MULTIPLE AND SUBMULTIPLE ANGLES

Class: VIII. Mathematics

IIT ADVANCED

SOLUTIONS

TEACHING TASK

01. $\dfrac{\sin 3A}{1+2\cos 2A} = \dfrac{3\sin A - 4\sin^3 A}{1+2(1-2\sin^2 A)} = \dfrac{\sin A(3-4\sin^2 A)}{(3-4\sin^2 A)}$

$= \sin A$

shortcut method

put $A = 30^\circ$.

$LHS = \dfrac{\sin 90^\circ}{1+2\cos 60^\circ} = \dfrac{1}{1+2(\frac{1}{2})} = \dfrac{1}{2}$

opt A: $\sin A = \sin 30^\circ = \dfrac{1}{2}$ Ans: A

02 $\sin 2\alpha + \sin 2\beta + \sin 2\gamma - \sin 2(\alpha+\beta+\gamma)$

put $\alpha = \beta = \gamma = 30^\circ$

$\therefore = \sin 60^\circ + \sin 60^\circ + \sin 60^\circ - \sin 180^\circ$

$= 3\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{3\sqrt{3}}{2}$

opt A: $4\sin(\alpha+\beta)\cdot\sin(\beta+\gamma)\cdot\sin(\gamma+\beta)$

$= 4\sin 60^\circ \cdot \sin 60^\circ \cdot \sin 60^\circ$

$= 4\left(\dfrac{\sqrt{3}}{2}\right)^3 = 4\times\dfrac{3\sqrt{3}}{8} = \dfrac{3\sqrt{3}}{2}$

Ans. A

03 $4\cos 16\theta \cdot \cos 4\theta \cdot \cos 2\theta$.

put $\theta = 0^\circ$

$\therefore 4 \cdot \cos 0^\circ \cdot \cos 0^\circ \cdot \cos 0^\circ = 4$.

opt: A: $\cos 12\theta + \cos 8\theta + \cos 4\theta + 1$

$= \cos 0^\circ + \cos 0^\circ + \cos 0^\circ + 1$

$= 1 + 1 + 1 + 1 = 4$.

Ans: A

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04 $\sin^2 2\alpha + \cos^2 2\alpha + \tan^2\alpha$

$= 1 + \tan^2\alpha$

$= \sec^2\alpha$

Ans: A

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05 $\dfrac{\cos A - \cos 3A}{\cos A} + \dfrac{\sin A + \sin 3A}{\sin A}$

$= \dfrac{\cos A - (4\cos^3 A - 3\cos A)}{\cos A} + \dfrac{\sin A + 3\sin A - 4\sin^3 A}{\sin A}$

$= 4\cancel{\cos A} - 4\cos^2 A + 4 - 4\sin^2 A$

$= 8 - 4(\cos^2 A + \sin^2 A) = 8 - 4 = 4$

Ans: D

Shortcut method

put $A = 45^\circ$.

$\therefore \dfrac{\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)} + \dfrac{\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)} = \dfrac{2\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)} + \dfrac{2\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$

$= 2 + 2 = 4$ Ans. D

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06. $(2\cos^2 3\theta - 1)(\cos 5\theta) = \cos 6\theta \cdot \cos 5\theta = \frac{1}{2}(2\cos 6\theta \cdot \cos 5\theta)$

$= \frac{1}{2}[\cos(11\theta) + \cos\theta]$

Ans. A

07. $(\cos 12^\circ + \cos 132^\circ) + (\cos 84^\circ + \cos 156^\circ)$ (4)

$= \cos 154^\circ + \cos 120^\circ + \cos 240^\circ + \cos 72^\circ$

$= -\cos 36^\circ - \cos 60^\circ - \sin 30^\circ + \sin 18^\circ$

$= -\frac{\sqrt{5}+1}{4} - \frac{1}{2} - \frac{1}{2} + \frac{\sqrt{5}-1}{4}$

Ans: C

$= -\frac{1}{2}$

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08 $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$ | $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$

$\sin^2 18^\circ = \frac{3-\sqrt{5}}{8}$ | $\Rightarrow \cos^2 36^\circ = \frac{3+\sqrt{5}}{8}$

Now, $\sin^2 18^\circ + \cancel{\sin}\cos^2 36^\circ = \frac{3}{4}$; $\sin^2 18^\circ . \cos^2 36^\circ = \frac{1}{16}$

Required. Q E.

$x^2 - \frac{3}{4}x + \frac{1}{16} = 0 \Rightarrow 16x^2 - 12x + 1 = 0$

Ans: A

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09. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = K\cos\theta$

put $\theta = 0^\circ$ since $\theta < \frac{\pi}{16}$

$\sqrt{2+\sqrt{2+\sqrt{2+2}}} = K$

Ans: A

$= K = 2$

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10. $\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$ put $x = 0^\circ$

$= \frac{1+6+15+10}{1+5+10} = \frac{32}{16} = 2$

opt: D: $2\cos x$

$= 2\times 1$

$= 2$

Ans-D

11. $\tan 7\frac{1}{2}^\circ = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$

$= (\sqrt{3} - \sqrt{2})(\sqrt{2} - 1)$

$= (\sqrt{2} - \sqrt{3})(1 - \sqrt{2})$

Ans. A, C, D

12. let $x = \tan A$, $y = \tan B$, $\tan C = z$

$xy + yz + zx = 1$

$\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A = 1 \longrightarrow (1)$

Now $\tan(A+B+C) = \dfrac{(\tan A + \tan B + \tan C) - \tan A \cdot \tan B \cdot \tan C}{1 - (TATB + TBTC + TCTA)}$

$=$ Not defined (Since from (1))

$\therefore \quad A + B + C = \frac{\pi}{2}$

$\Rightarrow 2A + 2B + 2C = \pi$

$\Rightarrow 2A + 2B = \pi - 2C$

$\Rightarrow \tan(2A + 2B) = \tan(\pi - 2C)$

$\Rightarrow \dfrac{\tan 2A + \tan 2B}{1 - \tan 2A \cdot \tan 2B} = -\tan 2C$

$\Rightarrow \tan 2A + \tan 2B + \tan 2C = \tan 2A \cdot \tan 2B \cdot \tan 2C$

$\Rightarrow \sum \tan 2A = \prod \tan 2A$

$\Rightarrow \sum \dfrac{2x}{1-x^2} = \prod\left(\dfrac{2x}{1-x^2}\right)$ or $\prod\left(\dfrac{2y}{1-y^2}\right)$ or $\left(\dfrac{\pi 2z}{1-z^2}\right)$

Ans: B, C, D

13 $\tan\theta = \dfrac{p^2-q^2}{2pq}$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} \Rightarrow \tan\theta = \dfrac{2\tan\theta/2}{1-\tan^2(\theta/2)}$

$\Rightarrow \dfrac{p^2-q^2}{2pq} = \dfrac{2x}{1-x^2}$ (let $\tan\frac{\theta}{2} = x$)

$\Rightarrow \dfrac{1-x^2}{x} = \dfrac{4pq}{p^2-q^2}$

$\Rightarrow \dfrac{1}{x} - x = \left(\dfrac{p+q}{p-q}\right) - \left(\dfrac{p-q}{p+q}\right)$

$\therefore x = \dfrac{p-q}{p+q}$ i.e $\tan\frac{\theta}{2} = \dfrac{p-q}{p+q}$ (or)

$\cot\dfrac{\theta}{2} = \dfrac{p+q}{p-q}$

Ans : A,D

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14 Statement A: Let $A = 30^\circ$, $B = 45^\circ$, $C = 60^\circ$

Satisfies $\cos 2B = \dfrac{\cos(A+C)}{\cos(A-C)}$

$\Rightarrow \cos 2\times 45^\circ = \dfrac{\cos(30^\circ+60^\circ)}{\cos(30^\circ-60^\circ)}$

$\Rightarrow 0 = \dfrac{0}{\left(\frac{\sqrt{3}}{2}\right)} \Rightarrow 0 = 0$

hence $\tan 30^\circ, \tan 45^\circ, \tan 60^\circ$

$= \frac{1}{\sqrt{2}}, 1, \sqrt{3}$ are in G.P. (True)

Statement B: (Conceptual) true

Ans: A

15. Statement I: $\dfrac{x}{\cos\theta} = \dfrac{y}{\cos\left(\theta - \frac{2\pi}{3}\right)} = \dfrac{z}{\cos\left(\theta + \frac{2\pi}{3}\right)}$

Let $\theta = 0^\circ$

$\Rightarrow \dfrac{x}{1} = \dfrac{y}{\left(-\frac{1}{2}\right)} = \dfrac{z}{-\frac{1}{2}} = K$

$\Rightarrow x = K,\ y = \dfrac{-K}{2},\ z = \dfrac{-K}{2}$

Now $x + y + z = K - \dfrac{K}{2} - \dfrac{K}{2} = K - K = 0.$

(False)

Statment II:

$\tan\theta + \tan\left(\dfrac{\pi}{4} + \theta\right) + \tan\left(\dfrac{\pi}{4} - \theta\right) = K.\sec 2\theta$

put $\theta = 0^\circ$

$\Rightarrow 0 + 1 + 1 = K.1$

$\Rightarrow K = 2$ (True)

Ans: D

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16. $\cos\theta = \cos\alpha \cdot \cos\beta$

$\Rightarrow \dfrac{\cos\beta}{1} = \dfrac{\cos\theta}{\cos\alpha}$

$\Rightarrow \dfrac{1 + \cos\beta}{\cos\beta - 1} = \dfrac{\cos\theta + \cos\alpha}{\cos\theta - \cos\alpha}$

$\Rightarrow \dfrac{\cancel{2}\cos^2\frac{\beta}{2}}{-\cancel{2}\sin^2\frac{\beta}{2}} = \dfrac{\cancel{2}\cos\left(\frac{\theta+\alpha}{2}\right)\cos\left(\frac{\theta-\alpha}{2}\right)}{-\cancel{2}\sin\left(\frac{\theta+\alpha}{2}\right).\sin\left(\frac{\theta-\alpha}{2}\right)}$

$\Rightarrow \cot^2\dfrac{\beta}{2} = \cot\left(\dfrac{\theta+\alpha}{2}\right)\cot\left(\dfrac{\theta-\alpha}{2}\right)$

Ans: $\cancel{B}$ A

17. $$\frac{\cos x}{\cos(x-2y)} = \frac{\lambda}{1}$$

$$\Rightarrow \frac{\cos(x-2y) - \cos x}{\cos(x-2y) + \cos x} = \frac{1-\lambda}{1+\lambda}$$

$$\Rightarrow \frac{-2\sin\left(\frac{x-2y+x}{2}\right)\sin\left(\frac{x-2y-x}{2}\right)}{2\cos\left(\frac{x-2y+x}{2}\right)\cos\left(\frac{x-2y-x}{2}\right)} = \frac{1-\lambda}{1+\lambda}$$

$$\Rightarrow -\tan(x-y) \cdot -\tan y = \frac{1-\lambda}{1+\lambda}$$

$$\therefore \frac{1-\lambda}{1+\lambda} = \tan(x-y) \cdot \tan y$$

Ans: B

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18. $$\cos(x-y) = 3 \cdot \cos(x+y)$$

$$\frac{\cos(x-y)}{\cos(x+y)} = \frac{3}{1} \Rightarrow \frac{\cos(x-y) + \cos(x+y)}{\cos(x-y) - \cos(x+y)} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{\cancel{2}\cos x \cos y}{\cancel{2}\sin x \cdot \sin y} = 2 \Rightarrow \cot x \cdot \cot y = 2$$

Ans: C

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19. $$\frac{\sin 65^\circ + \sin 25^\circ}{\cos 65^\circ + \cos 25^\circ} = \frac{2\sin(45^\circ) \cdot \cos 20^\circ}{2\cos 45^\circ \cdot \cos 20^\circ} = 1$$

Ans: 1

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20. $\cos\theta - \sin\theta = \frac{1}{5}$

$\Rightarrow 1 - \sin 2\theta = \frac{1}{25} \Rightarrow \sin 2\theta = \frac{24}{25}$

$\Rightarrow \cos 2\theta = \frac{7}{25}$

$\Rightarrow \cot 2\theta = \frac{7}{24}$

$(\cos\theta + \sin\theta)^2 = (\cos\theta - \sin\theta)^2 + 4\cos\theta\sin\theta$

$= \left(\frac{1}{5}\right)^2 + 2\times\frac{24}{25}$

$= \frac{1+48}{25} = \frac{49}{25}$

$\therefore \cos\theta + \sin\theta = \frac{7}{5}$

$\therefore$ a) $\cos\theta + \sin\theta = \frac{7}{5}$

b) $\sin 2\theta = \frac{24}{25}$

c) $\cos 2\theta = \frac{7}{25}$

d) $\cot 2\theta = \frac{7}{24}$

Ans : —, r, s, q

LEARNERS TASK

CUQ'S

01. $\sin\left(\frac{A+B}{2}\right) + \sin\left(\frac{A-B}{2}\right)$

$= 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)$

Ans: A

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02 $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

Ans: B

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03 $A+B+C = 180°$

$\Rightarrow 2A+2B = 360° - 2C$

$\Rightarrow \sin(2A+2B) = -\sin 2C$

Ans: D

(10)

04. $\sin 85^\circ - \sin 35^\circ - \cos 65^\circ$

$= 2\cos\left(\frac{85^\circ+35^\circ}{2}\right)\sin\left(\frac{85^\circ-35^\circ}{2}\right) - \cos 65^\circ$

$= 2\cos 60^\circ . \sin 25^\circ - \sin 25^\circ$

Ans: A

$= 0$

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05 $\cos 3A = 4\cos^3 A - 3\cos A = 4\theta^3 - 3\theta$ Ans: A

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06 $\sin\theta = 1 \Rightarrow \theta = 90^\circ$

Ans: B

$\cos 2\theta = \cos 180^\circ = -1.$

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07. $\cos\left(\frac{A}{2}\right) = \frac{4}{5} \Rightarrow \sin\left(\frac{A}{2}\right) = \frac{3}{5}$

$\sin A = 2\sin\frac{A}{2} . \cos\frac{A}{2}$

$= 2 . \frac{4}{5} . \frac{3}{5} = \frac{24}{25}$ Ans: B

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08 $\tan 22\frac{1}{2}^\circ + \cot 22\frac{1}{2}^\circ$

Ans: B

$= \sqrt{2} - 1 + \sqrt{2} + 1 = 2\sqrt{2}$

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09 $\sin 72^\circ . \cos 72^\circ$

$= \frac{1}{2}\left(2\sin 72^\circ . \cos 72^\circ\right)$

$= \frac{1}{2}\left(\sin 144^\circ\right)$

$= \frac{1}{2}\left(\sin(180^\circ - 36^\circ)\right)$

$= \frac{1}{2}\sin 36^\circ$

$= \frac{1}{2} . \frac{\sqrt{10 - 2\sqrt{5}}}{4}$ Ans: D

10. $\tan A = \frac{x}{2}$ (10)

$$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

$$= \frac{\frac{3x}{2} - \frac{x^3}{8}}{1 - \frac{3x^2}{4}} = \frac{12x - x^3}{2(4-3x^2)}$$ Ans: B

JEEMAINS LEVEL QUESTIONS

01. $\sin 78^\circ - \sin 18^\circ + \cos 132^\circ$

$= 2\cos 48^\circ . \sin 30^\circ - \cos 48^\circ = 0$ Ans: B

02 $1 + \cos 2\theta + \cos 4\theta + \cos 6\theta$

put $\theta = 0^\circ$

$\therefore\ 1 + 1 + 1 + 1 = 4.$

opt: B) $4\cos\theta . \cos 2\theta . \cos 3\theta$

$= 4 \times 1 \times 1 \times 1 = 4$ Ans: B

03 $\sqrt{2} . \operatorname{cosec} 20^\circ . \sec 20^\circ$

$$= \frac{\sqrt{2}}{\sin 20^\circ . \cos 20^\circ} = \frac{2\sqrt{2}}{\sin 40^\circ} = 4\sin 45^\circ . \operatorname{cosec} 40^\circ$$

Ans: D

04 $2(1 - 2\sin^2\theta) . \cos 4\theta$

$= 2 . \cos 2\theta . \cos 4\theta$

$= \cos 6\theta + \cos 2\theta$ Ans: C

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05 $\dfrac{\sin\alpha}{a} = \dfrac{\cos\alpha}{b}$ ~~$=K$~~ $\Rightarrow$ ~~$a = \dfrac{K}{\sin\alpha}$~~ , $b = \dfrac{K}{\sin}$ 12

let $\alpha = 45^\circ$

$\dfrac{1}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}b}$

12

05 $\dfrac{\sin\alpha}{a} = \dfrac{\cos\alpha}{b} \Rightarrow \tan\alpha = \dfrac{a}{b}$

$a\sin 2\alpha + b\cos 2\alpha = a\left(\dfrac{2\tan\alpha}{1+\tan^2\alpha}\right) + b\left(\dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}\right)$

$= a\left(\dfrac{2\left(\frac{a}{b}\right)}{1+\left(\frac{a}{b}\right)^2}\right) + b\left(\dfrac{1-\left(\frac{a}{b}\right)^2}{1+\left(\frac{a}{b}\right)^2}\right)$

$= b$

Ans: B

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06 $\tan 2\alpha - \tan\alpha\,(1+\sec 2\alpha)$

let $\alpha = 30^\circ$

$= \sqrt{3} - \dfrac{1}{\sqrt{3}}(1+2) = \sqrt{3} - \dfrac{3}{\sqrt{3}} = 0$

Ans: C

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07 $\dfrac{\cos A + \sin A}{\cos A - \sin A} - \dfrac{\cos A - \sin A}{\cos A + \sin A}$

let $A = 0^\circ$

$= \dfrac{1+0}{1-0} - \dfrac{1-0}{1+1} = 0.$

opt: A) $2\tan 2A = 2\tan 0^\circ = 0$

Ans: A

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08 $\sin 75^\circ + \sin 15^\circ$

$= 2\sin 45^\circ \cdot \cos 30 = 2 \times \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{\sqrt{2}}$

Ans: B

13

09 $\dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ}$

$= \dfrac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cdot \cos 10^\circ}$

$= \dfrac{2\left(\frac{1}{2}\cdot\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)}{\frac{1}{2}\left(2\sin 10^\circ\cdot\cos 10^\circ\right)} = 4\cdot\dfrac{\sin 20^\circ}{\sin 20^\circ} = 4$ Ans: A

10. $\cos^2 76^\circ + \cos^2 16^\circ - \cos 16^\circ\cdot\cos 76^\circ$

$= \frac{1}{2}\left[2\cos^2 76^\circ + 2\cos^2 16^\circ - 2\cos 16^\circ\cdot\cos 76^\circ\right)$

$= \frac{1}{2}\left(1 + \cos 152^\circ + 1 + \cos 32^\circ - \cos(92^\circ) - \cos(60^\circ)\right]$

$\cancel{= \frac{1}{2}\left[2 + \cos 92^\circ + \cos 60^\circ - \cos 92^\circ - \cos 60^\circ\right]}$

$= \frac{1}{2}\left(2 + 2\cos 92^\circ\cdot\cos 60^\circ - \cos 92^\circ - \frac{1}{2}\right)$

$= \frac{1}{2}\left(2 - \frac{1}{2}\right) = \frac{3}{4}$ Ans. D

11 $\dfrac{\sin 70^\circ + \cos 40^\circ}{\cos 70^\circ + \sin 40^\circ} = \dfrac{\sin 70^\circ + \sin 50^\circ}{\sin 20^\circ + \sin 40^\circ}$

$= \dfrac{\cancel{2}\sin 60^\circ\cdot\cancel{\cos 10^\circ}}{\cancel{2}\sin 30^\circ\cdot\cancel{\cos 10^\circ}}$

$= \dfrac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \sqrt{3}$ Ans: A

12. $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$

$\Rightarrow \tan\theta = \dfrac{\tan\alpha - 1}{\tan\alpha + 1} = \dfrac{\tan\alpha - \tan\frac{\pi}{4}}{1 + \tan\alpha \cdot \tan\frac{\pi}{4}} = \tan\left(\alpha - \frac{\pi}{4}\right)$

$\therefore \theta = \alpha - \frac{\pi}{4} \Rightarrow \alpha = \theta + \frac{\pi}{4}$

opt: A) $\sin\alpha - \cos\alpha$

$= \sin\left(\theta + \frac{\pi}{4}\right) - \cos\left(\theta + \frac{\pi}{4}\right)$

$= \left(\sin\theta \cos\frac{\pi}{4} + \cos\theta \sin\frac{\pi}{4}\right) - \left(\cos\theta\cos\frac{\pi}{4} - \sin\theta\sin\frac{\pi}{4}\right)$

$= \left(\dfrac{\sin\theta + \cos\theta}{\sqrt{2}}\right) - \left(\dfrac{\cos\theta - \sin\theta}{\sqrt{2}}\right)$

$= \dfrac{2\sin\theta}{\sqrt{2}} = \sqrt{2}\sin\theta.$

Similarly we can prove opt: B. Ans. A, B

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13 $\sin\alpha = \sin\beta$ | $\cos\alpha = \cos\beta$

$\sin\alpha - \sin\beta = 0$ | $\Rightarrow \cos\alpha - \cos\beta = 0$

$2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) = 0 \rightarrow (1)$ | $-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) = 0 \rightarrow (2)$

$\dfrac{(1)}{(2)} \Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right) = 0 \Rightarrow \sin\left(\frac{\alpha+\beta}{2}\right) = 0 \Rightarrow \frac{\alpha+\beta}{2} = 0$

$\Rightarrow \alpha + \beta = 0$

Ans: A, B

14. $\tan 11\frac{1}{4}^\circ = \frac{\sin 11\frac{1}{4}^\circ}{\cos 11\frac{1}{4}^\circ} \times \frac{2\sin 11\frac{1}{4}^\circ}{2\sin 11\frac{1}{4}^\circ} = \frac{2\sin^2 11\frac{1}{4}^\circ}{2\sin 11\frac{1}{4}^\circ \cdot \cos 11\frac{1}{4}^\circ}$

$$= \frac{1-\cos 22\frac{1}{2}^\circ}{\sin 22\frac{1}{2}^\circ} = \frac{1-\sqrt{\frac{1+\cos 45^\circ}{2}}}{\sqrt{\frac{1-\cos 45^\circ}{2}}}$$

$$= \frac{\sqrt{2}-\sqrt{1+\frac{1}{\sqrt{2}}}}{\sqrt{1-\frac{1}{\sqrt{2}}}}$$

$$\Rightarrow \frac{\sqrt{2\sqrt{2}}-\sqrt{\sqrt{2}+1}}{\sqrt{\sqrt{2}-1}} \times \frac{\sqrt{\sqrt{2}+1}}{\sqrt{\sqrt{2}+1}}$$

$$= \sqrt{2\sqrt{2}(\sqrt{2}+1)} - (\sqrt{2}+1)$$

$$= \sqrt{4+2\sqrt{2}} - (\sqrt{2}+1)$$ Ans: C

15.

15 $\tan 142\frac{1}{2}^\circ = \tan\left(180^\circ - 37\frac{1}{2}^\circ\right)$

$$= -\tan 37\frac{1}{2}^\circ$$

$$= -\frac{\sin 37\frac{1}{2}^\circ}{\cos 37\frac{1}{2}^\circ} \times \frac{2\sin 37\frac{1}{2}^\circ}{2\sin 37\frac{1}{2}^\circ}$$

$$= \frac{-2\sin^2 37\frac{1}{2}^\circ}{\sin 75^\circ}$$

$$= \frac{\cos 75^\circ - 1}{\sin 75^\circ} = \frac{\frac{\sqrt{3}-1}{2\sqrt{2}} - 1}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{\sqrt{3}-1-2\sqrt{2}}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}-1-2\sqrt{2})(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{\cancel{\sqrt{9}}\,3 - \sqrt{3} - \sqrt{3} + 1 - 2\sqrt{6} + 2\sqrt{2}}{2}$$

$$= \frac{4 - 2\sqrt{3} - 2\sqrt{6} + 2\sqrt{2}}{2} = 2 + \sqrt{2} - \sqrt{3} - \sqrt{6}$$

Ans: D

$$\cot 7\frac{1}{2}^\circ = \frac{\cos 7\frac{1}{2}^\circ}{\sin 7\frac{1}{2}^\circ} \times \frac{2\cos 7\frac{1}{2}^\circ}{2\sin 7\frac{1}{2}^\circ \cos 7\frac{1}{2}^\circ} = \frac{2\cos^2 7\frac{1}{2}^\circ}{\sin 15^\circ} = \frac{1+\cos 15^\circ}{\sin 15^\circ}$$

$$= \frac{1 + \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3} + \sqrt{3} + 1}{2}$$

$$= \frac{4 + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6}}{2}$$

$$= 2 + \sqrt{2} + \sqrt{3} + \sqrt{6}$$

Ans: A

17 $\cos^2 A(3-4\cos^2 A)^2 + \sin^2 A(3-4\sin^2 A)^2$

$= [\cos A(3-4\cos^2 A)]^2 + [\sin A(3-4\sin^2 A)]^2$

$= [3\cos A - 4\cos^3 A]^2 + [3\sin A - 4\sin^3 A]^2$

$= [-\cos 3A]^2 + [\sin 3A]^2 = \cos^2 3A + \sin^2 3A = 1$

<u>short cut method</u> : put $A = 0°$.

$\cos^2 0°(3-4\cos^2 0°)^2 + \sin^2 0°(3-4\sin^2 0°) = 1$

Ans : A

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19 a) $\frac{1}{\cos 290°} + \frac{1}{\sqrt{2}\sin 250°} = \frac{1}{\sin 20°} + \frac{1}{\sqrt{3}\cos 20°}$

$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sqrt{3}\sin 20°\cdot\cos 20°}$

$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)}{\frac{\sqrt{3}}{2}(2\sin 20°\cos 20°}$

$= \frac{2(\sin 60°\cos 20° - \cos 60°.\sin 20°)}{\frac{\sqrt{3}}{2}.\sin 40°} = \frac{4}{\sqrt{3}}$.

b) $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = \frac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10°.\cos 10°} = \frac{2\left(\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°\right)}{\frac{1}{2}(2\sin 10°.\cos 10°}$

$= 4$

c) $\frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°.\cos 20°} = \frac{2\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)}{\frac{1}{2}.\sin 40°}$

$= 4$

d) $6\sin 20° - 8\sin^3 20°$

$= 2(3\sin 20° - 4\sin^3 20°) = 2.\sin 3\times 20° = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$

Ans : S, P, P, Q

18) $\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)} + \frac{\cos(\theta_3+\theta_4)}{\cos(\theta_3-\theta_4)} = 0$ (16)(A)

$= \cancel{\cos(\theta_1 - \ldots}$

$= \frac{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2}{\cos\theta_1\cos\theta_2 - \sin\theta_1\cdot\sin\theta_2} + \frac{\cos\theta_3\cos\theta_4 - \sin\theta_3\sin\theta_4}{\cos\theta_3\cos\theta_4 + \sin\theta_3\sin\theta_4} = 0$

$\Rightarrow \frac{1+\tan\theta_1\tan\theta_2}{1-\tan\theta_1\tan\theta_2} + \frac{1-\tan\theta_3\tan\theta_4}{1+\tan\theta_3\tan\theta_4} = 0$

$\Rightarrow (1+\tan\theta_1\tan\theta_2)(1+\tan\theta_3\tan\theta_4) + (1-\tan\theta_3\tan\theta_4)(1-\tan\theta_1\cdot\tan\theta_2) = 0$

$= 1+\tan\theta_3\cancel{\tan\theta_4} + \tan\theta_1\cancel{\tan\theta_2} + x + 1 - \tan\theta_1\tan\theta_2$

$- \tan\theta_3\cancel{\tan\theta_4} + x = 0$

$\Rightarrow 2x+2=0$

$\Rightarrow x=-1. \Rightarrow \tan\theta_1\cdot\tan\theta_2\cdot\tan\theta_3\tan\theta_4 = -1$

20. $\cos 5\theta = \cos(2\theta + 3\theta)$

$= \cos 2\theta \cdot \cos 3\theta - \sin 2\theta \cdot \sin 3\theta$

$= \cos 2\theta \cdot \cos 3\theta - (2\sin\theta \cos\theta)(3\sin\theta - 4\sin^3\theta)$

$= \cos 2\theta \cdot \cos 3\theta - [6\sin^2\theta \cos\theta - 8\sin^4\theta \cos\theta]$

$= \cos 2\theta \cdot \cos 3\theta - [6(1-\cos^2\theta)\cdot\cos\theta - 8(1-\cos^2\theta)^2 \cdot \cos\theta]$

$= \cos 2\theta \cdot \cos 3\theta - [6\cos\theta - 6\cos^3\theta - 8(1 - 2\cos^2\theta + \cos^4\theta)\cos\theta]$

$= \cos 2\theta \cdot \cos 3\theta - [6\cos\theta - 6\cos^3\theta - 8\cos\theta + 16\cos^3\theta - 8\cos^5\theta]$

$= [(2\cos^2\theta - 1)(4\cos^3\theta - 3\cos\theta)] - [10\cos^3\theta - 2\cos\theta - 8\cos^5\theta]$

$= 8\cos^5\theta - 6\cos^3\theta - 4\cos^3\theta + 3\cos\theta - 10\cos^3\theta + 2\cos\theta + 8\cos^5\theta$

$= 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$

$= P_0 + P_1\cos\theta + P_2\cos^2\theta + P_3\cos^3\theta + P_4\cos^4\theta + P_5\cos^5\theta.$

$P_1 = 5, \quad P_3 = -20, \quad P_5 = 16$ Ans: —, S, P, Q

ADDITIONAL PRACTICE QUESTIONS

01. $A+B+C = 180°$. let $A=B=C=60°$

LHS $= \cos 120° + \cos 120° + \cos 120° = 3 \times \cos 120°$

$= 3 \times -\cos 60° = -\frac{3}{2}$

opt: D: $-1 - 4\cos A \cos B \cos C$

$= -1 - 4 \cdot \cos 60° \cdot \cos 60° \cdot \cos 60°$

$= -1 - 4 \times \left(\frac{1}{2}\right)^3 = -1 - \frac{4}{8} = -1 - \frac{1}{2} = -\frac{3}{2}$

Ans = D

19

02 $\underline{\sin 47^\circ - \sin 25^\circ} + \underline{\sin 61^\circ - \sin 11^\circ}$

$= 2\sin 11^\circ . \cos 36^\circ + 2\sin 25^\circ . \cos 36^\circ$

$= 2\cos 36^\circ \left(\sin 11^\circ + \sin 25^\circ\right)$

$= 2\cos 36^\circ . \; 2\sin 18^\circ . \cos 7^\circ$

$= 4 . \frac{\sqrt{5}+1}{4} . \frac{\sqrt{5}-1}{4} . \cos 7^\circ = \cos 7^\circ$

Ans.- A

03 $\cot 16^\circ . \cot 44^\circ + \cot 44^\circ . \cot 76^\circ - \cot 76^\circ . \cot 16^\circ$

We know $16^\circ + 44^\circ = 60^\circ$

$\Rightarrow \frac{\cot 16^\circ . \cot 44^\circ - 1}{\cot 16^\circ + \cot 44^\circ} = \frac{1}{\sqrt{3}}$

$\Rightarrow \cot 16^\circ . \cot 44^\circ = \frac{\cot 16^\circ + \cot 44^\circ}{\sqrt{3}} + 1 \rightarrow (1)$

Also $44^\circ + 76^\circ = 120^\circ$

$\Rightarrow \frac{\cot 44^\circ \cot 76^\circ - 1}{\cot 44^\circ + \cot 76^\circ} = -\frac{1}{\sqrt{3}}$

$\Rightarrow \cot 44^\circ . \cot 76^\circ = -\frac{(\cot 44^\circ + \cot 76^\circ)}{\sqrt{3}} + 1 \rightarrow (2)$

Also $76^\circ - 16^\circ = 60^\circ$

$\Rightarrow \frac{\cot 76^\circ . \cot 16^\circ + 1}{\cot 16^\circ - \cot 76^\circ} = \frac{1}{\sqrt{3}}$

$\Rightarrow \cot 76^\circ . \cot 16^\circ = \frac{\cot 16^\circ - \cot 76^\circ}{\sqrt{3}} - 1 \rightarrow (2)$

Now Given expression $= (1) + (2) - (3)$

$= \frac{\cot 16^\circ + \cot 44^\circ}{\sqrt{3}} + 1 - \frac{(\cot 44^\circ + \cot 76^\circ}{\sqrt{3}} + 1 - \frac{\cot 76^\circ - \cot 76^\circ}{\sqrt{3}} + 1$

$= 3$

Ans: 3

4) $\sec(\theta-\alpha)$, $\sec\theta$, $\sec(\theta+\alpha)$ are in A.P.

$$\Rightarrow 2\sec\theta = \sec(\theta-\alpha)+\sec(\theta+\alpha)$$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{1}{\cos(\theta-\alpha)} + \frac{1}{\cos(\theta+\alpha)}$$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{\cos(\theta+\alpha)+\cos(\theta-\alpha)}{\cos(\theta-\alpha)\cdot\cos(\theta+\alpha)}$$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{2\cos\theta\cdot\cos\alpha}{\cos^2\theta-\sin^2\alpha}$$

$$\Rightarrow \cos^2\theta-\sin^2\alpha = \cos^2\theta\cdot\cos\alpha$$

$$\Rightarrow \cos^2\theta - \cos^2\theta\cdot\cos\alpha = \sin^2\alpha$$

$$\Rightarrow \cos^2\theta\,(1-\cos\alpha) = (1-\cos^2\alpha) = (1+\cos\alpha)(1-\cos\alpha)$$

$$\Rightarrow \cos^2\theta = 1+\cos\alpha$$

$$\Rightarrow \cos^2\theta = 2\cos^2\frac{\alpha}{2}$$

$$\Rightarrow \cos\theta = \pm\sqrt{2}\cos\frac{\alpha}{2}$$

$$\Rightarrow \cos\theta\cdot\sec\frac{\alpha}{2} = \pm\sqrt{2}$$

Ans: C

$\Longrightarrow$ THE END $\Longleftarrow$