PROGRESSIONS-I

Arithmetic Progression upto nth term

Class: VIII, Mathematics

SOLUTIONS

TEACHING TASK

01. a, b, c AP $\rightarrow 2b = a + c$

$$a^3 + c^3 - 8b^3 = a^3 + c^3 - (2b)^3$$
$$= a^3 + c^3 - (a+c)^3$$
$$= a^3 + c^3 - [a^3 + c^3 + 3ac(a+c)]$$
$$= -3ac(2b)$$
$$= -6abc$$

Ans : D

02. a, q, c AP $\Rightarrow 2q = a + c$

$$\Rightarrow \frac{1}{p} + \frac{1}{r} = \frac{2}{p}$$ Since $\frac{1}{p}, q, \frac{1}{r} \rightarrow$ AP

$a, \frac{1}{p}, c \rightarrow$ AP

Ans : B

03. $F(n+1) = \frac{2F(n)+1}{2}$ Given $F(1) = 2$

$$F(1+1) = \frac{2F(1)+1}{2} = \frac{2(2)+1}{2} = \frac{5}{2} \quad \therefore F(2) = \frac{5}{2}$$

$$F(2+1) = \frac{2F(2)+1}{2} = \frac{2 \cdot \frac{5}{2} + 1}{2} = 3$$

$\therefore F(1), F(2), F(3) \ldots$ are in P
$2, \frac{5}{2}, 3, \ldots$

$a = 2$, $d = 1/2$ | $F(101) = a + 100d = 2 + \frac{1}{2} \times 100 = 52$

Ans : B.

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04. $a_3 + a_5 + a_8 = 11$

$\Rightarrow (a+2d) + (a+4d) + (a+7d) = 11$

$\Rightarrow 3a + 13d = 11 \rightarrow (1)$

$a_2 + a_4 = -2$

$\Rightarrow a+d+a+3d = -2$

$\Rightarrow 2a + 4d = -2$

$\Rightarrow a + 2d = -1 \rightarrow (2)$

$\Rightarrow$

$(1) \Rightarrow 3(-1-2d) + 13d = 11$

$\Rightarrow -3 - 6d + 13d = 11$

$\Rightarrow -3 + 7d = 11$

$\Rightarrow 7d = 14 \Rightarrow \boxed{d=2}$

$(2) \Rightarrow a + 2(2) = -1$

$\Rightarrow \boxed{a = -5}$

Now, $a_1 + a_6 + a_7 = a + a + 5d + a + 6d$

$= 3a + 11d$

$= 3(-5) + 11(2)$

$= 7$

Ans: C

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05 $4x^2 + 9y^2 + 16z^2 - 6xy - 12yz - 8zx = 0$

$\Rightarrow 8x^2 + 18y^2 + 32z^2 - 12xy - 24yz - 16zx = 0 \quad (\because \times 2)$

$\Rightarrow (2x-3y)^2 + (3y-4z)^2 + (4z-2x)^2 = 0$

$\Rightarrow 2x - 3y = 0,\ 3y - 4z = 0,\ 4z - 2x = 0$

$\Rightarrow 2x = 3y = 4z$

$\Rightarrow x = \frac{k}{2},\ y = \frac{k}{3},\ z = \frac{k}{4}$

$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z} = \frac{2}{k}, \frac{3}{k}, \frac{4}{k}$

$\rightarrow$ AP

Ans. A

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06 $\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \cdots + \frac{1}{a_n \cdot a_{n+1}}$ (3)

$= \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + \cdots + \frac{1}{a_n} - \frac{1}{a_{n+1}}\right)$

$= \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{n+1}}\right)$

$= \frac{1}{d}\left(\frac{1}{a} - \frac{1}{a+nd}\right)$

$= \frac{1}{d}\left(\frac{a+nd-a}{a(a+nd)}\right) = \frac{n}{a(a+nd)} = \frac{n}{a_1 \cdot a_{n+1}}$

Ans: D

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07 $t_p = a \mid t_q = b, \mid t_r = c$

$\Rightarrow x+(p-1)y = a \mid x+(q-1)y = b \mid x+(r-1)y = c$

Now $a(q-r) + b(r-p) + c(p-q)$

$= \sum a(q-r)$

$= \sum a\left[\left(\frac{b-x}{y}+1\right) - \left(\frac{c-x}{y}+1\right)\right]$

$= \sum a\left[\frac{b-c}{y}\right]$

$= \frac{a(b-c)}{y} + \frac{b(c-a)}{y} + \frac{c(a-b)}{y}$

$= \frac{0}{y}$

$= 0$

Ans: A

4

08 a, b, c, d, e form an AP.

let $a=1,\ b=2,\ c=3,\ d=4,\ e=5$

Now $a-4b+6c-4d+e$

$= 1-4(2)+6(3)-4(4)+5$

$= 1-8+18-16+5$

$= 0$

Ans: C

09. We observe

$a_1 + a_{2n} = a + a + (2n-1)d = 2a + (2n-1)d$

$a_2 + a_{2n-1} = a + d + a + (2n-2)d = 2a + (2n-1)d$

$\therefore$ let $a_1 + a_{2n} = a_2 + a_{2n-1} = \cdots\cdots = a_n + a_{n+1} = K$

$K\left[\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \cdots\cdots + \frac{1}{\sqrt{a_n}+\sqrt{a_{n+1}}}\right]$

$= K\left[\frac{\sqrt{a_1}-\sqrt{a_2}}{-d} + \frac{\sqrt{a_2}-\sqrt{a_3}}{-d} + \cdots + \frac{\sqrt{a_n}-\sqrt{a_{n+1}}}{-d}\right]$

$= K\left[\frac{\sqrt{a_1}-\sqrt{a_{n+1}}}{-d}\right] = K\left[\frac{a_1 - a_{n+1}}{-d} \times \frac{1}{\sqrt{a_1}+\sqrt{a_{n+1}}}\right]$

$= K\left[\frac{a - a - (n-1)d}{-d\left(\sqrt{a_1}+\sqrt{a_{n+1}}\right)}\right]$

$= \frac{K(n)}{\sqrt{a_1}+\sqrt{a_{n+1}}} = \frac{n[2a+(2n-1)d]}{\sqrt{a_1}+\sqrt{a_{n+1}}}$

$= \frac{n[a_1 + a_{2n}]}{\sqrt{a_1}+\sqrt{a_{n+1}}}$

Ans: B

10 $3+7+11+\cdots$ $\quad d = 7-3 = 4$

$1+6+11+\cdots$ $\quad d = 6-1 = 5$

5

$L.C.M \text{ of } \{4,5\} = 20$

10^{th} common term = first common term + $(10-1) \times$ L.C.M of $\{4,5\}$

$= 11 + 9 \times 20$

$= 191$

Ans: A

11. $\dfrac{1}{\sqrt{2}+\sqrt{5}} + \dfrac{1}{\sqrt{5}+\sqrt{8}} + \dfrac{1}{\sqrt{8}+\sqrt{11}} + \cdots$ n terms

Now, $2, 5, 8, \ldots$ AP $\Rightarrow t_n = 2+(n-1)3$
$= 3n-1$

$5, 8, 11 \ldots$ AP $\Rightarrow t_n = 5+(n-1)3$
$= 3n+2$

$\therefore \dfrac{1}{\sqrt{2}+\sqrt{5}} + \dfrac{1}{\sqrt{5}+\sqrt{8}} + \cdots + \dfrac{1}{\sqrt{3n-1}+\sqrt{3n+2}}$

$= \dfrac{\sqrt{2}-\sqrt{5}}{-3} + \dfrac{\sqrt{5}-\sqrt{8}}{-3} + \cdots + \dfrac{\sqrt{3n-1}-\sqrt{3n+2}}{-3}$

$= \dfrac{\sqrt{2}-\sqrt{3n+2}}{-3} = \dfrac{\sqrt{3n+2}-\sqrt{2}}{3} \rightarrow$ opt A

$\dfrac{\sqrt{3n+2}-\sqrt{2}}{3} \times \dfrac{\sqrt{3n+2}+\sqrt{2}}{\sqrt{3n+2}+\sqrt{2}} = \dfrac{3n+2-2}{3(\sqrt{3n+2}+\sqrt{2})} = \dfrac{n}{\sqrt{2+3n}+\sqrt{2}}$

Since $n < \sqrt{2+3n}+\sqrt{2}$

$\therefore \dfrac{n}{\sqrt{2+3n}+\sqrt{2}} < n$

Ans: A, B, C

12. $a_1 + a_3 + a_5 = -12$

$\Rightarrow a + a + 2d + a + 4d = -12$

$\Rightarrow 3a + 6d = -12$

$\Rightarrow a + 2d = -4 \rightarrow (1)$

$a_1 \cdot a_3 \cdot a_5 = 80$

$\Rightarrow a(a+2d)(a+4d) = 80$

$\Rightarrow a(-4)(a+4d) = 80$

$\Rightarrow a(a+4d) = -20 \rightarrow (2)$

$(2) \Rightarrow a(a + 8 - 2a) = -20$ Since $4d = -8 - 2a$

$\Rightarrow a(-8-a) = -20$

$\Rightarrow a(a+8) = 20$

$\Rightarrow \boxed{a = 2}$

Now $(1) \Rightarrow a + 2d = -4$

$\Rightarrow 2 + 2d = -4$

$\Rightarrow \boxed{d = -3}$

Now A) $a_1 = 2$

B) $a_2 = a + d = 2 - 3 = -1$

C) $a_3 = a + 2d = 2 + 2(-3) = 2 - 6 = -4$

D) $a_5 = a + 4d = 2 + 4(-3) = 2 - 12 = -10$

Ans : B, C

13. Let $t_{\cancel{q}} = p$, $t_m = q$, $t_n = r$

$\Rightarrow a + (l-1)d = p$ | $a + (m-1)d = q$ | $a + (n-1)d = r$

Now $\dfrac{r-q}{q-p} = \dfrac{[a+(n-1)d] - [a+(m-1)d]}{[a+(m-1)d] - [a+(l-1)d]} = \dfrac{n-m}{m-l}$,

which is a rational number since l, m, n ~~are~~ $\in Z^+$

Now $\sqrt{2}, \sqrt{3}, \sqrt{5} \Rightarrow \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ is not a rational number.

Hence Both the statements are True

Ans: A

14 **Statement I:** $9 \times t_9 = 4 \times t_4$

$\Rightarrow 9(a+8d) = 4(a+3d)$

$\Rightarrow 9a + 72d = 4a + 12d$

$\Rightarrow 5a + 60d = 0$

$\Rightarrow a + 12d = 0$

$\Rightarrow t_{13} = 0$ (True)

Statement II: Conceptual (True)

Ans: A

15 $2, 5, 8, 11, \ldots 59$

here $a=2$, $d=3$, $l=59$

5^{th} term from the end $= t_5 = l-(n-1)d$

$= 59-(5-1)\times 3$

$= 59 - 12$

$= 47$

Ans: B

16. $1, \frac{5}{2}, 4, \frac{11}{2}, \ldots 28$

here $a=1$, $d=\frac{3}{2}$, $l=28$, $n=10$

$\therefore t_n = l-(n-1)d$

$t_{10} = 28-(10-1)\times\frac{3}{2}$

$= 28 - \frac{27}{2}$

$= \frac{29}{2}$

Ans: C

17 Given $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$

$= 20, \frac{77}{4}, 18\frac{1}{2} \ldots$

$a = 20, \quad d = \frac{77}{4} - 20 = \frac{-3}{4}$

Now, $t_n = a + (n-1)d$

$= 20 + (n-1)\left(-\frac{3}{4}\right)$

$= -\frac{3n}{4} + \frac{83}{4} < 0$

$\Rightarrow 3n > 83$

$\Rightarrow n > \frac{83}{3}$

$\Rightarrow n > 27.\overline{}$

$\Rightarrow n \geq 28$

Ans. A

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18 $a = 4, \quad t_9 = 20$

$a + 8d = 20$

$\Rightarrow 4 + 8d = 20$

$\Rightarrow d = 2$

$t_{15} = a + 14d$

$= 4 + 14(2)$

$= 32$

Ans.. D

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19 The numbers which are divisible by 9 between 1 and 1000 is $9, 18, \ldots 999$.

here $a = 9, \quad d = 9$

$\therefore l = a + (n-1)d$

$999 = 9 + (n-1) \times 9$

$\Rightarrow 111 = 1 + n - 1$

$\Rightarrow \boxed{n = 111}$

Ans: 111

20 Given a, b, c, d are distinct integers in A.P. (9)

Let x be the common difference.

Now $a = a$, $b = a + x$, $c = a + 2x$, $d = a + 3x$

Given $d = a^2 + b^2 + c^2$

$\Rightarrow a + 3x = a^2 + (a+x)^2 + (a+2x)^2$

$\Rightarrow a + 3x = 3a^2 + 6ax + 5x^2$

$\Rightarrow 5x^2 + (6a-3)x + 3a^2 - a = 0 \rightarrow (1)$

This is a Q.E. in x

$\therefore x = \dfrac{-(6a-3) \pm \sqrt{(6a-3)^2 - 4 \cdot 5(3a^2 - a)}}{2 \times 5}$

$\Rightarrow x = \dfrac{-(6a-3) \pm \sqrt{-24a^2 - 16a + 9}}{10}$

We have $-24a^2 - 16a + 9 \geq 0$

$\Rightarrow 24a^2 + 16a - 9 \leq 0$

$\Rightarrow -\frac{1}{3} - \frac{\sqrt{70}}{3} \leq a \leq -\frac{1}{3} + \frac{\sqrt{70}}{12}$

$\Rightarrow a = -1, 0$ [since $a \in I$]

$\Rightarrow$ when $a = 0$ (1) $\Rightarrow 5x^2 - 3x = 0$

$\Rightarrow x = 0, \frac{3}{5}$ (Not possible)

When $\boxed{a = -1}$ (1) $\Rightarrow 5x^2 - 9x + 4 = 0$

$\Rightarrow x = 1, \frac{4}{5} \Rightarrow \boxed{x = 1}$

$\therefore a + b + c + d = -1 + 0 + 1 + 2 = 2$

Ans: 2

21) a) $-2, 0, 2, 4, \ldots$

$a = -2,\quad d = 2$

$\therefore\ t_{20} = a + 19d = -2 + 38 = 36$

b) $t_{19} = a + 18d$

c) $1, \frac{3}{2}, 2, \frac{5}{2}, \ldots\ t_{17}$.

$a = 1,\quad d = \frac{1}{2}\qquad \therefore\ t_{17} = a + 16d$

$= 1 + 16\left(\frac{1}{2}\right) = 9$

d) $14, 12, 10, \ldots$

$a = 14,\quad d = -2$

$\therefore\ t_{20} = a + 19d$

$= 14 + 19(-2) = -24$

Ans: t, r, s, q

~~22) a) $1, \frac{-1}{2}, 2, \ldots -26$~~

~~$a = 1,\ d = \frac{-3}{2}$~~

~~$\therefore\ t_{15} = a + 14d = 1 + 14\left(\frac{-3}{2}\right) =$~~

22 a) $1, -\frac{1}{2}, 2, \ldots -26$

$a = 1,\quad d = -\frac{3}{2},\quad l = -26,\quad m = 15$

$t_m = l - (m-1)d = -26 - (15-1)\left(-\frac{3}{2}\right) = -5$

b) $t_{18} = a + 17d = a - d + 17d = a + 16d$

c) $t_{12} = a + 11d = 0.1 + 11(0.01) = 0.21$

d) $t_{10} = a + 9d = \sqrt{2} + 9\sqrt{2} = 10\sqrt{2}$

Ans: p, q, r, s

LEARNERS TASK

CUQ'S

01. All the given terms form a sequence. Ans: D

02. All the given terms from a series Ans: D

03. m^{th} term from the end $= (n-m+1)^{th}$ term from the begining Ans: A

04. p, q, r AP $\Rightarrow 2q = p + r$

$$\Rightarrow q = \frac{p+r}{2}$$

Ans: D

05. Need not be in A.P. Ans: B

06. Common difference $= d + K$ Ans: A

07. $t_m = n,\ t_n = m \Rightarrow a + (m-1)d = n$

$a + (n-1)d = m$

$\cancel{t_{m+n} = a + (m+n-1)d}$

$\cancel{= 0}$

07. Change the question as follows

If m times the n^{th} term is equal to n times the m^{th} term, then $t_{m+n} = 0$

Given $m t_n = n \times t_m \Rightarrow m(a + (n-1)d) = n(a + (m-1)d)$

$$\Rightarrow a + (m+n-1)d = 0$$

$$\Rightarrow t_{m+n} = 0$$

Ans: A

08 All the given statements are true — Ans: D

09 Common difference = Kd — Ans: C

10 $t_n = 2n+3$

$t_{10} = 2(10)+3 = 23$ — Ans: B

JEE MAINS LEVEL

01. $t_n = 2n+5$ | $t_4 = 2(4)+5 = 13$

$t_2 = 2(2)+5 = 9$ | $t_5 = 2(5)+5 = 15$

$\therefore \dfrac{t_5 - t_4}{t_2} = \dfrac{15-13}{9} = \dfrac{2}{9}$ — Ans: C

02 $t_n = (-1)^{n-1} \cdot 2^n$

$t_5 = (-1)^{5-1} \cdot 2^5$

$= 32$ — Ans: A

~~03~~ $\cancel{a_n = n^3 - 6n^2 + 11n - 6}$

$\cancel{a_0 = 0^3 - 6.0^2 + 11.0 - 6 = -6}$

$\cancel{a_1 = 1^3 - 6.1^2 + 11.1 - 6 = 1 - 6 + 11 - 6 = 0}$

03 $a_n = n^3 - 6n^2 + 11n - 6$

$a_1 = 1^3 - 6.1^2 + 11.1 - 6 = 1 - 6 + 11 - 6 = 0$

$a_2 = 2^3 - 6.2^2 + 11.2 - 6 = 8 - 24 + 22 - 6 = 0$

$a_3 = 3^3 - 6.3^2 + 11.3 - 6 = 27 - 54 + 33 - 6 = 0$

$a_4 = 4^3 - 6.4^2 + 11.4 - 6 = 64 - 96 + 44 - 6 = 6 > 0$

$\therefore$ Ans C

04 $a_1 = 1, a_2 = 1$

$a_n = a_{n-1} + a_{n-2}$

$a_3 = a_2 + a_1 = 1+1 = 2$

$a_4 = a_3 + a_2 = 2+1 = 3$

$a_5 = a_4 + a_3 = 3+2 = 5$

$a_6 = a_5 + a_4 = 5+3 = 8$

Now $\frac{a_{n+1}}{a_n} = \frac{a_6}{a_5} = \frac{8}{5}$ Ans:—

05 $t_{m+1} = a + (m+1-1)d = a + md$ Ans: B

06 All the Expressions are linear in n.

Hence all are nth terms of an A.P. Ans: D

07 Option: B $1^2, 5^2, 7^2, \cancel{9^3}$

$1, 25, 49, \ldots$

$\Rightarrow$ common difference $= 24$. Ans: B

08 $9, 12, 15, 18, \ldots$

$a = 9,\ d = 12 - 9 = 3$

$\therefore t_n = a + (n-1)d$

$= 9 + (n-1)3$

$= 3n + 6$ Ans: B

09 $4, 9, 14, \ldots 254.$

$a = 4, \quad d = 5, \quad l = 254, \quad m = 10$

$\therefore t_m = l - (m-1)d \qquad l - (m-1)d$

$= 254 - (10-1) . 5$

$= 254 - 45$

$= 234. \qquad = 25$

$254 - 45 = \boxed{190} \quad \boxed{179} \qquad 254 - 45 = 209$

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09. $4, 9, 14, \ldots 254$ (14)

$a = 4, \quad d = 5, \quad l = 254, \quad m = 10$

$\therefore t_m = l - (m-1)d$

$= 254 - (10-1) \times 5$

$= 254 - 45$

$= 209$

Ans: A

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10 $-1, 3, 7, \ldots$

$a = -1, \quad d = 4 \quad l = 95$

$95 = -1 + (n-1)4$

$\Rightarrow n = 2\cancel{4}$ ~~th term.~~ $\therefore 25^{th}$ term

Ans: D