Teaching task

Jee mains level

1.Step 1: Identify given quantities

Let:

v_c = linear velocity of car

v_s = linear velocity of scooter

Given:

$v_c = v_s = v$ (same linear velocity)

Let:

r_c = radius of car's wheel

r_s = radius of scooter's wheel

Given:

$r_c = 2r_s$

Step 2: Relationship between linear and angular velocity

For any wheel rolling without slipping:

$v = r\omega$

where ω is the angular velocity.

So:

For car: $\omega_c = \frac{v}{r_c}$

For scooter: $\omega_s = \frac{v}{r_s}$

Step 3: Ratio of angular velocities

$$\frac{\omega_c}{\omega_s} = \frac{\frac{v}{r_c}}{\frac{v}{r_s}} = \frac{r_s}{r_c}$$

We know $r_c = 2r_s$, so:

$$\frac{r_s}{r_c} = \frac{r_s}{2r_s} = \frac{1}{2}$$

So: $\omega_c : \omega_s = 1:2$

Step 4: Match with options given

The problem's options are:

A) 2:1

B) 2:1 (looks like a misprint, but likely meaning another option differently stated in original, maybe intended as 1:2)

C) 1:4

D) 4:1

We found $1:21:2$ — not exactly listed unless B) was meant to be $1:2$

But if we write $\omega_c/\omega_s = 1/2$, that means **car's angular velocity : scooter's angular velocity = 1 : 2** or equivalently **scooter : car = 2 : 1**.

They might ask for "ratio of angular velocities of their wheels" in the order (car : scooter) — if so, that's 1:2; if they mean (scooter : car), that's 2:1.

Given the answer choices and typical multiple choice, they might want the *scooter to car* ratio $\omega_s/\omega_c = 2/1$, which is **2:1** — matches option A and B, but both A and B are printed as 2:1 here, probably a typo in the question's options.

Since they say A) 2:1 and B) 2:1 are same, likely answer intended is **2:1** meaning scooter's angular velocity : car's angular velocity = 2:1.

So angular velocity of scooter wheel is twice that of car's wheel.

Step 5: Final answer

From our calculation: $\frac{\omega_{scooter}}{\omega_{car}} = 2$

Thus ratio (scooter : car) = **2:1**.

2. Step 1: Determine the periods of the hands

The time period for one complete revolution of the minute hand is T_m= 60 minutes.
The time period for one complete revolution of the hour hand is T_h= 12 hours, which is 12×60=720 minutes.

Step 2: Calculate the angular speeds

The angular speed (ω) is given by $\omega = \frac{2\pi}{T}$.

The angular speed of the minute hand is $\omega_m = \frac{2\pi}{T_m} = \frac{2\pi}{60}$ radians per minute.

The angular speed of the hour hand is $\omega_h = \frac{2\pi}{T_h} = \frac{2\pi}{720}$ radians per minute.

Step 3: Calculate the ratio

The ratio of the angular speed of the minute hand to the hour hand is $\frac{\omega_m}{\omega_h}$:

$$\frac{\omega_m}{\omega_h} = \frac{\frac{2\pi}{60}}{\frac{2\pi}{720}} = \frac{2\pi}{60} \times \frac{720}{2\pi} = \frac{720}{60} = 12$$

The ratio is **12:1**.

3.

Step 1: Convert revolutions per minute to radians per second
First, convert the rotational speed from revolutions per minute (rpm) to radians per second (ω).

$$f = \frac{1200\text{rpm}}{60\text{s/min}} = 20\text{Hz}$$

$$\omega = 2\pi f = 2\pi \times 20\text{rad / s} = 40\pi\text{rad / s}$$

Step 2: Determine the radius of the flywheel

The diameter of the flywheel is given as 1 m. The radius (r) is half the diameter.

$$r = \frac{D}{2} = \frac{1m}{2} = 0.5m$$

Step 3: Calculate the centripetal acceleration

The acceleration of a point on the rim of a rotating object (centripetal acceleration) is calculated using the formula

$$a_c = \omega^2 r$$

$$a_c = (40\pi rad/s)^2 \times 0.5m$$

$$a_c = 1600\pi^2 \times 0.5m/s^2$$

$$a_c = 800\pi^2 m/s^2$$

The acceleration of a point on the rim of the flywheel is $800\pi^2 m/s^2$

4.

Step 1: Convert RPM to angular velocity

The relationship between revolutions per minute (RPM) and angular velocity (ω) in radians per second (rad/s) is given by the formula $\omega = \frac{2\pi \times \text{RPM}}{60}$.

The initial angular velocity is ω_1:

$$\omega_1 = \frac{2\pi \times 600}{60} = 20\pi \text{ rad/s}$$

The final angular velocity is ω_2:

$$\omega_2 = \frac{2\pi \times 1200}{60} = 40\pi \text{ rad/s}$$

Step 2: Calculate increase in angular velocity

The increase in angular velocity ($\Delta\omega$) is the difference between the final and initial velocities:

$$\Delta\omega = \omega_2 - \omega_1 = 40\pi - 20\pi = 20\pi \text{ rad/s}$$

Answer:

The increase in the fan's angular velocity is $\mathbf{20\pi}$ **rad/s**.

5.

Step 1: Interpret the given data

- Diameter $d = 3 \text{ m}$
- Radius $r = \frac{d}{2} = 1.5 \text{ m}$
- Linear velocity at rim $v = 18 \text{ m/s}$

Step 2: Relating linear velocity and angular velocity

For a point on the rim of a rotating wheel:

$$v = r\omega$$

where ω is the angular velocity in radians per second.

where ω is the angular velocity in radians per second.

Step 3: Solve for ω

$$\omega = \frac{v}{r} = \frac{18}{1.5} = 12$$

Units: v in m/s, r in m → ω in rad/s.

Step 4: Final answer

$$\boxed{12 \text{ rad/s}}$$

6.

Step 1: Interpret the problem

A belt passes over a wheel of radius $r = 25 \text{ cm}$.

Linear speed of the belt = linear speed of a point on the belt = $v = 5 \text{ m/s}$.

Since the belt is moving over the wheel without slipping (assuming ideal condition), the tangential speed of the wheel's rim is the same as the belt's speed: $v = 5 \text{ m/s}$.

Step 2: Convert radius to meters

$r = 25 \text{ cm} = 0.25 \text{ m}$

Step 3: Use relation between linear and angular velocity

For a rotating wheel without slipping:

$$v = r\omega$$

where ω is angular velocity in rad/s.

$$\omega = \frac{v}{r} = \frac{5}{0.25} = 20 \text{ rad/s}$$

7.

Step 1: Analyze the Hanging Mass

For the $100\ \mathrm{gm}$ mass ($\mathrm{m_2}$) to be in equilibrium, the tension (T) in the string must balance its weight ($\mathrm{m_2 g}$).

$$T = m_2 g$$

Using $\mathrm{m_2} = 0.1\ \mathrm{kg}$ and $\mathrm{g} = 9.8\ \mathrm{m/s^2}$:

$$T = 0.1\ \mathrm{kg} \times 9.8\ \mathrm{m/s^2} = 0.98\ \mathrm{N}$$

Some sources might use $g = 10\ \mathrm{m/s^2}$, which yields $T = 1\ \mathrm{N}$.

Step 2: Analyze the Rotating Mass

The tension in the string provides the centripetal force ($\mathrm{F_c}$) required for the $70\ \mathrm{gm}$ mass ($\mathrm{m_1}$) to rotate in a circle.

$$\mathrm{F_c} = T$$

The radius (r) of the circle is the total length of the string minus the hanging length:

$$\mathrm{r} = L - h = 1\ \mathrm{m} - 0.6\ \mathrm{m} = 0.4\ \mathrm{m}$$

Step 3: Relate Tension to Frequency

The centripetal force is also given by the formula $\mathrm{F_c} = m_1\omega^2 r$, where ω is the angular frequency.

$$T = m_1\omega^2 r$$

Rearranging to solve for ω^2:

$$\omega^2 = \frac{T}{m_1 r}$$

Using $T = 0.98\ \mathrm{N}$ (or $1\ \mathrm{N}$ if $g = 10\ \mathrm{m/s^2}$), $m_1 = 0.07\ \mathrm{kg}$, and $r = 0.4\ \mathrm{m}$:

$$\omega^2 = \frac{0.98}{0.07 \times 0.4} = \frac{0.98}{0.028} = 35\ \mathrm{rad^2/s^2}$$

$$\omega = \sqrt{35}\ \mathrm{rad/s}$$

If $T = 1\ \mathrm{N}$ was used: $\omega^2 = \dfrac{1}{0.07 \times 0.4} = \dfrac{1}{0.028} = \dfrac{1000}{28} = \dfrac{250}{7}\ \mathrm{rad^2/s^2}$.

Step 4: Calculate the Frequency

The linear frequency (f) is related to the angular frequency by $\omega = 2\pi f$:

$$f = \frac{\omega}{2\pi}$$

Using $\omega = \sqrt{35}$ rad/s:

$$f = \frac{\sqrt{35}}{2\pi} \text{ Hz} \approx \mathbf{0.9416} \text{ Hz}$$

Using the value with $g = 10 \text{ m/s}^2$:

$$f = \frac{\sqrt{250/7}}{2\pi} = \frac{5\sqrt{10/7}}{2\pi} = \frac{\pi}{\sqrt{140}} \text{ Hz}$$

Both numerical values are approximately the same (around 0.95 Hz).

8.

Step 1: Convert Rotational Speed to Angular Velocity

The given rotational speed is 600 revolutions per minute. First, convert this to revolutions per second, and then to radians per second.

- Frequency, $N = \frac{600 \text{ revolutions}}{1 \text{ minute}} = \frac{600 \text{ revolutions}}{60 \text{ seconds}} = 10 \text{ revolutions/second.}$
- Angular velocity, $\omega = 2\pi N = 2\pi \times 10 \text{ rad/s} = 20\pi \text{ rad/s.}$

Step 2: Calculate Centripetal Acceleration

The diameter of the flywheel is 2 m, so the radius is $r = \frac{2 \text{ m}}{2} = 1 \text{ m.}$

The centripetal acceleration (which is the total acceleration as the angular velocity is constant) is given by the formula $a_c = r\omega^2$.

- $a_c = 1 \text{ m} \times (20\pi \text{ rad/s})^2$
- $a_c = 1 \times (400\pi^2) \text{ m/s}^2$
- $a_c = 400\pi^2 \text{ m/s}^2$

9.

Step 1: Identify variables and formula

The given variables are the radius of the circle, $r = 20\text{ cm}$, and the centripetal acceleration, $a_c = 980\text{ cm/s}^2$. The relationship between centripetal acceleration (a_c), radius (r), and angular speed (ω) is given by the formula:

$$a_c = r\omega^2$$

Step 2: Calculate the angular speed

Rearrange the formula to solve for ω:

$$\omega = \sqrt{\frac{a_c}{r}}$$

Substitute the given values into the equation:

$$\omega = \sqrt{\frac{980\text{ cm/s}^2}{20\text{ cm}}} = \sqrt{49\text{ rad}^2/\text{s}^2}$$

The angular speed is: ω=7 rad/s

10.

Step 1: Calculate the radius

The radius (**r**) of the wheel is half of its diameter (**D**).

$$r = \frac{D}{2}$$

Substituting the given diameter of 400 cm:

$$r = \frac{400\text{ cm}}{2} = 200\text{ cm}$$

Step 2: Calculate the angular velocity

The linear velocity ($\mathbf{v}$) of a point on the rim is related to the angular velocity ($\boldsymbol{\omega}$) and the radius ($\mathbf{r}$) by the formula $v = \omega r$. Rearranging to solve for the angular velocity:

$$\omega = \frac{v}{r}$$

Substituting the given linear velocity of 1600 cm/s and the calculated radius of 200 cm:

$$\omega = \frac{1600 \text{ cm/s}}{200 \text{ cm}} = 8 \text{ rad/s}$$

The unit for angular velocity is radians per second.

JEE ADVANCED LEVEL QUESTIONS

11.

The assertion is false because velocity is a vector quantity that depends on both magnitude (speed) and direction, and while the speed is constant, the direction of motion in a circular path is continuously changing. The reason is also false because the direction of the velocity vector is always tangent to the circle, not away from the center.

- **Assertion:** "A body having constant speed in circular path has a constant velocity" is **False**.
    - **Explanation:** A body moving in a circle at a constant speed has a continuously changing velocity because velocity is a vector quantity that includes direction. Even if the speed ($|v|$) is constant, the direction of the velocity vector is not, so the velocity itself is not constant.
- **Reason:** "The direction of velocity is always away from centre" is **False**.
    - **Explanation:** The direction of velocity in circular motion is always tangent to the circle at that point, not directed radially outward.

12.

The Assertion is **True**, as constant speed in uniform circular motion means constant kinetic energy ($KE = \frac{1}{2} mv^2$). However, the Reason is **False**, because velocity (a vector) continuously changes direction, so momentum (also a vector, $p = mv$) constantly changes, requiring a centripetal force. Therefore, the correct option is that the Assertion is true, but the Reason is false.

Analysis:

- **Assertion (True):** In uniform circular motion (UCM), the *speed* (v) is constant, but the *direction* of velocity changes. Kinetic energy depends on speed (magnitude of velocity), not direction, so $KE = \frac{1}{2} mv^2$ remains constant.
- **Reason (False):** Momentum ($\vec{p} = m\vec{v}$) is a vector. Since velocity's direction changes in UCM, the momentum vector also continuously changes, even if its magnitude (speed) stays the same.

Conclusion:
The Assertion is correct, but the Reason provided is incorrect.

13.

In a uniform circular motion in the X-Y plane:

- **C) The magnitude of velocity remains constant at all instants.** Uniform circular motion is defined by constant speed (magnitude of velocity).
- **D) The magnitude of acceleration remains constant at all instants.** The acceleration is centripetal acceleration ($a = v^2/r$), and since both speed (v) and radius (r) are constant, the magnitude of acceleration is constant.

Options A and B are incorrect because the *speed* of the particle (the total magnitude of velocity) is constant. The *velocity components* along the X and Y axes change direction and magnitude continually, but the overall speed does not change.

14.

Explanation

- **Speed** is a scalar quantity (magnitude only) and is constant by the definition of uniform circular motion.
- **Kinetic energy** ($KE = \frac{1}{2} mv^2$) depends only on mass (m) and speed (v). Since both mass and speed are constant, the kinetic energy is also constant.
- **Angular momentum** ($\vec{L} = \vec{r} \times \vec{p}$) has a constant magnitude and a constant direction (perpendicular to the plane of motion and through the center of the circle) because the net external torque is zero.

Why other options are incorrect

- **B) momentum** is a vector quantity ($\vec{p} = m\vec{v}$). Although its magnitude (mass × speed) is constant, its direction is continuously changing as the object moves around the circle. Therefore, momentum is not constant.

15. In a uniform circular motion

- **B) acceleration and speed are constant but velocity changes.**
  - **Explanation:** Speed (magnitude of velocity) stays the same, but velocity is a vector, so its direction changes continuously, meaning velocity itself changes. The acceleration, called centripetal acceleration, is constant in magnitude but its direction (towards the center) continuously changes, acting as the force causing the circular path.

16. When a body moves with a constant speed along a circle

- **A) no work is done on it.**
  - **Explanation:** Work done by a force is $W = F \cdot d \cos \theta$. In uniform circular motion, the force (centripetal force) and acceleration are always directed towards the center, perpendicular to the instantaneous velocity (tangential). Since the angle between force and displacement is 90°, $\cos(90^{\circ}) = 0$, so no work is done.
  - **Why B and C are wrong:** There is *always* centripetal acceleration and a centripetal force causing it, even if speed is constant, to keep the object in a circle, so B and C are incorrect.

17.

- **A) Linear velocity (v)** is related to angular velocity (ω) and the radius (r) by $v = \omega r$. This corresponds to option s) ωr.
- B) Angular acceleration (α) is the rate of change of angular velocity, which is the second derivative of the angular displacement (θ) with respect to time. This corresponds to option r) $\frac{d^2\theta}{dt^2}$.
- C) Centripetal acceleration (a_c) is directed towards the center of the circle and can be expressed as $a_c = \omega^2 r$ or $a_c = \frac{v^2}{r}$. This corresponds to option p) $\frac{v^2}{r}$.
- D) Tangential acceleration (a_t) is directed along the tangent to the circle and is related to angular acceleration (α) and the radius (r) by $a_t = \alpha r$. This corresponds to option q) αr.

Therefore, the correct matching combination is A-s, B-r, C-p, D-q.

18.

The final angular velocity (ω_f) can be calculated using the first equation of rotational kinematics, which relates initial angular velocity (ω_i), angular acceleration (α), and time (t).

The formula is $\omega_f = \omega_i + \alpha t$.
Given $\omega_i = 0\,\text{rad/s}$, $\alpha = 3\,\text{rad/s}^2$, and $t = 10\,\text{s}$.

Substituting the values:

$$\omega_f = 0 + (3)(10) = 30\,\text{rad/s}$$

19.

The angle rotated through, or angular displacement (θ), can be calculated using the second equation of rotational kinematics:

The formula is $\theta = \omega_i t + \frac{1}{2}\alpha t^2$.
Using the same given values: $\omega_i = 0\,\text{rad/s}$, $\alpha = 3\,\text{rad/s}^2$, and $t = 10\,\text{s}$.

Substituting the values:

$$\theta = (0)(10) + \frac{1}{2}(3)(10)^2 = 0 + 1.5(100) = 150\,\text{rad}$$

20.

1. **Identify the given variables:**
   - Initial angular velocity (ω_0) = 10 rad/s
   - Final angular velocity (ω_f) = 0 rad/s (as it stops)
   - Angular acceleration (α) = $-2\,\text{rad/s}^2$ (negative because it is slowing down)

2. **Select the correct kinematic equation:**
   The equation that relates angular velocities, angular acceleration, and angular displacement ($\Delta\theta$) without time is:
   $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$
   This is analogous to the linear motion equation $v^2 = u^2 + 2as$.

3. **Solve for angular displacement in radians:**
   Rearrange the equation to solve for $\Delta\theta$:
   $$\Delta\theta = \frac{\omega_f^2 - \omega_0^2}{2\alpha}$$
   Substitute the known values:
   $$\Delta\theta = \frac{0^2 - (10\,\text{rad/s})^2}{2 \times (-2\,\text{rad/s}^2)}$$
   $$\Delta\theta = \frac{-100\,\text{rad}^2/\text{s}^2}{-4\,\text{rad/s}^2}$$
   $$\Delta\theta = 25\,\text{radians}$$

4. **Convert the angular displacement to revolutions:**
   One complete revolution is equal to 2π radians. To find the number of revolutions, divide the total angular displacement in radians by 2π:
   $$\text{Number of revolutions} = \frac{\Delta\theta}{2\pi}$$
   $$\text{Number of revolutions} = \frac{25\,\text{radians}}{2\pi\,\text{radians/revolution}} \approx 3.979\,\text{revolutions}$$

21.

Explanation

The average angular velocity (ω_{avg}) for an object with constant angular acceleration is the arithmetic mean of its initial (ω_i) and final (ω_f) angular velocities.

$$\omega_{avg} = \frac{\omega_i + \omega_f}{2}$$

Calculation

Substituting the given initial velocity of $15\ \text{rad/s}$ and final velocity of $25\ \text{rad/s}$:

$$\omega_{avg} = \frac{15\ \text{rad/s} + 25\ \text{rad/s}}{2} = \frac{40\ \text{rad/s}}{2} = 20\ \text{rad/s}$$

The time interval of 5 seconds is not needed for this calculation.

Answer:

The average angular velocity is **20 rad/s.**

LEARNERS TASK

CONCEPTUAL UNDERSTANDING QUESTIONS

1.

Explanation

In uniform circular motion, an object travels along a circular path at a constant speed (magnitude of velocity). However, because velocity is a vector quantity (having both magnitude and direction), and the direction of motion is continuously changing at every instant, the velocity itself is not constant; it is variable.

The change in the direction of velocity means there must be an acceleration, known as centripetal acceleration, which is directed towards the center of the circle. While the magnitude of this centripetal acceleration ($a = v^2/r$) remains constant because the speed (v) and radius (r) are constant, its direction also continuously changes as it always points to the moving center.

- **Velocity** changes because its direction changes.
- **Acceleration** changes because its direction changes.
- **Speed** and the magnitude of acceleration remain constant.

2.

The acceleration of a particle revolving around a circular path is **C) Along the radius**, directed towards the center of the circle.

Explanation

When an object moves in a circular path, its **velocity vector** is continuously changing direction, even if its speed remains constant. This change in direction requires a net force and thus an acceleration, known as **centripetal acceleration** ($\mathbf{a_c}$). This acceleration is always directed radially inward, toward the center of the circle.

- **Along the tangent** is the direction of the **tangential acceleration** ($\mathbf{a_t}$), which only exists if the particle's *speed* is changing.
- **Along the circumference** is not a specific vector direction for acceleration.
- **Zero** acceleration would mean the particle moves in a straight line at a constant speed (Newton's first law).

The magnitude of the centripetal acceleration is given by the formula $\mathbf{a_c} = \frac{v^2}{r}$, where v is the speed of the particle and r is the radius of the circular path. This acceleration vector is always along the radius and points inward [1].

3

The correct option is **C) angular velocity is same**.

Explanation

In the rotational motion of a rigid body, all particles rotate around a common, fixed axis. This means that every particle covers the same angular displacement in the same amount of time, resulting in the same angular velocity for all particles.

The linear velocity (v) of a particle in rotational motion is related to its angular velocity (ω) and its distance from the axis of rotation (r) by the formula $v = r\omega$.

4

The correct option is **D. Any of the above three**.

Explanation

Angular acceleration ($\boldsymbol{\alpha}$) is defined as the rate of change of angular velocity ($\boldsymbol{\omega}$) with respect to time ($\boldsymbol{\alpha} = d\boldsymbol{\omega}/dt$). Since angular velocity is a vector quantity (having both magnitude and direction), an angular acceleration can:

- **Change the magnitude of angular velocity (angular speed):** If the angular acceleration vector is parallel or antiparallel to the angular velocity vector, the object speeds up or slows down (e.g., a fan starting up or slowing down).
- **Change the direction of angular velocity:** If the angular acceleration vector is perpendicular to the angular velocity vector, the axis of rotation changes direction while the angular speed remains constant (e.g., precession of a gyroscope).
- **Change both magnitude and direction:** In the general case, an angular acceleration can be at an angle to the angular velocity vector, changing both its magnitude and direction simultaneously.

Therefore, angular acceleration can cause any of these changes to the angular velocity vector.

5.

Explanation

For a particle undergoing rotational motion with constant angular acceleration (α), the angular displacement (θ) over a time (t) is described by the kinematic equation:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

where ω_0 is the initial angular velocity.

The problem states the particle starts **from rest**, meaning its initial angular velocity is $\omega_0 = 0$.

Substituting this into the equation simplifies it to:

$$\theta = \frac{1}{2}\alpha t^2$$

Since α is a **constant angular acceleration**, the term $\frac{1}{2}\alpha$ is a constant value. The equation shows that the angular displacement θ is directly proportional to the square of the time (t^2), or $\theta \propto t^2$.

6.

The correct option is **C) The body is experiencing both normal and tangential acceleration**.

Explanation

- **Normal acceleration (centripetal acceleration):** A body moving in a circular path always experiences normal (or radial/centripetal) acceleration because the direction of its velocity is continuously changing, which requires a force directed towards the center of the circle. This acceleration is given by $a_c = v^2/r$ or $a_c = \omega^2 r$.
- **Tangential acceleration:** The body's angular velocity is decreasing, meaning its angular speed is changing. This change in speed (magnitude of velocity) results in a tangential acceleration (or deceleration). Tangential acceleration is in the direction opposite to the velocity vector when the speed is decreasing.

Since both the direction and the magnitude of the velocity are changing, the body experiences both normal and tangential acceleration.

7.

The correct option is **C) Both 1 and 2**, because a wheel rotating with constant angular velocity satisfies both conditions.

A **constant angular velocity (ω)** means the rate of rotation does not change over time.

- **Angular acceleration (α)**: Angular acceleration is defined as the time derivative of angular velocity, $\alpha = \frac{d\omega}{dt}$. If ω is constant, its derivative is zero. Thus, the angular acceleration is **zero** [B].
- **Angular displacement (θ)**: Angular velocity is defined as the time derivative of angular displacement, $\omega = \frac{d\theta}{dt}$. Integrating this constant relationship yields $\theta = \omega t + \theta_0$. Assuming the initial displacement θ_0 is zero, the angular displacement is directly **proportional to time** ($\theta \propto t$) [A].

Since both statements are correct implications of constant angular velocity, the best option is C.

8.

The angular acceleration of a clock's minute hand is **C) Zero**, because it moves at a constant rate (constant angular velocity), and angular acceleration is the rate of change of angular velocity; if velocity doesn't change, acceleration is zero.

- **Definition:** Angular acceleration (α) is the change in angular velocity (ω) over time (Δω/Δt).
- **Clock Motion:** The minute hand moves steadily, completing a circle in 60 minutes, so its angular velocity is constant.
- **Result:** Since there's no change in its constant speed, the acceleration is zero (0 rad/s^2).

9.

The direction of its angular acceleration is **A) Normally into the paper**.

Explanation

To determine the direction of angular velocity and acceleration, we use the **right-hand rule**.

- **Angular Velocity Direction ($\vec{\omega}$):** When curling the fingers of your right hand in the direction of the clockwise rotation (as the particle moves), your right thumb points **normally into the paper**.
- **Angular Acceleration Direction ($\vec{\alpha}$):** The angular acceleration vector is defined as the rate of change of angular velocity, $\vec{\alpha} = d\vec{\omega}/dt$. Since the magnitude of the angular velocity is gradually increasing and its direction (into the paper) remains constant, the angular acceleration vector must point in the **same direction** as the angular velocity vector.

Therefore, the angular acceleration is also directed normally into the paper.

10.

The correct option is **D) gets angular acceleration**.

Explanation

- Torque is the rotational analogue of force in linear motion.
- Newton's second law for rotation states that the net torque (τ) acting on a body is equal to its moment of inertia (I) multiplied by its angular acceleration (α), given by the formula $\tau = I\alpha$.
- If a steady (constant and non-zero) torque acts on a body, and the body has a non-zero moment of inertia, there must be a constant angular acceleration.
- This constant angular acceleration means the angular velocity (speed) of the body changes uniformly over time, it does not remain constant.

JEE MAINS LEVEL QUESTIONS

Multiple choice question type:

1.

The angular velocity (ω) is calculated using the formula $\omega = \frac{2\pi}{T}$, where T is the time period for one complete rotation.

- **Angular velocity of the hour hand of a clock** (ω_h): The hour hand completes one full rotation (2π radians) in 12 hours.
  - T_h = 12 hours.
  - $\omega_h = \frac{2\pi}{12} = \frac{\pi}{6}$ radians per hour.
- **Angular velocity of the Earth's self-rotation** (ω_e): The Earth completes one full rotation (2π radians) on its axis in approximately 24 hours (a day).
  - T_e = 24 hours.
  - $\omega_e = \frac{2\pi}{24} = \frac{\pi}{12}$ radians per hour.
- **Ratio of their angular velocities**: The ratio of the hour hand's angular velocity to the Earth's angular velocity is calculated as:
  - $\frac{\omega_h}{\omega_e} = \frac{\pi/6}{\pi/12} = \frac{12}{6} = \frac{2}{1}$.
  - The ratio is 2:1.

2.

Step 1: Unit Conversion and Variable Definition

First, convert the given values into consistent SI units (meters and seconds):

- **Velocity** (v): Given as 45 km/h.
  - Conversion: $v = 45 \times \frac{1000 \text{ m}}{3600 \text{ s}} = 12.5 \text{ m/s}$.
- **Diameter** (D): Given as 50 cm.
  - Conversion: $D = 50 \text{ cm} = 0.50 \text{ m}$.
- **Radius** (r):
  - Calculation: $r = \frac{D}{2} = \frac{0.50 \text{ m}}{2} = 0.25 \text{ m}$.

Step 2: Formula Application

The relationship between linear velocity (v) and angular velocity (ω) for a rolling object without slipping is given by the formula:

$$v = \omega r$$

To find the angular velocity, we rearrange the formula to solve for ω:

$$\omega = \frac{v}{r}$$

Step 3: Calculation

Substitute the converted values of linear velocity (v) and radius (r) into the rearranged formula:

$$\omega = \frac{12.5 \text{ m/s}}{0.25 \text{ m}}$$

$$\omega = 50 \text{ rad/s}$$

3.

Step 1: Convert speed to standard units

First, the speed must be converted from kilometers per hour (km/h) to meters per second (m/s) using the conversion factor $\frac{1000\,\text{m}}{3600\,\text{s}}$:

$$v = 36\,\text{km/h} \times \frac{1000\,\text{m}}{3600\,\text{s}} = 10\,\text{m/s}$$

Step 2: Calculate centripetal acceleration

The centripetal acceleration (a) required to negotiate a turn of radius r at speed v is given by the formula $a = \frac{v^2}{r}$. Using the converted speed $v = 10\,\text{m/s}$ and the given radius $r = 25\,\text{m}$:

$$a = \frac{(10\,\text{m/s})^2}{25\,\text{m}} = \frac{100\,\text{m}^2/\text{s}^2}{25\,\text{m}} = 4\,\text{m/s}^2$$

4.

Step 1: Define initial variables and formulas

Let the initial speed be v, the initial angular velocity be ω, and the initial radius be r. The centripetal acceleration a_c is given by the formulas:

$$a_c = \frac{v^2}{r}$$

$$a_c = \omega^2 r$$

The relationship between linear and angular velocity is $v = \omega r$.

Step 2: Define new variables and find the new radius

The new speed v' is double the old speed: $v' = 2v$. The new angular velocity ω' is half the old angular velocity: $\omega' = \frac{\omega}{2}$. Let the new radius be r'.

Using the relationship $v' = \omega' r'$, we can find how the radius changes:

$$2v = \frac{\omega}{2} r'$$

Substitute $v = \omega r$:

$$2(\omega r) = \frac{\omega}{2} r'$$

$$4\omega r = \omega r'$$

$$r' = 4r$$

The radius must quadruple to satisfy both conditions.

Step 3: Calculate the new centripetal acceleration

We can use either formula to find the new centripetal acceleration a_c'.

Using $a_c' = \frac{(v')^2}{r'}$:

$$a_c' = \frac{(2v)^2}{4r}$$

$$a_c' = \frac{4v^2}{4r}$$

$$a_c' = \frac{v^2}{r}$$

Thus, $a_c' = a_c$.

Using $a_c' = (\omega')^2 r'$:

$$a_c' = (\frac{\omega}{2})^2(4r)$$

$$a_c' = \frac{\omega^2}{4}(4r)$$

$$a_c' = \omega^2 r$$

Thus, $a_c' = a_c$. The centripetal acceleration remains unchanged.

5.

Step 1: Recall basic rotational kinematics for a point at distance r from axis

For circular motion:

$$v = \omega r$$

$$a_t = \alpha r$$

$$a_c = \omega^2 r$$

But here, "linear acceleration" means **total linear acceleration**? In pure rotational motion (rigid body), a point has:

Tangential acceleration: $a_t = \alpha r$
Centripetal acceleration: $a_c = \omega^2 r$
Magnitude of total linear acceleration:

$$a = \sqrt{a_t^2 + a_c^2}.$$

This is messy for ratio purposes unless motion is purely tangential acceleration related.

Step 2: Interpret the question

They likely mean **relation between** $\frac{a}{\alpha}$ and $\frac{v}{\omega}$.

From $v = \omega r$ and $a_t = \alpha r$,
we see:

$$\frac{a_t}{\alpha} = r, \quad \frac{v}{\omega} = r$$

$$\frac{a_t}{\alpha} = r, \quad \frac{v}{\omega} = r$$

Thus:

$$\frac{a_t}{\alpha} = \frac{v}{\omega}$$

So the **ratio** a/α equals the **ratio** v/ω (both equal r).

But if a means total linear acceleration magnitude, not just tangential, then it's not generally equal to v/ω.
However, **for tangential acceleration part**, yes.

In many exam contexts, they assume a = tangential acceleration for rotatory motion.

Step 3: Choose the exact relation

Given possible intended options (not listed here), the relation is:

$$\frac{a}{\alpha} = \frac{v}{\omega}$$

provided a is tangential acceleration.

If that's not an option, maybe they want "both equal r" or "proportional" etc.
But logically:
$\frac{a}{\alpha} = r$ and $\frac{v}{\omega} = r$, so they are equal.

Step 4: Final answer in words

$$\frac{\text{linear acceleration}}{\text{angular acceleration}} = \frac{\text{linear velocity}}{\text{angular velocity}}$$

(both equal the radius r).

$$\boxed{\frac{a}{\alpha} = \frac{v}{\omega}}$$

6.

When a body moves along a circular path with constant angular velocity, its velocity is variable.

Explanation:

- **Velocity is a vector quantity:** This means it has both magnitude (speed) and direction.
- **Constant angular velocity implies constant speed:** If the angular velocity is constant, the rate at which the angle changes is constant, which translates to a constant speed along the circular path.
- **Direction changes continuously:** In circular motion, even with constant speed, the direction of the body's motion is continuously changing as it follows the curve of the circle. The velocity vector is always tangential to the circular path at any given point
- **Variable velocity due to changing direction:** Since the direction component of the velocity vector is constantly changing, the overall velocity of the body is also continuously changing, even though its magnitude (speed) remains constant.

For a body on a circular path with *increasing* angular velocity (non-uniform circular motion), its acceleration **changes in both magnitude and direction** because it has a centripetal component (changing direction) and a tangential component (changing speed/magnitude), making option C the correct answer.

Here's why:

- **Centripetal Acceleration (a_c):** Always present in circular motion, directed towards the center, changing the *direction* of velocity.
- **Tangential Acceleration (a_t):** Occurs due to changing speed (increasing angular velocity), directed tangent to the path, changing the *magnitude* of velocity.
- **Total Acceleration (a):** The vector sum of a_c and a_t. Since both components vary (one in direction, the other in magnitude), the total acceleration's magnitude and direction both change continuously.

Therefore, the correct choice is **C) Changes both in magnitude and direction**.

8.

- **Acceleration formula:** The acceleration of a particle in a rotating disc is given by the formula $a = \omega^2 r$, where ω is the angular velocity and r is the distance from the center.
- **Same angular velocity:** For a rigid rotating disc, all particles have the same angular velocity (ω).
- **Comparing P and Q:**
    - Particle P is at a distance r_P from the center.
    - Particle Q is at a distance r_Q from the center.
    - We are given that $r_P > r_Q$.
- **Conclusion:** Since $a_P = \omega^2 r_P$ and $a_Q = \omega^2 r_Q$, and ω is the same for both particles, the acceleration is directly proportional to the radius. Therefore, because $r_P > r_Q$, it follows that $a_P > a_Q$. Thus, P has a greater acceleration than Q.

9.

The acceleration of the merry-go-round is $\mathbf{4\pi^2\ m/s^2}$.

Step 1: Identify given variables and formulas

The given variables are radius $R = 4$ m and period $T = 2$ s. For uniform circular motion, the acceleration is centripetal acceleration (a_c). We use the formula $a_c = \omega^2 R$

Step 2: Calculate the angular velocity

The angular velocity ω is calculated from the period T:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\,\text{s}} = \pi\ \text{rad/s}$$

Step 3: Calculate the centripetal acceleration

Using the angular velocity and radius in the acceleration formula:

$$a_c = \omega^2 R = (\pi\ \text{rad/s})^2 \times 4\ \text{m} = 4\pi^2\ \text{m/s}^2$$

10.

The angular velocity of the seconds hand in a watch is $\frac{\pi}{30}$ radians per second or approximately 0.105 rad/s. This is because it completes a full 2π radian rotation in 60 seconds.

Calculation

- **Formula:** Angular velocity (ω) is calculated by dividing the angular displacement (θ) by the time taken (t). The formula is $\omega = \frac{\theta}{t}$.
- **Angular displacement:** The seconds hand completes one full circle, which is 2π radians.
- **Time taken:** The time for one full rotation is 60 seconds.
- **Putting it together:** $\omega = \frac{2\pi \text{ radians}}{60 \text{ seconds}} = \frac{\pi}{30}$ rad/s.

11.

The assertion is false, but the reason is true. A body moving in a circular path with constant speed still has a force acting on it—the centripetal force—which is necessary to change its direction. The reason is true because the velocity vector's direction is constantly changing, as it is always tangent to the circle, and this change in velocity means there is acceleration, which in turn requires a force.

- **Assertion: False.** A body moving in a circular path at a constant speed is experiencing a change in direction, which means it is accelerating. According to Newton's laws, this acceleration requires a net force, known as the centripetal force, acting on the body to keep it on the circular path.
- **Reason: True.** The velocity of a body in circular motion is always changing because its direction is continuously changing. Since velocity is a vector, a change in direction means the velocity is not constant, even if its magnitude (speed) is.

12.

Step 1: Understanding the assertion

Assertion says: *There is no change in K.E of particle in uniform horizontal circular motion.*

Uniform circular motion means constant speed.
Kinetic energy: $K = \frac{1}{2}mv^2$, so if speed is constant, KE is constant → **Assertion is true.**

Step 2: Understanding the reason

Reason says: *Centripetal force acts normal to instantaneous velocity.*

In uniform circular motion, the centripetal force is directed toward the center, perpendicular to the instantaneous velocity (which is tangential). This perpendicular force does **no work**, so it changes only direction, not speed, hence KE remains constant. → **Reason is true**.

Step 3: Checking if reason explains assertion correctly

Does "centripetal force normal to velocity" explain "no change in K.E"?
Yes — perpendicular force ⇒ work done = 0 ⇒ KE constant.

Thus, both are true **and** reason is the correct explanation of assertion.

Final answer:

Both Assertion and Reason are true, and Reason is the correct explanation of the Assertion.

13.

Explanation

All three statements are correct for a body in uniform circular motion (constant angular velocity implies constant speed):

- **a) it experiences an acceleration which is not constant**: The body experiences centripetal acceleration, which is constant in magnitude but its **direction continuously changes** (always pointing towards the center of the circle). Since acceleration is a vector quantity, a change in direction means the acceleration itself is not a constant vector.
- **b) it is moving with variable velocity**: Velocity is a vector quantity, having both magnitude (speed) and direction. While the speed is constant, the **direction of the velocity vector is continuously changing** as it is always tangent to the circle. Therefore, the velocity is not constant, it is variable.
- **c) the work done by the centripetal force is zero**: The centripetal force is always directed towards the center, which is **perpendicular to the direction of the instantaneous velocity** (and thus, the displacement) at any point in time. The work done by a force is given by the dot product of force and displacement ($W = \vec{F} \cdot \vec{d} = Fd\cos\theta$). Since the angle θ between the centripetal force and the velocity (displacement) is always 90°, and $\cos(90^\circ) = 0$, the work done by the centripetal force is zero.

14.

Explanation

- **Linear Momentum (a):** According to Newton's second law, the net external force acting on a system is equal to the rate of change of its linear momentum ($\vec{F}_{\text{ext}} = d\vec{p}/dt$). If the net external force is zero, the rate of change of linear momentum is zero, meaning the linear momentum must remain constant.
- **Angular Momentum (c):** Similarly, the net external torque acting on a system is equal to the rate of change of its angular momentum ($\vec{\tau}_{\text{ext}} = d\vec{L}/dt$). Since the problem only mentions the absence of external *force* and does not mention the absence of external *torque*, we must infer the external torque may also be zero or that the forces are such that they produce no net torque (e.g., all internal). If there are no external forces, there are no external torques, so angular momentum must also remain constant.

15.

Explanation

- **Centrifugal force** is a fictitious (pseudo) force in a rotating frame of reference that appears to act outward, away from the center of rotation.
- **Centripetal force** is a real force required for circular motion; it acts inward, towards the center of rotation.
- **Tangential force** acts along the tangent to the circular path and is responsible for changing the magnitude of the angular velocity (i.e., causing angular acceleration).
- **Angular velocity** is a vector quantity whose direction is along the axis of rotation, following the right-hand rule.

16.

To determine the time t required for the disc to stop ($\omega_f = 0\,\text{rad/s}$), we use the first equation of angular kinematics:

$$\omega_f = \omega_i + \alpha t$$

where $\omega_i = 20\,\text{rad/s}$ is the initial angular velocity and $\alpha = -5\,\text{rad/s}^2$ is the angular acceleration.
Substituting the values:

$$0 = 20 + (-5)t$$

$$5t = 20$$

Solving for t:

$$t = \frac{20}{5} = 4\,\text{s}$$

17.

To determine the angular displacement θ during this time, we can use the third equation of angular kinematics:

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

Substituting the known values:

$$0^2 = 20^2 + 2(-5)\theta$$

$$0 = 400 - 10\theta$$

$$10\theta = 400$$

Solving for θ:

$$\theta = \frac{400}{10} = 40\,\text{rad}$$

18.

Step 1: Known quantities

Angular acceleration $\alpha = 3 \text{ rad/s}^2$

Initial angular velocity $\omega_0 = 0$

Angular displacement $\theta = 50 \text{ rev}$

Step 2: Convert revolutions to radians

$$\theta = 50 \times 2\pi = 100\pi \text{ rad.}$$

Step 3: Use rotational kinematic equation

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\omega^2 = 0 + 2 \times 3 \times 100\pi$$

$$\omega^2 = 600\pi$$

Step 4: Solve for ω

$$\omega = \sqrt{600\pi} \text{ rad/s}$$

Numerically:

$$\sqrt{600\pi} \approx \sqrt{1884.9556} \approx 43.42 \text{ rad/s.}$$

Step 5: Final answer

$$\boxed{\sqrt{600\pi} \text{ rad/s}}$$

(or approximately 43.4 rad/s).

19.

Step 1: Known quantities

Initial angular velocity $\omega_0 = 0$

Angular acceleration $\alpha = 4\ \mathrm{rad/s^2}$

Time $t = 8\ \mathrm{s}$

Step 2: Choose rotational kinematics equation

For angular displacement θ with constant angular acceleration:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

Step 3: Substitute values

$$\theta = 0 \cdot 8 + \frac{1}{2} \times 4 \times (8)^2$$

$$\theta = \frac{1}{2} \times 4 \times 64$$

$$\theta = 2 \times 64 = 128$$

Step 4: Unit

Angular displacement is in radians, so:

$$\theta = 128\ \mathrm{rad}$$

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