DERIVATIVES OF VARIOUS FUNCTIONS

Class: IX, Mathematics

SOLUTIONS (A)

TEACHING TASK

JEE MAINS LEVEL

01. Given $x^{2x} - 2x^{x}\cot y - 1 = 0 \rightarrow (1)$

when $x=1 \Rightarrow 1 - 2\cot y - 1 = 0$

$\Rightarrow \cot y = 0$ $\therefore (1, \frac{\pi}{2})$.

$\Rightarrow y = \frac{\pi}{2}$

We know, $\frac{d}{dx}(x^{2x}) = x^{2x}(2 + 2\log x)$

$\frac{d}{dx}(x^{x}) = x^{x}(1+\log x)$

$(1) \Rightarrow x^{2x}(2+2\log x) - 2\left[x^{x}(-\operatorname{cosec}^2 y \frac{dy}{dx}) + \cot y \, x^{x}(1+\log x)\right] - 0 = 0$

$\Rightarrow x^{2x}(1+\log x) + x^{x}\operatorname{cosec}^2 y \frac{dy}{dx} - \cot y \, x^{x}(1+\log x) = 0$

put $x=1$ & $y = \frac{\pi}{2}$

$\Rightarrow \frac{dy}{dx} = -1$

02 $f(x) = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$ ②

$= x(12x^2 - 6x^2) - x^2(6x - 0) + x^3(2-0)$

$= 6x^3 - 6x^3 + 2x^3$

$\therefore f(x) = 2x^3$

$\therefore f'(x) = 6x^2$

Ans: D

03. $y = \sqrt{\dfrac{1+\cos 2\theta}{1-\cos 2\theta}}$

$y = \sqrt{\dfrac{2\cos^2\theta}{2\sin^2\theta}}$

$y = \cot\theta$

$\dfrac{dy}{dx} = -\operatorname{cosec}^2\theta$

$= -\operatorname{cosec}^2\frac{3\pi}{4}$

$= -\operatorname{cosec}^2\frac{\pi}{4}$

$= -(\sqrt{2})^2 = -2$ Ans: A

04. $y = (1-x)(2-x)(3-x) \cdots\cdots (n-x)$

$\log y = \log(1-x) + \log(2-x) + \log(3-x) + \cdots + \log(n-x)$

$\therefore \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{1-x}(-1) + \dfrac{1}{2-x}(-1) + \dfrac{1}{3-x}(-1) + \cdots + \dfrac{1}{n-x}(-1)$

~~put x = 1~~

$\therefore \dfrac{dy}{dx} = -\left[\dfrac{y}{1-x} + \dfrac{y}{2-x} + \dfrac{y}{3-x} + \cdots + \dfrac{y}{n-x}\right]$

$= -\left[(2-x)(3-x)\cdots(n-x) + \dfrac{y}{2-x} + \dfrac{y}{3-x} + \cdots + \dfrac{y}{n-x}\right]$

at $x = 1$

$$\frac{dy}{dx} = -\left[1.2.3.4\ldots(n-1)! + \frac{0}{2-x} + \frac{0}{3-x} + \cdots + 0\right]$$

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$$\frac{dy}{dx} = -(n-1)!$$

Ans. C

05 $y = t^{10}+1$ | $x = t^{8}+1$

$\frac{dy}{dt} = 10t^{9}$ | $\frac{dx}{dt} = 8t^{7}$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{10t^{9}}{8t^{7}} = \frac{5}{4}\cdot t^{2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{5}{4}\cdot 2t\cdot\frac{dt}{dx}$$

$$= \frac{5}{2}t\cdot\frac{1}{8t^{7}} = \frac{5}{16t^{6}}$$

Ans. C

06 $f(x) = \cos x.\cos 2x.\cos 4x.\cos 8x$

$$f(x) = \frac{1}{2\sin x}(2\sin x\cos x).\cos 2x.\cos 4x.\cos 8x$$

$$= \frac{1}{2\sin x}.\sin 2x.\cos 2x.\cos 4x.\cos 8x$$

$$= \frac{1}{4\sin x}.\sin 4x.\cos 4x.\cos 8x$$

$$= \frac{1}{8\sin x}\sin 8x.\cos 8x$$

$$= \frac{\sin 16x}{16\sin x}.$$

$$f'(x) = \frac{16\sin x \cdot \cos 16x \cdot 16 - \sin 16x \cdot 16\cos x}{(16 \cdot \sin x)^2}$$

(4)

$$f'\left(\frac{\pi}{4}\right) = \frac{16\sin\frac{\pi}{4} \cdot \cos 4\pi \cdot 16 - \sin 4\pi \cdot 16 \cdot \cos\frac{\pi}{4}}{\left(16 \cdot \sin\frac{\pi}{4}\right)^2}$$

$$= \frac{(16)^2 \cdot \left(\frac{1}{\sqrt{2}}\right) \cdot 1 - 0}{(16)^2\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{2}$$

$$= \operatorname{cosec}\frac{\pi}{4}$$

Ans: D

07. $y = e^{2x}$

$\Rightarrow \frac{dy}{dx} = e^{2x} \cdot 2$

$\Rightarrow \frac{d^2y}{dx^2} = 4e^{2x}$

$y = e^{2x}$

$\Rightarrow \log y = 2x$

$\Rightarrow x = \frac{1}{2}\log y$

$\Rightarrow \frac{dx}{dy} = \frac{1}{2y}$

$\Rightarrow \frac{d^2x}{dy^2} = \frac{-1}{2y^2} = \frac{-1}{2 \cdot e^{4x}}$

$\therefore \frac{d^2y}{dx^2} \cdot \frac{d^2x}{dy^2} = 4e^{2x} \times \frac{-1}{2e^{4x}}$

$= -2e^{-2x}$

Ans: D

08 $f(x) = e^{\sin^{-1}x}$

$f'(x) = e^{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}$

$g(x) = \log x \Rightarrow g'(x) = \frac{1}{x}$

Derivative of $f(x)$ wrt $g(x) = \frac{f'(x)}{g'(x)}$

$= \frac{x e^{\sin^{-1} x}}{\cancel{x}\sqrt{1-x^2}}$

Ans : B

09. $x = \frac{3at}{1+t^2}$

$\Rightarrow \frac{dx}{dt} = 3a\left[\frac{(1+t^2).1 - (\cancel{1+}2t).t}{(1+t^2)^2}\right]$

$= 3a\left[\frac{1+t^2-2t^2}{(1+t^2)^2}\right] = 3a\left(\frac{1-t^2}{(1+t^2)^2}\right)$

Now, $y = \frac{3at^2}{1+t^2}$

$\frac{dy}{dt} = 3a\left[\frac{(1+t^2)(2t) - t^2(2t)}{(1+t^2)^2}\right]$

$= 3a\left[\frac{2t+2t^3-2t^3}{(1+t^2)^2}\right] = 3a\left(\frac{2t}{(1+t^2)^2}\right)$

$\therefore \frac{dy}{dx} = \frac{2t}{1-t^2} = \cancel{x}$

Ans. A.

10. $\frac{d}{dx}\left(\frac{3^x - 3^{-x}}{3^x + 3^{-x}}\right)$

$\therefore \frac{d}{dx}\Big($

$$= \frac{(3^x+3^{-x})(3^x.\log 3 + 3^{-x}.\log 3) - (3^x-3^{-x})(3^x.\log 3 - 3^{-x}.\log 3)}{(3^x+3^{-x})^2}$$

$$= \frac{4\log 3}{(3^x+3^{-x})^2}$$

Ans. A

11. $\frac{d}{dx}(\sinh^{-1}(3x)) = \frac{3}{\sqrt{1+9x^2}} = \frac{a}{\sqrt{1+bx^2}}$

$a=3,\ b=9,\ a+b=12,\ |a-b|=6$ Ans A,B,C,D

12. $y = \tan^{-1}\left(\frac{(3-x)\sqrt{x}}{1-3x}\right)$

$y = \tan^{-1}\left(\frac{3\sqrt{x} - x\sqrt{x}}{1-3x}\right)$ put $\sqrt{x} = \tan\theta$

$\Rightarrow x = \tan^2\theta$.

$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3(\tan\theta)^2}\right)$

$= \tan^{-1}(\tan 3\theta)$

$= 3\theta$

$= 3.\tan^{-1}(\sqrt{x})$

$\therefore y = 3.\tan^{-1}(\sqrt{x})$

$\frac{dy}{dx} = \frac{3}{1+(\sqrt{x})^2} \times \frac{1}{2\sqrt{x}}$

$a=3,\ b=2$

$a+b=5,\ a-b=1$

Ans. A,B,C,D

(7)

13. Statement I: $f(x) = a^x$ | $g(x) = \sin^{-1} x$

$f'(x) = a^x \log a$ | $g'(x) = \dfrac{1}{\sqrt{1-x^2}}$

$\therefore \dfrac{f'(x)}{g'(x)} = a^x \cdot \log a \cdot \sqrt{1-x^2}$ Ans

Statement: Conceptual (True)

Ans : A

14. Statement I: $y = \sec(\tan^{-1} x)$

$$\frac{dy}{dx} = \sec(\tan^{-1} x) \cdot \tan(\tan^{-1} x) \cdot \frac{1}{1+x^2}$$

at $x = 1$

$$\frac{dy}{dx} = \sec\frac{\pi}{4} \cdot \tan\frac{\pi}{4} \cdot \frac{1}{1+1^2}$$

$$= \sqrt{2} \cdot 1 \cdot \frac{1}{2} = \frac{1}{\sqrt{2}} \text{ (True)}$$

Ans: A

Statement II: Conceptual (True)

15. $x = \sin^{-1}(t)$ | $y = \log(1-t^2)$

$\dfrac{dx}{dt} = \dfrac{1}{\sqrt{1-t^2}}$ | $\dfrac{dy}{dt} = \dfrac{-2t}{1-t^2}$

$$\frac{dy}{dx} = \frac{-2t}{1-t^2} \times \sqrt{1-t^2}$$

at $t = 1/2$

$$= \frac{-2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} \cdot \sqrt{1-\frac{1}{4}}$$

$$= \frac{-4}{3} \cdot \frac{\sqrt{3}}{2} = \frac{-2}{\sqrt{3}}$$

Ans : C

8

16.

$$x = \frac{1+t}{t^3}$$

$$x = \frac{1}{t^3} + \frac{1}{t^2}$$

$$\frac{dx}{dt} = \frac{-3}{t^4} - \frac{2}{t^3}$$

$$\frac{dx}{dt} = \frac{-3-2t}{t^4}$$

$$y = \frac{3+4t}{2t^2}$$

$$y = \frac{3}{2t^2} + \frac{2}{t}$$

$$\frac{dy}{dt} = \frac{-3}{t^3} - \frac{2}{t^2}$$

$$= \frac{-3-2t}{t^3}$$

$$\therefore \frac{dy}{dx} = t$$

Now $x \cdot (y')^3 = \frac{1+t}{t^3} \times t^3 = 1+t = 1+y'$

Ans. B

17

$$\cos y = x \cos(a+y)$$

$$\Rightarrow x = \frac{\cos y}{\cos(a+y)}$$

Diff. w r t x on b|S

$$\Rightarrow 1 = \frac{\cos(a+y)(-\sin y)\frac{dy}{dx} - \cos y\left(-\sin(a+y)\right)\cdot\frac{dy}{dx}}{\cos^2(a+y)}$$ 9

$$\Rightarrow 1 = \frac{\left(\cos y \sin(a+y) - \cos(a+y)\sin y\right)\frac{dy}{dx}}{\cos^2(a+y)}$$

$$\Rightarrow 1 = \frac{\sin(a+y-y)}{\cos^2(a+y)}\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$ Ans:- A

18. $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 + x^2y = y^2 + xy^2$

$\Rightarrow x^2 - y^2 = xy^2 - x^2y$

$\Rightarrow (x+y)(x-y) = xy(y-x)$

$\Rightarrow x+y+xy = 0$

$\Rightarrow x+(1+x)y = 0$

$\Rightarrow y = \frac{-x}{1+x}$

$\therefore \frac{dy}{dx} = \frac{-1}{(1+x)^2}$ Ans:- C

19. $f(x) = \dfrac{x}{\sqrt{1-x^2}}$; $g(x) = \dfrac{x}{\sqrt{1+x^2}}$ (10)

$$fog(x) = f(g(x))$$
$$= f\left[\frac{x}{\sqrt{1+x^2}}\right]$$
$$= \frac{\left(\frac{x}{\sqrt{1+x^2}}\right)}{\sqrt{1-\frac{x^2}{1+x^2}}} = x$$

$\therefore \dfrac{d}{dx}(fog(x)) = 1$ Ans: 1

20. $y = \cos^{-1}(\cos x)$
$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-\cos^2 x}}(-\sin x) = \frac{\sin x}{|\sin x|}$$

At $x = \frac{5\pi}{4}$

$\sin x < 0$

$\Rightarrow |\sin x| = -\sin x$

$\sin\frac{5\pi}{4}$
$= \sin\left(\pi + \frac{\pi}{4}\right)$
$= -\sin\frac{\pi}{4}$

$\therefore \dfrac{dy}{dx} = \dfrac{\sin x}{-\sin x} = -1$

Ans: -1

21 (i) $y = \tan^{-1}(\sec x + \tan x)$
$$y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$$

(11)

$$y = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right) = \tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$$

$$\therefore y = \frac{\pi}{4}+\frac{x}{2} \quad \therefore \frac{dy}{dx} = \frac{1}{2}$$

(ii) $x^y = e^{x-y}$

$\Rightarrow y\log x = x-y$

$\Rightarrow y\log x + y = x$

$\Rightarrow y = \dfrac{x}{1+\log x}$

$$\frac{dy}{dx} = \frac{(1+\log x).1 - x\left(\frac{1}{x}\right)}{(1+\log x)^2}$$

$$= \frac{\log x}{(1+\log x)^2}$$

(iii) $\dfrac{d}{dx}\left[\log(\cosh x)\right] = \dfrac{1}{\cosh x}.\sinh x = \tanh x$

(iv) $x = a\cos^4 t$ | $y = b.\sin^4 t$

$\dfrac{dx}{dt} = a.4\cos^3 t.(-\sin t)$ | $\dfrac{dy}{dt} = b.4\sin^3 t.\cos t$

$$\frac{dy}{dx} = \frac{4b\sin^3 t.\cos t}{-4a\cos^3 t.\sin t} = -\frac{b}{a}.\tan^2 t$$

at $x = \dfrac{3\pi}{4}$: $\dfrac{dy}{dx} = \dfrac{-b}{a}$

Ans: t, p, r, q

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22 (i) $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

put $x = \tan\theta$

$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$y = \cos^{-1}(\cos 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = 1$

$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$= \tan^{-1}(\tan 2\theta)$

$= 2\theta$

$= 2.\tan^{-1}x$

(ii) $f(x) = \sin^{-1}x$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$g(x) = \cos^{-1}\sqrt{1-x^2}$

put $x = \sin\theta$

$g(x) = \cos^{-1}\sqrt{1-\sin^2\theta}$

$g(x) = \theta$

$g(x) = \sin^{-1}x$

$g'(x) = \frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{f'(x)}{g'(x)} = 1.$

(iii) $x+y = \sin(x+y)$ ①

put $x+y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{dt}{dx} - 1$

Now ① $\Rightarrow$ ~~$t = \sin t$~~

~~$\Rightarrow \frac{dt}{dx} = \cos t.$~~

(iii) $x+y = \sin(x+y)$

$\Rightarrow 1+\frac{dy}{dx} = \cos(x+y)\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \quad 1+\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -1$

(iv) $\sin(x+y) = \log(x+y)$

$\cos(x+y)\left(1+\frac{dy}{dx}\right) = \frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \quad 1+\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -1$

Ans: P, P, Q, Q

LEARNER'S TASK

CUQ'S

01. Conceptual

Ans: C

02

$y = \sqrt{f(x)+\sqrt{f(x)+\cdots\infty}}$

$y = \sqrt{f(x)+y}$

$\Rightarrow y^2 = f(x)+y$

$\Rightarrow 2y\cdot\frac{dy}{dx} = f'(x)+\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \frac{f'(x)}{2y-1}$

Ans. B

03.	Conceptual	Ans: B
04.	Conceptual	Ans: C
05.	Conceptual.	Ans: A
06.	Conceptual.	Ans: D
07	Conceptual	Ans. D
08.	conceptual	Ans: D
09.	Conceptual.	Ans: D
10	Conceptual.	Ans: D

JEE MAINS LEVEL

01. $x^y = \log x$

$\Rightarrow y \log x = \log(\log x)$

$$\Rightarrow y = \frac{\log(\log x)}{\log x}$$

$$\therefore \frac{dy}{dx} = \frac{\log x \cdot \frac{1}{\log x} \cdot \frac{1}{x} - \log(\log x) \cdot \frac{1}{x}}{(\log x)^2}$$

at $x = e$

$$\therefore \frac{dy}{dx} = \frac{1}{e}$$

Ans: D

15

02. $x = a\cos^3\theta$ | $y = a\sin^3\theta$

$\frac{dx}{d\theta} = a.\, 3\cos^2\theta.\,(-\sin\theta)$ | $\frac{dy}{d\theta} = a.\, 3\sin^2\theta.\, \cos\theta$

$\therefore \frac{dy}{dx} = \frac{3a\sin^2\theta.\,\cos\theta}{-3a\cos^2\theta\cdot\sin\theta} = -\tan\theta$

Ans: D

03. $f(x) = e^x$ | $g(x) = \sqrt{x}$

$f'(x) = e^x$ | $g'(x) = \frac{1}{2\sqrt{x}}$

$\frac{f'(x)}{g'(x)} = 2\sqrt{x}\, e^x$

Ans: B

04. $f(x) = \cos^{-1}x$ | $g(x) = \sin^{-1}x$

$f'(x) = \frac{-1}{\sqrt{1-x^2}}$ | $g'(x) = \frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{f'(x)}{g'(x)} = -1$

Ans: A

05 $\frac{d}{dx}\left[\tan^{-1}\frac{x}{1+x^2} + \tan^{-1}\frac{1+x^2}{x}\right]$

$= \frac{d}{dx}\left[\tan^{-1}\left(\frac{\frac{x}{1+x^2} + \frac{1+x^2}{x}}{1 - \frac{x}{1+x^2}\cdot\frac{1+x^2}{x}}\right)\right]$

$= \frac{d}{dx}\left[\tan^{-1}\left(\tan\frac{\pi}{2}\right)\right] = \frac{d}{dx}\left(\frac{\pi}{2}\right) = 0$

Ans: A

06. $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

$$\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\sqrt{\frac{1-x^2}{1-y^2}} \longrightarrow (1)$$

Now $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

$$\Rightarrow \frac{\pi}{2} - \cos^{-1}x + \sin^{-1}y = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}y = \cos^{-1}x$$

$$\Rightarrow \sin^{-1}y = \sin^{-1}\sqrt{1-x^2}$$

$$\Rightarrow y = \sqrt{1-x^2}$$

Similaly $x = \sqrt{1-y^2}$

$$(1) \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

Ans: C

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07. $\frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right]$

$$= \frac{d}{dx}\left[\tan^{-1}\sqrt{\frac{2\sin^2 x/2}{2\cos^2 x/2}}\right]$$

$$= \frac{d}{dx}\left[\tan^{-1}\left(\tan\frac{x}{2}\right)\right]$$

$$= \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2}$$

Ans: C

(17)

08

$$y = \sin x + e^x$$

$$\frac{dy}{dx} = \cos x + e^x \longrightarrow (1)$$

$$\frac{d^2y}{dx^2} = -\sin x + e^x = e^x - \sin x$$

~~Ans~~: D

$$(1) \Rightarrow \frac{dx}{dy} = \frac{1}{\cos x + e^x}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \left[\frac{-1}{(\cos x + e^x)^2} \cdot (-\sin x + e^x)\right] \frac{dx}{dy}$$

$$= \frac{\sin x - e^x}{(\cos x + e^x)^2} \times \frac{1}{(\cos x + e^x)}$$

$$= \frac{\sin x - e^x}{(\cos x + e^x)^3}$$

~~Ans~~:

Ans: —

09.

$$x = ct$$

$$\frac{dx}{dt} = c$$

$$y = \frac{c}{t}$$

$$\frac{dy}{dt} = -\frac{c}{t^2}$$

$$\frac{dy}{dx} = \frac{\left(-\frac{c}{t^2}\right)}{c} = -\frac{1}{t^2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{2}{t^3} \cdot \frac{dt}{dx}$$

$$= \frac{2}{t^3} \times \frac{1}{c}$$

at $t = \frac{1}{2}$

$$= \frac{16}{c}$$

Ans: D

10. $x^2 + xy + y^2 = 1$

$\Rightarrow 2x + x \cdot \frac{dy}{dx} + y \cdot 1 + 2y \frac{dy}{dx} = 0$

$= (x+2y)\frac{dy}{dx} = -(2x+y)$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{2x+y}{x+2y}\right)$

Ans : C

11. $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \cdots \infty}}}$

$\Rightarrow y = \sqrt{\tan x + y}$

$\Rightarrow y^2 = \tan x + y$

$\Rightarrow 2y \cdot \frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{\sec^2 x}{2y-1} = \frac{\sec^2 x}{ay+b}$

$\therefore a = 2, \quad b = -1$

Ans: A, D

12. $y = \cos^{-1}(\sqrt{1-x^2})$

$\therefore y = \sin^{-1} x$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} = \frac{a}{\sqrt{b+cx^2}}$

$a = 1, \quad b = 1, \quad c = -1$

Ans: A, B, C, D

13. **Statement I:** $y = x^{\frac{1}{x}}$

$$\Rightarrow \log y = \frac{1}{x} \cdot \log x = \frac{\log x}{x}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x}$$

$$\therefore \frac{dy}{dx} = x^{\frac{1}{x} - 1}(1 - \log x) \quad \text{(True)}$$

Statement II: Conceptual (true)

Ans: A

14. **Statement I:** $x = a(\theta - \sin\theta)$ | $y = a(1 - \cos\theta)$

$$\frac{dx}{d\theta} = a(1 - \cos\theta) \quad \Big| \quad \frac{dy}{d\theta} = a \sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{a \sin\theta}{a(1 - \cos\theta)} = \frac{2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}}{2 \sin^2\theta/2} = \cot\theta/2$$

$$\therefore \text{at } \theta = \frac{\pi}{2} \quad \therefore \frac{dy}{dx} = 1 \quad \text{(True)}$$

Statement II: Conceptual (True)

Ans: A

15. $f(x) = \sqrt{x}$ | $g(x) = \sin^{-1}x$

$$f'(x) = \frac{1}{2\sqrt{x}} \quad \Big| \quad g'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{f'(x)}{g'(x)} = \frac{\sqrt{1-x^2}}{2\sqrt{x}}$$

Ans: D

16. $f(x) = \log(\sec x)$ | $g(x) = \tan x$

$$f'(x) = \frac{1}{\sec x} \cdot \sec x \cdot \tan x \quad \Big| \quad \Rightarrow g'(x) = \sec^2 x$$

$$\therefore f'(x) = \tan x$$

Ans: $\frac{\tan x}{\sec^2 x} = \sin x \cos x$

Ans: A

17. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

$= \cos x\left(x^2 - 2x^2\right) - x\left(x\sin x - 2x\tan x\right) + 1\left(2x\sin x - x^2\tan x\right)$

$= -x^2\cos x - \cancel{2x\sin x} + 2x^2\tan x + \cancel{2x\sin x} - x^2\tan x$

$f(x) = x^2\tan x - x^2\cos x$

$f'(x) = \left[x^2 \cdot \sec^2 x + \tan x \cdot 2x\right] - \left[-x^2\sin x + \cos x \cdot 2x\right]$

$\lim_{x\to 0} \dfrac{f'(x)}{x} = \lim_{x\to 0} \dfrac{x^2 \sec^2 x + \tan x \cdot 2x + x^2\sin x - 2x\cdot\cos x}{x}$

$= \lim_{x\to 0} \left(x\sec^2 x + 2\tan x + x\sin x - 2\cos x\right)$

$= 0 + 0 - 2 = -2$

Ans : B

18. $f(x) = \begin{vmatrix} x^2 & 2x+1 & 2x^3 - 3 \\ \sin x & \cos x & -1 \\ 2 & 3 & x \end{vmatrix}$

$f(x) = x^2\left(x\cos x + 3\right) - (2x+1)\left(x\sin x + 2\right) + (2x^3 - 3)\left(3\sin x - 2\cos x\right)$

$\cancel{f(x) = x^3\cos x}$

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$$f(x) = (x^3\cos x + 3x^2) - (2x^2\sin x + 4x + x\sin x + 2)$$
$$+ 6x^3\sin x - 4x^3\cos x - 9\sin x + 6\cos x$$

$$f(x) = x^3\cos x + 3x^2 - 2x^2\sin x - 4x - x\sin x - 2 + 6x^3\sin x - 4x^3\cos x - 9\sin x + 6\cos x$$

$$f'(x) = x^3(-\sin x) + \cos x \cdot 3x^2 + 6x - 2(x^2 \cdot \cos x + \sin x \cdot 2x) - 4 - [x\cos x + \sin x \cdot 1]$$
$$- 0 + 6(x^3 \cdot \cos x + \sin x \cdot 3x^2) - 4(x^3(-\sin x) + \cos x \cdot 3x^2) - 9\cos x + 6(-\sin x)$$

$\therefore$ Coeff of $x = 6 - 4\sin x - \cos x$ Ans: —

19 $x^2 + y^2 + \sin y = 4$

$$\Rightarrow 2x + 2y \cdot \frac{dy}{dx} + \cos y \cdot \frac{dy}{dx} = 0 \longrightarrow (1)$$

$$\Rightarrow 2 + 2\left[y \cdot \frac{d^2y}{dx} + \frac{dy}{dx} \cdot \frac{dy}{dx}\right] + \cos y \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}(-\sin y)\frac{dy}{dx} = 0 \longrightarrow (2)$$

Now at $(-2, 0)$

$$(1) \Rightarrow 2(-2) + 2 \cdot 0 \cdot \frac{dy}{dx} + \cos 0 \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = 4$$

$$(2) \Rightarrow 2 + 2\left[0 \cdot \frac{d^2y}{dx^2} + 16\right] + \cos 0 \cdot \frac{d^2y}{dx^2} + 4 \cdot 4(-\sin 0) = 0$$

$$\Rightarrow 2 + 32 + \frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} = -34$$

Ans: -34

20. $\frac{d}{dx}\left[(x+1)(x^2+1)(x^4+1)(x^8+1)\right]$

$= \frac{d}{dx}\left[\frac{(x-1)(x+1)(x^2+1)(x^4+1)(x^8+1)}{x-1}\right]$

$= \frac{d}{dx}\left[\frac{x^{16}-1}{x-1}\right] = \frac{(x-1)(16x^{15})-(x^{16}-1)(1)}{(x-1)^2}$

$= (16x^{16}-16x^{15}-x^{16}+1)(x-1)^{-2}$

$= (15x^{16}-16x^{15}+1)(x-1)^{-2}$

$= (15x^{p}-16x^{q}+1)(x-1)^{-2}$

$p=16,\ q=15 \Rightarrow p+q=31$ Ans: 31

21 (i) $2x^2+3xy+y^2-4x-8y-20=0$

$\Rightarrow 4x+3\left[x\frac{dy}{dx}+y\cdot 1\right]+2y\cdot\frac{dy}{dx}-4-8\cdot\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx} = \frac{4-4x-3y}{3x+2y-8}$

(ii) $\sqrt{x}+\sqrt{y}=1$

$\Rightarrow \frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

at $\left(\frac{1}{4},\frac{1}{4}\right)$

$\therefore \frac{1}{1}+\frac{1}{1}\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-1$

(iii) $x = 3\cos\theta-\cos^3\theta$

$\frac{dx}{d\theta} = -3\sin\theta-3\cos^2\theta(-\sin\theta)$

$= -3\sin\theta+3\cos^2\theta\cdot\sin\theta$

$y = 3\sin\theta - \sin^3\theta$

$\frac{dy}{d\theta} = 3\cos\theta - 3\sin^2\theta \cdot \cos\theta$

$\therefore \frac{dy}{dx} = \frac{3\cos\theta - 3\sin^2\theta \cdot \cos\theta}{-3\sin\theta + 3\cos^2\theta \cdot \sin\theta} = -\theta$

$= \frac{-\cancel{\cos\theta\,(-3 + 3\sin^2\theta)}}{\cancel{\sin\theta\,(-3 + 3\sin}}$

$= \frac{3\cos\theta\,(1 - \sin^2\theta)}{-3\sin\theta\,(1 - \cos^2\theta)} = -\frac{\cos\theta \cdot \cos^2\theta}{\sin\theta \cdot \sin^2\theta}$

$= -\cot^3\theta$

(iv) $x = a(1-\cos\theta)$ | $y = a(\theta - \sin\theta)$

$\frac{dx}{d\theta} = a(0+\sin\theta) = a\sin\theta$ | $\frac{dy}{d\theta} = a(1-\cos\theta)$

Now, $\frac{dy}{dx} = \frac{a(1-\cos\theta)}{a\sin\theta} = \frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2} = \tan\theta/2$

$\therefore$ at $\theta = \frac{\pi}{2}$ $\therefore \frac{dy}{dx} = 1$

Ans: q, p, t, r

22 (i) $y = \log(\tan 5x)$

$\frac{dy}{dx} = \frac{1}{\tan 5x} \cdot \sec^2 5x \cdot 5 = \frac{5\sec^2 5x}{\tan 5x}$

(ii) $y = \sinh^{-1}\left(\frac{3x}{4}\right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{1+\left(\frac{3x}{4}\right)^2}} \times \frac{3}{4}$

$$= \frac{3}{\sqrt{16+9x^2}}$$

(iii) $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

put $x = \cos\theta$

$$y = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right)$$

$$y = \sec^{-1}(\sec 2\theta)$$

$$y = 2\theta$$

$$y = 2 \cdot \cos^{-1} x$$

$$\therefore \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

(iv) $y = \log(\cosh 2x)$

$$\frac{dy}{dx} = \frac{1}{\cosh 2x} \cdot \sinh 2x \cdot 2 = 2\tanh x$$

Ans: r, p, s, q.

$\Rightarrow$ THE END $\Leftarrow$