HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

1. $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)$

$=\left(1-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right)$

$\sin^2\frac{\pi}{10}\cdot\sin^2\frac{3\pi}{10}$

$=\left(\frac{\sqrt{5}-1}{4}\cdot\frac{\sqrt{5}+1}{4}\right)^2=\left(\frac{4}{16}\right)^2=\frac{1}{16}$

2. Since f(x) = sin x is an increasing function for $0<x<\pi/2$ and 1 rad is approximately 57°, we have $1°<1^R \Rightarrow \sin 1°<\sin 1$

3. On adding and subtracting

$x=\frac{3-\cos 4\theta+4\sin 2\theta}{2}$; $y=\frac{3-\cos 4\theta-4\sin 2\theta}{2}$

$x=\frac{4(1+\sin 2\theta)-(1+\cos 4\theta)}{2}$;

$y=\frac{4(1-\sin 2\theta)-(1+\cos 4\theta)}{2}$

$x=2(1+\sin 2\theta)-\cos^2 2\theta$; $y=2(1-\sin 2\theta)-\cos^2 2\theta$

$x=1+2\sin 2\theta+\sin^2 2\theta$; $y=1-2\sin 2\theta+\sin^2 2\theta$

$x=(1+\sin 2\theta)^2$; $y=(1-\sin 2\theta)^2$

$\Rightarrow \sqrt{x}+\sqrt{y}=2$

Alternate : Or put $\theta=\frac{\pi}{4}$ and verify

4. $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\tan A+\frac{n\sin A\cos A}{1-n\cos^2 A}}{1-\tan A\cdot\frac{n\sin A\cos A}{1-n\cos^2 A}}$

$=\frac{\sin A(1-n\cos^2 A)+n\sin A\cos^2 A}{\cos A(1-n\cos^2 A)-n\sin^2 A\cos A}$

$=\frac{\sin A-0}{\cos A(1-n\cos^2 A-n\sin^2 A)}=\frac{\sin A}{(1-n)\cos A}$

5. $A^2+B^2=3+2\left[\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right]$

$=3+2\left(-\frac{1}{2}\right)=2 \Rightarrow \sqrt{A^2+B^2}=\sqrt{2}$

6. Given expression reduce to

$\frac{\sin 22\cos 8+\cos 22\sin 8}{\sin 23\cos 7+\cos 23\sin 7}=\frac{\sin 30}{\sin 30}=1$

7. $\sin\theta\sec^7\theta+\cos\theta\,\text{cosec}^7\theta=\tan\theta\sec^6\theta+\cot\theta\,\text{cosec}^6\theta$

$=\sqrt{\frac{a}{b}}\left(1+\frac{a}{b}\right)^3+\sqrt{\frac{b}{a}}\left(1+\frac{b}{a}\right)^3$

$=(a+b)^3\left[\frac{\sqrt{a}}{b^{7/2}}+\frac{\sqrt{b}}{a^{7/2}}\right]=\frac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}$.

8. $\frac{\sin 2\alpha+\sin 4\alpha-\sin 3\alpha}{\cos 2\alpha+\cos 4\alpha-\cos 3\alpha}$

$=\frac{2\sin 3\alpha\cos\alpha-\sin 3\alpha}{2\cos 3\alpha\cos\alpha-\cos 3\alpha}$

$=\frac{\sin 3\alpha(2\cos\alpha-1)}{\cos 3\alpha(2\cos\alpha-1)}=\tan 3\alpha$

9. $\cos 20°+2\sin^2 55°-\sqrt{2}\sin 65°$

$=\cos 20°+1-\cos 110°-\sqrt{2}\sin 65°$

$=2\sin 65°\sin 45°+1-\sqrt{2}\sin 65°=1.$

10. Use $\cos 3\theta=4\cos^3\theta-3\cos\theta$.

11. cot 123° cot 147° cot 133° cot 137°

$=\frac{\cos 123°\cos 147°}{\sin 123°\sin 147°}\cdot\frac{\cos 133°\cos 137°}{\sin 133\sin 137°}$

$=\frac{\cos 270°+\cos 24°}{\cos 24°-\cos 270°}\cdot\frac{\cos 270°+\cos 24°}{\cos 24°-\cos 270°}=1$

12. $5\tan\theta = 4$ or $\tan\theta = \frac{4}{5}$

now, $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5\frac{\sin\theta}{\cos\theta} - 3}{5\frac{\sin\theta}{\cos\theta} + 2}$

$= \frac{5\tan\theta - 3}{5\tan\theta + 2}$

$= \frac{5\times\frac{4}{5} - 3}{5\times\frac{4}{5} + 2} = \frac{1}{6}$

13. $4x^2 - 2\sqrt{5}\,x + 1 = 0$

Let α and β be the roots, we have

$a + b = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}, \alpha\beta = \frac{1}{4}$

Since $\sin 18° = \frac{\sqrt{5}-1}{4}$, $\cos 36° = \frac{\sqrt{5}+1}{4}$

we have

$\sin 18° + \cos 36° = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$ $\sin 18° \cos 36°$

$= \frac{5-1}{16} = \frac{4}{16} = \frac{1}{4}$

Here, the required roots are sin18°, cos36°.

14. $\cos(A - B) = \frac{3}{5}$

or 5 cosA cosB + 5 sinA sinB = 3

From the second relation, we have

sinA sinB = 2 cosA cosB

$\Rightarrow$ $\cos A \cos B = \frac{1}{5}$ and $\sin A \sin B = \frac{2}{5}$

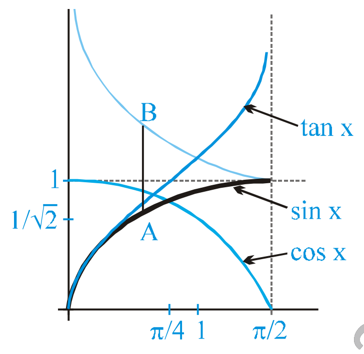
15. $\frac{1}{\cos\alpha} = \frac{2 - \cos\beta}{2\cos\beta - 1}$ Applying C/D

$\Rightarrow \frac{1 - \cos\alpha}{1 + \cos\alpha} = \frac{3(1 - \cos\beta)}{1 + \cos\beta}$

$\Rightarrow \tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2} \Rightarrow \tan^2\frac{\alpha}{2}\cot^2\frac{\beta}{2} = 3$

16. Given $\tan x = \cos x$

or $\sin x = \cos^2 x = 1 - \sin^2 x$ ....(1)



now, $\text{cosec}\, x - \sin x = \frac{1 - \sin^2 x}{\sin x} = 1$ (from (1))

17. On rationalizing ; we get

$\frac{1 - \sin x + 1 + \sin x + 2|\cos x|}{1 - \sin x - 1 + \sin x} = \frac{2(1 + |\cos x|)}{-2(\sin x)}$

$= \frac{1 + \cos x}{-(\sin x)} \Rightarrow$ **(D)**

18. $\cot x + \frac{\cos(60 + x)}{\sin(60 + x)} + \frac{\cos(x - 60)}{\sin(x - 60)}$

$= \frac{\cos x}{\sin x} + \frac{\sin(2x)}{\sin(x + 60)\sin(x - 60)}$

$= \frac{\cos x}{\sin x} + \frac{8\sin x \cos x}{4\sin^2 x - 3}$

$= \frac{4\sin^2 x\cos x - 3\cos x + 8\sin^2 x\cos x}{4\sin^3 x - 3\sin x}$

$= \frac{3[3\cos x - 4\cos^3 x]}{\sin^3 x} = 3\cot 3x$

$\Rightarrow \frac{3[1 - 3\tan^2 x]}{3\tan x - \tan^3 x}$

19. $\sin\frac{7\pi}{8} = \sin\left(\pi - \frac{\pi}{8}\right) = \sin\frac{\pi}{8}$

$\sin\frac{5\pi}{8} = \sin\left(\pi - \frac{3\pi}{8}\right) = \sin\frac{3\pi}{8}$

Therefore, the given value $= 2\left[\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8}\right]$

$= 2\left[\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right]$

$= 2(1) = 2$

$\left[\because \sin\frac{3\pi}{8} = \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \cos\frac{\pi}{8}\right]$

20. We have $\tan 90° - \tan 27° - \tan 63° + \tan 81°$

$= (\tan 9° + \tan 81°) - (\tan 27° + \tan 63)$

$= \frac{1}{\sin 9° \cos 9°} - \frac{1}{\sin 27° \cos 27°}$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$

$= 2\left[\frac{\sin 54° - \sin 18°}{\sin 54° \sin 18°}\right]$

$= 2\left[\frac{\cos 36° \sin 18°}{\sin 18° \cos 36°}\right] = 4$

EXERCISE - 2

Part # I : Multiple Choice

1. $\cos\frac{\pi}{10} \cdot \cos\frac{2\pi}{10} \cdot \cos\frac{4\pi}{10} \cdot \cos\frac{8\pi}{10} \cdot \cos\frac{16\pi}{10}$

$= \frac{\sin 2^5 \frac{\pi}{10}}{2^5 \sin\frac{\pi}{10}} = \frac{1}{32}\frac{\sin\frac{32\pi}{10}}{\sin\frac{\pi}{10}} = \frac{1}{32}\frac{\sin\left(3\pi + \frac{2\pi}{10}\right)}{\sin\left(\frac{\pi}{10}\right)}$

$= -\frac{1}{32}\ \mathrm{U}\ \frac{2\sin\frac{\pi}{10}\cos\frac{\pi}{10}}{\sin\frac{\pi}{10}}$

$= -\frac{1}{16}\cos\frac{\pi}{10} = -\frac{1}{64}\sqrt{10 + 2\sqrt{5}}$

2. $\cos^2 x + \cos^2 y + \cos^2 z - 2\cos x \cos y \cos z$

(Given x + y = z)

$= 1 + \cos(x + y)\cos(x - y) + \cos^2 z - 2\cos x \cos y \cos z$

$= 1 + \cos z\,[\cos(x - y) + \cos(x + y)] - 2\cos x \cos y \cos z$

$= 1 + \cos z \,.\, 2\cos x \cos y - 2\cos x \cos y \cos z$

$= 1$

$= \cos(x + y - z)$

4. $\tan A + \tan B + \tan C = 6$, $\tan A \tan B = 2$

In any Δ ABC,

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow$ $6 = 2\tan C$ $\Rightarrow$ $\tan C = 3$

$\therefore$ $\tan A + \tan B + 3 = 6$

$\Rightarrow$ $\tan A + \tan B = 3$ & $\tan A \tan B = 2$

Now $(\tan A - \tan B)^2 = (\tan A + \tan B)^2 - 4\tan A \tan B$

$= 9 - 8$

$= 1$

$\Rightarrow$ $\tan A - \tan B = \pm 1$

$\therefore$ $\tan A - \tan B = 1$ or $\tan A - \tan B = -1$

$\tan A + \tan B = 3$ $\qquad$ $\tan A + \tan B = 3$

on solving $\qquad$ on solving

$\tan A = 2$ $\qquad$ $\tan A = 1$

$\tan B = 1$ $\qquad$ $\tan B = 2$

5. $\sqrt{\sin\theta + \left(\sqrt{\sin\theta + \sqrt{\sin\theta + ... + \infty}}\right)} = (\sec^4\alpha - \sin\theta)$

$\sqrt{\sin\theta + (\sec^4\alpha - \sin\theta)} = \sec^4\alpha - \sin\theta$

$\sin\theta = \sec^4\alpha - \sec^2\alpha = \sec^2\alpha\tan^2\alpha$

(B) $\frac{2(2\sin^2\alpha)}{4(\cos^2\alpha)^2} = \tan^2\alpha\sec^2\alpha$

6. $\cos 4\theta - \cos 4\phi = 2\cos^2 2\theta - 2\cos^2 2\phi$

$= 2(\cos 2\theta + \cos 2\phi)(\cos 2\theta - \cos 2\phi)$

$= 2(2\cos^2\theta - 2\sin^2\phi)\, 2(\cos^2\theta - \cos^2\phi)$

$= 8(\cos\theta + \sin\phi)(\cos\theta - \sin\phi)(\cos\theta - \cos\phi)$

$(\cos\theta + \cos\phi)$

8. $\sqrt{\dfrac{1-\sin A}{1+\sin A}} + \dfrac{\sin A}{\cos A} = \dfrac{1}{\cos A}$

$\Rightarrow \sqrt{\dfrac{(1-\sin A)^2}{1-\sin^2 A}} + \dfrac{\sin A}{\cos A} = \dfrac{1}{\cos A}$

$\Rightarrow \dfrac{|1-\sin A|}{|\cos A|} + \dfrac{\sin A}{\cos A} = \dfrac{1}{\cos A}$ $(\because 1-\sin A \geq 0)$

$\Rightarrow \dfrac{1}{\cos A} = \dfrac{1}{\cos A}$ when $\cos A > 0$

$\Rightarrow$ A belongs to I[st] & IV[th] quadrant.

9. (A) $\dfrac{\cos 2\alpha \tan\left(\frac{\pi}{4}+\alpha\right)}{1+\cos\left(\frac{\pi}{2}-2\alpha\right)}$

$= \dfrac{(\cos^2\alpha - \sin^2\alpha)}{1+\sin 2\alpha} . \tan\left(\dfrac{\pi}{4}+x\right)$

$= \dfrac{\cos^2\alpha - \sin^2\alpha}{(\cos\alpha + \sin\alpha)^2} \tan\left(\dfrac{\pi}{4}+x\right)$

$= \dfrac{1-\tan\alpha}{1+\tan\alpha} . \dfrac{1+\tan\alpha}{1-\tan\alpha} = 1.$

(B) $\dfrac{\sin\alpha}{\sin\alpha - \cos\alpha \tan\frac{\alpha}{2}} - \cos\alpha$

$= \dfrac{\sin\alpha \cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} - \cos\alpha = \dfrac{\sin\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} = 1$

10. $\cos A + \cos B = 1$

$2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2} = 1 \quad \Rightarrow \cos\dfrac{A-B}{2} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \cos(A-B) = 2\cos^2\dfrac{A-B}{2} - 1 = \dfrac{2}{3} - 1 = -\dfrac{1}{3}$

& $|\cos A - \cos B| = \left|2\sin\dfrac{A+B}{2}\sin\dfrac{B-A}{2}\right| = \sqrt{\dfrac{2}{3}}$

(C) $\dfrac{1}{\sin^2 2\alpha} + \dfrac{(\cos^2\alpha - \sin^2\alpha)^2}{\sin^2 2\alpha}$

$= \dfrac{1+\cos^2 2\alpha}{\sin^2 2\alpha} \neq 1$

(D) $\dfrac{(\sin\alpha + \cos\alpha)^2}{(\sin\alpha + \cos\alpha)^2} = 1$

11. $f(x) = \dfrac{\sin x}{|\sec x|} + \dfrac{\cos x}{|\operatorname{cosec} x|} = \sin x\,|\cos x| + \cos x\,|\sin x|$

$\Rightarrow$ ***f*** (x) is constant when sinx and cosx are of opposite sign, i.e. ***f***(x) is in II[nd] or IV[th] quadrant.

12. $f_n(\theta) = \dfrac{\sin\frac{\theta}{2}(1+\cos\theta)(1+\cos 2\theta)(1+\cos 4\theta).....(1+\cos 2^n\theta)}{\cos\frac{\theta}{2}\cos\theta\cos 2\theta\cos 4\theta......\cos 2^n\theta}$

$= \dfrac{\sin\frac{\theta}{2}\left(2\cos^2\frac{\theta}{2}\right)(2\cos^2\theta)(2\cos^2 2\theta)...(2\cos 2^{n-1}\theta)}{\cos\frac{\theta}{2}\cos\theta\cos 2\theta\cos 4\theta...\cos 2^n\theta}$

$= \dfrac{2^{n+1}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\theta\cos 2\theta.....\cos 2^{n-1}\theta}{\cos 2^n\theta}$

$= \dfrac{\sin 2^n\theta}{\cos 2^n\theta} = \tan 2^n\theta$

$f_2\left(\dfrac{\pi}{16}\right) = \tan 2^2\left(\dfrac{\pi}{16}\right) = 1$

$f_3\left(\dfrac{\pi}{32}\right) = 1 \quad , f_4\left(\dfrac{\pi}{64}\right) = 1, f_5\left(\dfrac{\pi}{128}\right) = 1$

13. Let $\tan\dfrac{\alpha}{2} = t$

$(a+2)\,2t + (2a-1)(1-t^2) = (2a+1)(t^2+1)$

$\Rightarrow 2at + 4t + 2a - 2at^2 - 1 + t^2 = 2a + 1 + 2at^2 + t^2$

$\Rightarrow 4at^2 - 2t(2+a) + 2 = 0$

$\Rightarrow 2at^2 - 2t - at + 1 = 0$

$\Rightarrow 2t(at-1) - 1(at-1) = 0$

$\Rightarrow t = 1/2,\ t = 1/a$

$\Rightarrow \tan\alpha = \dfrac{2\tan\alpha/2}{1-\tan^2\alpha/2}$

$\Rightarrow \quad \tan\alpha = \dfrac{2\times 1/2}{1-1/4} = 4/3$

or $\quad \tan\alpha = \dfrac{2/a}{1-1/a^2} = \dfrac{2a}{a^2-1}$

14. $y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$

& $\quad \tan x = \dfrac{2b}{a-c}$

$z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$

$\Rightarrow \quad y + z = a + c$

and $y - z = (a-c)\left(\cos^2 x - \sin^2 x\right) + 4b\sin x\cos x$

$= (a-c)\cos 2x + 2b\sin 2x$

$(\because\ 2b = (a-c)\tan x)$

$= (a-c)\,[\cos 2x + \tan x.\sin 2x] = (a-c)$

$\left[2\cos 2x + \dfrac{\sin x}{\cos x}\sin 2x\right] = \dfrac{(a-c)\cos(2x-x)}{\cos x} = (a-c).$

15. $\sin^6 x + \cos^6 x = a^2$

$\Rightarrow \quad (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x) = a^2$

$\Rightarrow \quad (\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x = a^2$

$\Rightarrow \quad 1 - 3\sin^2 x\cos^2 x = a^2$

$\Rightarrow \quad 1 - \dfrac{3}{4}\sin^2 2x = a^2 \qquad \Rightarrow \quad \dfrac{4(1-a^2)}{3} = \sin^2 2x$

$\Rightarrow \quad 0 \le \dfrac{4}{3}(1-a^2) \le 1$

$1 - a^2 \ge 0 \qquad$ and $\qquad 4 - 4a^2 \le 3$

$a^2 \le 1 \qquad$ and $\qquad \dfrac{1}{4} \le a^2$

$-1 \le a \le 1 \qquad$ and $\qquad a \ge \dfrac{1}{2}$ or $a \le -\dfrac{1}{2}$

$a \in \left[-1, -\dfrac{1}{2}\right] \cup \left[\dfrac{1}{2}, 1\right]$

Part # II : Assertion & Reason

1. Statement-I: $\cos\alpha + \cos\left(\alpha + \dfrac{2\pi}{3}\right) + \cos\left(\alpha + \dfrac{4\pi}{3}\right) = 0$

$= \cos\alpha + 2\cos(\alpha+\pi)\cos\left(\dfrac{\pi}{3}\right) = 0$

$= \cos\alpha - \cos\alpha = 0$

i.e. $a + b + c = 0$ (say)

$\Rightarrow \quad a^3 + b^3 + c^3 = 3abc$

hence statement I is true

But statement II is false as vice versa is not true.

2. If A is obtuse than $0° < B + C < 90°$

$\Rightarrow \quad \tan(B+C) = \dfrac{\tan B + \tan C}{1 - \tan B\,\tan C} > 0$

as numerator is positive

$1 - \tan B\tan C > 0$

$\tan B\tan C < 1$

Statement II is obviously true & it explain I

3. $\cos^2\theta = \dfrac{(x+y)^2}{4xy} = \dfrac{x^2+y^2}{4xy} + \dfrac{1}{2}$

$\dfrac{x^2+y^2}{2} \ge xy \qquad (AM \ge GM)$

$(x-y)^2 \ge 0$ only when $x = y$

Statement-II is true.

4. $(\sin\theta - 1)^2 = 0$

$\Rightarrow \quad \sin\theta = 1$

$\Rightarrow \quad \theta = \dfrac{\pi}{2}$

$\Rightarrow$ Statement-I only hold for n = 1 hence false.

Statement-II is true.

6. Statement I is wrong as z can be written as $\frac{-(x+y)}{1-xy}$.

It implies that for any values of xy(xy ≠ 1), we get a value of z and statement II is correct.

7. In the first quadrant $\cos\theta > \sin\theta$ for $\theta \in (\pi/4, \pi/2)$

Hence, $\cos 1 < \sin 1$.

Also in the first quadrant, cosine is decreasing and sine is increasing, but this is not the correct reason for which $\cos 1 < \sin 1$. Thus, the correct answer is (B).

8. Statement II is true, because each trigonometric function has a principle period of π and 2π and hence 2π is one of the periods of every trigonometric function. Thus,

$f(2A) = f(2B)$.

$\Rightarrow$ $2A = 2n\pi + 2B$, for some $n \in Z$.

or $A = n\pi + B$

EXERCISE - 3

Part # I : Matrix Match Type

1. (A) $$\frac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10° \cos 10°} = \frac{2\left(\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°\right)}{\frac{2\sin 10° \cos 10°}{2}}$$

$$= \frac{4\cos 70°}{\sin 20°} = 4$$

(B) $$\frac{4\cos 20° \sin 20° - \sqrt{3}\cos 20°}{\sin 20°}$$

$$= \frac{2\sin 40° - \sqrt{3}\cos 20°}{\sin 20°}$$

$$= \frac{2\sin 90° \sin 40° - 2\cos 30° \cos 20°}{\sin 20°}$$

$$= \frac{\cos 50° - \cos 130° - \cos 50° - \cos 10°}{\sin 20°}$$

$$= -\frac{2\cos 70° \cos 60°}{\sin 20°} = -1.$$

(C) $$\frac{\cos 40° + \cos 40° - \cos 20°}{\sin 20°}$$

$$= \frac{\cos 40° - 2\sin 30° \sin 10°}{\sin 20°}$$

$$= \frac{\cos 40° - \sin 10°}{\sin 20°} = \frac{\sin 50° - \sin 10°}{\sin 20°}$$

$$= \frac{2\cos 30° \sin 20°}{\sin 20°} = \sqrt{3}$$

(D) $$2\sqrt{2}\,\sin 10° \left[\frac{1}{2\cos 5°} + \frac{\cos 40°}{\sin 5°} - 2\sin 35°\right]$$

$$= 2\sqrt{2}\,\sin 10° \left[\frac{\sin 5° + 2\cos 5° \cos 40° - 4\sin 35° \sin 5°.\cos 5°}{2\cos 5° \sin 5°}\right]$$

$$= 2\sqrt{2}\,[\sin 5° + \cos 45° + \cos 35° - \cos 25° + \cos 45°]$$

$$= 2\sqrt{2}\,[\sin 5° + 2\cos 45° - 2\sin 30° \sin 5°]$$

$$= 4.$$

Part # II : Comprehension

1. a. **2.** b **3.** c

1. **(a)**

$\sin\alpha = A\sin(\alpha+\beta) = A(\sin\alpha\cos\beta + \sin\beta\cos\alpha)$

$\Rightarrow \sin\alpha(1 - A\cos\beta) = A\sin\beta\cos\alpha$ ....(i)

$\Rightarrow \tan\alpha = \dfrac{A\sin\beta}{(1-A\cos\beta)}$ ....(ii)

2. **(b)**

$$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{(1-A\cos\beta)\tan\alpha}{A\cos\beta}$$

$$= \frac{(1-A\cos\beta)\sin\alpha}{A\cos\alpha\cos\beta}$$ [from eq. (i) and (ii)]

3. **(c)**

$\tan(\alpha+\beta)$

$$= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$= \frac{\dfrac{A\sin\beta}{1-A\cos\beta}+\dfrac{\sin\beta}{\cos\beta}}{1-\dfrac{A\sin\beta\sin\beta}{(1-A\cos\beta)\cos\beta}}$$

$$= \frac{A\sin\beta\cos\beta+\sin\beta-A\sin\beta\cos\beta}{\cos\beta-A\cos^2\beta-A\sin^2\beta}$$

$$= \frac{\sin\beta}{\cos\beta-A}$$

Also $\tan(\alpha+\beta)$

$$= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

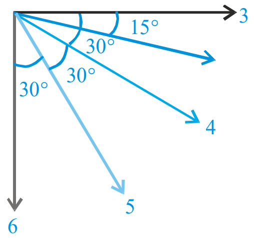
$$= \frac{\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\sin\alpha(1-A\cos\beta)}{A\cos\alpha\cos\beta}}{1-\dfrac{\sin^2\alpha(1-A\cos\beta)}{A\cos^2\alpha\cos\beta}}$$ [from eq. (ii)]

Comprehension # 2

1. **Hint :** $1^c = 57°$

$\Rightarrow \sin 1° < \sin 57°$ $\quad\Rightarrow \cos 1° > \cos 57°$

2.



$\Rightarrow$ Answer is $90° - 15° = 75°$

3. Let the number of side of a polygon be 5x and angle α.

& for other polygon number of side be 4x and angle β.

$$\alpha = \frac{(n-2)\pi}{n} = \frac{(5x-2)\pi}{5x}\ ;\quad \beta = \frac{(4x-2)\pi}{4x}$$

$$\alpha-\beta = \frac{\pi}{20} = \frac{\pi}{x}\left(\left(\frac{5x-2}{5}\right)-\left(\frac{4x-2}{4}\right)\right)$$

$\Rightarrow x = 2$ $\quad\Rightarrow$ side : 10, 8

4. $$\frac{4x}{3}\times\frac{90}{100}+3x+\frac{2\pi x}{75}\times\left(\frac{180°}{\pi}\right)=180°$$

$\Rightarrow x = 20$

Angles are 24°, 60°, 96°

Comprehension # 3

1. $$\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\left(\pi-\frac{\pi}{7}\right)$$

$$= -\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$$

$$\Rightarrow \alpha = \frac{\pi}{7} = \frac{\pi}{2^3-1}$$

Then value is $-\left(\dfrac{-1}{8}\right) = \dfrac{1}{8}$

2. $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$

Put $\cos\frac{7\pi}{15} = -\cos\frac{8\pi}{15}$ and arrange

$$= \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\left(-\cos\frac{8\pi}{15}\right)\cos\frac{3\pi}{15}$$

$$\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}$$

$$= -\left(\frac{\sin 2^4\frac{\pi}{15}}{2^4\sin\frac{\pi}{15}}\right)\cos 36^\circ.\cos 72^\circ.\frac{1}{2}$$

$$= -\frac{1}{2^4}\left(\frac{\sqrt{5}+1}{4}\times\frac{\sqrt{5}-1}{4}\right)\frac{1}{2} = 1/128.$$

3. $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}(1)\sin\frac{5\pi}{14}\sin\frac{3\pi}{14}\sin\frac{\pi}{14}$

$$= \sin^2\frac{\pi}{14}\sin^2\frac{3\pi}{14}\sin^2\frac{5\pi}{14}$$

$$= \left[\cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{14}\right)\right]^2$$

$$= \left[\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\right]^2$$

$$= \left[\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right]^2 = \left[\frac{\sin\frac{8\pi}{7}}{2^3\sin\frac{\pi}{7}}\right]^2 = \frac{1}{64}$$

EXERCISE - 4

Subjective Type

1. $\frac{m}{n} = \frac{\tan(\theta+120^\circ)}{\tan(\theta-30^\circ)}$

use C & D

$$\frac{m+n}{m-n} = \frac{\tan(\theta+120^\circ)+\tan(\theta-30)}{\tan(\theta+120)-\tan(\theta-30^\circ)}$$

$$= \frac{\sin(\theta+120^\circ)\cos(\theta-30^\circ)+\sin(\theta-30^\circ)\cos(\theta+120^\circ)}{\sin(\theta+120^\circ)\cos(\theta-30^\circ)-\sin(\theta-30^\circ).\cos(\theta+120^\circ)}$$

$$= \frac{\sin(2\theta+90^\circ)}{\sin 150^\circ}$$

$$\frac{m+n}{m-n} = 2\cos 2\theta$$

4. If $a = \sin(\theta+\alpha)$, $b = \sin(\theta+\beta)$

$\therefore\quad 2ab = 2\sin(\theta+\alpha)\sin(\theta+\beta)$

$2ab = \cos(\alpha-\beta) - \cos(2\theta+\alpha+\beta)$

Multiply both sides by $2\cos(\alpha-\beta)$

$\Rightarrow\quad 4ab\cos(\alpha-\beta) = 2\cos^2(\alpha-\beta)$

$\qquad - 2\cos(2\theta+\alpha+\beta).\cos(\alpha-\beta)$

$= 1 + \cos 2(\alpha-\beta) - \cos 2(\theta+\alpha) - \cos 2(\theta+\beta)$

$\Rightarrow\quad \cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta)$

$= \cos 2(\theta+\alpha) + \cos 2(\theta+\beta) - 1$

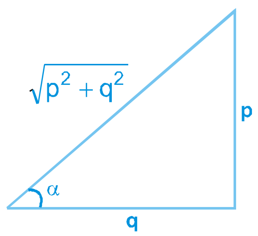
$= 1 - 2\sin^2(\theta+\alpha) + 1 - 2\sin^2(\theta+\beta) - 1$

$= 1 - 2a^2 - 2b^2$

6. $\tan\alpha = \frac{p}{q}$

$$\text{LHS} = \frac{1}{2}(p\ \mathrm{cosec}\ 2\beta - q\sec 2\beta)\times\frac{\sqrt{p^2+q^2}}{\sqrt{p^2+q^2}}$$

$$= \frac{1}{2}\left\{\frac{p}{\sqrt{p^2\times q^2}}\mathrm{cosec}\,2\beta - \frac{q}{\sqrt{p^2\times q^2}}\sec 2\beta\right\}\times\sqrt{p^2+q^2}$$



$\sin\alpha = \frac{p}{\sqrt{p^2+q^2}}$, $\cos\alpha = \frac{q}{\sqrt{p^2+q^2}}$

$$= \frac{1}{2}\left(\frac{\sin\alpha\cos 2\beta - \cos\alpha\sin 2\beta}{\sin 2\beta\cos 2\beta}\right)\times\sqrt{p^2+q^2}$$

$$= \frac{\sin(\alpha-2\beta)}{\sin 4\beta}\times\sqrt{p^2+q^2} = \frac{\sin 4\beta}{\sin 4\beta}\times\sqrt{p^2+q^2}$$

$(\because \ \alpha = 6\beta)$

7. **(i)** $\cot 7\frac{1}{2}^\circ = \tan 82\frac{1}{2}^\circ = \frac{\cos 7\frac{1}{2}^\circ}{\sin 7\frac{1}{2}^\circ} = \frac{2\cos^2 7\frac{1}{2}^\circ}{\sin 15^\circ}$

$$= \frac{1+\cos(45^\circ-30^\circ)}{\sin(45^\circ-30^\circ)} = \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$= \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(2\sqrt{2}+\sqrt{3}+1)(\sqrt{3}+1)}{3-1} = \sqrt{2}+\sqrt{3}+2+\sqrt{6}$$

$$= \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$$

$$= (\sqrt{2}+\sqrt{3})\ (\sqrt{2}+1)$$

(ii) $\tan 142\frac{1}{2}^\circ = -\cot 52\frac{1}{2}^\circ = \frac{-1}{\tan 52\frac{1}{2}^\circ}$

$$= \frac{-1}{\tan\left(45+7\frac{1}{2}^\circ\right)}$$

$$= -\frac{1-\tan 7\frac{1}{2}^\circ}{1+\tan 7\frac{1}{2}^\circ} = -\frac{\cos 7\frac{1}{2}^\circ - \sin 7\frac{1}{2}^\circ}{\cos 7\frac{1}{2}^\circ + \sin 7\frac{1}{2}^\circ}$$

$$= -\frac{\left(\cos 7\frac{1}{2}^\circ - \sin 7\frac{1}{2}^\circ\right)^2}{\cos 15^\circ} = -\frac{1-\sin 15^\circ}{\cos 15^\circ}$$

$$= -\left(\frac{1-\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}\right) = -\frac{(2\sqrt{2}-\sqrt{3}+1)(\sqrt{3}-1)}{2}$$

$$= -\frac{[2\sqrt{2}(\sqrt{3}-1)-(\sqrt{3}-1)^2]}{2}$$

$$= -\frac{[2\sqrt{2}(\sqrt{3}-1)-(4-2\sqrt{3})]}{2}$$

$$= -[\sqrt{2}(\sqrt{3}-1)-(2-\sqrt{3})]$$

$$= -\sqrt{6}+\sqrt{2}+2-\sqrt{3} = 2+\sqrt{2}-\sqrt{3}-\sqrt{6}$$

9. **(i)** $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$$= (\tan 9^\circ + \tan 81^\circ) - (\tan 27^\circ + \tan 63^\circ)$$

$$= \frac{\sin 90^\circ}{\cos 9^\circ\cos 81^\circ} - \frac{\sin 90^\circ}{\cos 27^\circ\cos 63^\circ}$$

$$= \frac{2}{2\sin 9^\circ\cos 9^\circ} - \frac{2}{2\sin 27^\circ\cos 27^\circ}$$

$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} = \frac{2}{\frac{\sqrt{5}-1}{4}} - \frac{2}{\frac{\sqrt{5}+1}{4}}$$

$$= \frac{8(\sqrt{5}+1-\sqrt{5}+1)}{4} = 4$$

(ii) $\text{cosec } 10^\circ - \sqrt{3}\ \sec 10^\circ = 2\left(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)$

$$\times\frac{1}{\sin 10^\circ\cos 10^\circ}\times\frac{2}{2} = 4$$

(iii) $2\sqrt{2}\ \sin 10^\circ\left(\frac{\sec 5^\circ}{2}+\frac{\cos 40^\circ}{\sin 5^\circ}-2\sin 35^\circ\right)$

$$= 2\sqrt{2}$$

$$\left(\frac{2\sin 5^\circ\cos 5^\circ\sec 5^\circ}{2}+\frac{2\sin 5^\circ\cos 5^\circ\cos 40^\circ}{\sin 5^\circ}-2\sin 35^\circ\sin 10^\circ\right)$$

$$= 2\sqrt{2}\ (\sin 5^\circ + 2\cos 45^\circ + \cos 35^\circ - \cos 25^\circ + \cos 45^\circ)$$

$$= 2\sqrt{2}\ (\sin 5^\circ + 2\cos 45^\circ + 2\sin 30^\circ\sin(-5^\circ))$$

$$= 2\sqrt{2}\ (\sqrt{2}) = 4$$

(iv) cot 70° + 4 cos 70°

$$= \frac{\cos 70^\circ}{\sin 70^\circ} + 4\cos 70^\circ = \frac{\cos 70^\circ + 4\cos 70^\circ \sin 70^\circ}{\sin 70^\circ}$$

$$= \frac{\cos 70^\circ + 2\sin 140^\circ}{\sin 70^\circ}$$

$$= \frac{(\cos 70^\circ + \sin 140^\circ) + \sin 140^\circ}{\sin 70^\circ} = \frac{(\sin 20^\circ + \sin 140^\circ) + \sin 140^\circ}{\sin 70^\circ}$$

$$= \frac{2\sin 80^\circ \times \cos 60^\circ + \sin 140^\circ}{\sin 70^\circ} = \frac{2\sin 120^\circ \times \cos 20^\circ}{\sin 70^\circ}$$

$$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

(v) tan 10° – tan 50° + tan 70°

= tan 10° – tan (60° – 10°) + tan (60° + 10°)

$$= \tan 10^\circ - \frac{\sqrt{3} + \tan 10^\circ}{1 - \sqrt{3}\tan 10^\circ} + \frac{\sqrt{3} - \tan 10^\circ}{1 + \sqrt{3}\tan 10^\circ}$$

$$= \frac{9\tan 10^\circ - 3\tan^3 10^\circ}{1 - 3\tan^2 10^\circ}$$

$$= 3\left(\frac{3\tan 10^\circ - \tan^3 10^\circ}{1 - 3\tan^2 10^\circ}\right) = 3\tan 30^\circ$$

$$= \sqrt{3}$$

13. $P_n - P_{n-2} = \cos^n\theta + \sin^n\theta - \cos^{n-2}\theta - \sin^{n-2}\theta$

$= \cos^{n-2}\theta\,(\cos^2\theta - 1) + \sin^{n-2}\theta\,(\sin^2\theta - 1)$

$= \cos^{n-2}\theta\,(-\sin^2\theta) + \sin^{n-2}\theta\,(-\cos^2\theta)$

$= (-\sin^2\theta\cos^2\theta)\,\{\cos^{n-4}\theta + \sin^{n-4}\theta\}$

$= (-\sin^2\theta\cos^2\theta)\,P_{n-4}$

put n = 4

⇒ $P_4 - P_2 = (-\sin^2\theta\cos^2\theta)\,P_0$

⇒ $P_4 = P_2 - 2\sin^2\theta\cos^2\theta$

$= 1 - 2\sin^2\theta\cos^2\theta$

similarly we can prove the other result also.

19. $\sqrt{13 - 18\tan x} = 6\tan x - 3$ ......(i)

⇒ $13 - 18\tan x = 36\tan^2 x + 9 - 36\tan x$

⇒ $\tan x = \frac{2}{3}, -\frac{1}{6}$

Put in (i)

⇒ $\tan x = \frac{2}{3}$ is correct

⇒ $x = n\pi + \tan^{-1}\frac{2}{3}$

$= n\pi + \alpha = \alpha, \pi + \alpha, -\pi + \alpha, -2\pi + \alpha$ in $(-2\pi, 2\pi)$

21. $\left(\sin 2\theta + \sqrt{3}\cos 2\theta\right)^2 - 5 = \cos\left(\frac{\pi}{6} - 2\theta\right)$

⇒ $4\left(\frac{1}{2}\sin 2\theta + \frac{\sqrt{3}}{2}\cos 2\theta\right)^2 - \cos\left(\frac{\pi}{6} - 2\theta\right) - 5 = 0$

⇒ $4\cos^2\left(\frac{\pi}{6} - 2\theta\right) - \cos\left(\frac{\pi}{6} - 2\theta\right) - 5 = 0$

⇒ $\cos\left(\frac{\pi}{6} - 2\theta\right) = \frac{5}{4}, -1$

⇒ $\cos\left(\frac{\pi}{6} - 2\theta\right) = -1 = \cos\pi$

⇒ $\frac{\pi}{6} - 2\theta = 2n\pi \pm \pi$ ⇒ $2\theta = \frac{\pi}{6} - 2n\pi \mp \pi$

⇒ $\theta = \frac{2n\pi}{2} + \frac{\pi}{12} \pm \frac{\pi}{2}$

⇒ $\theta = \frac{7\pi}{12}, \frac{19\pi}{12}$.

23. $a\cos 2\theta + b\sin 2\theta = c$

⇒ $\frac{a(1-t^2)}{1+t^2} + \frac{b(2t)}{1+t^2} = c$ where $t = \tan\theta$

⇒ $(c+a)t^2 - 2bt + (c-a) = 0$

⇒ $t_1 + t_2 = \frac{2b}{c+a}, \; t_1t_2 = \frac{c-a}{c+a}$

∴ $\cos^2\alpha + \cos^2\beta = \frac{1 + \cos 2\alpha + 1 + \cos 2\beta}{2} = 1 + \frac{1}{2}$

$[\cos 2\alpha + \cos 2\beta]$

$$= 1 + \frac{1}{2}\left[\frac{1-t_1^2}{1+t_1^2} + \frac{1-t_2^2}{1+t_2^2}\right]$$

simplifying and using values for t_1, t_2 we get

$$\cos^2\alpha + \cos^2\beta = 1 + \frac{ac}{a^2+b^2} = \frac{a^2+b^2+ac}{a^2+b^2}.$$

25. $1+2\operatorname{cosec} x=\dfrac{-\sec^2\left(\frac{x}{2}\right)}{2} \Rightarrow 1+\dfrac{2}{\sin x}=\dfrac{-1}{1+\cos x}$

$\Rightarrow (2+\sin x)(1+\cos x)=-\sin x$

$\Rightarrow 2+2\cos x+\sin x+\sin x\cos x=-\sin x$

$\Rightarrow 2(\sin x+\cos x)+\sin x\cos x+2=0$

Put $\sin x+\cos x=t$

$\Rightarrow 1+2\sin x\cos x=t^2$

$\therefore 2t+\dfrac{t^2-1}{2}+2=0 \Rightarrow t^2+4t+3=0$

$\Rightarrow t=-1,-3 \Rightarrow \sin x+\cos x=-1$

$\Rightarrow \cos\left(x-\dfrac{\pi}{4}\right)=\dfrac{-1}{\sqrt{2}}=\cos\dfrac{3\pi}{4}$

$\Rightarrow x-\dfrac{\pi}{4}=2n\pi\pm\dfrac{3\pi}{4} \Rightarrow x=2n\pi+\pi,\ 2n\pi-\dfrac{\pi}{2}$

$\Rightarrow x=2n\pi+\pi$ at which cosec x is not defined

$\therefore x=2n\pi-\dfrac{\pi}{2}$.

26. $\sin^2 4x+\cos^2x=2\sin 4x\,.\cos^4x$

$\Rightarrow \sin^2 4x-2\sin4x\cos^4x+\cos^2x=0$

$\Rightarrow (\sin4x-\cos^4x)^2+\cos^2x-\cos^8x=0$

$\Rightarrow (\sin4x-\cos^4x)^2+\cos^2x(1-\cos^6x)=0$

$\Rightarrow \sin4x-\cos^4x=0$ ......(i)

and $\cos^2x(1-\cos^6x)=0$ ......(ii)

From (ii) $\cos^2x=0, 1$

Case-I $\cos^2x=0 \Rightarrow x=n\pi\pm\dfrac{\pi}{2}$

$\Rightarrow 4x=4n\pi\pm2\pi$

$\therefore \sin 4x=0$

$\Rightarrow$ equation (1) is also true

Case-II $\cos^2x=1 \Rightarrow \sin^2x=0$

$\Rightarrow x=n\pi$ $\therefore$ equation (1) becomes

$0-1=0$ false $\therefore$ solution is $x=n\pi\pm\dfrac{\pi}{2}$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

1. since α is a root of

$25\cos^2\theta+5\cos\theta-12=0$

$\therefore 25\cos^2\alpha+5\cos\alpha-12=0$

$\Rightarrow (5\cos\alpha-3)(5\cos\alpha+4)=0$

$\Rightarrow \cos\alpha=-\dfrac{4}{5}$ and $\dfrac{3}{5}$

But $\dfrac{\pi}{2}<\alpha<\pi$ i.e., in second quadrant

$\therefore \cos\alpha=-\dfrac{4}{5}$

$\Rightarrow \sin\alpha=\dfrac{3}{5}$

now, $\sin2\alpha=2\sin\alpha\cos\alpha$

$=2\times\dfrac{3}{5}\times\left(-\dfrac{4}{5}\right)=-\dfrac{24}{25}$

3. $u=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}+\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$

$\Rightarrow u^2=a^2\cos^2\theta+b^2\sin^2\theta+a^2\sin^2\theta+b^2\cos^2\theta+$

$2\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}\times\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$

$\Rightarrow u^2=(a^2+b^2)+$

$2\sqrt{\{a^2+(b^2-a^2)\sin^2\theta\}\times\{a^2+(b^2-a^2)\cos^2\theta\}}$

$\Rightarrow u^2=(a^2+b^2)+$

$2\sqrt{a^4+a^2(b^2-a^2)+(b^2-a^2)^2\sin^2\theta\cos^2\theta}$

$\Rightarrow u^2=(a^2+b^2)+2\sqrt{a^2b^2+\left(\dfrac{b^2-a^2}{2}\right)^2\sin^2 2\theta}$.

$\therefore \min(u^2)=a^2+b^2+2ab=(a+b)^2$ and $\max(u^2)$

$=a^2+b^2+(a^2+b^2)=2(a^2+b^2)$

Now, $\max(u^2)-\min(u^2)=(a-b)^2$

4. $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$

squaring and adding, we get

$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha.\cos\beta$

$$= \left(-\frac{21}{65}\right)^2 + \left(-\frac{27}{65}\right)^2$$

$$\Rightarrow 2 + 2\cos(\alpha-\beta) = \frac{1170}{4225}$$

$$\Rightarrow \cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1170}{4\times 4225} = \frac{9}{130}$$

$$\Rightarrow \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}} \quad (\because \pi < \alpha-\beta < 3\pi$$

$$\Rightarrow \frac{\pi}{2} < \left(\frac{\alpha-\beta}{2}\right) < \frac{3\pi}{2})$$

5. $\because \tan\frac{P}{2}$ and $\tan\frac{Q}{2}$ are the roots of equation $ax^2 + bx + c = 0$

$$\because \tan\frac{P}{2} + \tan\frac{Q}{2} = -\frac{b}{a} \text{ and } \tan\frac{P}{2}\tan\frac{Q}{2} = \frac{c}{a}$$

$$\frac{P}{2} + \frac{Q}{2} + \frac{R}{2} = \frac{\pi}{2} \quad (\because P+Q+R=\pi)$$

$$\Rightarrow \frac{P+Q}{2} = \frac{\pi}{2} - \frac{R}{2} \Rightarrow \frac{P+Q}{2} = \frac{\pi}{4}$$

$(\because \angle R = \frac{\pi}{2})$

$$\Rightarrow \tan\left(\frac{P+Q}{2}\right) = 1 \quad \Rightarrow \frac{\tan\frac{P}{2} + \tan\frac{Q}{2}}{1 - \tan\frac{P}{2}.\tan\frac{Q}{2}} = 1$$

$$\Rightarrow \frac{-b/a}{1-c/a} = 1 \quad \Rightarrow c = a + b$$

6. $\cos x + \sin x = \frac{1}{2}$

$$\therefore \frac{1-\tan^2 x/2}{1+\tan^2 x/2} + \frac{2\tan x/2}{1+\tan^2 x/2} = \frac{1}{2}, \text{ Let } \tan\frac{x}{2} = t$$

$$\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} = \frac{1}{2} \Rightarrow 3t^2 - 4t - 1 = 0$$

$$\therefore t = \frac{2\pm\sqrt{7}}{3}$$

as $0 < x < \pi \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{2}$

$\therefore \tan\frac{x}{2}$ is positive

$$\therefore t = \tan\frac{x}{2} = \frac{2+\sqrt{7}}{3}$$

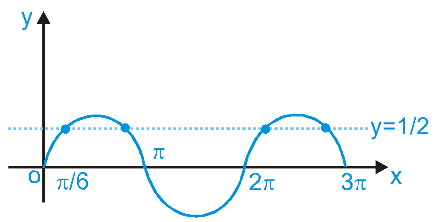
Now $\tan x = \frac{2\tan x/2}{1-\tan^2 x/2} = \frac{2t}{1-t^2}$

$$\Rightarrow \tan x = \frac{2\left(\frac{2+\sqrt{7}}{3}\right)}{1-\left(\frac{2+\sqrt{7}}{3}\right)^2} = -\left(\frac{4+\sqrt{7}}{3}\right)$$

7. Given equation is $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x + 3) = 0$

$$\Rightarrow \sin x = \frac{1}{2} \quad (\because \sin x \neq -3)$$



It is clear from figure that the curve intersect the line at four points in the given interval.

Hence, number of solutions are 4.

8. Given, $\cos x + \sin x = \frac{1}{2}$

$$\therefore \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1}{2}$$

Let $\tan\frac{x}{2} = t$

$$\Rightarrow \frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} = \frac{1}{2} \Rightarrow 3t^2 - 4t - 1 = 0$$

$$\Rightarrow t = \frac{2\pm\sqrt{7}}{3}$$

As $0 < x < \pi \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{2}$

$\therefore \tan\frac{x}{2}$ is positive.

$$\therefore t = \tan\frac{x}{2} = \frac{2+\sqrt{7}}{3}$$

Now, $\tan x = \dfrac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} = \dfrac{2t}{1-t^2}$

$$\Rightarrow \tan x = \frac{2\left(\frac{2+\sqrt{7}}{3}\right)}{1-\left(\frac{2+\sqrt{7}}{3}\right)^2}$$

$$\Rightarrow \tan x = -\frac{3\left(2+\sqrt{7}\right)}{1+2\sqrt{7}} \times \frac{1-2\sqrt{7}}{1-2\sqrt{7}}$$

$$\Rightarrow \tan x = -\left(\frac{4+\sqrt{7}}{3}\right).$$

11. $2\{\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)\}+3=0$

$(\cos\alpha+\cos\beta+\cos\gamma)^2+(\sin\alpha+\sin\beta+\sin\gamma)^2=0$

$\sum\cos\alpha = 0 = \sum\sin\alpha$

12. $\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta)) = \dfrac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$

$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}} = \frac{(9+5)4}{48-15} = \frac{14\times 4}{33} = \frac{56}{33}$$

Hence correct option is (1)

13. $A = \sin^2 x + \cos^4 x = \sin^2 x + (1-\sin^2 x)^2 = \sin^4 x - \sin^2 x + 1$

$$= \left(\sin^2 x - \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{3}{4} \le A \le 1$$

14. $3\sin P + 4\cos Q = 6$ .....(i)

$\Rightarrow 4\sin Q + 3\cos P = 1$ .....(ii)

Squaring and adding (i) & (ii) we get $\sin(P+Q) = \frac{1}{2}$

$$\Rightarrow P+Q = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \quad \Rightarrow R = \frac{5\pi}{6} \text{ or } \frac{\pi}{6}$$

If $R = \frac{5\pi}{6}$ then $0 < P, Q < \frac{\pi}{6}$

$$\Rightarrow \cos Q < 1 \text{ and } \sin P < \frac{1}{2} \quad \Rightarrow 3\sin P + 4\cos Q < \frac{11}{2}$$

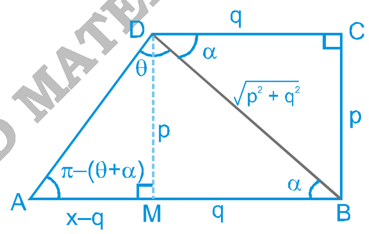
So $R = \frac{\pi}{6}$

15. Let $AB = x$

$$\tan(\pi-\theta-\alpha) = \frac{p}{x-q} \Rightarrow \tan(\theta+\alpha) = \frac{p}{q-x}$$

$\Rightarrow q - x = p\cot(\theta+\alpha)$

$\Rightarrow x = q - p\cot(\theta+\alpha)$



$$= q - p\left(\frac{\cot\theta\cot\alpha - 1}{\cot\alpha + \cot\theta}\right)$$

$$= q - p\left(\frac{\frac{q}{p}\cot\theta - 1}{\frac{q}{p} + \cot\theta}\right) = q - p\left(\frac{q\cot\theta - p}{q + p\cot\theta}\right)$$

$$= q - p\left(\frac{q\cos\theta - p\sin\theta}{q\sin\theta + p\cos\theta}\right)$$

$$\Rightarrow x = \frac{q^2\sin\theta + pq\cos\theta - pq\cos\theta + p^2\sin\theta}{p\cos\theta + q\sin\theta}$$

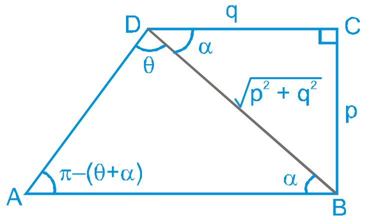
$$\Rightarrow AB = \frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}.$$

Alternative

From Sine Rule

$$\frac{AB}{\sin\theta} = \frac{\sqrt{p^2+q^2}}{\sin(\pi-(\theta+\alpha))}$$

$$AB = \frac{\sqrt{p^2+q^2}\,\sin\theta}{\sin\theta\cos\alpha+\cos\theta\sin\alpha}$$



$$= \frac{(p^2+q^2)\sin\theta}{q\sin\theta+p\cos\theta} \qquad \left(\because \cos\alpha = \frac{q}{\sqrt{p^2+q^2}}\right)$$

$$= \frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}.$$

16. Given expression

$$= \frac{\sin A}{\cos A}\times\frac{\sin A}{\sin A-\cos A}+\frac{\cos A}{\sin A}\times\frac{\cos A}{\cos A-\sin A}$$

$$= \frac{1}{\sin A-\cos A}\left\{\frac{\sin^3 A-\cos^3 A}{\cos A\sin A}\right\}$$

$$= \frac{\sin^2 A+\sin A\cos A+\cos^2 A}{\sin A\cos A} = 1+\sec A\,\mathrm{cosec}\,A$$

18. $f_k(x) = \frac{1}{k}(\sin^k x+\cos^k x)$, where $x \in R$ and $k \geq 1$

Now, $f_4(x) - f_6(x)$

$$= \frac{1}{4}(\sin^4 x+\cos^4 x) - \frac{1}{6}(\sin^6 x+\cos^6 x)$$

$$= \frac{1}{4}(1-2\sin^2 x\,.\,\cos^2 x) - \frac{1}{6}(1-3\sin^2 x\,.\,\cos^2 x)$$

$$= \frac{1}{4}-\frac{1}{6}=\frac{1}{12}$$

19. In ΔOA_1B_1,

$$\tan 45° = \frac{AB_1}{OB_1} \Rightarrow \frac{20}{OB_1}=1$$

$\Rightarrow$ $OB = 20$

In ΔOA_2, $\tan 30° = \frac{20}{OB_2}$

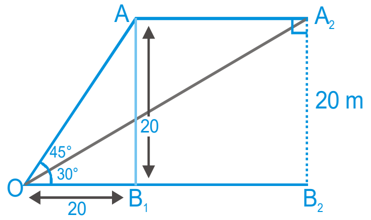
$\Rightarrow$ $OB_2 = 20\sqrt{3}$

$\Rightarrow$ $B_1B_2 + OB_1 = 20\sqrt{3}$

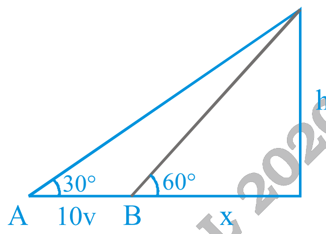
$\Rightarrow$ $B_1B_2 = 20\sqrt{3} - 20$

$\Rightarrow$ $B_1B_2 = 20(\sqrt{3}-1)$m

$\therefore$ $\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{20(\sqrt{3}-1)}{1}$



21.



Let speed = v units/min

$$\frac{h}{10v+x} = \tan 30°$$

$$\frac{h}{x} = \tan 60°$$

$\Rightarrow$ $\frac{x}{10v+x} = \frac{1}{3}$ $\Rightarrow$ $x = 5v$

So, time = 5 minutes.

Part # II : IIT-JEE ADVANCED

1. We are given that

$(\cot\alpha_1)(\cot\alpha_2) \ldots (\cot\alpha_n) = 1$

$\Rightarrow$ $(\cos\alpha_1)(\cos\alpha_2) \ldots (\cos\alpha_n)$

$= (\sin\alpha_1)(\sin\alpha_2) \ldots (\sin\alpha_n)$ ....(i)

Let $y = (\cos\alpha_1)(\cos\alpha_2) \ldots (\cos\alpha_n)$ (to be maximum)

squaring both sides, we get

$y_2 = (\cos^2\alpha_1)(\cos^2\alpha_2) \ldots (\cos^2\alpha_n)$

$= \cos\alpha_1\sin\alpha_1\cos\alpha_2\sin\alpha_2 \ldots \cos\alpha_n\sin\alpha_n$ [using eq. (i)]

$\frac{1}{2^{n/2}}[\sin 2\alpha_1 2\alpha_2 \ldots \sin 2\alpha_n \leq 1]$

As $0 \leq \alpha_1, \alpha_2, \ldots \alpha_n \leq \pi/2$, we have

$0 \leq \sin 2\alpha_1, \sin 2\alpha_2, \ldots \sin 2\alpha_n \leq 1$

$\therefore$ $y^2 \leq \frac{1}{2^n}\times 1$ or $y \times \frac{1}{2^{n/2}}$

2. Clearly $\theta = 30^\circ$ and $\phi \in (60^\circ, 90^\circ)$
Hence $\theta + \phi$ lies in $(90^\circ, 120^\circ)$.

3. Let $y = 2\sin t$

$$y = \frac{1-2x+5x^2}{3x^2-2x-1}$$

$(3y-5)x^2 - 2x(y-1) - (y+1) = 0$

$$x \in R - \left\{1, -\frac{1}{3}\right\}$$

$\therefore\ D \geq 0$
$\Rightarrow\ y^2 - y - 1 \geq 0$

$$\therefore\ y \geq \frac{1+\sqrt{5}}{2} \quad \text{or} \quad y \leq \frac{1-\sqrt{5}}{2}$$

$$\Rightarrow\ \sin t \geq \frac{1+\sqrt{5}}{4} \quad \text{or} \quad \sin t \leq \frac{1-\sqrt{5}}{4}$$

$$\therefore\ \text{range of t is } \left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$$

4. $\theta \in \left(0, \frac{\pi}{4}\right)$

$\because\ \tan\theta \uparrow$ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and $0 < \tan\theta < 1$

$\cot\theta \downarrow$ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and $\cot\theta > 1$

Let $\tan\theta = 1 - \lambda_1$ and $\cot\theta = 1 + \lambda_2$ where λ_1 and λ_2 are very small and positive, then

$t_1 = (1-\lambda_1)^{1-\lambda_1}$, $t_2 = (1-\lambda_1)^{1+\lambda_2}$,

$t_3 = (1+\lambda_2)^{1-\lambda_1}$, $t_4 = (1+\lambda_2)^{(1+\lambda_2)}$

$\therefore\ t_4 > t_3 > t_1 > t_2$

5. $$\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$$

$$\Rightarrow\ \frac{\sin^4 x}{2} + \frac{(1-\sin^2 x)^2}{3} = \frac{1}{5}$$

$$\Rightarrow\ \frac{\sin^4 x}{2} + \frac{1+\sin^4 x - 2\sin^2 x}{3} = \frac{1}{5}$$

$$\Rightarrow\ 5\sin^4 x - 4\sin^2 x + 2 = \frac{6}{5}$$

$\Rightarrow\ 25\sin^4 x - 20\sin^2 x + 4 = 0 \quad \Rightarrow\ (5\sin^2 x - 2)^2 = 0$

$$\Rightarrow\ \sin^2 x = \frac{2}{5}, \cos^2 x = \frac{3}{5} \quad \Rightarrow\ \tan^2 x = \frac{2}{3}$$

$$\text{and } \frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$$

6. $$f(\theta) = \frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$$

$$= \frac{1}{\frac{1-\cos 2\theta}{2} + \frac{3}{2}\sin 2\theta + \frac{5(1+\cos 2\theta)}{2}}$$

$$= \frac{2}{6 + 3\sin 2\theta + 4\cos 2\theta}$$

$$\therefore\ f(\theta)_{max} = \frac{2}{6-5} = 2$$

7. $$\frac{1}{\sin\frac{\pi}{n}} - \frac{1}{\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}}$$

$$\frac{2\cos\frac{2\pi}{n}\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}}$$

$$\sin\frac{4\pi}{n} = \sin\frac{3\pi}{n}$$

$$\frac{4\pi}{n} = (-1)^k\frac{3\pi}{n} + k\pi, k \in I$$

If $k = 2m \Rightarrow \frac{\pi}{n} = 2m\pi$

$\frac{1}{n} = 2m$, not possible

If $k = 2m+1 \Rightarrow \frac{7\pi}{n} = (2m+1)\pi$

$\Rightarrow\ n = 7,\ m = 0$

Ans. $n = 7$

8. $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\sin\theta = (\sqrt{2}+1)\cos\theta \quad \Rightarrow\ \tan\theta = \sqrt{2}+1$

$$\Rightarrow\ \theta = n\pi + \frac{3\pi}{8}; n \in I$$

$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\therefore\ \cos\theta = (\sqrt{2}-1)\sin\theta$

$$\Rightarrow\ \tan\theta = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$$

$$\Rightarrow\ \theta = n\pi + \frac{3\pi}{8}; n \in I \quad \therefore\ P = Q$$

9. As $\tan(2\pi-\theta)>0,\ -1<\sin\theta<-\frac{\sqrt{3}}{2},\ \theta\in[0,2\pi]$

$\Rightarrow\ \frac{3\pi}{2}<\theta<\frac{5\pi}{3}$

Now $2\cos\theta(1-\sin\phi)=\sin^2\theta(\tan\theta/2+\cot\theta/2)\cos\phi-1$

$\Rightarrow\ 2\cos\theta(1-\sin\phi)=2\sin\theta\cos\phi-1$

$\Rightarrow\ 2\cos\theta+1=2\sin(\theta+\phi)$

As $\theta\in\left(\frac{3\pi}{2},\frac{5\pi}{3}\right)$ $\Rightarrow\ 2\cos\theta+1\in(1,2)$

$\Rightarrow\ 1<2\sin(\theta+\phi)<2$ $\Rightarrow\ \frac{1}{2}<\sin(\theta+\phi)<1$

As $\theta+\phi\in[0,4\pi]$

$\Rightarrow\ \theta+\phi\in\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$ or $\theta+\phi\in\left(\frac{13\pi}{6},\frac{17\pi}{6}\right)$

$\Rightarrow\ \frac{\pi}{6}-\theta<\phi<\frac{5\pi}{6}-\theta$ or $\frac{13\pi}{6}-\theta<\phi<\frac{17\pi}{6}-\theta$

$\Rightarrow\phi\in\left(-\frac{3\pi}{2},\frac{-2\pi}{3}\right)\cup\left(\frac{2\pi}{3},\frac{7\pi}{6}\right)$

$\left(\because\ \theta\in\left(\frac{3\pi}{2},\frac{5\pi}{3}\right)\right)$

$\therefore$ correct option is (A, C, D)

10. $\cos x+\cos y+\cos z=0$

and $\sin x+\sin y+\sin z=0$

$\cos x+\cos y=-\cos z$ ....(i)

and $\sin x+\sin y=-\sin z$ ....(ii)

Squaring and adding we get,

$1+1+2(\cos x\cos y+\sin x\sin y)=1$

$\Rightarrow\ 2+2\cos(x-y)=1$

or $2\cos(x-y)=-1$ or $\cos(x-y)=-\frac{1}{2}$

or $2\cos^2\left(\frac{x-y}{2}\right)-1=-\frac{1}{2}$ or $\cos\left(\frac{x-y}{2}\right)=\frac{1}{2}$

11. $\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x$

$=\cos x\sin 2x\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$

or $\left[\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{\pi}{4}+x\right)\right]\cos 2x$

$=(\cos x\sin 2x\sec x-\sin x\sin 2x\sec x)$

or $\frac{2}{\sqrt{2}}\sin x\cos 2x=(\cos x-\sin x)\sin 2x\sec x$

or $\sqrt{2}\sin x\cos 2x=(\cos x-\sin x)\,2\sin x$

or $\frac{1}{\sqrt{2}}=\frac{1}{\cos x+\sin x}$

or $x=\frac{\pi}{4}$ $\Rightarrow$ $\sec x=\sec\frac{\pi}{4}=\sqrt{2}$

11. $x^2-2x\sec\theta+1=0$

$\Rightarrow\ x=\frac{2\sec\theta\pm\sqrt{4\sec^2\theta-4}}{2}$

$\Rightarrow\ x=\sec\theta+\tan\theta,\ \sec\theta-\tan\theta$

$\Rightarrow\ \alpha_1=\sec\theta-\tan\theta$

Now $x^2+2x\tan\theta-1=0$

$\Rightarrow\ x=\frac{-2\tan\theta\pm\sqrt{4\tan^2\theta+4}}{2}$

$\Rightarrow\ x=-\tan\theta\pm\sec\theta$ $\Rightarrow$ $\alpha_2=(\sec\theta-\tan\theta)$

$\Rightarrow\ \beta_2=-(\sec\theta+\tan\theta)$

$\therefore\ \alpha_1+\beta_2=-2\tan\theta$

13. $\sum_{k=1}^{13}\frac{\sin\left[\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)-\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right]}{\sin\frac{\pi}{6}\left(\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)\right)}$

$=2\sum_{k=1}^{13}\left(\cot\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)\right)$

$=2\left(\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right)=2\left(1-\cot\left(\frac{29\pi}{12}\right)\right)$

$=2\left(1-\cot\left(\frac{5\pi}{12}\right)\right)=2(1-(2-\sqrt{3}))=2(-1+\sqrt{3})$

$=2(\sqrt{3}-1)$

MOCK TEST

1. $2\cos x + \sin x = 1$ ....(i)

$4\cos^2 x = (1 - \sin x)^2$

$4 - 4\sin^2 x = 1 + \sin^2 x - 2\sin x$

$5\sin^2 x - 2\sin x - 3 = 0$

$(\sin x - 1)(5\sin x + 3) = 0$

$\Rightarrow \sin x = 1, \sin x = -\frac{3}{5}$

$\because \cos x = \frac{1-\sin x}{2}$ (from equation (i))

$\therefore$ when $\sin x = 1$

$7\cos x + 6\sin x = 7\left(\frac{1-\sin x}{2}\right) + 6\sin x$

$= 7\left(\frac{1-1}{2}\right) + 6 \times 1 = 6$ Ans.

and when $\sin x = \frac{-3}{5}$

then $7\cos x + 6\sin x = 7\left(\frac{1+\frac{3}{5}}{2}\right) - \frac{6\times 3}{5} = \frac{28-18}{5} = 2$

2. **(C)**

$\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$

$= \frac{1}{2}[1 + \cos 20^\circ - (\cos 60^\circ + \cos 40^\circ) + (1 + \cos 100^\circ)]$

$= \frac{1}{2}[1 + \cos 20^\circ - \frac{1}{2} - \cos 40^\circ + 1 - \cos 80^\circ]$

$= \frac{1}{2}\left[\frac{3}{2} + \cos 20^\circ - (2\cos 60^\circ \cos 20^\circ)\right] = \frac{3}{4}$

3. **(B)**

$$\frac{3 + \frac{\cos 80^\circ \cos 20^\circ}{\sin 80^\circ \sin 20^\circ}}{\frac{\cos 80^\circ}{\sin 80^\circ} + \frac{\cos 20^\circ}{\sin 20^\circ}}$$

$$= \frac{2\sin 80^\circ \sin 20^\circ + \cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ}{\sin 20^\circ \cos 80^\circ + \cos 20^\circ \sin 80^\circ}$$

$$= \frac{\cos 60^\circ - \cos 100^\circ + \cos 60^\circ}{\sin 100^\circ} = \frac{1 - \cos 100^\circ}{\sin 100^\circ}$$

$= \tan 50^\circ$

4. $\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + \ldots\ldots + \cos\frac{17\pi}{19}$

here $A = \frac{\pi}{19}, D = \frac{2\pi}{19}, n = 9$

$\because \cos A + \cos(A + D) + \cos(A + 2D) + \ldots\ldots + \cos(A + (n-1)D)$

$$= \frac{\sin\left(\frac{nD}{2}\right)}{\sin\frac{D}{2}} \cdot \cos\left(\frac{2A + (n-1)D}{2}\right)$$

$$= \frac{\sin 9 \times \frac{\pi}{19}}{\sin\frac{\pi}{19}} \times \cos\left(\frac{\frac{\pi}{19} + \frac{17\pi}{19}}{2}\right)$$

$$= \frac{\sin\frac{9\pi}{19}}{\sin\frac{\pi}{19}} \times \cos\frac{9\pi}{19}$$

$$= \frac{1}{2} \cdot \frac{\sin\left(\frac{18\pi}{19}\right)}{\sin\frac{\pi}{19}} = \frac{1}{2} \cdot \frac{\sin\frac{\pi}{19}}{\sin\frac{\pi}{19}} = \frac{1}{2}$$

5. **(C)**

$\cos x + \sin x = \sqrt{2}\cos x$

$\sin x = (\sqrt{2} - 1)\cos x$

$\cos x = \frac{1}{(\sqrt{2}-1)}\sin x$

$\cos x = (\sqrt{2} + 1)\sin x$

$\cos x - \sin x = \sqrt{2}\sin x$

6. $f(\theta) = \sin^2\theta + \sin^2\left(\theta + \frac{2\pi}{3}\right) + \sin^2\left(\theta + \frac{4\pi}{3}\right) = \frac{1}{2}$

$$\left[(1 - \cos 2\theta) + \left(1 - \cos\left(2\theta + \frac{4\pi}{3}\right)\right) + \left(1 - \cos\left(2\theta + \frac{8\pi}{3}\right)\right)\right]$$

$$= \frac{1}{2}\left[(1+1+1) - (\cos 2\theta + 2\cos(2\theta + 2\pi))\cos\left(\frac{2\pi}{3}\right)\right]$$

$= \frac{1}{2}[3 - (\cos 2\theta - \cos 2\theta)] = \frac{3}{2}$

$f(\theta) = \frac{3}{2}$ (constant) $\Rightarrow f\left(\frac{\pi}{15}\right) = \frac{3}{2}$

7. (A)

$\sin 2\beta = \sqrt{\sin\alpha.\cos\alpha}$

$\cos 4\beta = 1 - 2\sin^2 2\beta = 1 - 2\sin\alpha.\ \sin\alpha = (\sin\alpha - \cos\alpha)^2$

$= 2\sin^2\left(\alpha - \frac{\pi}{4}\right)$

$= 2\sin^2\left(\frac{\pi}{4} - \alpha\right)$

8. $0° < x < 90°$ & $\cos x = \frac{3}{\sqrt{10}}$

$\log_{10}\sin x + \log_{10}\cos x + \log_{10}\tan x$

$= \log_{10}(\sin x \cos x \tan x)$

$= \log_{10}(1 - \cos^2 x) = \log_{10}(1 - 9/10)$

$= \log_{10}\left(\frac{1}{10}\right) = -1$

9. $\sin\alpha \sin\beta - \cos\alpha \cos\beta + 1 = 0$

$\Rightarrow \cos(\alpha + \beta) = 1$

$\alpha + \beta = 2n\pi$

$\Rightarrow 1 + \cot\alpha \tan\beta = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

$= \frac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta} = 0$

10. $\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2}, \frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2}, 0 < A, B < \pi/2$

$\tan A = \frac{\sqrt{3}}{\sqrt{5}} \frac{\sin B}{\cos B}$

$\tan A = \frac{\sqrt{3}}{\sqrt{5}} \tan B$ ....(i)

$\frac{\sin A\cos A}{\sin B\cos B} = \frac{\sqrt{15}}{4}$

$\Rightarrow \frac{\tan A.\sec^2 B}{\tan B.\sec^2 A} = \frac{\sqrt{15}}{4}$

from (i)

$\Rightarrow \frac{\sqrt{3}}{\sqrt{5}} \frac{(1+\tan^2 B)}{(1+\tan^2 A)} = \frac{\sqrt{15}}{4}$

$\Rightarrow 4 + 4\tan^2 B = 5 + 5\tan^2 A$

$\Rightarrow -1 + 4\tan^2 B = 5 \times \frac{3}{5}\tan^2 B \quad \Rightarrow \tan B = \pm 1$

$\Rightarrow \tan B = +1 \qquad (\because 0 < B < \frac{\pi}{2})$

Now $\tan A + \tan B = \frac{\sqrt{3}}{\sqrt{5}} + 1 = \frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$

11. $2(\sec^2\alpha - \mathrm{cosec}^2\alpha) + (\mathrm{cosec}^2\alpha + \sec^2\alpha)$

$(\mathrm{cosec}^2\alpha - \sec^2\alpha) = \frac{15}{4}$

$\Rightarrow (\mathrm{cosec}^2\alpha - \sec^2\alpha)[\mathrm{cosec}^2\alpha + \sec^2\alpha - 2] = \frac{15}{4}$

$\Rightarrow 4(\cot^2\alpha - \tan^2\alpha)(\cot^2\alpha + \tan^2\alpha) = 15$

$\Rightarrow 4(\cot^4\alpha - \tan^4\alpha) = 15$

$\Rightarrow 4(1 - \tan^8\alpha) = 15\tan^4\alpha$

$\Rightarrow 4\tan^8\alpha + 15\tan^4\alpha - 4 = 0$

$\Rightarrow 4\tan^8\alpha + 16\tan^4\alpha - \tan^4\alpha - 4 = 0$

$\Rightarrow (4\tan^4\alpha - 1)(\tan^4\alpha + 4) = 0$

$\Rightarrow \tan^4\alpha = \frac{1}{4}$ or $\tan^4\alpha = -4$ (not possible)

$\Rightarrow \tan^2\alpha = \pm\frac{1}{2} \quad \Rightarrow \tan^2\alpha = +\frac{1}{2}$

$\left(\because \tan^2\alpha \neq -\frac{1}{2}\right)$

$\Rightarrow \tan\alpha = \pm\frac{1}{\sqrt{2}}$

12. $3\sin\beta = \sin(2\alpha + \beta)$, (Given)

$\tan(\alpha + \beta) - 2\tan\alpha$

$= \tan(\alpha + \beta) - \tan\alpha - \tan\alpha$

$= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} - \frac{\sin\alpha}{\cos\alpha} - \tan\alpha$

$= \frac{\sin(\alpha+\beta-\alpha)}{\cos\alpha\cos(\alpha+\beta)} - \tan\alpha = \frac{\sin\beta}{\cos(\alpha+\beta)\cos\alpha} - \frac{\sin\alpha}{\cos\alpha}$

$= \frac{\sin\beta - \sin\alpha\cos(\alpha+\beta)}{\cos(\alpha+\beta)\cos\alpha}$

$= \frac{2\sin\beta - [\sin(2\alpha+\beta) - \sin\beta]}{2\cos(\alpha+\beta)\cos\alpha}$

$= \frac{-[\sin(2\alpha+\beta) - 3\sin\beta]}{2\cos(\alpha+\beta)\cos\alpha} = 0$

13. (A, B, C, D)

For option (A)

$\sin^2 x - \cos^2 x = -\cos 2x \le 1$

For option (B)

$$\sqrt{\frac{6}{5}}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{3}}\cos x\right) = \frac{\sqrt{3}}{\sqrt{5}}\sin x + \frac{\sqrt{2}}{\sqrt{5}}\cos x$$

$= \sin x \cdot \sin\phi + \cos x \cos\phi$ where $\sin\phi = \sqrt{\frac{3}{5}}$, $\cos\phi = \sqrt{\frac{2}{5}}$

$= \cos(x - \phi) \le 1$

For option (C)

$\cos^6 x + \sin^6 x = (\cos^2 x)^3 + (\sin^2 x)^3$

$= 1 - 3\sin^2 x \cos^2 x = 1 - \frac{3}{4}(\sin 2x)^2 \le 1$

For option (D)

$$\cos^2 x + \sin^4 x = 1 - \frac{(\sin 2x)^2}{4} \le 1$$

15. $\cos 4x \cos 8x - \cos 5x \cos 9x = 0$

$\Rightarrow 2\cos 4x \cos 8x = 2\cos 5x \cos 9x$

$\Rightarrow \cos 12x + \cos 4x = \cos 14x + \cos 4x$

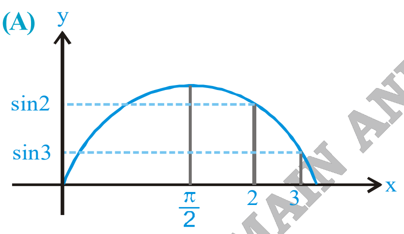
$\Rightarrow 14x = 2n\pi \pm (12x)$

$\Rightarrow 2x = 2n\pi$ or $26x = 2n\pi$

$\Rightarrow x = n\pi$ or $\frac{n\pi}{13}$

$\therefore \sin x = 0$ or $\sin 13x = 0$

16. (A)



17. (C)

Statement-I $7\cos x + 5\sin x = 2\lambda + 1$

$|2\lambda + 1| \le \sqrt{49 + 25}$

$\Rightarrow |2\lambda + 1| \le \sqrt{74}$ $\quad\Rightarrow -\sqrt{74} \le 2\lambda + 1 \le \sqrt{74}$

$\Rightarrow -8.6 \le 2\lambda + 1 \le 8.6,$ $\quad\Rightarrow -9.6 \le 2\lambda \le 7.6$

$\Rightarrow -4.8 \le \lambda \le 3.8$

$\Rightarrow \lambda$ **Integer** $= -4, -3, -2, -1, 0, 1, 2, 3$

Statement-II $a\cos\theta + b\sin\theta = c$ has no solution if $|c| > \sqrt{a^2 + b^2}$

18. From reason (R)

$$\prod_{i=1}^{n} \cos\alpha_i = \prod_{i=1}^{n} \sin\alpha_i$$

$$\Rightarrow \prod_{i=1}^{n} \cos^2\alpha_i = \prod_{i=1}^{n}\left(\frac{\sin 2\alpha_i}{2}\right) \qquad \text{.......(i)}$$

Now, $0 \le \alpha_i \le \frac{\pi}{2}$

$0 \le 2\alpha_i \le \pi$

then maximum value of $\sin 2\alpha_i$ is 1 for all i

$$\therefore \prod_{i=1}^{n} \cos^2\alpha_i \le \frac{1}{2^n}$$

$$\therefore \prod_{i=1}^{n} \cos\alpha_i \le \frac{1}{2^{n/2}}$$

19. $\sin\theta + \operatorname{cosec}\theta = 2$

$$\Rightarrow \frac{\sin\theta + \operatorname{cosec}\theta}{2} = 1$$

and $\sin\theta \operatorname{cosec}\theta = 1$

$\Rightarrow \sqrt{\sin\theta \operatorname{cosec}\theta} = 1$

i.e., AM = GM

$\Rightarrow \sin\theta = \operatorname{cosec}\theta = 1,$

then $\sin^n\theta + \operatorname{cosec}^n\theta = 1^n + 1^n = 2$

20. (D)

Statement-II $\alpha + \beta = \frac{\pi}{2} - \gamma$

$$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{1}{\tan\gamma}$$

$\Rightarrow \sum \tan\alpha\tan\beta = 1$

$\therefore$ Statement-2 is true.

Statement-I $\tan\alpha\tan\beta = \frac{a!}{6}$,

$\tan\beta\tan\gamma = \frac{b!}{2}$ and $\tan\alpha\tan\gamma = \frac{c!}{3}$

$$\frac{a!}{6} + \frac{b!}{2} + \frac{c!}{3} = 1$$

$\Rightarrow a! = 1 \quad b! = 1 \quad c! = 1$

$\Rightarrow \tan\alpha\tan\beta, \tan\gamma\tan\alpha$ and $\tan\beta\tan\gamma$ are not in A.P.

$\therefore$ **Statement-I** is false

21. (A) → (r), (B) → (p), (C) → (p), (D) → (q)

(A) $|\cot x| = \cot x + \dfrac{1}{\sin x}$

If $0 < x < \dfrac{\pi}{2} \Rightarrow \cot x > 0$

so $\cot x = \cot x + \dfrac{1}{\sin x} \Rightarrow \dfrac{1}{\sin x} = 0$ no solution

If $\dfrac{\pi}{2} < \cot x < \pi,\quad -\cot x = \cot x + \dfrac{1}{\sin x}$

$\Rightarrow \dfrac{2\cos x}{\sin x} + \dfrac{1}{\sin x} = 0$

$\Rightarrow 1 + 2\cos x = 0$ and $\sin x \neq 0 \Rightarrow x = \dfrac{2\pi}{3}$

(B) since $\sin\phi + \sin\theta = \dfrac{1}{2}$ ....**(i)**

and $\cos\theta + \cos\phi = 2$ ....**(ii)**

(ii) is true only if $\theta = \phi = 0$ or 2π but $\theta = \phi = 0$ or 2π do not satisfy **(i)**

Hence given system of equation has no solution.

(C) $\sin^2\alpha + \sin\left(\dfrac{\pi}{3} - \alpha\right) . \sin\left(\dfrac{\pi}{3} + \alpha\right)$

$= \sin^2\alpha + \sin^2\dfrac{\pi}{3} - \sin^2\alpha = \dfrac{3}{4}$

(D) $\tan\theta = 3\tan\phi$

$\tan(\theta - \phi) = \dfrac{\tan\theta - \tan\varphi}{1 + \tan\theta\tan\varphi} = \dfrac{2\tan\varphi}{1 + 3\tan^2\varphi}$

$= \dfrac{2}{\cot\varphi + 3\tan\varphi}$ Max if $\tan\phi > 0$

$\dfrac{\cot\varphi + 3\tan\varphi}{2} \geq \sqrt{3}$ (using AM ≥ GM)

$\Rightarrow (\cot\phi + 3\tan\phi)^2 \geq 12$

$\Rightarrow \left[\dfrac{2}{\tan(\theta - \varphi)}\right]^2 \geq 12$

$\Rightarrow \tan^2(\theta - \phi) \leq \dfrac{1}{3}$

22. (A) Let $y = \dfrac{7 + 6\tan\theta - \tan^2\theta}{(1 + \tan^2\theta)}$

$= 7\cos^2\theta + 6\sin\theta\cos\theta - \sin^2\theta$

$= 7\left(\dfrac{1 + \cos 2\theta}{2}\right) + 3\sin 2\theta - \left(\dfrac{1 - \cos 2\theta}{2}\right)$

$= 3\sin 2\theta + 4\cos 2\theta + 3$

$-\sqrt{(3^2 + 4^2)} + 3 \leq 3\sin 2\theta + 4\cos 2\theta + 3 \leq \sqrt{(3^2 + 4^2)} + 3$

$\therefore -2 \leq y \leq 8$

$\Rightarrow \lambda = 8, \mu = -2$

$\Rightarrow \lambda + \mu = 6, \lambda - \mu = 10$ (R, S)

(B) Let $y = 5\cos\theta + 3\cos(\theta + \pi/3) + 3$

$= 5\cos\theta + 3\left(\dfrac{1}{2}\cos\theta - \dfrac{\sqrt{3}}{2}\sin\theta\right) + 3$

$= \dfrac{13}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta + 3$

$\therefore 3 - \sqrt{\left(\dfrac{13}{2}\right)^2 + \left(\dfrac{-3\sqrt{3}}{2}\right)^2} \leq \dfrac{13}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin + 3$

$\leq 3 + \sqrt{\left(\dfrac{13}{2}\right)^2 + \left(\dfrac{-3\sqrt{3}}{2}\right)^2}$

$\Rightarrow 3 - 7 \leq y \leq 3 + 7$

$\Rightarrow -4 \leq y \leq 10$

$\therefore \lambda = 10, \mu = -4$

$\Rightarrow \lambda + \mu = 6, \lambda - \mu = 14$ (R, T)

(C) Let $y = 1 + \sin\left(\dfrac{\pi}{4} + \theta\right) + 2\cos\left(\dfrac{\pi}{4} - \theta\right)$

$= 1 + \cos\left(\dfrac{\pi}{2} - \left(\dfrac{\pi}{4} + \theta\right)\right) + 2\cos\left(\dfrac{\pi}{4} - \theta\right)$

$= 1 + \cos\left(\dfrac{\pi}{4} - \theta\right) + 2\cos\left(\dfrac{\pi}{4} - \theta\right)$

$= 1 + 3\cos\left(\dfrac{\pi}{4} - \theta\right)$

$\therefore \quad -1 \le \cos\left(\frac{\pi}{4}-\theta\right) \le 1$

$\Rightarrow \quad -3 \le 3\cos\left(\frac{\pi}{4}-\theta\right) \le 3$

$\Rightarrow \quad 1-3 \le 1+3\cos\left(\frac{\pi}{4}-\theta\right) \le 1+3$

$\therefore \quad -2 \le y \le 4$

$\Rightarrow \quad \lambda = 4, \mu = -2$

$\therefore \quad \lambda + \mu = 2, \lambda - \mu = 6$ (P, Q)

Comprehension Type

23. $P_0 = 2, P_2 = 1$

and $P_n - P_{n-2} = (\sin^n\theta + \cos^n\theta) - (\sin^{n-2}\theta + \cos^{n-2}\theta)$

$= -\sin^{n-2}\theta(1-\sin^2\theta) - \cos^{n-2}\theta(1-\cos^2\theta)$

$= -\sin^{n-2}\theta\cos^2\theta - \cos^{n-2}\theta\sin^2\theta$

$= -\sin^2\theta\cos^2\theta(\sin^{n-4}\theta + \cos^{n-4}\theta)$

$= -\sin^2\theta\cos^2\theta\, P_{n-4}$

$\therefore \quad P_n - P_{n-2} = -\sin^2\theta\cos^2\theta\, P_{n-4}$ ....(i)

for $n = 4$,

$P_4 - P_2 = -\sin^2\theta\cos^2\theta\, P_0$

$\Rightarrow \quad P_4 - 1 = -2\sin^2\theta\cos^2\theta$ $(\because P_2 = 1, P_0 = 2)$

$\therefore \quad P_4 = 1 - 2\sin^2\theta\cos^2\theta$ ....(ii)

for $n = 6$,

$\Rightarrow \quad P_6 - P_4 = -\sin^2\theta\cos^2\theta P_2$

$P_6 = P_4 - \sin^2\theta\cos^2\theta P_2$

$= 1 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$

$\therefore \quad P_6 = 1 - 3\sin^2\theta\cos^2\theta$ ....(iii)

1 $P_1 = m$

$\Rightarrow \quad P_1^2 = m^2$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = m^2$

$\Rightarrow \quad \sin\theta\cos\theta = \frac{(m^2-1)}{2}$

Now, from eq. (iii),

$P_6 = 1 - 3\sin^2\theta\cos^2\theta$

$\Rightarrow \quad (1-P_6) = 3(\sin\theta\cos\theta)^2$

$= \frac{3(m^2-1)^2}{4}$

or $\quad 4(1-P_6) = 3(m^2-1)^2$

2. $2P_6 - 3P_4 + 10$

$= 2(1-3\sin^2\theta\cos^2\theta) - 3(1-2\sin^2\theta\cos^2\theta) + 10$

(from eq. (ii) and (iii))

$= 2 - 3 + 10$

$= 9$

3. Let $\sin^2\theta\cos^2\theta = k$,

then from eq. (i),

$P_n - P_{n-2} = -k\,P_{n-4}$

from eq. (ii),

$P_4 = 1 - 2k$

and from eq. (iii)

$P_6 = 1 - 3k$

Put $n = 10$,

then $\quad P_{10} - P_8 = -k\,P_6 = -k(1-3k)$

$\therefore \quad P_{10} - P_8 = 3k^2 - k$ ....(iv)

and put $n = 8$,

then $\quad P_8 - P_6 = -k\,P_4 = -k(1-2k)$

$\therefore \quad P_8 = P_6 + 2k^2 - k$

$= 1 - 3k + 2k^2 - k$

$\Rightarrow \quad P_8 = 2k^2 - 4k + 1$

from eq. (iv), $P_{10} = 5k^2 - 5k + 1$

$\therefore \quad 6P_{10} - 15P_8 + 10P_6 + 7$

$= 6(5k^2 - 5k + 1) - 15(2k^2 - 4k + 1)$

$+ 10(1-3k) + 7$

$= 8$

24.

1. (D)

Angle subtended by two consecutive marks at centre = 30°

Hence at "half past 4", the angle is 45°

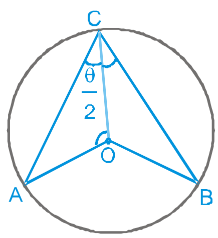
2. (A)

Distance covered in 1 second $= 5\left(2\pi\ .\frac{1}{2}\right) = 5\pi m$

Distance covered in 1 hour $= \frac{5\pi}{1000} \times 60 \times 60\text{ km} = 56.52\text{ km}$

3. (A)

Area of region ABC $= \frac{\pi r^2}{3}$



Area of OAB $= \frac{1}{2} r^2 . 2\theta = r^2 \theta$

Area of $\Delta OAC = \frac{1}{2} r^2 \sin\theta$ = Area of ΔOBC

$\therefore \quad \frac{1}{2} r^2 \sin\theta + \frac{1}{2} r^2 \sin\theta + r^2\theta = \frac{\pi r^2}{3}$

$\Rightarrow \quad 3\sin\theta + 3\theta = \pi$

25.

1. (C)

$\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14} = \sin\frac{5\pi}{14}\sin\frac{3\pi}{14}.\sin\frac{\pi}{14}$

$= \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7} = -\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

$= -\frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}} = \frac{1}{8}$

2. (B)

$\cos 2^3\frac{\pi}{10}\cos 2^4\frac{\pi}{10}.\cos 2^5\frac{\pi}{10}\,.........\,\cos 2^{10}\frac{\pi}{10}$

$= \frac{\sin 2^{11}\frac{\pi}{10}}{256\sin 2^3\frac{\pi}{10}} = \frac{1}{256}$

3. (C)

$\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\,.........\,\cos\frac{11\pi}{11}$

$= \left(\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}\right)^2$

$= \left(\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{4\pi}{11}\cos\frac{8\pi}{11}\cos\frac{5\pi}{11}\right)^2$

$= \left(\frac{\sin 16\frac{\pi}{11}}{16\sin\frac{\pi}{11}}.\cos\frac{5\pi}{11}\right)^2 = \left(\frac{2\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}}{32\sin\frac{\pi}{11}}\right)^2 = \frac{1}{1024}$

26. (4)

L.H.S. $= 2\sin 5(A+B)\cos 5(A-B) + 2\sin 5C\cos 5C$.

$= 2\sin(5\pi - 5C)\cos 5(A-B) + 2\sin 5C\cos 5C$

$= 2\sin 5C\,[\cos(5A-5B) + \cos 5C]$

$= 2\sin 5C\,[\cos(5A-5B) + \cos 5(\pi - \overline{A+B})]$

$= 2\sin 5C\,[\cos(5A-5B) - \cos(5A+5B)]$

$= 4\sin 5A\sin 5B\sin 5C$ = R.H.S.

27. Let $\theta = \frac{\pi}{16}$; $8\theta = \frac{\pi}{2}$

$\therefore \quad y = \tan\theta + \tan 5\theta + \tan 9\theta + \tan 13\theta$

$\therefore \quad y = (\tan\theta - \cot\theta) + (\tan 5\theta - \cot 5\theta)$

[$\tan 13\theta = \tan(8\theta + 5\theta) = -\cot 5\theta$; $\tan 9\theta = \tan(8\theta + \theta) = -\cot\theta$]

$= (\tan\theta - \cot\theta) + (\cot 3\theta - \tan 3\theta)$

$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2 3\theta - \sin^2 3\theta}{\sin 3\theta\cos 3\theta}$

$y = 2\left[\frac{\cos 6\theta}{\sin 6\theta} - \frac{\cos 2\theta}{\sin 2\theta}\right]$

$= 2\left[\frac{\sin 2\theta\cos 6\theta - \cos 2\theta\sin 6\theta}{\sin 6\theta\sin 2\theta}\right] = -2\cdot\frac{\sin 4\theta}{\cos 2\theta\sin 2\theta}$

$= -4$

Hence absolute value = 4

28. $\frac{\tan\alpha + \tan\gamma}{1 + \tan\alpha\tan\gamma} = \frac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}$

$\therefore \sin 2\beta = \frac{2\tan\beta}{1+\tan^2\beta} = \frac{2\frac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}}{1+\frac{\sin^2(\alpha+\gamma)}{\cos^2(\alpha-\gamma)}}$

$= \frac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{\cos^2(\alpha-\gamma) + \sin^2(\alpha+\gamma)}$

$= \frac{\sin 2\alpha + \sin 2\gamma}{\frac{1+\cos 2(\alpha-\gamma)}{2} + \frac{1-\cos 2(\alpha+\gamma)}{2}}$

$= \frac{\sin 2\alpha + \sin 2\gamma}{1 + \frac{1}{2}(\cos 2(\alpha-\gamma) - \cos 2(\alpha+\gamma))}$

$= \frac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha\sin 2\gamma}$

= R.H.S.

29. 6

30. (9)

We have, $f(\theta) = \frac{(\sin\theta)^x}{(\cos\theta)^x + (\sin\theta)^x} \Rightarrow f(\theta) + f\left(\frac{\pi}{2} - \theta\right) = 1.$

$\therefore \quad S = \sum_{\theta=1^\circ}^{89^\circ} f(\theta) = f(1^\circ) + f(2^\circ) + ... + f(88^\circ) + f(89^\circ)$

$= \left(\underbrace{1+1+1+.....+1}_{44\text{ times}}\right) + \frac{1}{2} = 44 + \frac{1}{2} = \frac{89}{2}$

$\therefore \quad 2S - 8 = 81$