Chapter : Elimination of Angle (θ) by Using Trignometric Identifies

TEACHING TASK

1. Given $\sin\theta+\cos\theta=p$ and $\tan\theta+\cot\theta=q$ let $\theta=45^0$

$\sin 45^0+\cos 45^0 = P$ and $\tan 45^0+\cot 45^0=q$

$$P=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$$

q = 1 + 1 = 2

Now $q(p^2-1) = (2)\left(\left(\sqrt{2}\right)^2-1\right)=2(2-1) = 2$

Ans : B

2. Given $\theta=45^0$

$a=x\cos\theta+y\sin\theta$ $\quad$ $b=x\sin\theta-y\cos\theta$

$a=x\cos 45^0+y\sin 45^0$ $\quad$ $b=x\sin 45^0-y\cos 45^0$

$$a=x\left(\frac{1}{\sqrt{2}}\right)+y\left(\frac{1}{\sqrt{2}}\right) \quad b=x\left(\frac{1}{\sqrt{2}}\right)-y\left(\frac{1}{\sqrt{2}}\right)$$

$$a=\frac{x+y}{\sqrt{2}} \quad b=\frac{x-y}{\sqrt{2}}$$

Now $a-b=\frac{x+y}{\sqrt{2}}-\frac{x-y}{\sqrt{2}}=\frac{2y}{\sqrt{2}}=\sqrt{2}y$

Ans: B

3. Given $\sin\theta=a+\frac{1}{4a}$

$$\Rightarrow 2\sec\theta=2a+\frac{1}{2a}$$

$$\Rightarrow (\sec\theta+\tan\theta)+(\sec\theta-\tan\theta)=2a+\frac{1}{2a}$$

$$\therefore \sec\theta+\tan\theta=2a$$

Ans: B

4. Given $m=a\cos^3\theta+3a\cos\theta.\sin^2\theta$

Let $\theta=0^0$

$$\therefore m=a\cos^3 0^0+3a\cos 0^0.\sin^2 0^0$$

$$\Rightarrow m=a$$

Given $n=a\sin^3\theta+3a\sin\theta.\cos^2\theta$

$Let\ \theta=0^0$

$\therefore\ n = a\sin^3 0^0 + 3a\sin 0^0.\cos^2 0^0$

$\Rightarrow n = 0$

Now $(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}}$

$= (a+0)^{\frac{2}{3}} + (a-0)^{\frac{2}{3}}$

$= 2a^{\frac{2}{3}}$

Ans: D

5. Given $x = a\cos^3\theta.\sin^2\theta$ $\qquad$ $y = a\sin^3\theta.\cos^2\theta$

$x^2 = a^2\cos^6\theta.\sin^4\theta$ $\qquad$ $y^2 = a^2\sin^6\theta.\cos^4\theta$

Now $x^2 + y^2 = a^2\cos^6\theta.\sin^4\theta + a^2\sin^6\theta.\cos^4\theta$

$= a^2\cos^4\theta.\sin^4\theta(\cos^2\theta + .\sin^2\theta)$

$= a^2\cos^4\theta.\sin^4\theta$

Now $xy = a\cos^3\theta.\sin^2\theta a\sin^3.\cos^2\theta$

$= a^2\cos^5\theta.\sin^5\theta$

$(x^2+y^2)^{\frac{1}{4}} = a^{\frac{1}{2}}\cos\theta.\sin\theta$

Also $(xy)^{\frac{1}{5}} = a^{\frac{2}{5}}.\cos\theta.\sin\theta$

$\therefore\ \dfrac{(x^2+y^2)^{\frac{1}{4}}}{(xy)^{\frac{1}{5}}} = \dfrac{a^{\frac{1}{2}}\cos\theta.\sin\theta}{a^{\frac{2}{5}}\cos\theta.\sin\theta} = a^{\frac{1}{10}}$, which is independent of θ

$\therefore P = \frac{1}{4}, q = \frac{1}{5}$

$\Rightarrow 4P = 5q$

Ans: A

6. Given $x = \cot\theta + \tan\theta$, $\qquad$ $y = \sec\theta - \cos\theta$

Let $\theta = 45^0$

$\therefore\ x = \cot 45^0 + \tan 45^0$ $\qquad$ $y = \sec 45^0 - \cos 45^0$

x = 1+1 $\qquad$ $y = \sqrt{2} - \frac{1}{\sqrt{2}}$

x = 2 $\qquad$ $y = \frac{1}{\sqrt{2}}$

Now $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}}$

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$$=\left(4.\frac{1}{\sqrt{2}}\right)^{\frac{2}{3}}-\left(2.\frac{1}{2}\right)^{\frac{2}{3}}$$

$$=\left(2\sqrt{2}\right)^{\frac{2}{3}}-1$$

$$=\left(2^{\frac{3}{2}}\right)^{\frac{2}{3}}-1$$

= 2-1

= 1

Ans: C

7. Given $x=a\left(\cos ec\alpha+\cot\alpha\right)$ $\qquad y=\dfrac{b(1-\cos\alpha)}{\sin\alpha}$

Let $\alpha=45^0$

$\therefore x=a\left(\cos ec45^0+\cot 45^0\right)$ $\qquad y=\dfrac{b(1-\cos 45^0)}{\sin 45^0}$

$\Rightarrow x=a\left(\sqrt{2}+1\right)$ $\qquad y=\dfrac{b\left(1-\frac{1}{\sqrt{2}}\right)}{\frac{1}{\sqrt{2}}}$

$y=b\left(\sqrt{2}-1\right)$

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Now $xy=a\left(\sqrt{2}+1\right).b\left(\sqrt{2}-1\right)$

$\Rightarrow xy=ab\left(2-1\right)$

$\Rightarrow xy=ab$

Ans: D

8. Given $a\sec\theta+b\tan\theta+c=0$ .... (1)

$p\sec\theta+q\tan\theta+r=0$ .... (2)

(1) x q $\Rightarrow aq\sec\theta+bq\tan\theta+cq=0$

(2) x b $\Rightarrow bp\sec\theta+bq\tan\theta+br=0$

$\Rightarrow\left(aq-bq\right)\sec\theta+\left(cq-br\right)=0$

$$\Rightarrow\sec\theta=\frac{br-cq}{aq-bp}$$

Similarly $\Rightarrow\tan\theta=\dfrac{pc-ar}{aq-bp}$

Now, $\sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow \left(\frac{br-qc}{aq-bp}\right)^2 - \left(\frac{pc-ar}{aq-bp}\right)^2 = 1$$

$$\Rightarrow (br-qc)^2 - (pc-ar)^2 = (aq-bp)^2$$

Ans: C

9. Given $x = a\sec\theta.\cos\phi$ $\quad y = b\sec\theta.\sin\phi$ $\quad z = c\tan\theta$

$\Rightarrow \frac{x}{a} = \sec\theta.\cos\phi$ $\quad \Rightarrow \frac{y}{b} = \sec\theta.\sin\phi$ $\quad \Rightarrow \frac{z}{c} = \tan\theta$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = \sec^2\theta.\cos^2\phi + \sec^2\theta.\sin^2\phi - \tan^2\theta$$

$$= \sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta$$

$$= \sec^2\theta - \tan^2\theta$$

$$= 1$$

Ans: 1

10. Given $x = a\sec\theta$; $y = b\tan\theta$

$$\Rightarrow \frac{x}{a} = \sec\theta \quad ; \quad \frac{y}{b} = \tan\theta$$

$$\Rightarrow \frac{x^2}{a^2} = \sec^2\theta, \quad \frac{y^2}{b^2} = \tan^2\theta$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Ans : B

11. Given SinA+cosA = p, $\quad \sin^3A+\cos^3A = q$

Let A = 0^0

$\therefore$ $\sin0^0+\cos0^0 = p$, $\quad \sin^30^0+\cos^30^0 = q$

$\therefore$ p = 1, $\quad$ q = 1

Now $p^3-3p+2q = (1)^3-3(1)+2(1)$

= 1-3+2

= 0

Ans: C

12. Given $\sec\theta+\tan\theta = x$ .... (1)

$$\Rightarrow \sec\theta-\tan\theta = \frac{1}{x} \quad \text{............ (2)}$$

(1)+(2) $\Rightarrow 2\sec\theta = x + \frac{1}{x}$

$$\Rightarrow \sec\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$$

(1)-(2) $\Rightarrow 2\tan\theta = x - \frac{1}{x}$

$$\Rightarrow \tan\theta = \frac{1}{2}\left(x - \frac{1}{x}\right)$$

13. Given $(\cos ec\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

Let $\theta = 45^0$

$(\cos ec45^0 - \sin 45^0)(\sec 45^0 - \cos 45^0)(\tan 45^0 + \cot 45^0)$

$= \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)(1+1)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(2) = 1$

14. $(1+\tan\theta+\sec\theta)(1+\cot\theta\text{-}\mathrm{cosec}\theta)$

Let $\theta = 45^0$

$= (1+\tan45^0+\sec45^0)(1+\cot45^0\text{-}\mathrm{cosec}45^0)$

$= (1+1+\sqrt{2})(1+1-\sqrt{2})$

$= (2+\sqrt{2})(2-\sqrt{2})$

= (4-2) = 2

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15. **Statement - I**

Given $\sin\theta + \cos\theta = x$

Let $\theta = 0^0$

$\therefore \sin 0^0 + \cos 0^0 = x$

$\therefore x = 1$

$\sin^6\theta + \cos^6\theta = \sin^6 0^0 + \cos^6 0^0$

$= 1$

$\frac{4-3(x^2-1)}{4} = \frac{4-3(1^2-1)}{4}$

$= 1$

$\therefore$ Statement - I is correct

Statement - II $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta.\cos^2\theta$

Let $\theta = 0^0$

$\sin^6 0^0 + \cos^6 0^0 = 1$

$1 - 3\sin^2 0^0.\cos^2 0^0 = 1$

$\therefore$ Statement - II is correct

16. **Statement - I**

Given $x = a\sec\theta + b\tan\theta$

$\Rightarrow\ x^2 = a^2 \sec^2\theta + b^2 \tan^2\theta + 2ab\sec\theta\tan\theta$

Again $y = a\tan\theta + b\sec\theta$

$\Rightarrow y^2 = a^2\tan^2\theta + b^2\sec^2\theta + 2ab\tan\theta\sec\theta$

Now $x^2 - y^2 = a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$

$\Rightarrow x^2 - y^2 = a^2 - b^2$

Statement-I is correct

Statement-II

Given $\sec\theta + \tan\theta = a$

$$\Rightarrow \sec\theta - \tan\theta = \frac{1}{a}$$

$$\Rightarrow \tan\theta - \sec\theta = \frac{-1}{a}$$

Statement - II is correct

17. Given $\ell = \operatorname{cosec}\theta - \sin\theta$ $\quad m = \sec\theta - \cos\theta$

Let $\theta = 45^0$

Statement - I

$\ell = \operatorname{cosec}45^0 - \sin 45^0$ $\quad m = \sec 45^0 - \cos 45^0$

$\Rightarrow \ell = \sqrt{2} - \frac{1}{\sqrt{2}}$ $\quad m = \sqrt{2} - \frac{1}{\sqrt{2}}$

$\Rightarrow \ell = \frac{1}{\sqrt{2}}$ $\quad m = \frac{1}{\sqrt{2}}$

Now $\ell^2 m^2\left(\ell^2 + m^2 + 3\right)$

$$= \left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2\left(\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 3\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2} + \frac{1}{2} + 3\right)$$

$$= \frac{1}{4}(4) = 1$$

$\therefore$ Statement - I is true

Statement - II

$$\frac{\cos\theta}{1-\sin\theta} = \frac{\cos\theta}{1-\sin\theta} \text{ x } \frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$\frac{\sin\theta}{1+\sin\theta} = \frac{\cos\theta}{1+\cos\theta}$$

Let $\theta = 0^0$ $\quad \frac{\sin 0^0}{1+\sin 0^0} = \frac{\cos 0^0}{1+\cos 0^0}$

$$\frac{0}{1+0} = \frac{1}{1+1}$$

$0 = \frac{1}{2}$ (False)

$\therefore$ Statement - II False

Given $\cos A+\cos^2 A = 1$

$$\Rightarrow \cos A = 1-\cos^2 A$$

$$\Rightarrow \cos A = \sin^2 A$$

18. $\sin^2 A+\sin^4 A$

$$\Rightarrow \cos A + \cos^2 A = 1$$

19. Now, $\sin^4 A-2\sin^2 A-\sin^8 A$

= $\cos^2 A-2\cos A-\cos^4 A$

= $\cos^2 A-\cos^4 A-2\cos A$

= $\cos^2 A(1-\cos^2 A)-2\cos A$

= $\cos^2 A(\cos A)-2\cos A$

= $\cos A(\cos^2 A-2)$

= $\cos A(1-\cos A-2)$

= $\cos A(-1-\cos A) = -\cos A-\cos^2 A=-1$

20. Now, $\cos^6 A+4\cos^3 A$

= $\cos^3 A(\cos^3 A+4)$

= $\cos^2 A.\cos A(\cos^2 A.\cos A+4)$

= $(1-\cos A)\cos A[(1-\cos A)\cos A+4]$

= $(\cos A-\cos^2 A)(\cos A-\cos^2 A+4)$

= $(\cos A-1+\cos A)(\cos A-1+\cos A+4)$

= $(2\cos A-1)(2\cos A+3)$

= $4\cos^2 A+4\cos A-3$

= $4(1-\cos A)+4\cos A-3$

= $4-4\cos A+4\cos A-3=1$

21. Given $2\sec\theta = x+\frac{1}{x}$, $\quad 2\tan\theta = x-\frac{1}{x}$

$$\Rightarrow 4\sec^2\theta = \left(x+\frac{1}{x}\right)^2 \qquad \Rightarrow 4\tan^2\theta = \left(x-\frac{1}{x}\right)^2$$

$$\Rightarrow 4\left(\sec^2\theta-\tan^2\theta\right) = \left(x+\frac{1}{x}\right)^2 - \left(x-\frac{1}{x}\right)^2$$

$$\therefore \left(x+\frac{1}{x}\right)^2 - \left(x-\frac{1}{x}\right)^2 = 4$$

22. a) Given $m = \cot\theta+\tan\theta$ $\quad n = \sec\theta-\cos\theta$

$\theta = 45^0$

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$\therefore\ m = \cot 45^0 + \tan 45^0$

$m = 1 + 1$

$m = 2$

$\therefore\ n = \sec 45^0 - \cos 45^0$

$n = \sqrt{2} - \dfrac{1}{\sqrt{2}}$

$n = \dfrac{1}{\sqrt{2}}$

Now $\left(m^2 n\right)^{\frac{2}{3}} - \left(mn^2\right)^{\frac{2}{3}}$

$= \left(4.\dfrac{1}{\sqrt{2}}\right)^{\frac{2}{3}} - \left(2.\dfrac{1}{2}\right)^{\frac{2}{3}}$

$= \left(2.\sqrt{2}\right)^{\frac{2}{3}} - (1)^{\frac{2}{3}}$

$= \left(2^{\frac{3}{2}}\right)^{\frac{2}{3}} - 1$

= 2-1

= 1

b) Given $m = \operatorname{cos}ec\theta - \sin\theta$ $\quad$ $n = \sec\theta - \cos\theta$

Let $\theta = 45^0$ $\quad$ Let $\theta = 45^0$

$\Rightarrow m = \sqrt{2} - \dfrac{1}{\sqrt{2}}$ $\quad$ $n = \sqrt{2} - \dfrac{1}{\sqrt{2}}$

$\Rightarrow m = \dfrac{1}{\sqrt{2}}$ $\quad$ $n = \dfrac{1}{\sqrt{2}}$

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Now $\left(m^2 n\right)^{\frac{2}{3}} + \left(mn^2\right)^{\frac{2}{3}}$

$= \left(\dfrac{1}{2}.\dfrac{1}{\sqrt{2}}\right)^{\frac{2}{3}} + \left(\dfrac{1}{\sqrt{2}}.\dfrac{1}{2}\right)^{\frac{2}{3}}$

$= \left(2^{\frac{1}{3/2}}\right)^{\frac{2}{3}} + \left(\dfrac{1}{2^{3/2}}\right)^{\frac{2}{3}}$

$= \dfrac{1}{4} + \dfrac{1}{2} = 1$

c) Given $a = x\cos^2\alpha + y\sin^2\alpha$

Now $(x - \alpha)(y - a) + (x - y)^2 \sin^2\alpha.\cos^2\alpha$

$= \left(x - x\cos^2\alpha - y\sin^2\alpha\right)\left(y - x\cos^2\alpha - y\sin^2\alpha\right) + (x - y)^2 \sin^2\alpha.\cos^2\alpha$

$= \left(x(1-\cos^2\alpha) - y\sin^2\alpha\right)\left(y(1-\sin^2\alpha) - x\cos^2\alpha\right) + \left(x-y\right)^2 \sin^2\alpha.\cos^2\alpha$

$= \left(x\sin^2\alpha - y\sin^2\alpha\right)\left(y\cos^2\alpha - x\cos^2\alpha\right) + \left(x-y\right)^2 \sin^2\alpha.\cos^2\alpha$

$= (x-y)\sin^2\alpha\left(y-x\right)\cos^2\alpha + \left(x-y\right)^2 \sin^2\alpha.\cos^2\alpha$

$= -(x-y)^2\sin^2\alpha.\cos^2\alpha + \left(x-y\right)^2 \sin^2\alpha.\cos^2\alpha$

$= 0$

d) Given $x = r\cos\alpha.\cos\beta, \cos\gamma, y = r\cos\alpha\cos\beta\sin\gamma$

$\mu = r\sin\beta \; and \; z = r\sin\alpha.\cos\beta$

$Let \; \alpha = \beta = \gamma = 45^0$

$\therefore x = r\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$x = \frac{r}{2\sqrt{2}}$ similarly $y = \frac{r}{2\sqrt{2}}, \mu = \frac{r}{\sqrt{2}}, z = \frac{r}{2}$

Now

$$x^2 + y^2 + z^2 + \mu^2 = \frac{r^2}{8} + \frac{r^2}{8} + \frac{r^2}{2} + \frac{r^2}{4}$$

$$= \frac{r^2 + r^2 + 4r^2 + 2r^2}{8}$$

$$= \frac{8r^2}{8}$$

$$= r^2$$

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Ans : a-r, b-r, c-q, d-s

LEARNERS TASK

1. $x = m\sec\theta$, $\quad y = n\tan\theta$

$\Rightarrow \frac{x}{m} = \sec\theta, \quad \frac{y}{n} = \tan\theta$

$\Rightarrow \frac{x^2}{m^2} - \frac{y^2}{n^2} = \sec^2\theta - \tan^2\theta$

$\Rightarrow \frac{x^2}{m^2} - \frac{y^2}{n^2} = 1$

2. Given $x = \text{cosec}\,\theta + \cot\theta$, $\quad y = \text{cosec}\,\theta - \cot\theta$

$\therefore \; xy = \left(\cos ec\theta + \cot\theta\right)\left(\cos ec\theta - \cot\theta\right)$

$xy = \text{cosec}^2\theta - \cot^2\theta$

$xy = 1$

3. Given $a = x\cos ec\theta$ *and* $b = y\cot\theta$

$$\Rightarrow \frac{a}{x} = \cos ec\theta, \qquad \frac{b}{y} = \cot\theta$$

Now, $\frac{a^2}{x^2} - \frac{b^2}{y^2} = \cos ec^2\theta - \cot^2\theta$

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$$

4. Given $x = a(\sec\theta + \tan\theta)^2$

$$y = b(\sec\theta - \tan\theta)^2$$

$$xy = a(\sec\theta + \tan\theta)^2\, b(\sec\theta - \tan\theta)^2$$

$$xy = ab(\sec^2\theta - \tan^2\theta)^2$$

$$xy = ab \Rightarrow x^2y^2 = a^2b^2$$

5. Given $x = a(\text{cosec}\,\theta + \cot\theta)$

$$y = b(\text{cosec}\,\theta - \cot\theta)$$

$$\Rightarrow \quad xy = ab\,(\text{cosec}^2\theta - \cot^2\theta)$$

$$\Rightarrow \quad xy = ab$$

6. Given $x = \cot\theta + \cos\theta$, $\quad y = \cot\theta - \cos\theta$

$x + y = 2\cot\theta$, $\quad x - y = 2\cos\theta$

$$(x+y)(x-y) = 4\cot\theta.\cos\theta$$

$$(x^2 - y^2)^2 = 16\cot^2\theta.\cos^2\theta$$

$$= 16\,\frac{\cos^2\theta}{\sin^2\theta}(1 - \sin^2\theta)$$

$$= 16\left(\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta\right)$$

$$= 16\,(\cot^2\theta - \cos^2\theta)$$

$$= 16\,(\cot\theta + \cos\theta)(\cot\theta - \cos\theta)$$

$$= 16xy$$

7. $x = a\sin\theta$ and $y = b\tan\theta$

$$\Rightarrow \frac{a}{x} = \cos ec\theta, \ \frac{b}{y} = \cot\theta$$

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$$\Rightarrow \frac{a^2}{x^2} - \frac{b^2}{y^2} = \cos ec^2\theta - \cot^2\theta$$

$= 1$

8. Given $x = 2\cos ec\theta$, $y = 2\cot\theta$

$$\Rightarrow \frac{x}{2} = \cos ec\theta, \qquad \Rightarrow \frac{y}{2} = \cot\theta$$

$$\Rightarrow \frac{x^2}{4} - \frac{y^2}{4} = \cos ec^2\theta - \cot^2\theta = 1$$

$$\Rightarrow x^2 - y^2 = 4$$

9. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$= 2(\sin^2\theta + \cos^2\theta)$

$= 2(1)$

$= 2$

10. Given $x = at$, $y = \frac{a}{t}$

$$\Rightarrow xy = (at)\left(\frac{a}{t}\right)$$

$$\Rightarrow xy = a^2$$

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JEE MAIN LEVEL QUESTIONS

1. $x = a\tan\theta$, $y = b\sec\theta$

$$\Rightarrow \frac{x}{a} = \tan\theta, \qquad \frac{y}{b} = \sec\theta$$

$$\Rightarrow \frac{y^2}{b^2} - \frac{x^2}{a^2} = \sec^2\theta - \tan^2\theta$$

$$\Rightarrow \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1 \qquad \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

2. $x = p(\cos ec\theta + \cot\theta)$, $y = q(\cot\theta - \cos ec\theta)$

$$\Rightarrow xy = -pq(\cos ec\theta + \cot\theta)(\cos ec\theta - \cot\theta)$$

$$\Rightarrow xy = -pq(\cos ec^2\theta - \cot^2\theta)$$

$$\Rightarrow xy = -pq$$

$$\Rightarrow \frac{x}{p} = -\frac{-q}{y}$$

3. $p = \cos\theta + \sin\theta$, $q = \sin\theta - \cos\theta$

$p^2 + q^2 = (\cos\theta + \sin\theta)^2 + (\sin\theta - \cos\theta)^2$

$= 2(\cos^2\theta + \sin^2\theta)$

$= 2$

$p^2 - 2 = -q^2$

4. $x = p\cos\theta + q\sin\theta$, $\quad y = p\sin\theta - q\cos\theta$

$x^2 + y^2 = p^2\cos^2\theta + q^2\sin^2\theta + 2pq\cos\theta\sin\theta + p^2\sin^2\theta + q^2\cos^2\theta - 2pq\sin\theta\cos\theta$

$\therefore\ x^2 + y^2 = p^2 + q^2$

$\Rightarrow\ x^2 - p^2 = q^2 - y^2$

5. $x^2 = a^2\cos^2\theta$; $\quad y^2 = b^2\sin^2\theta$

$\Rightarrow \dfrac{x^2}{a^2} = \cos^2\theta$, $\quad \Rightarrow \dfrac{y^2}{b^2} = \sin^2\theta$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

6. $x^2 + y^2 + z^2 = r^2\cos^2\theta.\cos^2\phi + r^2\cos^2\theta.\sin^2\phi + r^2\sin^2\theta$

$= r^2\cos^2\theta(\cos^2\phi + \sin^2\phi) + r^2\sin^2\theta$

$= r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2(\cos^2\theta + \sin^2\theta)$

$= r^2$

7. $x = a\sin^4\theta$, $\quad y = b\cos^4\theta$

$\dfrac{x}{a} = \sin^4\theta$, $\quad \dfrac{y}{b} = \cos^4\theta$

$\Rightarrow \sqrt{\dfrac{x}{a}} + \sqrt{\dfrac{y}{b}} = \sin^2\theta + \cos^2\theta = 1$

8. $a = x\sec\theta + y\tan\theta$, $\quad b = x\tan\theta + y\sec\theta$

$a^2 = x^2\sec^2\theta + y^2\tan^2\theta + 2xy\sec\theta.\tan\theta$

$b^2 = x^2\tan^2\theta + y^2\sec^2\theta + 2xy\tan\theta.\sec\theta$

$a^2 - b^2 = x^2(\sec^2\theta - \tan^2\theta) - y^2(\sec^2\theta - \tan^2\theta)$

$\therefore a^2 - b^2 = x^2 - y^2$

9. Let $\phi = \psi = \delta = 45^0$

$\therefore a = \cos 45^0.\cos 45^0 + \sin 45^0.\sin 45^0.\cos 45^0$

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$\Rightarrow a = \frac{1}{\sqrt{2}} . \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} . \frac{1}{\sqrt{2}} . \frac{1}{\sqrt{2}}$

$\Rightarrow a = \frac{1}{2} + \frac{1}{2\sqrt{2}}$

similarly $b = \frac{1}{2} - \frac{1}{2\sqrt{2}}$ and $c = \frac{1}{2}$

Now $a^2 + b^2 + c^2 = \left(\frac{1}{2} + \frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2} - \frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$

$= 2\left(\frac{1}{4} + \frac{1}{8}\right) + \frac{1}{4}$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

10. Given $x = a\cos^2\theta.\sin\theta$, $y = a\sin^2\theta.\cos\theta$

Let $\theta = 45^0$

$\therefore \; x = a\cos^2 45^0.\sin 45^0$, $y = a\sin^2 45^0.\cos 45^0$

$\Rightarrow x = a\left(\frac{1}{\sqrt{2}}\right)^2.\left(\frac{1}{\sqrt{2}}\right)$, $\Rightarrow y = a\left(\frac{1}{\sqrt{2}}\right)^2.\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow x = \frac{a}{2\sqrt{2}}$, $\Rightarrow y = \frac{a}{2\sqrt{2}}$

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Now $x^2 + y^2 = \frac{a^2}{4}$ $\Rightarrow$ $\left(x^2 + y^2\right)^3 = \frac{a^6}{64}$

$x^2 y^2 = \frac{a^4}{64}$

$\therefore \frac{\left(x^2 + y^2\right)^3}{x^2 y^2} = \frac{a^6}{a^4} = a^2$

ADVANCED LEVEL

11. Given $x = \sqrt{a}\cos ec\theta$, $y = \sqrt{a}\cot\theta$

$\Rightarrow \frac{x}{\sqrt{a}} = \cos ec\theta$, $\Rightarrow \frac{y}{\sqrt{a}} = \cot\theta$

$\Rightarrow \frac{x^2}{a} = \cos ec^2\theta$, $\frac{y^2}{a} = \cot^2\theta$

$$\Rightarrow \frac{x^2}{a} - \frac{y^2}{a} = 1$$

$$\Rightarrow x^2 - y^2 = a$$

$$(x+y)(x-y) - a = 0$$

12. **Statement I**

Given $\ell = p\sec\theta.\cos\phi$

$$\Rightarrow \frac{\ell}{p} = \sec\theta.\cos\phi$$

Similarly $\frac{m}{q} = \sec\theta.\sin\phi,\ \frac{n}{r} = \tan\theta$

Now, $\Rightarrow \frac{\ell^2}{p^2} + \frac{m^2}{q^2} + \frac{n^2}{r^2} = \sec^2\theta.\cos^2\phi + \sec^2\theta.\sin^2\phi + \tan^2\theta$

$$= \sec^2\theta(\cos^2\phi + \sin^2\phi) + \tan^2\theta$$

$$= \sec^2\theta + \tan^2\theta \neq 1$$

Satement-I is false.

Statement-II

i) $\frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta}{1-\cos\theta}.\frac{1+\cos\theta}{1+\cos\theta} = \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

$$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} = \frac{1+\cos\theta}{\sin\theta}$$

ii) We know $\sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$$

$$\Rightarrow \tan\theta + \sec\theta = \frac{1}{\sec\theta - \tan\theta}$$

Statement-II is false

13. Given $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1,\ \frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$

$$\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right)^2 = (1)^2$$

$$\Rightarrow \frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{2xy}{ab}\cos\theta\sin\theta = 1 \quad \text{.................. (1)}$$

Again $\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right)^2 = (1)^2$

$$\Rightarrow \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - \frac{2xy}{ab}\sin\theta\cos\theta = 1 \quad \text{.................. (2)}$$

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(1)+(2) $\Rightarrow \dfrac{x^2}{a^2}\left(\cos^2\theta+\sin^2\theta\right)+\dfrac{y^2}{b^2}\left(\sin^2\theta+\cos^2\theta\right)=2$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=2$$

$$\Rightarrow a^2y^2+b^2x^2=2a^2b^2$$

14. We have $a^2y^2+b^2x^2=2a^2b^2$

Given a=2, b=2

$$4y^2+4x^2=2.4.4$$

$$\Rightarrow x^2+y^2=8$$

15. $\dfrac{x}{a}\cos\theta+\dfrac{y}{b}\sin\theta=1$ Given $\theta=45^0$, a=b=2

$$\Rightarrow \frac{x}{2}\cos 45^0+\frac{y}{2}\sin 45^0=1$$

$$\Rightarrow \frac{x}{2\sqrt{2}}+\frac{y}{2\sqrt{2}}=1$$

$$\Rightarrow x+y=2\sqrt{2}$$

16. Give $u=\ell\tan\theta$ $\quad$ $\text{v}=m\sin\theta$

$\Rightarrow \cot\theta=\dfrac{\ell}{u}$ $\quad$ $\Rightarrow \operatorname{cosec}\theta=\dfrac{m}{\text{v}}$

Now, $\dfrac{m^2}{\text{v}^2}-\dfrac{\ell^2}{\text{u}^2}=\operatorname{cosec}^2\theta-\cot^2\theta$

$= 1$

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17. a) x = at², y = 2at

$$\Rightarrow y^2=4a^2t^2$$

$$=4a\left(at^2\right)$$

$$=4ax$$

$$\therefore y^2=4ax$$

b) $x=at,\ y=\dfrac{a}{t}$

$$\Rightarrow xy=(at)\left(\frac{a}{t}\right)$$

$$\Rightarrow xy=a^2$$

c) $x=a\sec\theta,\qquad y=a\tan\theta$

$$\Rightarrow \frac{x}{a} = \sec\theta, \qquad \frac{y}{a} = \tan\theta$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

$$\Rightarrow x^2 - y^2 = a^2$$

d) $x = a\sin\theta_1, \quad y = b\cos\theta$

$$\Rightarrow \frac{x}{a} = \sin\theta_1, \quad \frac{y}{b} = \cos\theta$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

18. a) $(x-h)^2 + (y-k)^2 = r^2$

Let $x = h + r\cos\theta, \qquad y = k + r\sin\theta$

$\Rightarrow (x-h) = r\cos\theta, \qquad y - k = r\sin\theta$

Now, $(x-h)^2 + (y-k)^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$$= r^2(\cos^2\theta + \sin^2\theta)$$

$$= r^2$$

b) $(x+h)^2 + (y+k)^2 = r^2$

Let $x = -h + r\cos\theta, \qquad y = -k + r\sin\theta$

$\Rightarrow x + h = r\cos\theta, \qquad y + k = r\sin\theta$

$\Rightarrow (x+h)^2 + (y+k)^2 = r^2$

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c) $(x-k)^2 + (y-h)^2 = r^2$

Let $x = k + r\cos\theta, \qquad y = h + r\sin\theta$

$\Rightarrow x - k = r\cos\theta, \qquad y - h = r\sin\theta$

$\Rightarrow (x-k)^2 + (y-h)^2 = r^2$

d) $(x+k)^2 + (y+h)^2 = r^2$

Let $x = -k + r\sin\theta, \qquad y = -h + r\cos\theta$

$\Rightarrow (x+k) = r\sin\theta, \qquad y + h = r\cos\theta$

$\Rightarrow (x+k)^2 + (y+h)^2 = r^2$

ADDITIONAL PRACTICE QUESTIONS FOR STUDENT

1. Given $x = a^2(\sec\theta + \tan\theta)^2, \qquad y = b^2(\sec\theta - \tan\theta)^2$

$$\Rightarrow xy = a^2b^2\left(\sec^2\theta - \tan^2\theta\right)^2$$

$$\Rightarrow xy = a^2b^2$$

2. Given a = b = 1, $\theta = 60^0$

$$x = 1^2\left(\sec 60^0 + \tan 60^0\right)^2$$

$$x = \left(2+\sqrt{3}\right)^2 = 7+4\sqrt{3}$$

Similarly $y = \left(2-\sqrt{3}\right)^2 = 7-4\sqrt{3}$

Now $x^2 + y^2 = \left(7+4\sqrt{3}\right)^2 + \left(7-4\sqrt{3}\right)^2$

$$= 2\left(7^2 + \left(4\sqrt{3}\right)^2\right)$$

= 194

3. Given $a = b = \frac{1}{2}$, $\theta = 45^0$

$$x = \left(\frac{1}{2}\right)^2\left(\sqrt{2}+1\right)^2 = \frac{3+2\sqrt{2}}{4}$$

Similarly $y = \frac{3-2\sqrt{2}}{4}$

Now

$$x^2 - y^2 = \left(\frac{3+2\sqrt{2}}{4}\right)^2 - \left(\frac{3-2\sqrt{2}}{4}\right)^2$$

$$= \frac{4.3.2\sqrt{2}}{16} = \frac{3\sqrt{2}}{2}$$

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