

WS-4 6th advanced

Task 1

①

Given $\vec{A} = 4\vec{i} - 3\vec{j}$ and $\vec{B} = 6\vec{i} - 5\vec{j}$

$$\begin{aligned}\vec{B} - \vec{A} &= 6\vec{i} - 5\vec{j} - (4\vec{i} - 3\vec{j}) = 6\vec{i} - 5\vec{j} - 4\vec{i} + 3\vec{j} \\ &= 2\vec{i} - 2\vec{j}\end{aligned}$$

$$|\vec{B} - \vec{A}| = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

unit vector parallel to $\vec{B} - \vec{A} = \frac{\vec{B} - \vec{A}}{|\vec{B} - \vec{A}|}$

$$\begin{aligned}&= \frac{2\vec{i} - 2\vec{j}}{2\sqrt{2}} \\ &= \frac{2(1\vec{i} - 1\vec{j})}{2\sqrt{2}} = \frac{1\vec{i} - 1\vec{j}}{\sqrt{2}}\end{aligned}$$

②

let the vector be $\vec{A} = 6\vec{i} - 8\vec{j}$

let $\vec{B}$ is a vector parallel to $\vec{A}$ and same magnitude

$$\vec{B} = \frac{-\vec{A}}{|\vec{A}|} \times 10 \quad \text{as that of } \vec{A}$$

$$\Rightarrow \vec{B} = \frac{-(6\vec{i} - 8\vec{j})}{\sqrt{6^2 + (-8)^2}} \times 10 = -\frac{(6\vec{i} - 8\vec{j})}{10} \times 10$$

$$\Rightarrow \vec{B} = -6\vec{i} + 8\vec{j}$$

③

Let $\vec{A} = 3\hat{i} + 4\hat{j}$; $\vec{B} = \hat{j} + \hat{k}$

$\vec{C}$ is a vector which is parallel to $\vec{A}$ and same magnitude as that of $\vec{B}$.

$$\vec{C} = \lambda |\vec{B}|$$

$$|\vec{B}| = \sqrt{y^2 + z^2}$$

$$= \sqrt{1^2 + 1^2}$$

$$= \sqrt{2}$$

$$\Rightarrow \vec{C} = \frac{\vec{A}}{|\vec{A}|} |\vec{B}|$$

$$\Rightarrow \vec{C} = \frac{3\hat{i} + 4\hat{j}}{5} (\sqrt{2})$$

$$|\vec{A}| = \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16} = 5$$

$$\Rightarrow \vec{C} = \frac{\sqrt{2}(3\hat{i} + 4\hat{j})}{5}$$

④

The Given two vectors are

$$\vec{A} = 2\hat{i} - 4\hat{j} + 3\hat{k} \quad \text{and} \quad \vec{B} = 8\hat{i} + 3\hat{j} + 12\hat{k}$$

$$A_x \quad A_y \quad A_z \quad B_x \quad B_y \quad B_z$$

When $\vec{A}$ and $\vec{B}$ are parallel then

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

$$\Rightarrow \frac{2}{8} = \frac{-4}{3} = \frac{3}{12} \Rightarrow y = -\frac{3}{4}$$



(5)

Given $\vec{A} = 3\hat{i} + 4\hat{j}$; $\vec{B} = 5\hat{i} + 12\hat{j}$.

Let $\vec{C}$ is a vector parallel to $\vec{A}$ and same magnitude

as that of $\vec{B}$

Magnitude of $\vec{B} = \sqrt{5^2 + 12^2}$

$$\therefore \vec{C} = \hat{A} |\vec{B}|$$

$$\therefore \vec{C} = \frac{\vec{A}}{|\vec{A}|} \times |\vec{B}|$$

$$\therefore \vec{C} = \frac{3\hat{i} + 4\hat{j}}{5} \times 13$$

$$\therefore \vec{C} = 13 \frac{(3\hat{i} + 4\hat{j})}{5}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13$$

Magnitude of $\vec{A} = \sqrt{3^2 + 4^2}$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5$$

(6)

Let $\vec{A} = 6\hat{i} - 8\hat{j}$; $\vec{B}$ is a vector opposite to $\vec{A}$

$$\vec{B} = -\hat{A} |\vec{B}| \quad \text{and given } |\vec{B}| = 5$$

$$= -\frac{(6\hat{i} - 8\hat{j})}{\sqrt{6^2 + (-8)^2}} (5)$$

$$= -\frac{(6\hat{i} - 8\hat{j})}{10} \times 5$$

$$= -\frac{1}{2} (6\hat{i} - 8\hat{j})$$

$$= -3\hat{i} + 4\hat{j}$$

(7)

$$\text{let } \vec{A} = 3\hat{i} - 2\hat{j} + 4\hat{k}$$

$$A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\text{and } \vec{B} = -6\hat{i} + p\hat{j} - 8\hat{k}$$

$$B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

If

 $\vec{A}$ and $\vec{B}$ are opposite

$$(\vec{A}) = -(\vec{B})$$

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

$$\Rightarrow \frac{3}{-6} = \frac{-2}{p} = \frac{4}{-8} \Rightarrow$$

$$\text{Take any two } \frac{3}{-6} = \frac{4}{-8}$$

$$\Rightarrow \frac{1}{2} = \frac{2}{p} \Rightarrow p = 4$$

(8)

$$\text{Given } \vec{A} = \sqrt{2} \hat{i} + \sqrt{2} \hat{j} \quad \vec{B} = \hat{i} + \hat{j}$$

let $\vec{C}$ be the vector parallel to $\vec{A}$ and same magnitudeas that of $\vec{B}$ Magnitude of $\vec{B}$

$$|\vec{B}| = \sqrt{x^2 + y^2}$$

$$= \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$|\vec{A}| = \sqrt{x^2 + y^2}$$

$$= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$$

$$= 2$$

$$\vec{C} = \frac{\sqrt{2} \hat{i} + \sqrt{2} \hat{j}}{2} \times \sqrt{2}$$

$$\Rightarrow \frac{\sqrt{2} (\hat{i} + \hat{j}) \times \sqrt{2}}{2}$$

$$\vec{C} = \hat{i} + \hat{j}$$

③

Given vector $0.2\hat{i} + 0.3\hat{j} + 2\hat{k}$ is a unit vector

(ie) $|0.2\hat{i} + 0.3\hat{j} + 2\hat{k}| = 1$

$$\Rightarrow \sqrt{(0.2)^2 + (0.3)^2 + 2^2} = 1$$

Squaring on both sides we get

$$\Rightarrow \sqrt{0.04 + 0.09 + 4} = 1^2$$

$$\Rightarrow 0.13 + 4 = 1 \Rightarrow 4 = 1 - 0.13 = 0.87$$

⑩

If $\vec{A}$ and $\vec{B}$ are two parallel vectors then

$$\vec{A} = k\vec{B} \quad \text{let } \vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

or $\vec{B} = k\vec{A}$

For option D $\vec{B} = 6\hat{i} + 9\hat{j} - 24\hat{k}$

$$= 3(2\hat{i} + 3\hat{j} - 8\hat{k})$$

$$\Rightarrow \vec{B} = 3\vec{A}$$

⑬

$$\vec{A} = 3\hat{i} + 4\hat{j}, \quad \vec{B} = 12\hat{i} + 5\hat{j}$$

$$|\vec{A}| = \sqrt{3^2 + 4^2} \quad ; \quad |\vec{B}| = \sqrt{12^2 + 5^2}$$

$$= \sqrt{9 + 16} = 5 \quad = \sqrt{144 + 25} = 13$$

$$\text{unit vector } \vec{A} = \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{3\hat{i} + 4\hat{j}}{5}$$

$\vec{B}$ is not a unit vector because its magnitude is not 1

(14), (15), (16)

Given $\vec{A} = 2\hat{i} + 2\hat{j}$; $\vec{B} = 2\hat{i} - 4\hat{j}$

Magnitude of $\vec{B} = |\vec{B}| = \sqrt{x^2 + y^2} = \sqrt{2^2 + (-4)^2}$

$$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Magnitude of $\vec{A} = |\vec{A}| = \sqrt{x^2 + y^2} = \sqrt{2^2 + 2^2}$

$$= 2\sqrt{2}$$

Unit vector of $\vec{A} = \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{2\hat{i} + 2\hat{j}}{2\sqrt{2}} = \frac{2(\hat{i} + \hat{j})}{2\sqrt{2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

(17)

Given $\vec{A} = 4\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} - 8\hat{k}$

Let $\vec{C}$ is the resultant of $\vec{A}$ and $\vec{B}$ Then

$$\begin{aligned}\vec{C} = \vec{A} + \vec{B} &= 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k} \\ &= 3\hat{i} + 6\hat{j} - 2\hat{k}\end{aligned}$$

vector Parallel to resultant of $\vec{A}$ and $\vec{B}$ (i.e) to $\vec{C}$ is

$$= \frac{\vec{C}}{|\vec{C}|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}}$$

$$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{9 + 36 + 4}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{49}}$$

$$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

(18)

let $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} = 2\hat{i} - 3\hat{j} + 8\hat{k}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k} = -\hat{i} + 1.5\hat{j} + c\hat{k}$

condition for two vectors to be parallel is

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} \Rightarrow \frac{2}{-1} = \frac{-3}{1.5} = \frac{-8}{c}$$

Take any two i.e. $\frac{2}{-1} = \frac{-8}{c} \Rightarrow c = 4$

Task see main's level

(1)

let the given vector is $\vec{A} = \frac{1}{2}\hat{i} + \frac{1}{2}\hat{j} + c\hat{k}$

Given $\vec{A}$ is a unit vector $|\vec{A}| = 1$

$$= \left| \frac{1}{2}\hat{i} + \frac{1}{2}\hat{j} + c\hat{k} \right| = 1$$

$$\Rightarrow \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + c^2} = 1 \quad \text{on squaring both sides}$$

$$\Rightarrow \left(\frac{1}{4} + \frac{1}{4} + c^2 \right) = 1^2$$

$$\Rightarrow \frac{2}{4} + c^2 = 1 \Rightarrow \frac{1}{2} + c^2 = 1 \Rightarrow c^2 = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow c = \frac{1}{\sqrt{2}}$$

(2)

let given $P = (1, 2, -1)$ and $Q = (3, 2, 3)$
 $x_1, y_1, z_1 \quad x_2, y_2, z_2$

vector $\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$
 $= (3-1)\hat{i} + (2-2)\hat{j} + (3-(-1))\hat{k}$
 $= 2\hat{i} + 0\hat{j} + 4\hat{k} = 2\hat{i} + 4\hat{k}$

length of $|\vec{PQ}| = \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20}$

(3)

Initial position $r_1 = 2\hat{i} + 3\hat{j} + 5\hat{k}$ m

Final position $r_2 = \hat{i} - 2\hat{j} + \hat{k}$ m

$$\begin{aligned}\text{Displacement} &= \vec{r}_2 - \vec{r}_1 = \hat{i} - 2\hat{j} + \hat{k} - (2\hat{i} + 3\hat{j} + 5\hat{k}) \\ &= \hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k} \\ &= -\hat{i} - 5\hat{j} - 4\hat{k}\end{aligned}$$

(4)

Given

$$0.5\hat{i} + 0.8\hat{j} + 0.6\hat{k} \text{ is a unit vector}$$

$$\Rightarrow |0.5\hat{i} + 0.8\hat{j} + 0.6\hat{k}| = 1$$

$$\Rightarrow \sqrt{(0.5)^2 + (0.8)^2 + (0.6)^2} = 1 \text{ on squaring both sides}$$

$$\Rightarrow \sqrt{0.25 + 0.64 + 0.36} = 1$$

$$\Rightarrow 0.25 + 0.64 + 0.36 = 1 \Rightarrow 1.25 = 1 \Rightarrow 0.25 = 0 \Rightarrow 0.5 = 0 \Rightarrow 0.5 = 1$$

(5)

Let $\vec{A}$ is a vector having magnitude 5 and opposite to $3\hat{i} + 4\hat{j}$ (let be $\vec{B}$)

$$\begin{aligned}\therefore \vec{A} &= 5 \times (-\text{unit vector of given vector}) \text{ (i.e.) } \vec{B} = \frac{\vec{B}}{|\vec{B}|} \\ &= -5 \times \frac{(3\hat{i} + 4\hat{j})}{\sqrt{3^2 + 4^2}} = -5 \times \frac{(3\hat{i} + 4\hat{j})}{5} \\ &= -(3\hat{i} + 4\hat{j})\end{aligned}$$

⑤

⑥

Let the given vector be $\vec{A} = 1\hat{i} + 1\hat{j} + 1\hat{k}$.
unit vector Parallel to $\vec{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{1\hat{i} + 1\hat{j} + 1\hat{k}}{\sqrt{1^2 + 1^2 + 1^2}}$

$$= \frac{1\hat{i} + 1\hat{j} + 1\hat{k}}{\sqrt{3}}$$

⑦

Given $\vec{r}_1 = 4\hat{i} + 3\hat{j} + 8\hat{k}$ & $\vec{r}_2 = 2\hat{i} + 10\hat{j} + 5\hat{k}$

Position of a vector 1 with respect to 2 is $=(\vec{r}_2 - \vec{r}_1)$

$$= -(2\hat{i} + 10\hat{j} + 5\hat{k} - (4\hat{i} + 3\hat{j} + 8\hat{k}))$$

$$= -[2\hat{i} + 10\hat{j} + 5\hat{k} - 4\hat{i} - 3\hat{j} - 8\hat{k}]$$

$$= -[-2\hat{i} + 7\hat{j} - 3\hat{k}] = 2\hat{i} - 7\hat{j} + 3\hat{k}$$

⑧

Given

$$\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7t\hat{k} \text{ m}$$

$$\text{At } t = 10 \text{ s} \quad \vec{r} = 300\hat{i} + 400\hat{j} + 70\hat{k}$$

$$\text{Displacement} \Rightarrow |\vec{r}| = \sqrt{(300)^2 + (400)^2 + 7^2}$$

$$= \sqrt{90000 + 160000 + 49}$$

$$= \sqrt{250049} \approx 500 \text{ m}$$

(9)

$$\text{let } \vec{A} = \begin{matrix} 6\hat{i} - 4\hat{j} + 8\hat{k} \\ A_x \quad A_y \quad A_z \end{matrix} \quad \text{and } \vec{B} = \begin{matrix} -2\hat{i} + p\hat{j} - 16\hat{k} \\ B_x \quad B_y \quad B_z \end{matrix}$$

For $\vec{A}$ and $\vec{B}$ to be opposite condition is

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} \Rightarrow \frac{6}{-12} = \frac{-4}{p} = \frac{8}{-16}$$

$$\text{Take any two } \frac{6}{-12} = \frac{-4}{p} = \frac{-16}{8} \Rightarrow \frac{-1}{4} = \frac{-2}{p}$$

$$\Rightarrow p = 8$$

(10)

Given

$$\text{let } \vec{A} = 2\hat{i} + 2\hat{j} \quad ; \quad \vec{B} = \hat{i} + \hat{j} + \hat{k}$$

let $\vec{C}$ is a vector parallel to $\vec{A}$

$$\text{Given } |\vec{C}| = |\vec{B}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\therefore \vec{C} = |\vec{C}| \hat{A} = \sqrt{3} \frac{\vec{A}}{|\vec{A}|} = \sqrt{3} \frac{(2\hat{i} + 2\hat{j})}{\sqrt{2^2 + 2^2}}$$

$$\vec{C} = \frac{\sqrt{3} \times 2(\hat{i} + \hat{j})}{2\sqrt{2}} = \sqrt{\frac{3}{2}} (\hat{i} + \hat{j})$$

(15), (16)

$$\text{Given } \vec{F}_1 = 8\hat{i} + 6\hat{j} \text{ N} ; \vec{F}_2 = 4\hat{i} - 8\hat{j} \text{ N}$$

$$\text{then } \vec{F}_1 + \vec{F}_2 = 8\hat{i} + 6\hat{j} + 4\hat{i} - 8\hat{j} \Rightarrow 12\hat{i} - 2\hat{j}$$

$$\text{and } \vec{F}_1 - \vec{F}_2 = 8\hat{i} + 6\hat{j} - [4\hat{i} - 8\hat{j}] = 8\hat{i} + 6\hat{j} - 4\hat{i} + 8\hat{j}$$

$$= 4\hat{i} + 14\hat{j}$$

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02

magnitude of resultant of $\vec{F}_1$ & $\vec{F}_2 = |\vec{F}_1 + \vec{F}_2|$

$$\begin{aligned} &= |3\hat{i} - 2\hat{j}| \\ &= \sqrt{(3)^2 + (-2)^2} \\ &= \sqrt{9+4} = \sqrt{13} \text{ N} \end{aligned}$$

(18)

Given $\vec{A} = 3\hat{i} + 2\hat{j}$ & $\vec{B} = 4\hat{i} - 5\hat{j}$

Resultant of $\vec{A}$ and $\vec{B}$ is $\vec{C} = \vec{A} + \vec{B}$

$$\Rightarrow \vec{C} = 3\hat{i} + 2\hat{j} + 4\hat{i} - 5\hat{j} \Rightarrow 7\hat{i} - 3\hat{j}$$

unit vector parallel to $\vec{C}$ is $\Rightarrow \frac{\vec{C}}{|\vec{C}|} = \frac{7\hat{i} - 3\hat{j}}{\sqrt{49+9}}$

$$= \frac{7\hat{i} - 3\hat{j}}{\sqrt{58}} = \frac{7\hat{i} - 3\hat{j}}{\sqrt{58}} = \frac{7(3\hat{i} - \hat{j})}{\sqrt{58}}$$

$$= \frac{3\hat{i} - \hat{j}}{\sqrt{5.8}}$$