NUMBER THEORY

SQUARES AND SQUARE ROOTS

What is special about the numbers 4, 9, 25, 64 and other such numbers?
Since, 4 can be expressed as 2 × 2 = 22, 9 can be expressed as 3 × 3 = 32, all such numbers can be expressed as the product of the number with itself.
such numbers like 1, 4, 9, 16, 25, ... are known as square numbers.

Square number :

A rational number *m* can be expressed as n^2, where *n* is also a rational, then m is a square number.

Is 32 a square number?

We know that $5^2 = 25$ and $6^2 = 36$. If 32 is a square number, it must be the square of a natural number between 5 and 6. But there is no natural number between 5 and 6.Therefore 32 is not a square number.

Perfect squares :

A rational number that is equal to the square of another rational number.

Ex : $9 = 3 \text{ X } 3 = 3^{2.}$

3,5,6,7 are not a perfect squares as it cannot be expressed as the product of two equal factors.

Properties of Perfect squares :

1. The number which cannot be written as square of any other rational number is called non-perfect squares.
2. All primes are not perfect squares.
3. Composite numbers may or may not perfect squares.
4. Every perfect square is a composite number.
5. The square of an even number is always an even number.
6. The square of an odd number is always an odd number.
7. The square of integers end with the digits 1,4,5,6,9,0 and not with 2,3,7,8.
8. The number of zero's at the end of square is always even.
9. The numbers ending with an even number of zeros may or may not be a perfect square.
10. For every natural number n, $(n+1)^2 - n^2 = (n+1) + n$ i.e., the difference of squares of two consecutive numbers is equal to their sum.
11. The square of a natural number is equal to the sum of first 'n' odd natural numbers.
12. If 1 is added to the product of two consecutive odd natural numbers, it is

equal to the square of the even natural number between them.

Eg: $3 \times 5 + 1 = 16 = 4^2$

$7 \times 9 + 1 = 64 = 8^2$

13. If 1 is added to the product of two consecutive even natural numbers, it is equal to the square of the odd natural number between them.

Eg: $4 \times 6 + 1 = 25 = 5^2$

$8 \times 10 + 1 = 81 = 9^2$

14. The square of any natural number 'n' can be expressed as the sum of two consecutive natural numbers, $\left(\frac{n^2-1}{2}+\frac{n^2+1}{2}\right)$

Eg: $52 = 12 + 13 = \left(\frac{5^2-1}{2}+\frac{5^2+1}{2}\right)$

Palindrome : A palindrome is a word; phrase, a sentence or numerical that reads the same forward or backward.

Ex : NOON ,MALAYALAM,MADAM,15651

Square Root : It is the inverse operation of square.

let the number be n, then the square root of n is that number which when multiplied by itself gives n as the product.

Ex : $\sqrt{4} = 2$

Properties of a square roots:

1. If the units digit of number is 2,3,7 or 8 then it does not have a square root in natural numers.
2. If a number ends in an odd numer of zero's then it does not have a square root in natural numbers.
3. The square root on a even square number is even and that square root of an odd square number is odd.
4. If a number has a square root in natural number then its units digit must be 0,1,4,5,6 or 9.
5. i) $\sqrt{ab} = \sqrt{a}\ X\ \sqrt{b}\ \ (a>0, b>0)$

   ii) $\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$ iii) $\sqrt{a-b} \neq \sqrt{a}-\sqrt{b}$

   iv) $\sqrt{a^2+b^2} \neq a+b$ v) $\sqrt{a^2-b^2} \neq a-b$ vi) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}(a>0, b>0)$

Pythageorean triplet :

A triplet (a,b,c) of three numbers a,b and c is called a pythagorean triplet if

$$a^2 + b^2 = c^2$$

Methods :

There are three methods to find the square root of a number. They are

i) Subtraction of successive odd numbers

ii) Prime factorisation method

iii) Division method

Finding square root through repeated subtraction

We remember that the sum of the first n odd natural numbers is n^2 . Every square number can be expressed as a sum of successive odd natural numbers starts from 1.

Consider 81 . Then,

(i) 81 – 1 = 80 (ii) 80 – 3 = 77 (iii) 77 – 5 = 72 (iv) 72 – 7 = 65

(v) 65 – 9 = 56 (vi) 56 – 11 = 45 (vii) 45 – 13 = 32 (viii) 32 – 15 = 17

(ix) 17 – 17 = 0

From 81 we have subtracted successive odd numbers starting from 1 and obtained 0 at 9th step. Therefore $\sqrt{81}$ = 9.

Finding square root through prime factorisation

Consider the prime factorisation of the following numbers and their squares.

Prime factorisation of a Number	Prime factorisation of its Square
$6 = 2 \times 3$	$36 = 2 \times 2 \times 3 \times 3$
$8 = 2 \times 2 \times 2$	$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$12 = 2 \times 2 \times 3$	$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$15 = 3 \times 5$	$225 = 3 \times 3 \times 5 \times 5$

How many times does 2 occur in the prime factorisation of 6? Once.

How many times does 2 occur in the prime factorisation of 36? Twice.

Similarly, observe the occurrence of 3 in 6 and 36 of 2 in 8 and 64 etc.

You will find that each prime factor in the prime factorisation of the square of a number, occurs twice the number of times it occurs in the prime factorisation of the number itself. Let us use this to find the square root of a given square number, say 324.

2	324
2	162
3	81
3	27
3	9
	3

we know that the prime factorisation of324 is 324 = 2 x 2 x 3 x 3 x 3 x 3

Finding square root by division method :

• Consider the following steps to find the square root of 529.

Step 1 Place a bar over every pair of digits starting from the digit at one's place.

If the number of digits in it is odd, then the left-most single digit too will have a bar.

Thus we have, 5 29 .

Step 2 Find the largest number whose square is less than or equal to the number under the extreme left bar ($22 < 5 < 32$). Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend(here 5). Divide and get the remainder (1 in this case).

Step 3 Bring down the number under the next bar (i.e., 29 in this case) to the right of the remainder. So the new dividend is 129.

Step 4 Double the divisor and enter it with a blank on its right.

	2
2	5 29
	- 4
4	1 29

Step 5 Guess a largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new quotient the product is less than or equal to the dividend.

In this case $42 \times 2 = 84$. As $43 \times 3 = 129$ so we choose the new digit as 3.

Get the remainder.

	23
2	5 29
	- 4
43	1 29
	-129
	0

Step 6 Since the remainder is 0 and no digits are left in the given number, therefore, $\sqrt{529} = 23$

TEACHING TASK

I. MCQ's with single correct answers.

1. $\sqrt{\frac{25}{81}-\frac{1}{9}}=...$

   A) $\frac{2}{3}$ B) $\frac{4}{9}$ C) $\frac{16}{81}$ D) $\frac{25}{81}$

2. If $\frac{52}{x}=\sqrt{\frac{169}{289}}$, the value of x is....

   A) 52 B) 58 C) 62 D) 68

3. If $\sqrt{18x14x\,a}=84$, then a =...

   A) 22 B) 24 C) 28 D) 32

4. $\sqrt{\frac{x}{169}}=\frac{54}{39}$ then the value of x is...

   A) 108 B) 324 C) 2916 D) 4800

5. If $\sqrt{1+\frac{x}{169}}=\frac{14}{13}$, then x =....

   A) 1 B) 13 C) 27 D) None

II Solve the following:

1. What will be the unit digit of the squares of the following numbers?
   (i) 81 (ii) 1234 (iii) 52698 (iv)26387
2. Without adding, find the sum.
   (i) 1 + 3 + 5 + 7 + 9 (ii) 1 + 3 + 5 + 7 + 9 + I1 + 13 + 15 + 17 +19
   (iii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23
3. Find the squares of the following numbers
   (i)36 (ii)45 (iii)27 (iv)60
4. What could be the possible 'ones' digits of the square root of each of the following numbers?
   (i) 4761 (ii) 99856 (iii) 178929 (iv) 657666025
5. Find the square roots of 100 and 169 by the method of repeated subtraction.
6. Find the square roots of the following numbers by the Prime Factorisation Method.
   (i) 729 (ii) 1936 (iii) 4624 (iv) 7921 (v) 7744
7. Find the square root of each of the following numbers by Division method.
   (i) 2401 (ii) 4489 (iii) 3481 (iv) 7.29 (v) 31.36

8. Area of a square plot is 2916 m^2. Find the side of the square plot.
9. Find the least number of four digits number which is a perfect squares.
10. The figure is made up of squares of the same size.The area of the figure is 324 square meters. Find the side length of the squares.

LEARNER TASK

Beginners (Level - I)

I MCQ with single correct answers:

1. Which of the following number is a perfect square?
   A) 136 B) 121 C) 234 D) 240
2. Which of the following number is not a perfect square?
   A) 625 B) 1024 C) 1369 D) 2161
3. How many two-digit numbers satisfy this property : The last digit (unit's digit) of the square of the two - digit number is 8.
   A) 1 B) 0 C) 3 D) None
4. What will be the possible units digits for the square number?
   A) 2 B) 9 C) 7 D) 8
5. The value of $5^2 + 6^2 + 30^2$ is...
   A) 25^2 B) 30^2 C) 31^2 D) 38^2
6. Which of the following are consecutive square numbers?
   A) p^2 , $(p+2)^2$ B) $(p+1)^2$, $(p+3)^2$ C) p^2 , $(p+1)^2$ D) $(p+1)^2$, $(p+3)^2$
7. 1^2, 11^2 , 111^2 , 1111^2 , ... are the examples for...
   A) square root B) cube root C) palindrome D) perfect numbers
8. 370 , 5000 , 1000 are all....
   A) perfect squares B) non-perfect squares C) square root D) none
9. Which of the following have 6 in units place....
   A) 144^2 B) 251^2 C) 118^2 D) 299^2
10. The square of a natural number n is equal to the sum of first n ..... numbers.
    A) even natural B) odd natural C) natural D) none

Achievers (Level II)

MCQ with one or more than one correct answers.

1. Which of the following end with digit 1?
   A) 161^2 B) 77^2 C) 109^2 D) 123^2
2. Which of the following is a pythagorean triplet?
   A) $(2p, p^2-1, p^2+1)$ B) (3,4,5) C) (5,12,13) D) (8,15,17)
3. Which of the followign is a palindrome?
   A) 1234321 B) 11111 C) 1231 D) 10000

4. Non-perfect square numbers generally ends with...
   A) 2 B) 3 C) 8 D) 7

5. $\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)^2$ simplifies to...

   A) $\frac{4}{3}$ B) $\frac{4}{\sqrt{3}}$ C) $\frac{12}{3\sqrt{3}}$ D) $\frac{\sqrt{48}}{3\sqrt{3}}$

6. What should be divided to $\sqrt{27}$ to make it a natural number?

   A) $\sqrt{3}$ B) $\sqrt{27}$ C) $3\sqrt{3}$ D) 3

7. The smallest number by which 5808 should be multiplied so that the product becomes a perfect square is...
   A) 6 B) 30 C) 4 D) 3

8. Match the following.

Column - I		***Column-II***
a. $1^2+2^2+3^2$=...	[ ]	1) 49
b. $2^2+3^2+6^2$=...	[ ]	2) 13^2
c. $3^2+4^2+12^2$ = ...	[ ]	3) 7^2
d. $4^2+5^2+20^2$ = ...	[ ]	4) 3^2
		5) 21^2

   A) a-4, b-1, c-2, d-5 B) a-4, b-3, c-2, d-5
   C) a-3, b-1, c-5, d-2 D)a-3, b-, c-4, d-5

Explorers (Level -III)

Solve the following.

1. Find the least perfect square with four digits.
2. Find the smallest number which must be added to 2300 so that it becomes a perfect square.
3. A gardenar wishes to plant 6096 plants in the form of a square and found that there were 12 plants left. How many plants were planted in each row.
4. Find the smallest number by which 3645 must be multiplied to get a perfect square.
5. Find the smallest number by which 6000 should be divided to get a perfect square.
6. A man plants 15376 apple trees in his garden and arranges them so that there are as many rows as there are apple trees in each row. Then find the number of rows are?
7. Find the square roots of the following numbers by division method.
   i) 286225 ii) 44521 iii) 29929 iv) 3136
8. Using prime factorization method, find the square root of
   i) 15876 ii) 32400 iii) 19044 iv) 5184

9. Find the square root of $\frac{256}{441}$, $\frac{625}{1296}$

10. Find the length of a side of a square playground whose area is equal to the area of rectangular field of dimensions 72 m and 338 m.

CUBE AND CUBE ROOTS

Introduction

This is a story about one of India's great mathematical geniuses, S. Ramanujan. Once another famous mathematician Prof. G.H. Hardy came to visit him in a taxi whose number was 1729. While talking to Ramanujan, Hardy described this number "a dull number". Ramanujan quickly pointed out that 1729 was indeed interesting. He said it is the smallest number that can be expressed as a sum of two cubes in two different ways:

$1729 = 1728 + 1 = 12^3 + 1^3$ $1729 = 1000 + 729 = 10^3 + 9^3$

Cubes : A natural number 'a' is called a perfect cubes if there exists another number 'b' such that $a = b \times b \times b = b^3$

In a simple language, if we multiply a number by itself three times , we get the cube of a number.

Squares and Cubes and their unit's digit:

Number(x)	Square(x^2)	Cubes(x^3)
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375
16	256	4096
17	289	4913
18	324	5832
19	381	6859
20	400	8000

Properties of cubes :

1. If a number can be expressed as a product of three equal factors then it is said to be a perfect cube or cubic number.

2. If the number has 1 in the units place, then its cube ends with 1.
3. If the number has 0,4,5,6 or 9 in the units place then its cube ends with 0,4,5,6 or 9.
4. The cube of 2 ends with 8 and cube of 8 ends with 2.
5. The cube of 3 ends with 7 and cube of 7 ends with 3.
6. Cubes of all even natural numbers are even.
7. Cubes of all odd natural numbers are odd.
8. Cube of a negative rational number is always negative.Also $\sqrt[3]{-a} = -\sqrt[3]{a}$
9. Cubes of all odd number are odd.
10. Cubes of all even number are even.
11. Cubes of a natural numbers of the form 3n is a multiple of 27.
12. Cubes of a natural numbers of the form 3n+1 is also number of the form 3n+1.
13. Cubes of a natural numbers of the form 3n+2 is also is in the form 3n+2.
14. Cubes of a number which ends in a zero, ends in three zeros.
15. Cubes of a negative number always ends in negative.

Note : The sum of cubes of first 'n' natural numbers is equal to the square of their sum. i.e., $1^3+2^3+3^3+.....+n^3 = (1+2+3+....n)^2$

Some interesting patterns :

1. Adding consecutive odd numbers Observe the following pattern of sums of odd numbers.

$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

$13 + 15 + 17 + 19 = 64 = 4^3$

$21 + 23 + 25 + 27 + 29 = 125 = 5^3$

How many consecutive odd numbers will be needed to obtain the sum as 103?

2. Cubes and their prime factors

Consider the following prime factorisation of the numbers and their cubes.

Prime factorisation	Prime factorisation of a number of its cube
$4 = 2 \times 2$	$4^3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3$
$6 = 2 \times 3$	$6^3 = 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$
$15 = 3 \times 5$	$15^3 = 3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3^3 \times 5^3$
$12 = 2 \times 2 \times 3$	$12^3 = 1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 2^3 \times 3^3$

Cube root :

Finding the square root, as you know, is the inverse operation of squaring. Similarly, finding the cube root is the inverse operation of finding cube.

We know that $2^3 = 8$; so we say that the cube root of 8 is 2.

We write $\sqrt[3]{8} = 2$. The symbol $\sqrt[3]{\ }$ denotes 'cube-root.'

TEACHING TASK

I. MCQ's with single correct answers.

1. $\sqrt[3]{-17576} = ...$

   A) -24 B) -26 C) -16 D) -36

2. What will be the unit digit of the cube of the 27?

   A) 2 B) 7 C) 3 D) 9

3. $\sqrt[3]{\frac{8}{125000}} = ...$

   A) 0.4 B) 0.04 C) 0.8 D) 0.6

4. The value of $1^3+2^3+3^3+4^3$ =

   A) 50 B) 100 C) 150 D) 75

5. $\sqrt[3]{4\frac{12}{125}} = ...$

   A) $\frac{1}{5}$ B) $\frac{1}{8}$ C) $\frac{8}{5}$ D) $\frac{5}{8}$

II. Solve the following.

1. Consider the following pattern.

   $2^3 - 1^3 = 1 + 2 \times 1 \times 3$

   $3^3 - 2^3 = 1 + 3 \times 2 \times 3$

   $4^3 - 3^3 = 1 + 4 \times 3 \times 3$

   Using the above pattern, find the value of the following.

   (i) $7^3 - 6^3$ (ii) $12^3 - 11^3$ (iii) $20^3 - 19^3$ (iv) $51^3 - 50^3$

2. Is 392 a perfect cube? If not, find the smallest natural number by which 392 must be multiplied so that the product is a perfect cube.

3. Which of the following numbers are not perfect cubes?

   (i) 216 (ii) 512 (iii) 8000 (iv) 256 (v) 4665

4. Find the one's digit of $(9)^3$.

5. Find the cube root of each of the following numbers by prime factorisation method.

   (i) 512 (ii) 15625 (iii) 13824

   (iv) 19683 (v)32768 (vi)125000

6. Find the smallest number by which each of the following numbers must be multiplied

   to obtain a perfect cube.

   (i) 243 (ii) 256 (iii) 72 (iv) 675

7. Can we find the cube root of (-10648) ? If yes find the integer.

8. Find the value of $\sqrt[3]{392} X \sqrt[3]{448}$

9. The volume of cubical box is 46656 cubic meters. Find the length of each side of the box.

LEARNER Task:

Beginners (Level-I)

MCQ's with single correct answers.

1. How many perfect cubes are present between 1 to 500...
   A) 5 B) 6 C) 7 D) 8
2. If $2^x = \sqrt[3]{32}$ then x=...
   A) $\frac{5}{2}$ B) $\frac{5}{3}$ C) $\frac{125}{75}$ D) $\frac{3}{5}$
3. Which of the following one is a perfect cube?
   A) 225 B) 900 C) 27000 D) 3025
4. What will be the unit digit of the cube pf the 27?
   A) 2 B) 7 C) 3 D) 9
5. The cube of 2013 ends with (i.e units digit is)
   A) 3 B) 9 C) 1 D) 7
6. The digit in the unit place of 13^3+25^3 is ...
   A) 7 B) 2 C) 5 D) 3

Achievers (Level-II)

MCQs with one or more than one correct answer.

1. Which of the following are perfect cubes?
   A) 1331 B) 512 C) 2500 D) 216
2. Which of the following are not perfect cubes?
   A) 700 B) 516 C) 1000 D) 729
3. The value of $1^3+2^3+3^3$ is...
   A) $(1+2+3)^3$ B) $(1+2+3)^2$ C) 36 D) 216
4. If the number has 0,4,5,6 in the units place, then its cube ends with...
   A) 0 B) 4 C) 5 D) 6
5. What is the smallest number by which 1323 is to multipled so that the product is a perfect cube?
   A) 7 B) 49 C) $\sqrt[3]{343}$ D) $\sqrt{49}$
6. **Matching:**

	Column - I		**Column - II**
a.	By what least number 675 be multiplied to obtain a number which is a perfect cube	[ ]	1) 17
b.	What is the smallest number by which 8640 must be divided so that the quotient is a perfect cube	[ ]	2) 450

c. What smallest number should 7803 be multiplied with so that the produt becomes a perfect cube. [ ] 3) 5

d. What is the smallest number by which 3600 be divided to make it a perfect cube [ ] 4) 45 x 10

5) $(4913)^{\frac{1}{3}}$

A) a-3,b-3,c-1,d-2 B) a-3,b-3,c-5,d-4
C) a-3,b-1,c-5,d-2 D)a-3,b-1,c-4,d-5

7. Comprehension questions:
Consider the following pattern

$2^3 - 1^3 = 1 + 2x1x3 = 7$

$3^3 - 2^3 = 1 + 3x2x3 = 19$

$4^3 - 3^3 = 1 + 4x3x3 = 37$ *and so on...*

i) The value of 10^3-9^3 by using above pattern is...
A) 281 B) 271 C) 217 D) 218

ii) The value of 15^3-14^3 by using above pattern is...
A) 631 B) 613 C) 531 D) 513

iii) The value of 26^3-25^3 by using the above pattern is...
A) 1981 B) 1951 C) 1915 D) 1918

iv) Sum of the values of the above three questions is...
A) 2835 B) 2358 C) 2853 D) 2538

Explorers (Level - III)

Solve the following.

1. By what least number should 288 be multiplied to get a perfect cube? What is this perfect cube? Find its cube root?
2. By what least number should 1029 be divided to get a perfect cube? What is this perfect cube? Find its cube root?
3. Find the cube root of
i) 3375 ii) 13824 iii) 91125 iv) 9261
4. Evaluate $\sqrt[3]{1372}X\sqrt[3]{1458}$ and $\sqrt[3]{\frac{125}{216}}$
5. Find the cube root of 1728 , 4096 through prime factorization method.

Researchers (*Level - IV*)

1. The denominator of a fraction is 3 more than the numerator. If the numerator as

well as the denominator is increased by 4, the fractions becomes $\frac{4}{5}$. What was the orginal fraction. **[SBIPO'S-99]**

A) $\frac{8}{11}$ B) $\frac{5}{8}$ C) $\frac{10}{13}$ D) $\frac{7}{10}$

2. The value of $\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{5}}}}$ **[AMTI-2007]**

A) $\frac{77}{60}$ B) $\frac{68}{157}$ C) $\frac{2007}{2008}$ D) None

3. If P=2x3x5x7 and Q=11x13x17x19. What is the integer part of $\frac{Q}{P}$? **[Ramanujan-14]**

A) 220 B) 219 C) 218 D) 217

4. It is given that $5\frac{3}{a} X\, b\frac{1}{2} = 19$ (where the two fractions are mixed fractions) then a+b=... **[AMTI-07]**

A) 10 B) 12 C) 9 D) 15

5. If 1.5a=0.04y then the value of $\left(\frac{b-a}{b+a}\right)$ is... **[NSO-12]**

A) $\frac{730}{770}$ B) $\frac{73}{77}$ C) $\frac{7.3}{7.7}$ D) $\frac{730}{77}$

6. Express $0.\overline{001}$ as a fraction in simplest form. **[AMT-10]**

A) $\frac{9}{999}$ B) $\frac{111}{999}$ C) $\frac{1}{999}$ D) $\frac{11}{999}$

7. Sum of $\frac{0.1}{0.01}+\frac{0.01}{0.1}+\frac{0.001}{0.01}+\frac{0.01}{0.001}$ is... **[APAMT-10]**

A) 2.2 B) 22 C) 20.2 D) 11.11

8. The value of $\sqrt{53824}$ =...

A) 202 B) 232 C) 242 D) 252

9. a x b=a+b+$\sqrt{ab}$, the value of 6 x 24 is...

A) 41 B) 42 C) 43 D) 44

10. What is the square root of 0.16 **[S.S.C-98]**

A) 0.004 B) 0.04 C) 0.4 D) 4

11. $\sqrt{0.00004761}$ =... **[CBI-98]**

A) 0.00069 B) 0.0069 C) 0.0609 D) 0.069

12. If $\sqrt{3^n}$ =729, then the value of n is... **[S.S.C-08]**

A) 6 B) 8 C) 10 D) 12

13. The square root of $\left(7+3\sqrt{5}\right)\left(7-3\sqrt{5}\right)$ is... **[S.S.C-04]**

A) $\sqrt{5}$ B) 2 C) 4 D) $3\sqrt{5}$

14. Given $\sqrt{2}$ =1.414. The value of $\sqrt{8}+2\sqrt{32}-3\sqrt{128}+4\sqrt{50}$ is... **[S.S.C-03]**

A) 8484 B) 8.484 C) $\frac{84.84}{100}$ D) $\frac{8484}{1000}$

15. $\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)^2$ simplifies to...

A) $\frac{4}{3}$ B) $\frac{4}{\sqrt{3}}$ C) $\frac{12}{3\sqrt{3}}$ D) $\frac{\sqrt{48}}{3\sqrt{3}}$

SQUARE ROOT
TEACHINGTASK

I. 1) B 2) D 3) C 4) B 5) C

II. 1) i) 1 ii) 6 iii) 4 iv) 9

2) i) 25 ii)100 iii) 144

3) i) 1296 ii) 2025 iii) 729 iv) 3600

4) i) 1 or 9 ii) 4 or 6 iii) 3 or 7 iv) 5 5) 10 and 13

6) i) 27 ii) 44 iii) 68 iv) 89 v) 88

7) i) 49 ii) 67 iii) 59 iv) 2.7 v) 5.6

8) 54 9) 1024 10) 18

LEARNER'S TASK

Beginners(LEVEL- I) 1) B 2) D 3) B 4) B
5) C 6) C 7) C 8) B 9) A 10) B

Achievers (LEVEL- II) 1) A , C 2) A , B, C, D 3) A , B 4) A , B , C , D
5) A , D 6) A, B, C 7) D 8) A, B

Explorers (LEVEL- III) 1) 1024 2) 4 3) 78 4) 5 5) 20 6)124
7) i) 535 ii) 211 iii) 173 iv) 56

8) i) 126 ii) 180 iii) 138 iv) 72 9) $\frac{16}{21}$ *and* $\frac{25}{36}$

10) 156

CUBE ROOT
TEACHING TASK

I 1) B 2) C 3) B 4) B 5) C

II 1) i) 127 ii) 397 iii) 1141 iv) 7551
2) 7 3) i) 6 ii) 8 iii) 20 iv) not perfect cube
v) not perfect cube
4) 9 5) i) 8 ii) 25 iii) 24 iv) 27 v) 32 vi) 50
6) i) 3 ii) 2 iii) 3 iv) 5
7) -22 8) 56 9) 36

LEARNER'S TASK

Beginners(LEVEL- I) 1) B 2) B 3) C 4) C 5) D 6) B

Achievers (LEVEL- II) 1) A , C, D 2) A , B 3) B 4)
A,B,C,D 5) A,C,D6) A, B 7) i) B ii) A iii) B iv) D

Explorers (LEVEL- III) 1) 6, 1728 , 12 2) 3 , 343 , 7 3) i) 15
ii) 24 iii) 45 iv) 21 4) 126 and $\frac{5}{6}$ 5) 12 and 16

RESEARCHERS (LEVEL-IV)

1) A 2) B 3) B 4) A 5) A,B,C 6) C 7) C 8) B
9) B 10) C 11) B 12) D 13) B 14) B , D
15) A,D

DIVISIBILITY RULES

Tests for divisibility of numbers :

Divisibility by 2

Rule : A number is divisible by 2 if its units digit is 0 or an even digit. i.e 2,4,6,...

Divisibility by 3

Rule : If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

Divisibility by 4

Rule : If the last two digits of a number is divisible by 4, then the number is divisible by 4.

Divisibility by 5

Rule : A number is divisible by 5, it its units digit is 0 or 5.

Divisibility by 6

Rule : If a number is divisible by both 2 and 3,the number is also divisible by 6.

Divisibility by 8

Rule : If the last three digits of a number is divisible by 8, then the number is divisible by 8.

Divisibility by 9

Rule : If the sum of all the digits of a number is divisible by 9, then the number is divisible by 9.

Divisibility by 10

Rule : A number is divisible by 10, if its units digit is zero.

Divisibility by 11

Rule : If the difference of digits of odd and even places is either zero or a multiple of 11, then the number is divisible by 11.

Divisibility by 12

Rule : If a number is divisible by both 4 and 3, then it is divisible by 12.

FACTORS AND MULTIPLES

Factor : A factor of a number is an exact divisor of that number.

In other words, a factor of a number is that number which completely divides number without leaving a remainder.

Ex. 1, 2, 3, 6 divides '6' exactly

$\therefore$ 1, 2, 3, 6 are called factors or divisors of the number '6'

Multiple : "A multiple of a number is a number obtained by multiplying it by a natural number".

Thus, if we multiply 4 by 1,2,3,4,5... we get 4x1=4, 4x2=8, 4x3=12, 4x4=16, 4x5=20.....

Thus, 4,8,12,16,20.... are multiples of 4.

Factor and Multiple: If a number 'a' divides another number 'b' exactly, we say that 'a' is a factor of 'b' and 'b' is multiple of 'a'.

Eg. The numbers 3, 6, 9, 12, 15, 18,..... are called the multiples of '3'.

Consider the product 3x7=21

clearly, each of the numbers 3 and 7 divides 21 exactly.

so, 3 and 7 are factors of 21 and 21 isa multiple of 3 and 7

Properties of Factors and multiple :

i) '1' is a factor of every number.

ii) Every non-zero number is a factor of itself.

iii) The factor of a non-zero number is always less than or equal to the number. The greatest factor of a non-zero number is the number itself.

iv)The number of factors of a number are finite.

v) Every number is a multiple of itself.

vi) Every number is a multiple of 1.

vii) Every multiple of the number is either greater than or equal to that number.

viii) The multiples of a number are infinite.

ix) If 'a' is a factor of b and 'b' is a factor of 'a', then a = b.

Perfect Numbers: If the sum of all the factors of a number is two time the numer, then the number is called a perfect number.

Eg: The factors of 6 are 1,2,3 and 6

Also 1+2+3+6=12

i.e sum of all factors of 6 = 2 times of 6 = 12

so, 6 is a perfect number

Even Number : The number which is divisible by 2 is known as an even number.

Ex : 2, 4, 8, 12, 24,.....

It is also of the form 2n (where n = whole number)

Odd number : The number which is not divisible by 2 is known as an odd number.

Ex : 3, 9, 11, 17,....

Prime number : A number is called a prime number if it has no factor other than 1 and the number itself.

Ex : 2, 3, 5, 7, 11, 13, 17, 19,.....

Composite numbers : A number is a called a composite number if it has atleast one factor other than 1 and the number itself. (or) A number which has more than two factors is called a composite number.

Ex : 4, 6, 8, 9, 12, 15, ...

Some important facts:

i) Number 1 is neither prime nor composite.

ii) 2 is the smallest prime number.

iii) Every prime number other than 2 is odd number.

iv) 2 is the only prime number. All other even numbers are compostie numbers.

v) If a number is not divisible by any one of the primes less than half of it, then it is prime otherwise it is a composite number.

Sieve of Eratosthenes: A method for finding the prime numbers was found by the Greek Mathematician Eratoshenes. This method is known as Sieve of Eratosthenes.

In this method, we prepare a table of natural numbers, say from 1 to 100.

Step 1: We know that 1 is not a prime number, so we separate it by cross out.

Step 2: The first prime number is 2, leave 2, round off and cross out all the multiples of 2 in the remaining numbers.

They are 4,6,8....

Step 3: The next highest number is 3. Round off and cross out all the multiples of 3 in the remaining numbers.

They are 6,9,...

Step 4: The next highest number is 5. Round off and cross out all the multiples of 5 in the remaining numbers.

They are 10,15....

Step 5: The next highest number is 7. Round off and cross out all the multiples of 7 in the remaining numbers.

They are 14,21...

Step 6: This process continues till the highest prime which does not exceed $\sqrt{n}$.

SIEVE OF ERATOSTHENES TABLE:

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97 are prime numbers below 100.

Note: The number of prime numbers below 100 are 25.

Twin Primes : A pair of prime numbers that differ by 2 are known as Twin primes.

Pairs of twin primes between 1 and 100 are.

$(3,5),(5,7),(11,13),(17,19),(29,31),(41,43),(59,61)\text{and }(71,73)$

Prime Triplet : A set of three consecutive prime numbers differing by 2 is called a prime triplet.

The only prime triplet so far known is (3, 5, 7)

Co - primes (or) Relatively Prime numbers : Two numbers are said to be co-prime if they have only 1 as a common factor. [(or) If H.C.F of two numbers is 1, then they are co-prime or relatively prime].

For example : (2, 3), (3, 4), (4, 5), (4, 9), (8, 15) are pairs of co-primes.

Note : i) Two prime numbers are always co - prime.

ii) Two co-primes need not be both prime numbers.

iii) Composite numbers need not be even. 9 is the smallest odd composite number.

iv) 90, 91, 92, 93, 94, 95, 96 are seven consecutive numbers, less then 100, which are all composite, having no prime number between them.

v) There is no number which has no factors.

vi) There are inifinitely many primes.

vii) There is no largest prime number.

Palindrome Number: The number which on reversing their digits givs the same number is known as polyndrome number.

Eg: 131, 1221, 201102 etc.

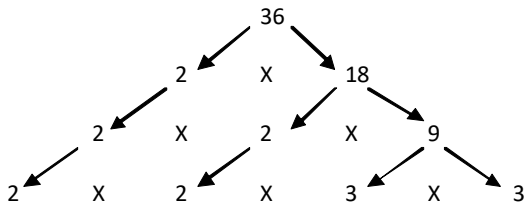
Note: Every polyndrome number with even number of digits is always divisible by 11.

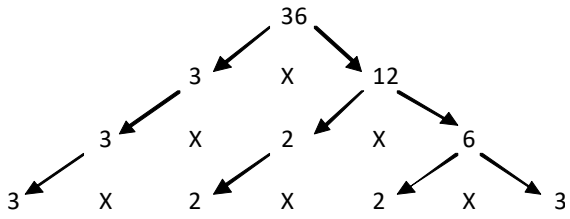
Armstrong number: Armstrong number is a number that is equal to the sum of cubes of its digits.

Eg: $153 = 1^3+5^3+3^3+ = 1+125+27=153$

PRIME FACTORISATION, HCF & LCM

Prime factorization : When a number is expressed as a product of prime factors, is called prime factorisation of that number.





Fundamental theorem of Arithmetic or Prime factorisation property :

Every composite number can be factorized into prime factors in one way.

Methods of Prime Factorization:

1. **Division method:** To find the prime factorization of 42 using division method we proceed as follow.

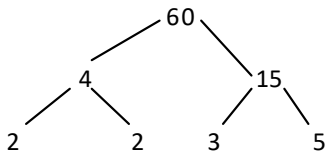
2	42
3	21
7	7
	1

Start dividing by the least prime factor. Continue division till the resulting number to be divided is 1.

Prime factorization of 42 is 2 x 3 x 7

Factor Tree method: We can find the prime factorization of 60 by drawing a factor tree. To find the prime factorization of 60 using factor tree method,

we proceed as follows.



Step 1: Express 60 as a product of two numbers.

Step 2: Factorize 4 and 15 fruther, since they are composite numbers.

Step 3: Continue till all the factors are prime numbers.

Commmon factors:A common factor of two or more numbers is a number which divides each of the given number exactly.

Eg:The factors of 6 are 1,2,3 and 6

Common multiple : A natural number is said to be a common multiple of two or more numbers, if it is a multiple of each of them.

Highest Common Factor (HCF) : The highest common factors of two or more numbers is called their highest common factor (H.C.F) of given numbers.

H.C.F is also called Greatest Common Divisor (G.C.D)

Methods of finding H.C.F:

1. Common factors method 2. Prime factorisation method

1. Common factors method:

1. Find all the factors of each given number.
2. Find common factors of the given number
3. The highest of all the factors obtained in step 2, is the required H.C.F.

Ex :Find HCF of 12 and 16

Factors of 12 are 1 ,(2),(3), 4 ,(6),12

Factors of 16 are 1 ,(2),(4),(8),16

Common factors of 12 and 16 are 1, 2, 4

so Required HCF = 4

2. Prime factorisation method:

1. Break each given number into its prime factors.

2. Find the product of all the prime factors common to all the numbers to get the required H.C.F.

Eg: Find the HCF of 84 and 105

$84 = 2 \times 2 \times (3) \times (7)$

$105 = (3) \times 5 \times (7)$

The required HCF = 3 x 7 = 21

3. Division method (0r) continued divison method:

This method of divison was invented by famous Greek

$$
\begin{array}{r}
56\overline{)\ 64\ (1} \\
-56 \\
\hline
8\overline{)\ 56\ (7} \\
-56 \\
\hline
0
\end{array}
$$

Mathematician Eculid.

1. Divide the greater number by the smaller number.
2. By the remainder of step 1, divide the smaller number.
3. Continue in the same way, until the remainder is 0.

The last divisor is the requied H.C.F

Last divisor is 8 whose remainder becomes 0. Thus, HCF of 56 and 64 is 8.

To find the HCF of more than two numbers :

Rule : Find the HCF of any two of the numbers and then find the HCF of this HCF and the third number and so on. The last HCF will be the required HCF.

Least common multiple (LCM) : The smallest of the common multiples of two or more numbers is called their lowest common multiple (L.C.M).

Ex : Multiples of 12 : 12, 24, 36, 48, 60, 72, 84, 96,.....

Multiples of 16 : 16, 32, 48, 64, 80, 96, 112,.....

Common multiples of 12 and 16 : 48, 96,.....

Smallest common multiple of 12 and 16 is 48

To find the LCM of two or more given numbers :

Rule : The least common multiple of two or more given numbers is the least number which is exactly divisible by each of the given numbers.

Methods of finding L.C.M

1. Common multiples method:

1. Write common multiples of given numbers.
2. Identify the common multiples of given number.
3. The least of these multiples is our required L.C.M

Eg: Find the LCM of 6 and 12

The multiples of 6 are : 6, 12, 18, 24, 30, 36....etc.

The multiples of 12 are : 12, 24, 36, 48, ...

The common multiples of 6 and 12 are : 12 , 24 , 36 ....

The least of these common multiples is 12

Hence the required LCM = 12

Prime factor method:

1. Resolve each given number into its prime factors.
2. The product of all different prime factors of these numbers, using the greatest power of each common prime factor, gives the LCM.

Eg:Find the LCM of 18, 24 and 60

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

LCM of 18,24 and $60 = 2^3 \times 3^2 \times 5 = 360$

3. Common division method:

1. Writ the given number is horozontal line, separate them by commas.
2. Divide them by suitable prime number, whcih exactly divide atleast two of the given numbers.
3. Write down the quotients and undivided numbers step 2, in a line below the first.
4. Repeat the process until we get a line of number which are prime to one another.
5. The product of all divisors and the numbers in the last line will be the required LCM

Eg: Find the LCM of 18,24 and 60

2	18 , 24 , 60
2	9 , 12 , 30
3	9 , 6 , 15
	3 , 2 , 5

Relationship between LCM and HCF:

First number x second number = L.C.M X H.C.F

Let us consider the two numbers 32 and 48.

LCM of 32 and 48

32 = 2 x 2 x 2 x 2 x 2

48 = 2 x 2 x 2 x 2 x 3 $\therefore$ LCM = 96

HCF of 32 and 48 = 16

Now LCM x HCF = 16 x 96 = 1536

and 1st number x 2nd number = 32 x 48 = 1536

Therefore, We can say that First number x second number = LCM x HCF

H.C.F and L.C.M of Fractions:

i) H.C.F of given fractions = $\frac{H.C.F\ of\ numerators}{L.C.M\ .of\ deno\min ators}$

ii) L.C.M of given fractions = $\frac{L.C.M\ \ of\ numerators}{H.C.F\ of\ deno\min ators}$

TEACHING TASK

I. MCQ's with Single correct answer type:

1. The products we get, when a number is multiplied by the numbers 1, 2, 3 ... are called the .....of the given number.

   A) Divisors B) Factors C) Multiples D) Remainders

2. Sum of factors of 24 is

   A) 24 B) 60 C) 72 D) 48

3. Number of prime factors of 666 are.....

   A)3 B)4 C)12 D)5

4. $\frac{-2}{5}+\frac{3}{5}+\frac{4}{10}$=.....

   A) $\frac{-2}{5}$ B) $\frac{3}{5}$ C) $\frac{3}{10}$ D) $\frac{2}{15}$

5. Number of 2-digit numbers whose digit sum is divisible by 6=.....

   A)13 B)8 C)7 D)4

6. Which of the following are divisible by 25
   A)6125 B)50105 C)32960 D)None
7. How many multiples of 500 which are ≤ 500
   A)1 B)2 C)3 D)4
8. Number of primes between 1 to 30
   A)11 B)9 C)10 D)12
9. $\frac{22}{7}$ =.....(approximate to 4 decimal places)
   A)3.1415 B)3.1416 C)3.1428 D)3.1429
10. HCF of 28,84
    A)7 B)14 C)28 D)84
11. Number of factors of 120
    A)16 B)12 C)20 D)24
12. Sum of factors of 64
    A)127 B)128 C)64 D)None
13. The least number which is nearest to 6444 and exactly divisible by 64 is.....
    A)6400 B)6440 C)6464 D)None
14. Which of the following are in ascending order

    A) $\frac{1}{6},\frac{1}{5},\frac{1}{4},\frac{1}{3},\frac{1}{2}$ B) $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6}$

    C) $\frac{5}{2},\frac{4}{3},\frac{3}{4},\frac{2}{5},\frac{1}{6}$ D)None
15. LCM of 2016,2017 is.....
    A)2016 x 2017 B)2016 C)2017 D)2017-2016

II MCQ's with Multiple correct answer type:

1. The following is a factor of 'Ramanjan number'
   A) 19 B) 17 C) 13 D) 7
2. Fractional form of 0.056

   A) $\frac{56}{100}$ B) $\frac{28}{500}$ C) $\frac{7}{125}$ D) $\frac{56}{1000}$
3. Which of the following are in descending order.

   A) $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6}$ B) $\frac{2013}{5},\frac{2014}{5},\frac{2015}{5},\frac{2016}{5},\frac{2017}{5}$

   C) $\frac{2017}{7},\frac{2016}{7},\frac{2015}{7},\frac{2014}{7},\frac{2013}{7}$ D) $\frac{1}{5},\frac{1}{3},\frac{2}{5},\frac{1}{2},\frac{3}{4}$

4. Which of the following is sum of two odd primes
   A)35 B)42 C)84 D)100
5. The number '111111111111' is always divisible by.....
   A)3 B)7 C)13 D)11
6. The highest and lowest common factor of 36 and 84 is.....
   A)7 B)1 C)4 D)12
7. Five digit number a679b is multiple of 72 then the value of a+b
   A)3 B)5 C)6 D)7
8. The H.C.F and L.C.M of two numbers are 21 and 84 respectively. I f th ratio of the two numbers is 1 : 4, then the larger of the two numbers is.....
   A)21 B)48 C)84 D)108
9. The H.C.F of $2^2 \times 3^3 \times 5^5$, $2^3 \times 3^2 \times 5^2 \times 7$ and $2^4 \times 3^4 \times 5 \times 7^2 \times 11$ is .....
   A)$2^2 \times 3^2 \times 5$ ` B)$2^2 \times 3^2 \times 5 \times 7 \times 11$
   C)$4 \times 9 \times 5$ D)$2^4 \times 3^4 \times 5^5$
10. which of following are factors of 2016?
    A)8 B)16 C)32 D)64

III Matrix matching type:

1.

Column-I	**Column-II**
i)factors or divisors of a nuber are	p)co-primes
ii)a number and successor are	q)finite
iii)a number having only two factors is	r)(3,5,7)
iv)prime triplet is	s)1
v)number of even primes is	t)prime

A)i-q,ii-p,iii-t,iv-s,v-r B)i-q,ii-t,iii-p,iv-s,v-r
C)i-q,ii-p,iii-t,iv-r,v-s D)i-q,ii-t,iii-p,iv-r,v-s

LEARNER'S TASK
Beginners (LEVEL I)

I MCQ's with Single correct answer type:

1. A number which is a factor of every number is
   A) 0 B) 1 C) 2 D) None
2. The greatest factor of a number is the .....
   A) 0 B) 1 C) number itself D) none
3. Factors of 91 are
   A) 3 B) 13 C) 7 D) None
4. Every number is a multiple of
   A) 1 B) 0 C) 3 D) 2

5. The largest multiple of a number is
   A) 1 B) 0 C) 3 D) Does not exist
6. .....is a factor of every number.
   A) 1 B) 0 C) 3 D) 2
7. The factors of 2 are...
   A) only 1 B) only 2 C) 1 and 2 D) No factors
8. The number of factors of 2005 is...
   A) 1 B) 2 C) 3 D) 4
9. The number of multiples of a given number are ...
   A) 1 B) finite C) infinite D) None
10. The L.C.M and H.C.F of two numbers is 12 and 84 , on of the numbers is 36 then the other number is.....
    A)12 B)28 C)36 D)48

Achievers (LEVEL II)

I MCQ's with Multi correct answer type:

1. If $\frac{1}{3}+\frac{1}{2}+\frac{1}{x}=4$ then x=.....

   A) $\frac{5}{18}$ B) $\frac{6}{19}$ C) $\frac{12}{38}$ D) $\frac{24}{11}$

2. Which of the following improper fractions ?

   A) $\frac{8}{6}$ B) $\frac{10}{25}$ C) $\frac{15}{10}$ D) $\frac{4}{5}$

3. The value of $2\frac{1}{10}$ is.....

   A)0.21 B)2.1 C)21 D) $\frac{21}{10}$

4. Multiples of least prime number are.....
   A)even natural numbers B)infinite
   C)negative integers D)none
5. The relation between L.C.M and H.C.F of two numbers is.....

   A) $\text{L.C.M}=\frac{\text{product of numbers}}{\text{H.C.F}}$ B) $\text{H.C.F}=\frac{\text{product of numbers}}{\text{L.C.M}}$

   C) $\text{H.C.F}=\frac{\text{L.C.M}}{\text{product of numbers}}$ D) $\text{L.C.M}=\frac{\text{H.C.F}}{\text{product of numbers}}$

6. Which of the following pairs are not a pair of co-primes?
   A)(18,25) B)(23,92) C)(16,62) D)(21,35)
7. Which of the following are factors of 2460
   A)6 B)12 C)40 D)17
8. In which of the following * has least value 3 if it is divisible by '9'.
   A)47*4 B)6*10575 C)8*711 D)234*17

II Matrix matching type:

1.

Column-I	**Column-II**
i) $\frac{3}{10}$	p)0.07
ii) $\frac{7}{100}$	q)0.0037
iii) $\frac{4}{1000}$	r)0.3
iv) $\frac{37}{10000}$	s)0.004

A)i-r,ii-p,iii-s,iv-q B)i-q,ii-r,iii-p,iv-s
C)i-r,ii-p,iii-q,iv-s D)i-r,ii-s,iii-p,iv-q

2.

Column-I	**Column-II**
i) $2\frac{4}{5}+3\frac{5}{6}$	p) $\frac{199}{30}$
ii) $8\frac{1}{4}-2\frac{5}{6}$	q) $-\frac{2}{27}$
iii) $\frac{7}{27}-\frac{3}{9}$	r) $\frac{1}{12}$
iv) $\frac{11}{12}-\frac{5}{6}$	s) $\frac{65}{12}$

A)i-p,ii-s,iii-r,iv-q B)i-p,ii-s,iii-q,iv-r
C)i-p,ii-r,iii-s,iv-q D)i-p,ii-r,iii-q,iv-s

III Comprehension type:

1. If a number 'x' divides another number 'y' exactly, we say that 'x' is a factor of 'y' and 'y' is a maultiple of 'x'.

i) The factor 2 is .....

A)1 B)2 C)Both A and B D)none

ii) The multiple of 6 is.....

A)5 B)12 C)2 D)3

iii) The number of factors of 2005 is .....

A)1 B)2 C)3 D)4

2. Given two different prime numbers P and Q then find the number of divisors for the following

i) PQ

A)2 B)4 C)6 D)8

ii) P^2Q

A)2 B)4 C)6 D)8

iii) P^3Q^2

A)2 B)4 C)6 D)12

3. The L.C.M and H.C.F of fractions as follows

i) H.C.F of given fractions = $\frac{H.C.F\ of\ numerators}{L.C.M.of\ deno\min ators}$

ii) L.C.M of given fractions = $\frac{L.C.M\ \ of\ numerators}{H.C.F\ of\ deno\min ators}$

i) The H.C.F of $\frac{2}{3},\frac{8}{9},\frac{16}{81},\frac{10}{27}$ is.....

A) $\frac{2}{81}$ B) $\frac{81}{2}$ C) $\frac{3}{80}$ D) $\frac{80}{3}$

ii) The L.C.M of $\frac{2}{3},\frac{8}{9},\frac{16}{81},\frac{10}{27}$ is.....

A) $\frac{3}{80}$ B) $\frac{80}{3}$ C) $\frac{2}{81}$ D) $\frac{81}{2}$

Explorers (LEVEL - III)

Descriptive answers type:

1. Find HCF of following numbers using prime factorisation method.
   i) 2^2x3x5^2; 2^3x3^2x7; $2x3^3x5x7$
   ii) 5^2x7x2; $5x7x3^2$; 2^2x3x11
2. Find HCF of following by prime factorisation method
   i) 324;208; 360 ii) 630;1134 iii) 577; 792; 1512
3. Find the HCF of 324,576,1080 by using divison method.
4. Find the least natural number which is divisible by 15,20,24,32 and 36.
5. For each of the following paris of numbers, verify the property (product of two numbers = product of LCM and HCF)
6. The product of two numbers is 2560 and their LCM is 320. Find their HCF.
7. Find LCM if HCF of two numbers is 12 and their product is 2880.
8. Find the LCM of $\frac{1}{3},\frac{5}{6},\frac{2}{9},\frac{4}{27}$
9. Find the HCF of $\frac{9}{10},\frac{12}{25},\frac{18}{35},\frac{21}{40}$
10. If $\frac{3}{4}=\frac{x}{48}$, then x =
11. If $\frac{2}{4}=\frac{72}{x}$, then x=...
12. If $\frac{1}{3}+\frac{1}{2}+\frac{1}{x}$ =4, then x=?
13. Find the difference of i) $\frac{13}{24}and\frac{7}{16}$ ii) $\frac{5}{18}and\frac{4}{15}$ iii) $\frac{1}{12}and\frac{3}{4}$ iv) $\frac{2}{3}and\frac{6}{7}$
14. Evaluate i) $\frac{5}{6}\times\frac{3}{4}$ ii) $\frac{3}{4}\times\frac{4}{3}$ iii) $\frac{15}{13}\times\frac{3}{16}$

    iv) $\frac{5}{13}\times\left(-\frac{1}{3}\right)$ v) $\frac{1}{2}\div 2$ vi) $\frac{3}{4}\div\frac{5}{6}$

vii) $\frac{5}{8} \div \frac{8}{6}$ vii) $\frac{5}{3} \div \frac{4}{3}$ ix) $\frac{5}{10} \div \frac{2}{4}$

HINTS AND SOLUTIONS

1.i) $a = 2^2 \times 3 \times 5^2$
$b = 2^3 \times 3^2 \times 7$
$c = 2 \times 3^2 \times 5 \times 7$

H.C.F=2 X3X5=30

II) $a = 2 \times 5^2 \times 7$
$b = 3^2 \times 5 \times 7$
$c = 2^2 \times 3 \times 11$

H.C.F=1

2) $324 = 2^2 \times 3^4$
$208 = 2^4 \times 13$
H.C.F=4

II) $630 = 2 \times 3^2 \times 5 \times 7$
$1134 = 2 \times 3^4 \times 7$
H.C.F=2X3X3=18

III. $577 = 1 \times 577$
$792 = 2^3 \times 3 \times 11$
$1512 = 2^3 \times 3^3 \times 7$
H.C.F=1

3) H.C.F of a b c=36
4) L.C.M=1440
6) 2560=320xH.C.F
H.C.F=2560/320=8
7) 12XL.C.M=2880
8) L.C.M(1,5,2,4)/H.C.F(3,6,9,27)=20/3
9) H.C.F(9,12,18,21)/L.C.M(10,25,35,40)=3/1400
10) X=36

11) X=36X4=144

12) 5X+6=24X,X=6/19

13) $\frac{13}{24}-\frac{7}{16}=\frac{26-21}{48}=\frac{5}{48}$

II) $\frac{5}{18}-\frac{4}{15}=\frac{25-24}{90}=\frac{1}{90}$

III) $\frac{1}{12}-\frac{3}{4}=\frac{1-9}{12}=-\frac{2}{3}$

IV) $\frac{2}{3}-\frac{6}{7}=\frac{14-18}{21}=-\frac{4}{21}$

14)i)

$\frac{5}{6}\times\frac{3}{4}=\frac{5}{8}$

$ii)\frac{3}{4}\times\frac{4}{3}=1$

$iii)\frac{15}{13}\times\frac{3}{16}=\frac{45}{208}$

$iv)\frac{5}{13}\times(-\frac{1}{3})=-\frac{5}{39}$

$v)\frac{1}{2}/2=\frac{1}{4}$

$vi)\frac{3}{4}/\frac{5}{6}=\frac{9}{10}$

$vii)\frac{5}{8}/\frac{8}{6}=\frac{15}{32}$

$viii)\frac{5}{3}/\frac{4}{3}=\frac{5}{4}$

$ix)\frac{5}{10}/\frac{2}{4}=1$

LEARNER'S TASK

1) Factors of 91

2) 2 factors=1,91

H.C.F=1

2) $324 = 2^2 \times 3^4$

$208 = 2^4 \times 13$

H.C.F=4

II) $630 = 2 \times 3^2 \times 5 \times 7$

$1134 = 2 \times 3^4 \times 7$

H.C.F=2X3X3=18

III. $577 = 1 \times 577$

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H.C.F=1

3) H.C.F of a b c=36

4) L.C.M=1440

6) 2560=320xH.C.F

H.C.F=2560/320=8

7) 12XL.C.M=2880

8) L.C.M(1,5,2,4)/H.C.F(3,6,9,27)=20/3

9) H.C.F(9,12,18,21)/L.C.M(10,25,35,40)=3/1400

10) X=36

11) X=36X4=144

8) $2005 = 5^1 \times 401^1$

$d(N) = (n_1 + 1)(n_2 + 1) = (1+1)(1+1) = 4$

$12 \times 84 = x \times 36$

10) $x = \frac{84}{3} = 28$

$\frac{1}{3} + \frac{1}{2} + \frac{1}{x} = 4$

DoL.C.Mtofindtheanswer

$[x = \frac{6}{19}]$

II. i) $\frac{8}{6}=\frac{3}{2}$ Im *properfraction*

iii) $2\frac{1}{10}=\frac{21}{10}=2.1$

iii) $\frac{8}{6}=\frac{3}{2}$ Im *properfraction*)

comprehension

1) $\frac{H.C.F(2,8,16,10)}{L.C.M(3,9,81,27)}=\frac{2}{81}$

2) $\frac{L.C.M(2,8,16,10)}{H.C.F(3,9,81,27)}=\frac{80}{3}$