12- Gravitation — 9th foundation

Task

(1)

Given $m_1 = m_2 = 1\,gm = 10^{-3}\,kg$

distance $r = 1\,m$

According to Newton's law of gravitation

$$F = G\frac{m_1 m_2}{r^2} = \frac{6.67\times10^{-11}\times10^{-3}\times10^{-3}}{1^2}$$

$$= 6.67\times10^{-17}\,N. \rightarrow C$$

(2)

Given $m_1 = 2\,kg$: $m_2 = 0.5\,kg$: $F = 2\,N$.

distance b/w them $r_1 = r$.

If distance is doubled ie $r_2 = 2r$, Force $= F'$

we know $F \propto \frac{1}{r^2}$

$$\Rightarrow \frac{F'}{F} = \left[\frac{r}{r'}\right]^2 \Rightarrow \frac{F'}{F} = \left(\frac{r}{2r}\right)^2 = \frac{1}{4}$$

$$\Rightarrow F' = \frac{F}{4}\,N. = \frac{2}{4} = \frac{1}{2}\,N. \rightarrow A$$

(3)

Given two masses are m_1 & m_2

F_{12} is a force on m_2 due to m_1 = action

F_{21} is force on m_1 due to m_2 = Reaction

According to Newton's 3rd law action = - Reaction

$$\therefore F_{12} = -F_{21} \rightarrow C$$

4

we know that $F \propto m_1 m_2$

For given options if $m_1 = \frac{M}{2}$; $m_2 = \frac{M}{2}$.

$F \propto \frac{M}{2} \frac{M}{2} \Rightarrow F \propto \frac{M^2}{4}$ maximum $\rightarrow$ D

(a) $m_1 = \frac{M}{4}$; $m_2 = \frac{3M}{4}$: $F \propto \frac{M}{4}\left(\frac{3M}{4}\right) \Rightarrow F \propto \frac{3M^2}{16}$

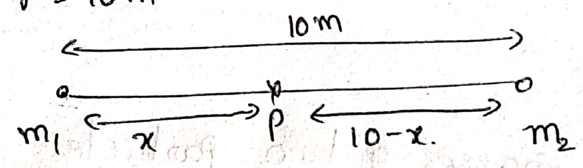
(b) $m_1 = \frac{M}{3}$; $m_2 = \frac{2M}{3} \Rightarrow F \propto \frac{M}{3}\left(\frac{2M}{3}\right) \Rightarrow F \propto \frac{2M}{9}$

(c) $m_1 = \frac{M}{5}$; $m_2 = \frac{4M}{5} \Rightarrow F \propto \frac{M}{5}\left(\frac{4M}{5}\right) \Rightarrow F \propto \frac{4M^2}{25}$.

5

let $m_1 = 900$ kg ; $m_2 = 1600$ kg

$r = 10$m.



let 'p' is the point where field is zero.

$F_1 = F_2$

$\Rightarrow \frac{G m_1}{r_1^2} = \frac{G m_2}{r_2^2}$

$\Rightarrow \frac{900}{x^2} = \frac{1600}{(10-x)^2}$

$\Rightarrow \left(\frac{10-x}{x}\right)^2 = \frac{1600}{900} \Rightarrow \left(\frac{10-x}{x}\right)^2 = \frac{16}{9}$

$\Rightarrow \frac{10-x}{x} = \frac{4}{3}$

$\Rightarrow 30 - 3x = 4x$

$\Rightarrow 7x = 30$

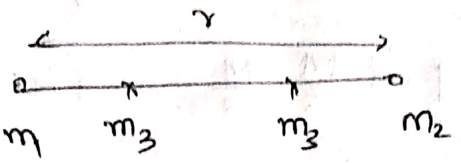
$x = \frac{30}{7}$

From larger mass distance $= 10 - x \Rightarrow 10 - \frac{30}{7} = \frac{40}{7}$ m $\rightarrow$ A

⑥

we know According to Newton's law of gravitation

$$F = G\frac{m_1 m_2}{r^2} \qquad F \propto \frac{m_1 m_2}{r^2}$$



Here force changes either mass change, distance between them changes or, both changes.

we are keeping m_3 near to m_1 & m_2 without changinging m_1 & m_2 values and distance between them.

so force b/w m_1 & m_3 , m_2 & m_3 is different

but b/w m_1 & m_2 does not changes ∴ it is equal to F only

→ c

⑦

Given masses of two particles is $m_1 = m_2 = m$

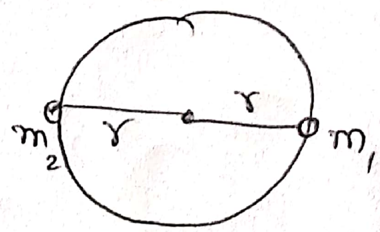
∴ From law of gravitation $F = G\frac{m_1 m_2}{r^2}$

$$\Rightarrow F = \frac{G\,m\,m}{(2r)^2} = \frac{G m^2}{4r^2}$$

two particles are revolving in a circular path under the action of gravitational force of attraction which provides necessary centripetal force.

$$\therefore F_c = F$$

$$\Rightarrow \frac{mv^2}{r} = \frac{Gm^2}{4r^2}$$



$$\Rightarrow v^2 = \frac{Gm}{4r}$$

distance b/w the two bodies = 2r.

$$\Rightarrow v = \sqrt{\frac{Gm}{4r}}$$

8

let original masses of two bodies are m_1, m_2

distance between them $= r_1$

$\therefore$ Force $F = \frac{G m_1 m_2}{r_1^2}$

when masses are doubled ie, $2m_1$, $2m_2$

distance between them $r_2 = \frac{r_1}{2}$.

Force $F' = \frac{G(2m_1)(2m_2)}{\left(\frac{r_1}{2}\right)^2}$

$\Rightarrow F' = 4 \frac{G m_1 m_2}{\frac{r_1^2}{4}}$

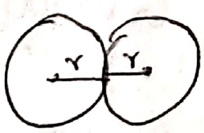
$\Rightarrow F' = 16 \frac{G m_1 m_2}{r_1^2} \Rightarrow F' = 16F \rightarrow C$

9

let m = mass of each sphere.

density $d = m/vol \Rightarrow$ ~~d~~ mass $= d$ vol

$m = d \times \frac{4\pi}{3} r^3 = \frac{4\pi d r^3}{3}$



Distance between $= 2r$

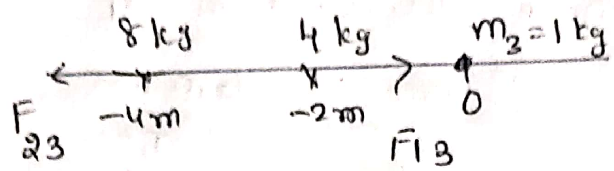
Force $= \frac{G m_1 m_2}{r^2} = \frac{G\left(\frac{4\pi}{3} d r^3\right)\left(\frac{4\pi}{3} d r^3\right)}{(2r)^2}$

$F = \frac{\left(\frac{4\pi}{3}\right)^2 G\, d^2 r^6}{4 r^2}$

$F \propto d^2 r^4 \rightarrow B$

(10)

let $m_1 = 4\,kg$; $m_2 = 8\,kg$



FIG

Net force on $m_3 = \frac{G m_1 m_3}{r_1^2} - \frac{G m_2 m_3}{r_2^2}$

$$m_3 = G\left[\frac{4\times1}{2^2} - \frac{8\times1}{4^2}\right]$$

$$= G\left[1 - \frac{8}{16}\right] = G\left[1-\frac{1}{2}\right] = \frac{G}{2} \rightarrow c$$

(11)

we know

$$F = \frac{G m_1 m_2}{r^2}$$

$A \rightarrow$ $F = \frac{G(20)(30)}{1^2} = 600\,G$

$B \rightarrow$ $F = \frac{G(40)(60)}{(0.5)^2} = \frac{G(2400)}{(\frac{1}{2})^2} = 9600\,G$

$C \rightarrow$ $F = \frac{G(30)(50)}{2^2} = \frac{G\times1500}{4} = 375\,G$

$D \rightarrow$ $F = \frac{G(10)(40)}{(2.5)^2} = \frac{G\times400\times4}{25} = 64\,G$

Descending order B, A, C, D $\rightarrow$ c

(12)

let original masses are m_1 & m_2.

distance between them $r_1 = r$.

$$\therefore F = \frac{G m_1 m_2}{r_1^2} = \frac{G m_1 m_2}{r^2}$$

After masses doubled ie) $m_1' = 2m_1$; $m_2' = 2m_2$

Distance $r_2 = 2r$.

$$\therefore F' = \frac{G m_1' m_2'}{r_2^2} = \frac{G (2m_1)(2m_2)}{(2r)^2}$$

$$F' = \frac{\cancel{4} G m_1 m_2}{\cancel{4} r^2} = \frac{G m_1 m_2}{r^2} = F.$$

No change in force.

(14)

we know the relation between g and G is

$$g = \frac{G M_e}{R_e^2} \rightarrow ① \quad \text{Mass} = \text{density} \times \text{vol}$$

$$= \rho \times \frac{4\pi}{3} R_e^3$$

$$\Rightarrow g = \frac{G \rho (\frac{4\pi}{3}) R_e^3}{R_e^2}$$

$$\Rightarrow g = \frac{4\pi}{3} G \rho R_e \Rightarrow G = \frac{3g}{4\pi \rho R_e}$$

From ① $G = \frac{g R_e^2}{M_e} \rightarrow B$

(15)

If earth stops rotating, then there will be no centripeta force acting on the body and gravitational force increases

$\therefore$ The value of 'g' will increase in some places.

At centre $g = 0$, it is more at poles

$$g_{depth} = \frac{4\pi}{3} \rho (R-d) G \quad \text{At centre } R = d$$

15th continuation

$g_{depth} = \frac{gr}{R}$ $r = R-d$

$\Rightarrow g_{depth} \propto r$.   so g_d increases with increase in r

(16)

The chandrasekhar limit is accepted to be approximately 1.4 times the mass of the sun. A star that exceeds this mass is destined to end its life in a most violent from of explosion.

The maximum mass of white dwarf star is 1.44 times the mass of the sun.

(17), (18)

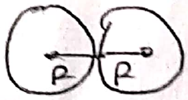
let m_1, m_2 are the masses of two identical spheres.

$m_1 = m_2 = \text{density} \times \text{vol} = \rho \frac{4\pi}{3} R^3$

$R \to$ radius $\to$ Diameter $d = 2R$

. when two spheres are kept in contact

$F = \frac{G m_1 m_2}{r^2} = \frac{G\left(\rho\left(\frac{4\pi}{3}\right)R^3\right)^2}{(2R)^2} \to (1)$



(or) $F = \frac{Gm^2}{d^2} \Rightarrow m^2 \propto F d^2$

Since $F \propto d^4$ $\Rightarrow m^2 \propto d^4 d^2 \Rightarrow m \propto d^3$

From (1) $F \propto = \frac{G\rho^2\left(\frac{4\pi}{3}\right)^2 R^6}{d^2} = \frac{G\rho^2\left(\frac{4\pi}{3}\right)^2 \frac{d^6}{2^6}}{d^2}$

$F = G\frac{1}{2^6}\rho^2\left(\frac{4\pi}{3}\right)^2 d^4$

$F \propto d^4$ As diameter $= \frac{d}{2}$ $F' \propto \left(\frac{d}{2}\right)^4$

$\Rightarrow F' \propto \frac{d^4}{16} = \frac{1}{16}F$

L Task

CQ's

(5)

Gravitational force is a non-contact force. this means that it is not necessary that the bodies should be in contact for the gravitational force to act.

so we can call ~~ab~~ it as a force at a distance

(6)

Newton's law of gravitation is universal because it is applicable universally for all masses at all distances and it is also does not depend on medium.

(8)

on the earth's surface at the poles we find gravity will be the strongest. the earth is an oblate spheroid, and that means it bulges out in the middle (the equator). That also means the poles end up a little closer to the centre of gravity. so at the poles the intensity of gravity is maximum.

(9)

we know weight of a body is nothing but the force of attraction exerted by the earth on a body and is equal to the product of mass of the body and acceleration due to gravity (g)

If the earth loses its power of attraction then the value of 'g' will become zero and as a result weight will also become zero.

(10)

we know $g = \frac{GM}{R^2}$.

$\Rightarrow \frac{g}{G} = \frac{M}{R^2}$

unit of $\frac{g}{G} = kg/m^2$ (or) $kg\, m^{-2}$

Jee main level

(11)

As the planet revolves around the sun Gravitational force between planet and sun provides necessary centripetal force.

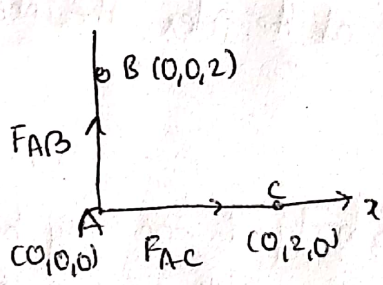
$F = mR\omega^2$ Given $F \propto \frac{1}{R^m}$.

$\Rightarrow \frac{GM\cancel{m}}{R^m} = \cancel{m}R\omega^2$ $F = \frac{GMm}{R^m}$

$\Rightarrow \omega^2 = \frac{GM}{R^{m+1}} \Rightarrow \left[\frac{2\pi}{T}\right]^2 = \frac{GM}{R^{m+1}}$

$\Rightarrow T^2 \propto R^{M+1} \Rightarrow T \propto R^{\frac{m+1}{2}} \rightarrow D$

(12)



$\vec{F}_{AC} = \frac{G m_A m_C}{r_{AC}^2} = \frac{6.67\times10^{-11}\times1\times1}{(0.2)^2}\hat{i}$ $\left[r = 2cm = 0.2m\right)$

$\vec{F}_{AC} = 1.67\times10^{-9}\,\hat{i}\,N$

Similarly $\vec{F}_{AB} = \frac{G m_A m_B}{r_{AB}^2} = \frac{6.67\times10^{-11}\times1\times1}{(0.2)^2}\hat{j}$

$\vec{F}_{AB} = 1.67\times10^{-9}\,\hat{j}\,N$

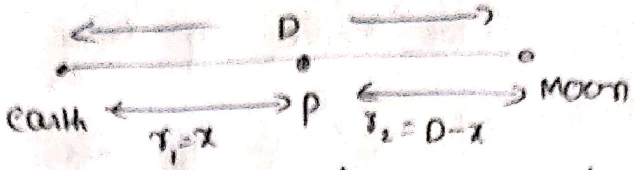
$\therefore$ Net force on particle $A\vec{F} = \vec{F}_{AC} + \vec{F}_{AB}$

$= 1.67\times10^{-9}\,\hat{i} + 1.67\times10^{-9}\,\hat{j}$

$= 1.67\times10^{-9}(\hat{i}+\hat{j})\,N \rightarrow C$

(13)

$d = D \qquad M_e = 81\, M_m$



P is the point where Gravitational field is zero.

ie
Field due to earth at P = Field due to moon at P

$$\Rightarrow \frac{G m_e}{r_1^2} = \frac{G m_m}{r_2^2}$$

$$\Rightarrow \frac{81\, M_m}{x^2} = \frac{M_m}{(D-x)^2}$$

$$\Rightarrow 81 = \left[\frac{x}{D-x}\right]^2 \Rightarrow 9 = \frac{x}{D-x}$$

$$\Rightarrow 9D - 9x = x \Rightarrow 10x = 9D$$

$$x = \frac{9D}{10}\ m \rightarrow D$$

(14)

Given $\frac{R_1}{R_2} = \frac{2}{1}$

we know $g = \frac{GM}{R^2} = \frac{\frac{4\pi}{3} \rho G R^3}{R^2}$

$$g = \frac{4\pi}{3} \rho G R$$

As both are made up of same material ρ = constant

$$\frac{g_1}{g_2} = \frac{R_1}{R_2} = \frac{2}{1}$$

(15)

$g_p = \frac{1}{6} g_e$; $R_p = \frac{1}{3} R_e$

we know $g = \frac{GM}{R^2}$

$\Rightarrow \frac{g_p}{g_e} = \frac{M_p}{M_e}\left[\frac{R_e}{R_p}\right]^2$

$\Rightarrow \frac{1}{6} = \frac{M_p}{M_e}[3]^2$

$\Rightarrow \frac{1}{6} = \frac{M_p}{M_e} 9 \Rightarrow \frac{M_p}{M_e} = \frac{1}{54} \rightarrow 0$

(16)

Given $M_p = \frac{1}{64} M_e$ $g_p = \frac{1}{9} g_e$.

we know $g = \frac{GM}{R^2}$

$\Rightarrow \frac{g_p}{g_e} = \frac{M_p}{M_e}\left[\frac{R_e}{R_p}\right]^2$

$\Rightarrow \frac{1}{9} = \frac{1}{64}\left[\frac{R_e}{R_p}\right]^2$

$\Rightarrow \frac{R_e}{R_p} = \left[\frac{64}{9}\right]^{1/2} = \frac{8}{3}$

$\Rightarrow R_e = \frac{8}{3} R_p$.

(17)

Given $m =$ constant ; $R_e' = 2R$

we know $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$

$\Rightarrow \frac{g'}{g} = \left[\frac{R}{R'}\right]^2$

$\Rightarrow \frac{g'}{g} = \left[\frac{R}{2R}\right]^2 = \frac{1}{4}$

$\Rightarrow g' = \frac{g}{4} \rightarrow c$

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$M_m = \frac{1}{9} M_e$ $R_m = \frac{1}{2} R_e$ $W_e = 90N = m g_e$

we know $g = \frac{GM}{R^2}$

$\Rightarrow \frac{g_m}{g_e} = \frac{M_m}{M_e} \left[\frac{R_e}{R_m} \right]^2$

$\Rightarrow \frac{g_m}{g} = \frac{1}{9} [2]^2$

$\Rightarrow g_m = \frac{4}{9} g$

weight on mars $= W_m = m g_m = \frac{4}{9} mg$

$= \frac{4}{9} \times 90^{10}$

$= 40N \rightarrow C$

19

let masses of two bodies is m_1 and m_2 and distance between them $r_1 = r$.

$\therefore F_1 = \frac{G m_1 m_2}{r_1^2} = \frac{G m_1 m_2}{r^2} \rightarrow (1)$

If masses are doubled ie $m_1' = 2m_1$, $m_2' = 2m_2$

distance is halved ies $r_2 = \frac{r}{2}$

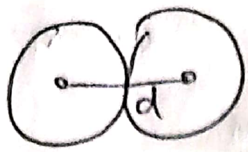
$\therefore F' = \frac{G m_1' m_2'}{r_2^2} = \frac{G (2m_1)(2m_1)}{\left(\frac{r}{2}\right)^2}$

$\Rightarrow F' = 4 \frac{G m_1 m_2}{\frac{r^2}{4}}$

$\Rightarrow F' = 16 \frac{G m_1 m_2}{r^2}$

$\Rightarrow F' = 16 F_1$

(20)



∴ From law of Gravitation

$r = d$ = diameter $\quad F = \dfrac{G m_1 m_2}{r^2}$

$m_1 = m_2 = m$ $\qquad m = \rho \times vol$

$$F = \frac{G m^2}{d^2} \qquad = \rho \times \frac{4\pi}{3} r^3$$

$$= \frac{4\pi}{3} \rho \left(\frac{d}{2}\right)^3$$

$$\Rightarrow F = \frac{G\left[\frac{4\pi \rho d^3}{3 \times 8}\right]^2}{d^2} \qquad = \frac{4\pi}{3} \rho \frac{d^3}{8}$$

$$\Rightarrow F \propto \frac{d^6}{d^2} \Rightarrow \underline{F \propto d^4} \rightarrow B$$

(21)

we know $\quad g = \dfrac{GM}{R^2} \Rightarrow g \propto \dfrac{1}{R^2}$

we know Earth is Bulged at equator so Radius is maximum as compared at poles

∴ g is minimum at equator and maximum at poles

g also varies with rotation of earth because latitude changes.

24

If G starts to decrease, then gravity starts decrease. ∴ This period of revolution around ~~Earth~~ Sun increases due to increase in R [$T^2 \propto R^3$]

∴ Duration of the year will increase.

Since the radius of circular path of earth will increase or the earth will follow a spiral path of increasing radius. ∴ P.E will increase but P.E is always negative. so the magnitude of P.E will decrease and hence K.E will also decrease.

25

$g_m = \frac{1}{6} g_e.$

we know $g \propto \frac{1}{R^2}$ if density = constant

$g \propto R$

$\Rightarrow \frac{g_m}{g_e} = \frac{R_m}{R_e}.$

$\Rightarrow \frac{1}{6} = \frac{R_m}{R_e}.$

If $\frac{\rho_e}{\rho_m} = \frac{5}{3}$ $g \propto \rho R$

$\Rightarrow \frac{g_e}{g_m} = \frac{\rho_e}{\rho_m} \frac{R_m}{R_e}$

$\Rightarrow \frac{R_m}{R_e} = \frac{g_e}{g_m} \frac{\rho_m}{\rho_e} \Rightarrow 6 \times \frac{3}{5}$

$\frac{R_m}{R_e} = \frac{18}{5}$

(26) , (27)

Given $m_1 = 1\,kg$; $m_2 = 1\,kg$; $d = 1\,m$.

$$F = G\frac{m_1 m_2}{r^2} = G\times\frac{1\times1}{1} = G\ N \rightarrow A$$

If $m_1 = \frac{m_1}{2}$. $m_2 = \frac{1}{2}$; $d = \frac{1}{2}d$

$$\therefore F' = G\frac{m_1 m_2}{d^2}$$

$$= G\frac{(\frac{1}{2})(\frac{1}{2})}{(\frac{1}{2})^2} = G\frac{\cancel{\frac{1}{4}}}{\cancel{\frac{1}{4}}} = G\,N \rightarrow C$$