FOUNDATION

Class: IX . MATHEMATICS

8. LIMITS OF TRIGONOMETRIC FUNCTIONS

Teaching Task. (Jee Mains)

01. $\lim_{x\to 0} \frac{\sin x}{x} = 1$ Ans: B

02. $\lim_{x\to 0} \frac{\sin 5x}{x} = 5$ Ans.. B

03. $\lim_{x\to 0} \frac{1-\cos x}{x}$

$= \lim_{x\to 0} \frac{2\sin^2\left(\frac{x}{2}\right)}{x}$

$= \lim_{x\to 0} \frac{2\sin^2\left(\frac{x}{2}\right)}{x^2} \times x$

$= 2 \cdot \lim_{x\to 0} \frac{\sin^2\left(\frac{x}{2}\right)}{x^2} \times \lim_{x\to 0} 0$

$= 2 \times \left(\frac{1}{2}\right)^2 \times 0 = 0$ Ans: A

04. $\lim_{x\to 0} \frac{\tan 3x}{\sin 4x}$

$= \frac{\lim_{x\to 0} \frac{\tan 3x}{x}}{\lim_{x\to 0} \frac{\sin 4x}{x}}$

$= \frac{3}{4}$

Ans.. A

05. $\lim_{x\to a} f(x) = l$ Ans. A

06. $\lim_{x\to \frac{\pi}{2}} \frac{-\sin\left(x-\frac{\pi}{2}\right)}{\left(x-\frac{\pi}{2}\right)} = -1$ Ans: A

②

07. We know $x \to 0^+$ $\Rightarrow x > 0$ $\Rightarrow |x| = x$

$\therefore \lim_{x \to 0^+} \frac{\sin x}{x} = 1$

$x \to 0^-$ $\Rightarrow x < 0$ $\Rightarrow |x| = -x$

$\therefore \lim_{x \to 0^-} \frac{\sin x}{-x} = -1.$

$\therefore$ Both Right hand side Limit and Left hand side Limits are not Equal

$\therefore \lim_{x \to 0} \frac{\sin x}{|x|}$ does not exists

Ans: C

08 $\lim_{x \to 0} \frac{\sin 2x \cdot \cos 3x}{x}$

$= \lim_{x \to 0} \frac{\sin 2x}{x} \cdot \lim_{x \to 0} \cos 3x$

$= 2 \times \cos 3 \times 0$

$= 2 \times 1$

$= 2$

Ans. C

09 $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

$= \lim_{x \to 0} \frac{2 \sin^2\left(\frac{x}{2}\right)}{x^2}$

$= 2\left(\frac{1}{2}\right)^2$

$= \frac{1}{2}$

Ans: B

10. $\lim_{x \to 0} \frac{\sin(x^2)}{x \cdot \sin x}$

$= \lim_{x \to 0} \frac{\sin(x^2)}{x^2 \cdot \left(\frac{\sin x}{x}\right)}$

$= \frac{\lim_{x \to 0} \frac{\sin(x^2)}{x^2}}{\lim_{x \to 0} \frac{\sin x}{x}}$

$= \frac{1}{1} = 1$

Ans: B

ADVANCED

01. Given $\underset{x \to 0}{Lt}\ x^n \cdot \sin\left(\frac{1}{x^2}\right)$.

A) Let $n = 2 > 0$.

$\therefore \underset{x \to 0}{Lt}\ x^2 \cdot \sin\left(\frac{1}{x^2}\right)$

We have $-1 \le \sin\left(\frac{1}{x^2}\right) \le 1$

$$-x^2 \le x^2 \sin\left(\frac{1}{x^2}\right) \le x^2$$

$$\therefore \underset{x \to 0}{Lt}\,(-x^2) = \underset{x \to 0}{Lt}\,(x^2) = 0$$

$\therefore \underset{x \to 0}{Lt}\ x^2 \sin\left(\frac{1}{x^2}\right) = 0$.

$\therefore \underset{x \to 0}{Lt}\ f(x)$ exists

B) Let $n = -2 < 0$

$\underset{x \to 0}{Lt}\ x^{-2} \cdot \sin\left(\frac{1}{x^2}\right)$

$$\boxed{\cancel{\underset{x \to 0}{Lt}\ \frac{\sin\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)}}}$$

We have

$$-1 \le \sin\left(\frac{1}{x^2}\right) \le 1$$

$$-x^{-2} \le x^{-2} \sin\left(\frac{1}{x^2}\right) \le x^{-2}$$

$$\therefore \underset{x \to 0}{Lt}\left(\frac{-1}{x^2}\right) = -\infty$$

$$\underset{x \to 0}{Lt}\ \frac{1}{x^2} = \infty$$

$\therefore$ LHL $\neq$ RHL

Limit does not exists

Ans.. A, B

(4)

02. $L = \lim_{x \to 0} \frac{|2\sin x - 1|}{2\sin x - 1}$

A) when $a = \frac{\pi}{6}$

$L = \lim_{x \to \frac{\pi}{6}} \frac{|2\sin x - 1|}{2\sin x - 1}$

$LHL = x \to \frac{\pi}{6}^-$

$\Rightarrow x < \frac{\pi}{6}$

$\Rightarrow \sin x < \sin\frac{\pi}{6}$

$\Rightarrow \sin x < \frac{1}{2}$

$\Rightarrow 2\sin x - 1 < 0$

$\Rightarrow |2\sin x - 1| = -(2\sin x - 1)$

$= 1 - 2\sin x$

$\therefore \lim_{x \to \frac{\pi}{6}^-} \frac{1 - 2\sin x}{2\sin x - 1} = -1$

$RHL = x \to \frac{\pi}{6}^+$

$\Rightarrow x > \frac{\pi}{6}$

$\Rightarrow \sin x > \sin\frac{\pi}{6}$

$\Rightarrow \sin x > \frac{1}{2}$

$\Rightarrow 2\sin x - 1 > 0$

$\therefore |2\sin x - 1| = (2\sin x - 1)$

$\therefore \lim_{x \to \frac{\pi}{6}^+} \frac{2\sin x - 1}{2\sin x - 1} = 1$

$LHL \neq RHL$

$\therefore \lim_{x \to 0} f(x)$ Does not exists

B) $\lim_{x \to \pi} \frac{|2\sin x - 1|}{2\sin x - 1} = \frac{|2\sin\pi - 1|}{2\sin\pi - 1} = \frac{|-1|}{-1} = -1$

C) $\lim_{x \to \frac{\pi}{2}} \frac{|2\sin x - 1|}{2\sin x - 1} = \frac{|2\sin\frac{\pi}{2} - 1|}{2\sin\frac{\pi}{2} - 1} = \frac{|2.1 - 1|}{2 - 1} = 1$

D) $\lim_{x \to 0} \frac{|2\sin 0 - 1|}{2\sin 0 - 1} = \frac{|-1|}{-1} = -1$

Ans: A, B, C

3. Statement I: Conceptual (True)
Statement II: Conceptual (True)

Ans. A

4. Statement I: Conceptual (False)
Statement II: Conceptual (True)

Ans. D

(5)

5 **Assertion:** $\lim_{x\to 0} \frac{\sqrt{1-\cos 2x}}{x}$

$= \lim_{x\to 0} \frac{\sqrt{2\sin^2 x}}{x}$

$= \sqrt{2} \cdot \lim_{x\to 0} \frac{|\sin x|}{x}$

$LHL = \sqrt{2} \cdot \lim_{x\to 0^-} \frac{-\sin x}{x} = -\sqrt{2}$

$RHL = \sqrt{2} \cdot \lim_{x\to 0^+} \frac{\sin x}{x} = \sqrt{2}$

$LHL \neq RHL$

$\lim_{x\to 0} f(x)$ Does not exists

Reason: $\lim_{x\to 0} \frac{|x|}{x}$.

$LHL = \lim_{x\to 0^-} \frac{(-x)}{x} = -1$

$RHL = \lim_{x\to 0^+} \frac{x}{x} = 1$

$LHL \neq RHL$

Limit Does not exists

Ans: B

06. **Assertion:** $\lim_{x\to 0} \frac{\sin x - x}{x^2}$

Differentiate both Nr and Dr

$= \lim_{x\to 0} \frac{\cos x - 1}{2x}$

$= \lim_{x\to 0} \frac{-\sin x}{2} = -\frac{\sin 0}{2} = 0$ (False)

Reason: $\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$

Differentiate both Nr and Dr

$= \lim_{x\to 0} \frac{e^{x^2} \cdot 2x + \sin x}{2x}$

$= \lim_{x\to 0} \left(e^{x^2} + \frac{\sin x}{2x}\right)$

$= 1 + \frac{1}{2}$

$= \frac{3}{2}$ (True)

Ans: D

07. **Assertion:** $\lim_{x \to 0} \frac{\sqrt{1-\cos 2x}}{x}$

$= \lim_{x \to 0} \frac{\sqrt{2\sin^2 x}}{x}$

$= \sqrt{2} \cdot \lim_{x \to 0} \frac{|\sin x|}{x}$

$LHL = \sqrt{2} \cdot \lim_{x \to 0^-} \frac{-(\sin x)}{x} = -\sqrt{2}$

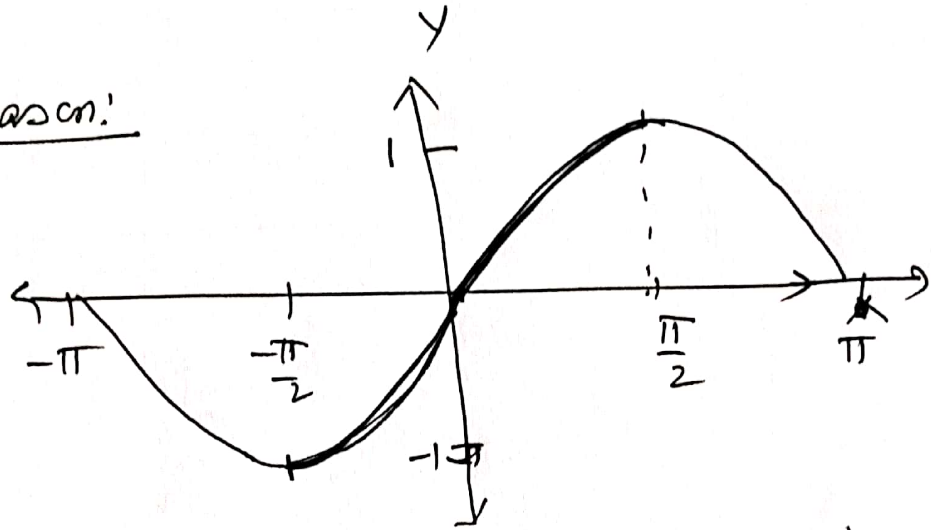
$RHL = \sqrt{2} \cdot \lim_{x \to 0^+} \frac{\sin x}{x} = \sqrt{2}$

$LHL \neq RHL$

$\lim_{x \to 0} f(x)$ doesnot exists

(True)

Reason:



clearly $0 < x < \frac{\pi}{2} \Rightarrow \sin x > 0 \Rightarrow |\sin x| = \sin x$

$-\frac{\pi}{2} < x < 0 \Rightarrow \sin x < 0 \Rightarrow |\sin x| = -\sin x$

(True)

Ans: A

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08 $-1 \leq \cos\left(\frac{2}{x}\right) \leq 1$

$-x^3 \leq x^3 \cos\left(\frac{2}{x}\right) \leq x^3$

$\therefore \lim_{x \to 0} (-x^3) = \lim_{x \to 0} x^3 = 0$

$\therefore \lim_{x \to 0} x^3 \cos\left(\frac{2}{x}\right) = 0$

Ans: C

09. $L = \lim_{x \to \infty} \dfrac{x^2(2+\sin^2 x)}{x+100}$

$= \lim_{x \to \infty} \dfrac{x^2(2+\sin^2 x)}{x^2\left(\frac{1}{x}+\frac{100}{x^2}\right)}$

$L = \lim_{x \to \infty} \dfrac{2+\sin^2 x}{\left(\frac{1}{x}+\frac{100}{x^2}\right)}$

clearly $2+\sin^2 x$ is a finite value

As $x \to \infty$, $\left(\frac{1}{x}+\frac{100}{x^2}\right) \to 0$.

$\therefore L = \dfrac{\text{finite value}}{0} = \infty$

Ans: C

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10 $-1 \le \sin\left(\frac{4}{x}\right) \le 1$

$-x^2 \le x^2 \sin\left(\frac{4}{x}\right) \le x^2$

$\lim_{x \to 0} (-x^2) = \lim_{x \to 0} (x^2) = 0$

$\therefore \lim_{x \to 0} x^2 \sin\left(\frac{4}{x^2}\right) = 0$

Ans: A

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11 Conceptual ~~Ans: B~~

12 Conceptual Ans: B

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13. $\lim_{x \to 2} \dfrac{(x+2)(x-2)}{(x-2)} = \lim_{x \to 2} (x+2) = 2+2 = 4$

Ans: 4

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14 $\lim_{x \to 2} (3x+1) = 3.2+1 = 7$

Ans: 7

15 a) $\lim_{x\to 0} \frac{\sin x^\circ}{x}$

$= \lim_{x\to 0} \frac{\sin \frac{\pi x}{100}}{x} = \frac{\pi}{180}$ (s)

b) $\lim_{x\to 0} \frac{\tan 2x - x}{3x - \sin x}$

$= \lim_{x\to 0} \frac{\frac{\tan 2x}{x} - 1}{3 - \frac{\sin x}{x}}$

$= \frac{2-1}{3-1}$

$= \frac{1}{2}$ (p)

c) $\lim_{x\to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$

$= \lim_{x\to 0} \frac{\sin[\pi(1-\sin^2 x)]}{x^2}$

$= \lim_{x\to 0} \frac{\sin(\pi - \pi \sin^2 x)}{x^2}$

$= \lim_{x\to 0} \frac{\sin(\pi \sin^2 x)}{x^2}$

$= \lim_{x\to 0} \frac{\sin(\pi \sin^2 x)}{x^2} \times \frac{\pi \sin^2 x}{\pi \sin^2 x}$

$= \pi \lim_{x\to 0} \frac{\sin(\pi \sin^2 x)}{\pi \sin^2 x} \times \lim_{x\to 0} \frac{\sin^2 x}{x^2}$

$= \pi \times 1 \times 1 = \pi$ (q)

d) $\lim_{x\to \infty} 2^{x-1} \cdot \tan\left(\frac{a}{2^x}\right)$

$= \frac{1}{2} \cdot \lim_{x\to\infty} \frac{\tan\left(\frac{a}{2^x}\right)}{\left(\frac{1}{2^x}\right)} = \frac{1}{2} \times a = \frac{a}{2}$ (r)

Ans: s, p, q, r

16 a) $\lim_{x\to 0} \frac{\sqrt{1+\tan x} - \sqrt{1-\tan x}}{\sin x} \times \frac{\sqrt{1+\tan x} + \sqrt{1-\tan x}}{\sqrt{1+\tan x} + \sqrt{1-\tan x}}$

$= \lim_{x\to 0} \frac{1+\tan x - 1 + \tan x}{\sin x} \times \frac{1}{\sqrt{1+\tan x} + \sqrt{1-\tan x}}$

$= 2 \cdot \lim_{x\to 0} \frac{\tan x}{\sin x} \cdot \frac{1}{2} = 2 \times 1 \times \frac{1}{2} = 1$ (t)

b) $\lim_{x \to 0} \frac{1-\cos 2x^\circ}{x^2}$

$= \lim_{x \to 0} \frac{2\sin^2\left(\frac{x^\circ}{2}\right)}{x^2}$

$= 2 \cdot \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)^2}{x^2}$

$= 2 \cdot \left(\frac{\pi}{180}\right)^2$ (p)

c) $\lim_{x \to 0} \frac{1-\cos x}{x \log(1+x)}$

$= \lim_{x \to 0} \frac{2\sin^2\left(\frac{x}{2}\right)}{x^2 \cdot \frac{\log(1+x)}{x}}$

$= 2 \cdot \left(\frac{1}{2}\right)^2 \times \frac{1}{1} = \frac{1}{2}$ (r)

d) $\lim_{x \to 0} \frac{e^x - e^{\sin x}}{2(x - \sin x)}$

$= \frac{1}{2} \cdot \lim_{x \to 0} \frac{e^{\sin x}\left(e^{x - \sin x} - 1\right)}{(x - \sin x)}$

$= \frac{1}{2} \cdot e^{\sin 0}$

$= \frac{1}{2}$ (r)

Ans: t, p, r, r

LEARNERS TASK (CCUQ's)

01 $\lim_{x \to a} \frac{\tan(x-a)}{x-a} = 1$ Ans: A

02 $\lim_{x \to 0} \frac{\sin 3x}{x} = 3$ Ans: C

03 $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$ Ans: D

04 $\lim_{x \to 0} \frac{1-\cos 4x}{1-\cos 6x}$

Differentiate w.r.t x

$\lim_{x \to 0} \frac{\sin 4x}{\sin 6x} = \frac{4}{6} = \frac{2}{3}$ Ans: A

05 1 Ans: B

06 Conceptual Ans: C

07. 1 Ans: C

08 $\cos 0 = 1$ Ans: D

09 $\lim_{x \to 0} \frac{\tan^{-1} x}{x} = \lim_{x \to 0} \frac{\left(\frac{1}{1+x^2}\right)}{1} = \frac{1}{1+0^2} = 1$ Ans: C

10 Conceptual Ans: A

JEE MAINS LEVEL

01. $\lim_{x \to 0} \frac{\tan x}{x} = 1$ Ans: B

02 $\lim_{x \to 0} \frac{\sin 3x}{3x} = 1$ Ans: C

03 $\lim_{x \to 0} \frac{\sin 5x}{x} = 5$ Ans: A

04 $\lim_{x \to 0} \frac{\sin x}{\tan x} = \frac{\lim_{x \to 0} \frac{\sin x}{x}}{\lim_{x \to 0} \frac{\tan x}{x}} = \frac{1}{1} = 1$ Ans: A

05 $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

Differentiating wrt x on ~~6t~~ Nr & Dr

$= \lim_{x \to 0} \frac{1 - \cos x}{3x^2}$

Again differentiate wrt x on

$= \lim_{x \to 0} \frac{\sin x}{6x}$

$= \frac{1}{6} \lim_{x \to 0} \frac{\sin x}{x}$

$= \frac{1}{6} \times 1 = \frac{1}{6}$

Ans: C

06 $\lim_{x\to 0} \frac{\sin(ax)}{bx} = \frac{a}{b}$ Ans: A

07 $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$

$= \lim_{x\to 0} \frac{\sec^2 x - \cos x}{3x^2}$

$= \lim_{x\to 0} \frac{2\sec x \cdot \sec x \tan x + \sin x}{6x}$

$= \frac{2}{6} \cdot \lim_{x\to 0} \frac{\tan x}{x} + \frac{1}{6} \cdot \lim_{x\to 0} \frac{\sin x}{x}$

Ans: D

$= \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

08 $\lim_{x\to 0} \frac{\tan x - x}{x^3}$

$= \lim_{x\to 0} \frac{\sec^2 x - 1}{3x^2}$

$= \lim_{x\to 0} \frac{2\sec x \cdot \sec x \cdot \tan x}{6x}$

$= \frac{2}{6} \cdot \lim_{x\to 0} \frac{\tan x}{x}$

$= \frac{1}{3}$

Ans: B

09 $\lim_{x\to 0} \frac{x}{\sin x} = 1$ Ans: A

10\. $\lim_{x\to 0} \frac{\sin 4x}{x} = 4$ Ans: A

JEE ADVANCED

1\. $\lim_{x\to 0} \frac{\sin x - 2\sin 3x + \sin 5x}{x} = 1 - 2\times 3 + 5 = 0$

Also, $\lim_{x\to \frac{\pi}{2}} \cos x = \cos\frac{\pi}{2} = 0$, $\lim_{x\to 0} \frac{x^3 - 2x}{\cos x} = 0$ Ans: A, B, C

02 $\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

$= \lim_{x \to \frac{\pi}{2}}$ Let $x - \frac{\pi}{2} = y \Rightarrow x = \frac{\pi}{2} + y$

As $x \to \frac{\pi}{2}$, $y \to 0$

$\therefore \lim_{y \to 0} \frac{\tan 2\left[\frac{\pi}{2} + y\right]}{y}$

$= \lim_{y \to 0} \frac{\tan(\pi + 2y)}{y}$

$= \lim_{y \to 0} \frac{\tan 2y}{y}$

$= 2$

~~Ans:~~

Also, $\lim_{x \to 0} \frac{\sin 2x}{x} = 2$

Ans.. A, D

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03 Statement I: Conceptual (True)

Statement II: Conceptual (False)

Ans: C

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04. Statement I: Conceptual (True)

Statement II: Conceptual (True)

Ans: A

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05 Assertion (A):

$\lim_{x \to 0} \frac{\sin^{-1} 3x}{\tan^{-1} 6x}$

$= \frac{\lim_{x \to 0} \frac{\sin^{-1} 3x}{x}}{\lim_{x \to 0} \frac{\tan^{-1} 6x}{x}} = \frac{3}{6} = \frac{1}{2}$ (True)

~~Ans:~~

Ans. A

Reason: Conceptual (True)

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06. Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans.. A

07. $\lim_{x \to 0} \dfrac{\sin^{-1}5x + \sin^{-1}x}{\sin^{-1}x + \sin^{-1}3x} = \dfrac{5+1}{1+3} = \dfrac{6}{4} = \dfrac{3}{2}$ Ans: D

08 $\lim_{x \to 0} \dfrac{\sin 3x + \tan 4x}{\tan 3x + \tan x} = \dfrac{3+4}{3+1} = \dfrac{7}{4}$ Ans: A

09. $\lim_{x \to 0} \dfrac{\sin^{-1}3x - \tan 4x}{\tan 3x + \sin^{-1}x} = \dfrac{3-4}{3+1} = \dfrac{-1}{4}$ Ans. A

10. $\lim_{x \to 0} \dfrac{\sin 3x + \sin 7x}{\tan 3x + \tan 7x} = \dfrac{3+7}{3+7} = \dfrac{10}{10} = 1$ Ans: C

11. Conceptual Ans: B

12 Conceptual Ans. B

13 $\lim_{x \to 1} \dfrac{x^2-1}{x-1} = \lim_{x \to 1} \dfrac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 1+1 = 2$ Ans: 2

14. $\lim_{x \to 0} \dfrac{\sin(kx)}{x} = k = 5$ Ans: 5

15 a) $\lim_{x \to 0^-} \dfrac{-\sin x}{x} = -1$ $\therefore \lim_{x \to 0} \dfrac{|\sin x|}{x}$ = Does not exist (r)

$\lim_{x \to 0^+} \dfrac{\sin x}{x} = 1$

b) $\lim_{x \to 0} \dfrac{\sin\left(\frac{\pi x}{100}\right)}{x} = \dfrac{\pi}{100}$ (p)

c) $\lim_{x \to 0^-} \dfrac{-x}{x} = -1$

$\lim_{x \to 0^+} \dfrac{x}{x} = 1$

$\therefore \lim_{x \to 0} \dfrac{|x|}{x}$ does not exist (r)

d) Conceptual (q)

Ans: r, p, r, q

16 a) $\lim_{x\to 0} \frac{\sin^2 2x}{\sin^2 4x}$

$= \frac{\lim_{x\to 0} \frac{\sin^2 2x}{x^2}}{\lim_{x\to 0} \frac{\sin^2 4x}{x^2}}$

$= \frac{(2)^2}{(4)^2} = \frac{4}{16} = \frac{1}{4}$

b) $\lim_{x\to \pi} \frac{\sin x}{x-\pi}$

$= \lim_{x\to\pi} \frac{\cos x}{1}$

$= \cos\pi$

$= -1$ (p)

c) $\lim_{x\to 0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}} \times \frac{\sqrt{x+1}+\sqrt{1-x}}{\sqrt{x+1}+\sqrt{1-x}}$

$= \lim_{x\to 0} \frac{\sin x}{\cancel{x}+\cancel{1}-\cancel{1}+x} \times \left(\sqrt{x+1}+\sqrt{1-x}\right)$

$= \frac{1}{2} \cdot \lim_{x\to 0} \frac{\sin x}{x} \times \lim_{x\to 0} \left(\sqrt{x+1}+\sqrt{1-x}\right)$

$= \frac{1}{2} \times 1 \times 2 = 1$ (r)

Ans: —, p, r

$\Longrightarrow$ THE END