THEORY OF EQUATIONS - II
Transformation of Equations
TEACHING TASK
Jee Mains Level Questions

1. Given $f(x) = ax^2 + bx + c = 0$

Required equation is f(x-2) = 0

$\Rightarrow a(x-2)^2 + b(x-2) + c = 0$

$\Rightarrow a(x^2 - 4x + 4) + b(x-2) + c = 0$

$\Rightarrow ax^2 - 4ax + 4a + bx - 2b + c = 0$

$\Rightarrow ax^2 + (b-4a)x + 4a - 2b + c = 0$

Ans : D

2. Let $f(x) = x^2 + x + 1 = 0$

Required equation is $f(\sqrt{x}) = 0$

$\therefore (\sqrt{x})^2 + \sqrt{x} + 1 = 0$

$\Rightarrow x + 1 = -\sqrt{x}$

$\Rightarrow x^2 + 2x + 1 = x$

$\Rightarrow x^2 + x + 1 = 0$

Ans : C

3. Let $f(x) = 9x^2 + 6x + 1 = 0$

Required equation is $f\left(\frac{1}{x}\right) = 0$

$\therefore 9\left(\frac{1}{x}\right)^2 + 6\left(\frac{1}{x}\right) + 1 = 0$

$\Rightarrow \frac{9}{x^2} + \frac{6}{x} + 1 = 0$

$\Rightarrow x^2 + 6x + 9 = 0$

Ans : B

4. Let α, α^2 be the roots of $x^2 + px + q = 0$

We have $\alpha + \alpha^2 = -p$ and $\alpha.\alpha^2 = q$

$\Rightarrow \alpha^3 = q$

$\Rightarrow \alpha = (q)^{\frac{1}{3}}$

Now $(q)^{\frac{2}{3}} + p(q)^{\frac{1}{3}} + q = 0$

$\Rightarrow (q)^{\frac{2}{3}} + pq^{\frac{1}{3}} = -q$ .... (i)

cubing on both sides, we get

$\Rightarrow q^2 + p^3q + 3q^{\frac{2}{3}}.p.q^{\frac{1}{3}}\left(q^{\frac{2}{3}} + pq^{\frac{1}{3}}\right) = -q^3$

$\Rightarrow q^2 + p^3q + 3pq(-q) = -q^3$ [from(i)]

$\Rightarrow q + p^3 - 3pq = -q^2$

$\Rightarrow p^3 + q^2 = 3pq - q$

$\Rightarrow p^3 + q^2 = q(3p-1)$

Ans : C

5. Let $f(x) = x^3 + 2x^2 - 4x - 3 = 0$

The required equation is f(3x) = 0

$\Rightarrow (3x)^3 + 2(3x)^2 - 4(3x) - 3 = 0$

$\Rightarrow 27x^3 + 18x^2 - 12x - 3 = 0$

$\Rightarrow 9x^3 + 6x^2 - 4x - 1 = 0$

Ans : B

6. Let $f(x) = x^3 + x^2 + 2x + 3 = 0$

$f(x) = x^3 + px^2 + qx + r = 0$

The required equation is f(-p-x) = 0

i.e. f(-1-x) = 0

$(-1-x)^3 + (-1-x)^2 + 2(-1-x) + 3 = 0$

$\Rightarrow -(1 + 3x + 3x^2 + x^3) + (1 + x^2 + 2x) - 2 - 2x + 3 = 0$

$\Rightarrow x^3 + 2x^2 + 3x - 1 = 0$

Ans : A

7. Given α, β, γ are the roots of $x^3 + 3x^2 - 4x + 2 = 0$

We have $\alpha + \beta + \gamma = -3$

$\alpha\beta + \beta\gamma + \gamma\alpha = -4$

$\alpha\beta\gamma = -2$

Now $s_1 = \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$

$$= \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma}$$

$$= \frac{-3}{-2} = \frac{3}{2}$$

$$s_2 = \frac{1}{\alpha^2\beta\gamma} + \frac{1}{\beta^2\alpha\gamma} + \frac{1}{\gamma^2\alpha\beta}$$

$$= \frac{1}{\alpha\beta\gamma}\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$$

$$= \frac{1}{\alpha\beta\gamma}\left(\frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}\right)$$

$$= \frac{1}{-2}\left(\frac{-4}{-2}\right) = -1$$

$$s_3 = \frac{1}{\alpha^2\beta^2\gamma^2} = \frac{1}{(-2)^2} = \frac{1}{4}$$

The required equation is

$$x^3 - s_1x^2 + s_2x - s_3 = 0$$

$$\Rightarrow x^3 - \frac{3}{2}x^2 - x - \frac{1}{4} = 0$$

$$\Rightarrow 4x^3 - 6x^2 - 4x - 1 = 0$$

Ans : D

8. Let $f(x) = x^3 + 3x^2 + 2 = 0$

Required equation is $f\left(\sqrt[3]{x}\right) = 0$

$$\Rightarrow \left(\sqrt[3]{x}\right)^3 + 3\left(\sqrt[3]{x}\right)^2 + 2 = 0$$

$$\Rightarrow x + 3x^{\frac{2}{3}} + 2 = 0$$

$$\Rightarrow (x+2) = -3.x^{\frac{2}{3}}$$

Cubing on both sides

$$\Rightarrow x^3 + 3x^2.2 + 3.x.2^2 + 2^3 = -27.x^2$$

$$\Rightarrow x^3 + 33x^2 + 12x + 8 = 0$$

Ans : A

9. We have $\alpha+\beta+\gamma=0$

$$\alpha\beta+\beta\gamma+\gamma\alpha=4$$

$$\alpha\beta\gamma=-1$$

Now $\frac{\alpha^2}{\beta+\gamma},\frac{\beta^2}{\gamma+\alpha},\frac{\gamma^2}{\alpha+\beta}$

$\Rightarrow \frac{\alpha^2}{-\alpha},\frac{\beta^2}{-\beta},\frac{\gamma^2}{-\gamma}$

$\Rightarrow -\alpha,-\beta,-\gamma$

Let $f(x)= x^3+4x+1=0$

The required equation is f(-x) = 0

$\Rightarrow -x^3-4x+1=0$

$\Rightarrow x^3+4x-1=0$

Ans : B

10. Let $f(x)=ax^4+bx^3+cx^2+dx+e=0$

The roots of f(x) = 0 are 1, 3, 4, 0.

Given equation is f(x+3) = 0

i.e the roots must be 3 less than the roots of f(x) = 0

$\therefore$ Required roots = 1-3, 3-3, 4-3, 0-3

= -2, 0, 1, -3

Ans : D

JEE ADVANCED LEVEL QUESTIONS

Multi correct answer questions

11. Let $f(x)=x^4-3x^3+5x^2-2=0$

The required equation is f(-x) = 0

$\therefore x^4+3x^3+5x^2-2=0$

$\Rightarrow 2x^4+6x^3+10x^2-4=0$

Ans : A, D

12. $f(x)=ax^4+bx^3+cx^2+dx+e=0$

Given $f(-x)=ax^4-bx^3+cx^2-dx+e=0$

If 2, 5, 7, -4 are the roots of f(x) = 0, then -2, -5, -7, 4 are the roots of f(-x) = 0

Ans : B

Statement type:

13. Statement-I : $f(x)=2x^3+3x^2-4x+5=0$

Required equation is f(x-2) = 0

$\therefore 2(x-2)^3+3(x-2)^2-4(x-2)+5=0$

$\Rightarrow 2x^3-9x^2+8x+9=0$

Hence, statement-I is TRUE.
Statement-II : Clearly statement-II is TRUE.
Also, statement-II is the correct explanation of statement-I

Ans : A

14. Statement-I :

Let $f(x)=x^4+x^3+2x^2+x+1=0$

The required equation is $f(\sqrt{x})=0$

i.e $(\sqrt{x})^4+(\sqrt{x})^3+2(\sqrt{x})^2+\sqrt{x}+1=0$

$\Rightarrow x^2+x\sqrt{x}+2x+\sqrt{x}+1=0$

$\Rightarrow x^2+2x+1=-\sqrt{x}(x+1)$

squaring on both sides

$\Rightarrow x^4+4x^2+1+4x^3+4x+2x^2=x(x^2+2x+1)$

$\Rightarrow x^4+3x^3+4x^2+3x+1=0$

Hence, Statement-I is TRUE
Statement-II : Clearly the statement-II is TRUE.
Also, statement-II is the correct explanation of statement-I

Ans : A

Comprehension-I

15. Let $f(x)=x^3+x^2-2x+1=0$, The required equation is f(x-2) = 0

$\Rightarrow (x-2)^3+(x-2)^2-2(x-2)+1=0$

$\Rightarrow x^3-5x^2+6x+1=0$

Ans : D

16. Let $f(x)=x^2+px+q=0$

The required equation is f(x-1) = 0

$\Rightarrow (x-1)^2+p(x-1)+q=0$

$\Rightarrow x^2+(p-2)x+1-p+q=0$

Ans : A

Comprehension-II

17. Replace α,β,γ in place of p,q,r respectively, we will get the answer.

The required equation is $f\left(\frac{\gamma}{x-\beta}\right)=0$

Ans : D

18. $f(x)=x^3+2x^2+x+1=0$

$f(x)=x^3+px^2+qx+r=0$

Required equation is $f\left(\frac{r}{x-q}\right)=0$

$\Rightarrow f\left(\frac{1}{x-1}\right)=0$

Ans : C

Integer answer type questions

19. Given $f(x)=ax^4+bx^3+cx^2+dx+e=0$

$f(-x)=ax^4-bx^3+cx^2-dx+e=0$

If 2, 5, 7, -4 are the roots of f(x) = 0 then -2, -5, -7, 4 are the roots of f(-x) = 0

$\therefore$ Sum of the roots = -2, -5, -7+4

= -10

Ans : -10

20. Given $f(x)=x^4-5x^3+11x^2+3=0$

Required equation is $f(-x)=x^4+5x^3+11x^2+3=0$

$=x^4+px^3+qx^2+r=0$

Now, p+q+r = 5+11+3 = 19

Ans : 19

21. **Matrix Matching:**

a) The roots of f(x) = 0 are α *and* β f(-x) = 0 roots are $-\alpha, -\beta$

b) f(x-h) = 0 roots are $\alpha+h, \beta+h$

c) f(x+h) = 0 roots are $\alpha-h, \beta-h$

d) f(hx) = 0 roots are $\frac{\alpha}{h}, \frac{\beta}{h}$

Ans : a-t, b-p, c-r, d-q

22.

a) The roots of f(x) = 0 are α, β, γ

Now, the equation having roots $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ is f(-p-x) = 0

b) The equation whose roots are $\alpha\beta, \beta\gamma, \gamma\alpha$ is $f\left(\frac{-r}{x}\right)=0$

c) The equation whose roots are $\alpha(\beta+\gamma), \beta(\gamma+\alpha), \gamma(\alpha+\beta)$ is $f\left(\frac{-r}{x-q}\right)=0$

d) The equation whose roots are $\alpha+h,\ \beta+h,\ \gamma+h$ is $f(x-h)=0$

Ans : a-r, b-q, c-p, d-s

LEARNER'S TASK

CUQ'S

1. $f(x)=ax^2+bx+c=0$

   Given $-\alpha,\beta$ are the roots of $f(x)=0$

   The equation whose roots are $\alpha,-\beta$ is $f(-x)=0$

   $\therefore\ ax^2-bx+c=0$

Ans : D

2. Given $\frac{\alpha}{k}$ and $\frac{\beta}{k}$ are the roots of $f(x)=0$, then the equation whose roots are α and β is $f\left(\frac{x}{k}\right)=0$

Ans : A

3. Given α^2,β^2 are the roots of f(x) = 0, then the roots of $f(x^2)=0$ are α and β

Ans : B

4. Let $f(x)=x^3+x^2+x+1=0$

   $f(x)=x^3+px^2+qx+r=0$

   Given α,β,γ are the roots of $f(x)=0$

   The equation whose roots are $\alpha\beta,\ \beta\gamma,\ \gamma\alpha$ is $f(-p-x)=0$

   i.e $f(-1-x)=0$

   i.e $(-1-x)^3+(-1-x)^2+(-1-x)+1=0$

   $\Rightarrow x^3-x^2+x-1=0$

Ans : D

5. Let $f(x)=x^2+x+1=0$

   Given α,β are the roots of $f(x)=0$

   The equation whose roots are $2\alpha+1$ and

   $2\beta+1$ is $f\left(\frac{x-1}{2}\right)=0$

$$\Rightarrow \left(\frac{x-1}{2}\right)^2 + \left(\frac{x-1}{2}\right) + 1 = 0$$

$$\Rightarrow x^2 + 3 = 0$$

Ans : C

6. Let $f(x) = x^3 - 1 = 0$

Given α, β, γ are the roots of $f(x) = 0$

Then, the equation whose roots are

$2\alpha+1,\ 2\beta+1,\ 2\gamma+1$ is $f\left(\frac{x-1}{2}\right) = 0$

i.e $\left(\frac{x-1}{2}\right)^3 - 1 = 0$

$$\Rightarrow x^3 - 3x^2 + 3x - 9 = 0$$

Ans : B

7. Let $f(x) = x^4 - 2x^2 + x + 1 = 0$

The required eqation is $f(x-1) = 0$

i.e. $(x-1)^4 - 2(x-2)^2 + (x-1) + 1 = 0$

$$\Rightarrow x^4 - 4x^3 + 4x^2 + x - 1 = 0$$

Ans : A

8. Let $f(x) = ax^2 + bx + c = 0$

Given α, β are the roots of $f(x) = 0$

The equation whose roots are $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$ is $f\left(\frac{1}{x+1}\right) = 0$

i.e $a\left(\frac{1}{x+1}\right)^2 + b\left(\frac{1}{x+1}\right) + c = 0$

$$\Rightarrow c(x+1)^2 + b(x+1) + a = 0$$

$$\Rightarrow cx^2 + (2c+b)x + a + b + c = 0$$

But given $px^2 + qx + r = 0$

We have r = a + b + c

Ans : B

9. Given $ax^2 + bx + c = 0$

Let the quantity be k.

Given $(a+k)x^2 + (b+k)x + c = 0$

$\Rightarrow ax^2 + bx + c + kx^2 + kx = 0$

$\Rightarrow kx^2 + kx = 0$

$\Rightarrow x^2 + x = 0$

$\Rightarrow x^2 + x + 1 = 1$

Ans : A

10. Given α, β are the roots of $x^2 - px + r = 0$

We have $\alpha + \beta = p,\ \alpha\beta = r$

Similarly $\frac{\alpha}{2} + 2\beta = q$ and $\frac{\alpha}{2}.2\beta = r \Rightarrow \alpha\beta = r$

Now $\frac{\alpha}{2} + 2\beta = q$

$\Rightarrow \alpha + 4\beta = 2q$

$\Rightarrow p - \beta + 4\beta = 2q$

$\Rightarrow p + 3\beta = 2q$

$\Rightarrow \beta = \frac{2q - p}{3}$

since β is a root of $x^2 - px + r = 0$

$\Rightarrow \left(\frac{2q-p}{3}\right)^2 - p\left(\frac{2q-p}{3}\right) + r = 0$

$\Rightarrow r = p\left(\frac{2q-p}{3}\right) - \left(\frac{2q-p}{3}\right)^2$

$\Rightarrow r = \frac{2}{9}(2q-p)(2q-p)$

Ans : D

JEE MAINS LEVEL QUESTIONS

1. Let $f(x) = ax^2 + bx + c = 0$

Given α, β are the roots of $f(x) = 0$. The equation whose roots are $\alpha + h, \beta + h$ is $f(x-h) = 0$.

$\therefore a(x-h)^2 + b(x-h) + c = 0$

$\Rightarrow ax^2 + (b - 2ah)x + ah^2 - bh + c = 0$

but given the equation $px^2 + qx + r = 0$ both equations represent the same

$\therefore \frac{a}{p} = \frac{b - 2ah}{q}$

$$\Rightarrow b-2ah=\frac{aq}{p}$$

$$\Rightarrow 2ah=b-\frac{aq}{p}$$

$$\Rightarrow h=\frac{1}{2a}\left(b-\frac{aq}{p}\right)$$

$$\Rightarrow h=\frac{1}{2}\left(\frac{b}{a}-\frac{q}{p}\right)$$

Ans : C

2. The required equation is $f\left(\frac{r}{x-q}\right)=0$

$$\left(\frac{r}{x-q}\right)^3+p\left(\frac{r}{x-q}\right)^2+q\left(\frac{r}{x-q}\right)+r = 0$$

$$\Rightarrow r^3+pr^2(x-q)+qr(x-q)^2+r(x-q)^3 = 0$$

$$\Rightarrow r^2+pr(x-q)+q(x-q)^2+r(x-q)^3 = 0$$

The coefficient of x in the above equation is

$= pr-2q^2+3q^2$

$= pr+q^2$

Ans : B

3. Let $f(x)=ax^3+bx^2+cx+d=0$

We have $f(\sqrt{x})=ax\sqrt{x}+bx+c\sqrt{x}+d=0$

If α,β,γ are the roots of $f(x)=0$, then $\alpha^2,\beta^2,\gamma^2$ are the roots of $f(\sqrt{x})=0$

Hence the roots are 1,4,9.

Ans : B

4. Let $f(x)=x^4+x^3+2x^2+x+1=0$

The required equation is $f(\sqrt{x})=0$

$$\therefore \left(\sqrt{x}\right)^4=\left(\sqrt{x}\right)^3+2\left(\sqrt{x}\right)^2+\sqrt{x}+1=0$$

$$\Rightarrow x^2+x\sqrt{x}+2x+\sqrt{x}+1=0$$

$$\Rightarrow x^2+2x+1=-\sqrt{x}(x+1)$$

squaring on both sides, we get

$$\Rightarrow x^4 + 4x^2 + 1 + 4x^3 + 4x + 2x^2 = x\left(x^2 + 2x + 1\right)$$

$$\Rightarrow x^4 + 3x^3 + 4x^2 + 3x + 1 = 0$$

Ans : B

5. Let $f\left(x\right) = x^5 - 2x^4 + 3x^3 - 4x^2 + 5x - 6 = 0$

The required equation is f(x+2) = 0

i.e $\left(x+2\right)^5 - 2\left(x+2\right)^4 + 3\left(x+2\right)^3 - 4\left(x+2\right)^2 + 5\left(x+2\right) - 6 = 0$

The above expansion can be easily done by using Horner's method of synthetic division.

$x = 2$	1	-2	3	-4	5	-6
	0	2	0	6	4	18
$x = 2$	1	0	3	2	9	12
	1	2	4	14	32	
$x = 2$	1	2	7	16	41	
	0	2	8	30		
$x = 2$	1	4	15	46		
	0	2	12			
$x = 2$	1	6	27			
	0	2				
	1	8				

$$f\left(x+2\right) = x^5 + 8x^4 + 27x^3 + 46x^2 + 41x + 12 = 0$$

Ans : B

6. $f\left(x-3\right) = 5\left(x-3\right)^3 + 4\left(x-3\right)^2 - 13\left(x-3\right) - 25 = 0$

In the above expansion, the constant term is

= 5(-27)+4(9)-13(-3)-25

= -135+36+39-25

= -95

Ans : B

7. $f\left(x+5\right) = \left(x+5\right)^3 + \left(x+5\right)^2 + \left(x+5\right) + 1$

The coefficient of $x = 3.5^2 + 10 + 1$

$= 75 + 10 + 1$
$= 86$

Ans : A

8. When a quadratic equation posses roots of opposite sign, we have product of the roots < 0

$\Rightarrow \alpha\beta < 0$

$\Rightarrow \frac{c}{a} < 0$

$\Rightarrow \frac{a^2 - 3a + 2}{a^2 + 1} < 0$

$\Rightarrow (a^2 - 3a + 2)(a^2 + 1) < 0$

$\Rightarrow a^2 - 3a + 2 < 0$ since $a^2 + 1 > 0$

$\Rightarrow (a-1)(a-2) < 0$

$\Rightarrow 1 < a < 2$

Ans : C

9. The equation whose roots are reciprocals of the roots of $ax^2 + bx + c = 0$ is

$cx^2 + bx + a = 0$.

But given it is $px^2 + qx + r = 0$

$\therefore cx^2 + bx + a = 0$ and

$px^2 + qx + r = 0$ are same

we have $\frac{c}{p} = \frac{b}{q} = \frac{a}{r}$

$\Rightarrow \frac{c}{p} = \frac{b}{q}$ and $\frac{a}{r} = \frac{b}{q}$

Multiplying both the equations, we get

$\frac{ac}{pr} = \frac{b^2}{q^2}$

$\Rightarrow acq^2 = prb^2$

Ans : A

10. $f(x) = ax^2 + bx + c = 0$

The required equation is

$f(x+1) = a(x+1)^2 + b(x+1) + c = 0$

$\Rightarrow ax^2 + (2a+b)x + a + b + c = 0$

Given $2x^2+8x+2=0$

$\Rightarrow x^2+4x+1=0$

$\therefore a=1$	$2a+b=4$	$a+b+c=1$
	$\Rightarrow 2(1)+b=4$	$1+2+c=1$
	$\Rightarrow b=2$	$c=-2$
		Hence $b=-c$

Ans : B

JEE ADVANCED LEVEL QUESTIONS

Multi correct answer type questions

11. The equation whose roots are the reciprocals of the roots of $f(x)=ax^2+bx+c=0$ is

$cx^2+bx+a=0$ also $f\left(\frac{1}{x}\right)=0$

Ans : A, D

12. Required euqtion is $f(x-1)=0$

$\therefore a(x-1)^2+b(x-1)+c=0$

$\Rightarrow ax^2+(b-2a)x+a-b+c=0$ .... (i)

But given $2x^2+8x+2=0$

$\Rightarrow x^2+4x+1=0$ .... (ii)

Both equations (i) and (ii) are same

$a=1$	$b-2a=4$	$a-b+c=1$
	$\Rightarrow b-2(1)=4$	$1-6+c=1$
	$\Rightarrow b=6$	$\Rightarrow c=6$

Hence b = c

Ans : C

Statement Type

13. **Statement-I :** The equation whose roots are the reciprocals of the roots of $f(x)=0$ is $f\left(\frac{1}{x}\right)=0$

Given $f(x)=2x^2-3x+5=0$

$\Rightarrow f\left(\frac{1}{x}\right)=2\left(\frac{1}{x}\right)^2-3\left(\frac{1}{x}\right)+5=0$

$\Rightarrow \frac{2}{x^2}-\frac{3}{x}+5=0$

$\Rightarrow 5x^2-3x+2=0$

Hence, Statement-I is TRUE.
Statement-II: Clearly the statement-II is also TRUE.
Also, Statement-II is the correct explanation of statement-I
Ans : A

14. **Statement-I:**
Let $f(x) = x^3 - 2x + 1 = 0$
The required equation is $f(x-1) = 0$
$\therefore (x-1)^3 - 2(x-1) + 1 = 0$
$\Rightarrow x^3 - 3x^2 + 3x - 1 - 2x + 2 + 1 = 0$
$\Rightarrow x^3 - 3x^2 + x + 2 = 0$
Hence, statement-I is TRUE
Statement-II :
Clearly the statement-II is TRUE.
Also, Statement-II is the correct explanation of statement-I
Ans : A

Comprehension-I

15. $f(x) = x^4 - 3x^2 + 2x + 5 = 0$
The required equation is $f(\sqrt{x}) = 0$
$\Rightarrow (\sqrt{x})^4 - 3(\sqrt{x})^2 + 2(\sqrt{x}) + 5 = 0$
$\Rightarrow x^2 - 3x + 2\sqrt{x} + 5 = 0$
$\Rightarrow x^2 - 3x + 5 = -2\sqrt{x}$
squaring on both sides
$\Rightarrow x^4 + 9x^2 + 25 - 6x^3 - 30x + 10x^2 = 4x$
$\Rightarrow x^4 - 6x^3 + 19x^2 - 34x + 25 = 0$
Ans : A

16. Let $f(x) = x^3 - 2x + 1 = 0$
The required equation is $f(\sqrt{x}) = 0$
$\therefore (\sqrt{x})^3 - 2\sqrt{x} + 1 = 0$
$\Rightarrow x\sqrt{x} - 2\sqrt{x} + 1 = 0$
$\Rightarrow \sqrt{x}(x-2) = -1$
squaring on both sides
$\Rightarrow x(x^2 - 4x + 4) = 1$

$\Rightarrow x^3 - 4x^2 + 4x - 1 = 0$

Ans : C

17. Let $f(x) = x^3 - 2x^2 + 3x + 1 = 0$

The required equation is $f\left(\frac{x}{2}\right) = 0$

$\therefore \left(\frac{x}{2}\right)^3 - 2\left(\frac{x}{2}\right)^2 + 3\left(\frac{x}{2}\right) + 1 = 0$

$\Rightarrow x^3 - 4x^2 + 12x + 8 = 0$

Ans : D

18. Let $f(x) = x^2 - 2x + 1 = 0$

The required equation is $f(2x) = 0$

$\therefore (2x)^2 - 2(2x) + 1 = 0$

$\Rightarrow 4x^2 - 4x + 1 = 0$

Ans : C

Integer answer type questions

19. Let $f(x) = ax^3 + bx^2 + cx + d = 0$ .... (i)

Given $ax^3 + 2bx^2 + 4cx + 8d = 0$

$\Rightarrow a\left(\frac{x}{2}\right)^3 + b\left(\frac{x}{2}\right)^2 + c\left(\frac{x}{2}\right) + d = 0$

$\Rightarrow f\left(\frac{x}{2}\right) = 0$

Since 1,2,3 are the roots of $f(x) = 0$

2,4,6 are the roots of $f\left(\frac{x}{2}\right) = 0$

$\therefore \alpha + \beta + \gamma = 2 + 4 + 6 = 12$

Ans : 12

20. Let $f(x) = ax^3 + bx^2 + cx + d = 0$

Given $ax\sqrt{x} + bx + c\sqrt{x} + d = 0$

i.e $\Rightarrow a\left(\sqrt{x}\right)^3 + b\left(\sqrt{x}\right)^2 + c\left(\sqrt{x}\right) + d = 0$

$\Rightarrow f\left(\sqrt{x}\right) = 0$

Given 0,-1,2 are the roots of $f(x) = 0$

There fore 0,1,4 are the roots of $f\left(\sqrt{x}\right)=0$

$\therefore \alpha^2+\beta^2+\gamma^2=0+1+16=17$

Ans : 17

21. **Matrix matcing**

a) $-\alpha, -\beta$ are the roots of $f(-x)=0$

i.e $a(-x)^2+b(-x)+c=0$

$\Rightarrow ax^2-bx+c=0$

b) α^2, β^2 are the roots of $f\left(\sqrt{x}\right)=0$

$\therefore a\left(\sqrt{x}\right)^2+b\left(\sqrt{x}\right)+c=0$

$\Rightarrow ax+b\left(\sqrt{x}\right)+c=0$

$\Rightarrow ax+c=-b\left(\sqrt{x}\right)$

squaring on both sides

$\Rightarrow a^2x^2+2acx+c^2=b^2x$

$\Rightarrow a^2x^2+\left(2ac-b^2\right)x+c^2=0$

c) $\alpha+1, \beta+1$ are the roots of $f(x-1)=0$

$\therefore a(x-1)^2+b(x-1)+c=0$

$\Rightarrow a\left(x^2-2x+1\right)+b(x-1)+c=0$

$\Rightarrow ax^2+(b-2a)x+a-b+c=0$

d) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $f\left(\frac{1}{x}\right)=0$

$\therefore a\left(\frac{1}{x}\right)^2+b\left(\frac{1}{x}\right)+c=0$

$\Rightarrow \frac{a}{x^2}+\frac{b}{x}+c=0$

$\Rightarrow cx^2+bx+a=0$

Ans : a-s, b-t, c-p, d-q

22.a) Given 1,2,3 are the roots of $x^3+px^2+qx+r=0$

Now given roots are

3 = 1 + 2 = $\alpha+\beta$

5 = 2 + 3 = $\beta+\gamma$

4 = 1 + 3 = $\gamma+\alpha$

The required equation is $f(-p-x)=0$

b) 2 = 1 x 2 = $\alpha\beta$

6 = 2 x 3 = $\beta\gamma$

3 = 3 x 1 = $\gamma\alpha$

The required equation is $f\left(\frac{-r}{x}\right)=0$

c) 5 = 1(2+3) = $\alpha(\beta+\gamma)$

8 = 2(3+1) = $\beta(\gamma+\alpha)$

9 = 3(1+2) = $\gamma(\alpha+\beta)$

The required equation is $f\left(\frac{r}{x-q}\right)=0$

d) 2 = 1 + 1 = $\alpha+1$

3 = 2 + 1 = $\beta+1$

4 = 3 + 1 = $\gamma+1$

The required equation is $f(x-1)=0$

or $f(x-h)=0$

Ans : a-r, b-q, c-p, d-s