+x}\right)}{1 + \left(\frac{x}{1+x}\right)} = \frac{x}{1+2x}$
 $\therefore g(x) = \frac{x}{1+2x}$
 $g'(x) = \frac{(1+2x) \cdot 1 - x \cdot (2)}{(1+2x)^2} = \frac{1}{(1+2x)^2}$ Ans: D

07. $\frac{d}{dx}(ax^2 + bx + c) = 2ax + b$ Ans: C

08. $\frac{d}{dx}[\log(\cos(x^3))] = \frac{1}{\cos(x^3)} \cdot (-\sin(x^3)) \cdot 3x^2$

Ans: B

09. $\frac{d}{dx} \left(\frac{1}{\sqrt{x}} + \sqrt{x} + 2x \right)$ (13)

$$= \frac{-1}{2x\sqrt{x}} + \frac{1}{2\sqrt{x}} + 2$$

Ans: A

10. $\frac{d}{dx} \left(\log x + \frac{1}{x} + \frac{1}{x^2} \right)$

$$= \frac{1}{x} - \frac{1}{x^2} - \frac{2}{x^3}$$

Ans: D

11. $f(x) = (x^2 - 3)(4x^3 + 1)$

$$f(x) = 4x^5 + x^2 - 12x^3 - 3$$

$$f'(x) = 20x^4 + 2x - 36x^2$$

$$= 20x^4 - 36x^2 + 2x$$

$$ax^4 + bx^2 + cx$$

$\therefore a = 20, b = -36, c = 2$

$\therefore a + b + c = -14$

Ans: A, B, C

12. $f(x) = e^x + \sin x \cdot \cos x$

$$f(x) = e^x + \frac{1}{2} \sin 2x$$

$$f'(x) = e^x + \frac{1}{2} \cdot \cos 2x \cdot 2$$

$\therefore f'(x) = e^x + \cos 2x$

Ans: A, C

13. Statement I: $f(x) = xe^x \sin x$

$$f'(x) = 1 \cdot e^x \sin x + x \cdot e^x \sin x + xe^x \cos x$$

~~$- xe^x (\sin x + \cos x + \sin x) (\text{True})$~~

Statement II: (True)

(14)

Ans. A

14. Statement I: $y = \cos(\log x + e^x)$

$$\frac{dy}{dx} = -\sin(\log x + e^x) \cdot \left(\frac{1}{x} + e^x\right) \quad (\text{True})$$

Statement II: Conceptual (True)

Ans. A

15. $f(x) = \frac{2 \sin x + 3}{4 \sin x - 3}$

$$f'(x) = \frac{2 \cdot (-3) - (3) \cdot 4}{(4 \sin x - 3)^2} \times \frac{d}{dx}(\sin x)$$

$\frac{12}{6}$

$$= \frac{-18 \cos x}{(4 \sin x - 3)^2}$$

Ans. B

16. $f(x) = \frac{3e^x - 2}{6e^x - 4}$

$$f'(x) = \frac{3(-4) - (-2) \cdot 6}{(6e^x - 4)^2} \cdot \frac{d}{dx}(e^x)$$

$$= 0$$

Ans. D

17. $f(x) = 2x^2 + 3x - 5$

$$f'(x) = 4x + 3$$

$$\text{Now, } f'(0) + 3f'(-1)$$

$$= [4(0) + 3] + 3[4(-1) + 3]$$

$$= 3 - 12 + 9 = 0$$

Ans. D

18. $f(x) = 1 + x + x^2 + \dots + x^{100}$ (15)

$f'(x) = 1 + 2x + 3x^2 + \dots + 100x^{99}$

$f'(1) = 1 + 2 + 3 + \dots + 100$

$$= \frac{100(100+1)}{2}$$

$$= 5050$$

$S_n = n \frac{(n+1)}{2}$

Ans: D

19. $f(x) = e^x (x^2 + 1)$

$f'(x) = e^x (2x) + (x^2 + 1) \cdot e^x$

$f'(0) = e^0 (2 \cdot 0) + (0^2 + 1) \cdot e^0$

$$= 1$$

Ans: 1

20. $f(x) = \frac{\cos x}{\sin x + \cos x}$

$f'(x) = \frac{(\sin x + \cos x)(-\sin x) - \cos x (\cos x - \sin x)}{(\sin x + \cos x)^2}$

$f'(0) = \frac{(\sin 0 + \cos 0)(-\sin 0) - \cos 0 (\cos 0 - \sin 0)}{(\sin 0 + \cos 0)^2}$

$$= \frac{-1}{1} = -1$$

Ans: -1

21. (i) $\frac{d}{dx} \left(\frac{3x+1}{6x-1} \right) = \frac{3(-1) - (1)(6)}{(6x-1)^2} = \frac{-9}{(6x-1)^2}$

(ii)

$$(ii) \quad y = (1+x^{\frac{1}{4}})(1+x^{\frac{1}{2}})(1-x^{\frac{1}{4}})$$

(16)

$$y = \left[(1)^2 - (x^{\frac{1}{4}})^2 \right] (1+x^{\frac{1}{2}})$$

$$= (1-x^{\frac{1}{2}})(1+x^{\frac{1}{2}})$$

$$y = 1-x$$

$$\therefore \frac{dy}{dx} = -1$$

$$(iii) \quad y = 10^{-x}$$

$$\frac{dy}{dx} = 10^{-x} \cdot \log_{10}(-1)$$

$$= 10^{-x} \log\left(\frac{1}{10}\right)$$

$$(iv) \quad y = \sin^{-1}(x^2)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{d}{dx}(x^2) = \frac{2x}{\sqrt{1-x^4}}$$

Ans: t, p, q, r

$$22 (i) \quad e^x + e^y = e^{x+y}$$

$$\Rightarrow e^x + e^y \cdot \frac{dy}{dx} = e^{x+y} \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow e^x + e^y \frac{dy}{dx} = e^{x+y} + e^{x+y} \frac{dy}{dx}$$

$$\Rightarrow (e^y - e^{x+y}) \frac{dy}{dx} = e^{x+y} - e^x$$

$$\Rightarrow e^y (1 - e^x) \frac{dy}{dx} = e^x (e^y - 1)$$

$$\frac{dy}{dx} = \frac{e^x(e^y - 1)}{e^y(1 - e^x)}$$

(17)

$$= \frac{e^x(e^y - 1)}{e^y(1 - e^x)} = \frac{e^{x+y} - e^x}{e^y - e^{x+y}} = \frac{e^x + e^y - e^x}{e^y - e^x - e^y} = \frac{e^y}{-e^x} = -e^{y-x}$$

$$(ii) \quad 2^x + 2^y = 2^{x+y}$$

$$\Rightarrow 2^x \cdot \log 2 + 2^y \cdot \log 2 \frac{dy}{dx} = 2^{x+y} \cdot \log 2 \left(1 + \frac{dy}{dx}\right)$$

$$\text{when } x=y=1 \quad 2 + 2 \frac{dy}{dx} = 4 \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = -1$$

$$(iii) \quad x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$$

$$\Rightarrow x^2(1+y) = y^2(1+x)$$

$$\Rightarrow x^2 + x^2y = y^2 + xy^2$$

$$\Rightarrow x^2 - y^2 = xy^2 - x^2y$$

$$\Rightarrow (x^2 - y^2) = xy(y - x)$$

$$\Rightarrow (x+y)(x-y) = xy(y-x)$$

$$\Rightarrow x+y+xy=0$$

$$\Rightarrow x + (1+x)y = 0$$

$$\Rightarrow y = \frac{x}{x+1}$$

$$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

$$(iv) \quad y = x \sin(a+y)$$

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$$\Rightarrow x = \frac{y}{\sin(a+y)}$$

$$\Rightarrow 1 = \frac{\sin(a+y) \cdot \frac{dy}{dx} - y \cdot \cos(a+y) \frac{dy}{dx}}{\sin^2(a+y)}$$

$$\Rightarrow 1 = \frac{[\sin(a+y) - y \cos(a+y)] \frac{dy}{dx}}{\sin^2(a+y)}$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y \cos(a+y)}$$

Ans: P, Q, S, -

$\Rightarrow$ THE END $\Leftarrow$