9^{th} CLASS

MATHEMATICS

IIT FOUNDATION

STUDY MATERIAL

3. EXPONENTS AND POWERS

TEACHING TASK

JEE MAINS LEVEL QUESTIONS

① $\frac{a^{15}b^4}{a^6b^2} = a^{15}b^4 \times a^{-6} \times b^{-2}$ (use laws of exponents)

$\Rightarrow a^{(15-6)} \times b^{(4-2)} \Rightarrow a^9 \times b^2$

Ans: B

② $x+y=5$, $x^2+y^2=10$

$x+y=5$ (Squaring on both sides)

$\Rightarrow (x+y)^2 = 5^2 \Rightarrow x^2+y^2+2xy=25$

Substitute values,

$10+2xy=25$

$2xy=25-10 \Rightarrow 2xy=15$

$\Rightarrow xy=\frac{15}{2}=7\frac{1}{2}$.

Ans: A

(3) $\left(a^{4n} \cdot a^{3n} \cdot a^{2n}\right)^2$

$\Rightarrow \left(a^{4n+3n+2n}\right)^2$

$= \left(a^{9n}\right)^2 = a^{18n}$

Ans: C

(4) $\left[5\left[8^{1/3} + (27)^{1/3}\right)^3\right]^{1/4}$

$= 5\left(\left(2^{\cancel{3}}\right)^{1/\cancel{3}} + \left(3^{\cancel{3}}\right)^{1/\cancel{3}}\right)^3\right)^{1/4}$

$= 5\left((2+3)^3\right)^{1/4}$

$= 5\left((5)^3\right)^{1/4} \Rightarrow \left(5 \times 5^3\right)^{1/4} \Rightarrow \left(5^{3+1}\right)^{1/4}$

$\Rightarrow \left(5^{\cancel{4}}\right)^{1/\cancel{4}} = 5$

Ans: B

(5) $\left\{\left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right] \div \left(\frac{1}{4}\right)^{-3}\right\}$

$= \left\{\left\{(3)^3 - (2)^3\right\} \div (4)^3\right\} \quad \left(\because \left(\frac{1}{a}\right)^{-n} = a^n\right)$

$= \left\{(27-8) \div 64\right\}$

$= (19 \div 64) = \frac{19}{64}$

Ans: A

⑥ $\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$

$\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = 2^{-5}$

Take logarithm on both sides (Base 2):

$\log_2\left(\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}}\right) = \log_2\left(2^{-5}\right)$

$\log_2\left(\sqrt[3]{4}\right) = \log_2\left(4^{1/3}\right) = \frac{1}{3}\cdot 2 = \frac{2}{3}$

$\left(2x+\frac{1}{2}\right)\times\frac{2}{3} = -5$

$\Rightarrow 2x+\frac{1}{2} = -5\times\frac{3}{2}$

$\Rightarrow 2x+\frac{1}{2} = \frac{-15}{2} \Rightarrow 2x = \frac{-15}{2}-\frac{1}{2}$

$\Rightarrow 2x = \frac{-16}{2} = -8$

$\Rightarrow x = \frac{-8}{2} \Rightarrow \boxed{x = -4}$ Ans: A

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⑧ $\frac{\cancel{6}^{3}a^9b^4c^5}{\cancel{2}ab^2c^3} = \frac{3a^9b^4c^5}{ab^2c^3}$

$\Rightarrow 3a^9b^4c^5 \times a^{-1}b^{-2}\times c^{-3}$

$\Rightarrow 3a^{(9-1)}\times b^{(4-2)}\times c^{(5-3)}$

$= 3a^8b^2c^2$ Ans: D

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(9) $\left[\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^3\right]\times 2^6 \Rightarrow \left[\left(\frac{1}{2^2}\right)^2-\left(\frac{1}{2^2}\right)^3\right]\times 2^6$

$\Rightarrow \left[(2^{-2})^2-(2^{-2})^3\right]\times 2^6$

$\Rightarrow (2^{-4}-2^{-6})\times 2^6$

$\Rightarrow 2^{-4}\times 2^6 - 2^{-6}\times 2^6$

$\Rightarrow 2^{6-4} - 2^{6-6}$

$\Rightarrow 2^2 - 2^0 \Rightarrow 2^2 - 1$

$\Rightarrow 4-1=3$. Ans: C

(10)

$$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \cdot \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$$

$$\left(x^{(a-b)}\right)^{a^2+ab+b^2} \left(x^{(b-c)}\right)^{b^2+bc+c^2} \left(x^{(c-a)}\right)^{c^2+ac+a^2} \quad \left(\because \frac{x^n}{x^m} = x^{n-m}\right)$$

$$\Rightarrow x^{(a-b)a^2+ab+b^2+(b-c)(b^2+bc+c^2)+(c-a)(c^2+ac+a^2)}$$

Let simply the exponent expression.

$$(a-b)(a^2+ab+b^2)+(b-c)(b^2+bc+c^2)+(c-a)(c^2+ac+a^2)$$

each expand term is

$$(a-b)(a^2+ab+b^2) = a^3+\cancel{a^2b}+\cancel{ab^2}-\cancel{a^2b}-\cancel{ab^2}-b^3 = a^3-b^3$$

$$(b-c)(b^2+bc+c^2) = b^3+\cancel{b^2c}+\cancel{bc^2}-\cancel{b^2c}-\cancel{bc^2}-c^3 = b^3-c^3$$

$$(c-a)(c^2+ac+a^2) = c^3+\cancel{c^2a}+\cancel{ca^2}-\cancel{c^2a}-\cancel{ca^2}-a^3 = c^3-a^3$$

combining all these

$$a^3-b^3+b^3-c^3+c^3-a^3 = 0$$

$\therefore x^0 = 1$

Ans: A

(11) $2^{m+1} \times 2^{2m-3}$

$\Rightarrow$ Bases are same power should be added.

$\therefore 2^{m+1+2m-3}$

$= 2^{3m-2}$ Ans: A

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(12) If $a^2 = 16$

$a = \sqrt{16} \Rightarrow 4$

$a = 4$

$\therefore a^{-1} = \frac{1}{4}$ Ans: A

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(13) $x^{-2} = \frac{1}{25} \Rightarrow x^2 = 25$

$\therefore x = \sqrt{25} \Rightarrow \sqrt{5 \times 5} = \sqrt{5^2} = 5.$

$\therefore x = 5.$ Ans: A

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$x^2y^3 = z^5$

Take logarithms of both sides

$\log(x^2y^3) = \log(z^5)$

$\Rightarrow 2\log x + 3\log y = 5\log z$

Let $\log x = a$, $\log y = b$

then, $2a + 3b = 5\log z$

divid the eqn with 2

$a + \frac{3}{2}b = \frac{5}{2}\log z$

$a+b = \frac{5}{2}\log - \frac{1}{2}b$

$\therefore z^{5/2}$

Ans: A

15

$\frac{a^p}{a^q} = a^{p-q}$; $\frac{a^8}{a^2} = a^p$

$\Rightarrow a^8 . a^{-2} = a^p$

$\Rightarrow a^{8-2} = a^p$

$\Rightarrow a^6 = a^p$ $\therefore \boxed{P = 6}$

Ans: B

16

$m^n \times m^{-2n} = m^{-3}$

$m^{n-2n} = m^{-3}$

$m^{-n} = m^{-3}$

$-n = -3$

$\boxed{n = 3}$

Ans:

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(17) ☆ $\left(\frac{m^3}{n^2}\right)^{-2} \times \left(\frac{n^5}{m}\right)^{-1}$

$\Rightarrow \left(\frac{m^3}{n^2}\right)^{-2} \times \left(\frac{m}{n^5}\right)^{1} \Rightarrow \frac{m^{-6}}{n^{-2}} \times \frac{m^1}{n^5}$

$\Rightarrow \frac{m^{-6+1}}{n^{-4+5}} = \frac{m^{-5}}{n^{1}} = n^{-1} m^{-5}$

Ans: No right answer

(18) $x^{-3}y^2 = \frac{1}{4} \Rightarrow \frac{1}{x^3}y^2 = \frac{1}{4}$

$\Rightarrow y^2 = \frac{x^3}{4}$

let $xy = k$, $y = \frac{k}{x}$, substitute

$\left(\frac{k}{x}\right)^2 = \frac{x^3}{4}$

$\frac{k^2}{x^2} = \frac{x^3}{4} \Rightarrow k^2 = \frac{x^3 \cdot x^2}{4} \Rightarrow k^2 = \frac{x^5}{4}$

$k^2 = \frac{1}{4} \Rightarrow k = \frac{1}{\sqrt{4}} \Rightarrow k = \frac{1}{2}$

$\therefore xy$ is $\frac{1}{2}$

Ans: A

(19) $(a^3b^{-2})^2 \div a^{-4}b^3$

$\Rightarrow \dfrac{(a^3b^{-2})^2}{a^{-4}b^3} = \dfrac{a^6b^{-4}}{a^{-4}b^3} = a^6.a^4b^{-4}.b^{-3}$

$\Rightarrow a^{6+4}.b^{-4-3} \Rightarrow a^{10}\times b^{-7}$

$\Rightarrow a^{10}\times b^{-7} = \dfrac{a^{10}}{b^7}$

Ans: No right option.

(20) $(a^3b)^3\times(ab^{-2})^2 = a^mb^n$

$\Rightarrow (a^9b^3)\times a^2b^{-4} \Rightarrow a^{11}.b^{-1} = a^mb^n$

$\therefore m = 11,\ b = -1$

$\therefore m+n = 11+(-1) = 11-1 = 10.$

Ans: No right option.

JEE ADVANCED LEVEL QUESTIONS

MULTI CORRECT ANSWER TYPE:

① Through options

A) $a^0 = 1$, B) $a^1 = a$ C) $= a^2 = a \times a$ D) $a^{-1} = \frac{1}{a}$

All Statement are non zero number for a.

Ans: A, B, C, D

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② $$\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$$

$$9^n \times 3^5 \times (27)^3 = 27 \times 3 \times (81)^4$$

$$9^n = \frac{\cancel{27} \times \cancel{3} \times \cancel{81} \times \cancel{81}^{3} \times \cancel{81}^{3} \times 81}{\cancel{3 \times 3 \times 3 \times 3 \times 3} \times \cancel{27} \times \cancel{27}_{1} \times \cancel{27}_{1}}$$

$$9^n = 3 \times 3 \times 81 \Rightarrow 9^1 \times 9^2$$

$$9^n = 9^{1+2} \Rightarrow 9^n = 9^3$$

$\therefore$ Bases are equal power should be equal

$\therefore n = 3$.

Ans: B and D

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(3) $\sqrt{3^0+3^1+3^2+3^3+3^4} = \sqrt{1+3+9+27+81}$

$= \sqrt{121} = \sqrt{11^2} = 11.$

Ans: B and C.

(4) Equivalent to $a^{3/2}$

A) $\sqrt{a^3} = (a^3)^{1/2} \Rightarrow a^{3/2}$

B) $\sqrt[3]{a^2} = (a^2)^{1/3} \Rightarrow a^{2/3}$

C) $a.\sqrt{a} = a^1 \cdot a^{1/2} \Rightarrow a^{1+\frac{1}{2}} \Rightarrow a^{3/2}$

D) $a.a^{1/2} = a^{1+\frac{1}{2}} \Rightarrow a^{3/2}$

Ans: A, C and D

(5) $m^3.m^{-5}.n^2 = P$ (Bases are equal power should be added)

$m^{3-5}.n^2 = P$

$\therefore P = m^{-2}n^2$

Ans: C

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REASON AND ASSERTION TYPE

⑥ Assertion: When dividing two numbers with the same base and different exponents, the result is equal to the base raised to the difference of the exponents.

This statement is true. Example $\frac{a^m}{a^n} = a^{m-n}$

Reason: This can be explained by the quotient of powers rule, which states that $\frac{a^m}{a^n} = a^{m-n}$ for any non zero number 'a'. and integers 'm' and n.

Ans: A

⑦ Assertion: The zero exponent rule holds for all real numbers, including zero, this statement is false. The zero exponent rule states that any non zero number raised to the power of zero is 1. However, 0^0 is indeterminate and not universally defined as 1.

Reason: This statement is false. while any nonzero number raised to the power of zero is indeed defined to be 1. 0^0 is generally considered inderterminate and not universally accepted as 1.

Ans: D

(8) Assertion: This statement is true.

Example $a^m \cdot a^n = a^{(m+n)}$

Reason: This statement also true. The product of powers rule indeed states that multiplying two powers with the same base results in the base raised to the sum of the exponents.

Ans: A

(9) Assertion: This statement is true.

Example. $a^{-n} = \frac{1}{a^n}$

Reason: This statement also true. The definition of negative exponents indeed states that a negative exponent indicates the reciprocal of the base raised to the corresponding positive exponent.

Ans: A

(10) Assertion: This statement is true.

Example: $a^{-n} = \frac{1}{a^n}$

Reason: This statement also true. The negative exponent rule indeed states that a negative exponent indicates the reciprocal of the base raised to the corresponding positive exponents.

Ans: A

(11) Assertion: This statement is true. For example $-a^{2n}$ is positive because the product of an even number of negative factors is positive.

Reason: This statement is true and explains why the assertion is true. An even exponent means the negative number is multiplied an even number of times, resulting in a positive value.

Ans: A

(12) Assertion: This statement is true. The power of a power rule states that $(a^m)^n = a^{mn}$, and this rule can be applied to rational exponents as well.

Reason: This statement is true and explains why the Assertion is true. Rational exponents can be written as fractions, and the power of a power rule still holds. Ans: A

(13) Assertion: This statement is true. For example $a^{-n} = \frac{1}{a^n}$ when a is non zero.

Reason: This statement is also true. The rule states that a negative exponent signifies the reciprocal of the base raised to the corresponding positive exponent.

Ans: A

(14) Assertion: This statement is true. For example $\frac{a^m}{a^n} = a^{m-n}$, where the exponent in the denominator is subtracted from the exponent in the numerator.

Reason: This statement is true, for exponential expressions with the same base, division corresponds to subtracting the exponent of the divisor from the exponent of the dividend.

Ans: A.

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(15) Assertion: This statement is true. According to the power of a power rule, $(a^m)^n = a^{m \times n}$, which involves multiplying the exponents.

Reason: This statement is true but not entirely accurate as an explanation. The power of a power rule is more directly related to the rule of repeated multiplication rather than being an extension of the multiplication rule for exponents.

Ans: B

STATEMENT TYPE:

(16) Statement-I: $x^{2017} \times \frac{1}{x^{2016}} = 1$

$x^{2017} \times x^{-2016} \Rightarrow x^{(2017-2016)} \Rightarrow x^1 = x.$

Statement-II: $\frac{x^m}{x^n} = x^{m-n}$

$\Rightarrow \frac{x^m}{x^n} \Rightarrow x^m \times x^{-n} \Rightarrow x^{m-n}.$

Ans: D

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(17) Statement-1: $a^m = a^n$ then $m=n$.

It is true, because, bases are equal then power should be equal.

Statement-II: $8^{x+2} = 2^{4x-3}$; $x=9.$

$\Rightarrow 8^{x+2} = 2^{4x-3}$

$2^{3(x+2)} = 2^{4x-3}$ (Bases are equal, power should be equal)

$3(x+2) = 4x-3$

$3x+6 = 4x-3$

$6+3 = 4x-3x$

$\therefore \boxed{x=9}$

Ans: A

COMPREHENSION TYPE

Comprehension-1:

(18) $\frac{3^{n+2} \cdot 3^{2n-1}}{3^{4n-3}} = 3^{n+2} \cdot 3^{2n-1} \cdot 3^{-(4n-3)}$

$= 3^{n+2+2n-1-4n+3}$ $= 3^{3n+1-4n+3}$

3^{n+4} (or) $\frac{3^4}{3^n}$ Ans: No correct option.

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(19) $\frac{2^{a+3} \cdot 3^{2a-1}}{6^{a-2}} = \frac{2^{a+3} \cdot 3^{2a-1}}{(2\times 3)^{a-2}} = \frac{2^{a+3} \cdot 3^{2a-1}}{2^{a-2} \cdot 3^{a-2}}$

$\Rightarrow 2^{a+3} \cdot 3^{2a-1} \times 2^{-(a-2)} \cdot 3^{-(a-2)}$

$\Rightarrow 2^{\cancel{a}+3-\cancel{a}+2} \cdot 3^{2a-1-a+2}$ $\Rightarrow 2^5 \cdot 3^{a+1}$

Ans: No correct option.

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(20) $2^{3x-1} \cdot 4^{x+2} = 8^{2x}$

$\Rightarrow 2^{3x-1} \cdot 2^{2(x+2)} = 2^{3(2x)}$

$\Rightarrow 2^{3x-1} \cdot 2^{2x+4} = 2^{6x}$

$\Rightarrow 2^{3x-1+2x+4} = 2^{6x}$

Bases are equal then powers should be equal

$3x-1+2x+4 = 6x$

$5x+3 = 6x$

$3 = 6x-5x$

$\boxed{x = 3}$

Ans: No correct option

Comprehension:

(21) $(x^{a-b})^{a+b} \cdot (x^{b-c})^{b+c} \cdot (x^{c+a})^{c-a}$

$\Rightarrow x^{(a-b)(a+b)} \cdot x^{(b-c)(b+c)} \cdot x^{(c+a)(c-a)}$ $\left[\because (x^n)^m = x^{nm}\right]$

$\Rightarrow x^{(a-b)(a+b)+(b-c)(b+c)+(c+a)(c-a)}$

$\Rightarrow x^{\cancel{a^2}+\cancel{ab}-\cancel{ba}-\cancel{b^2}+\cancel{b^2}+\cancel{bc}-\cancel{bc}-\cancel{c^2}+\cancel{c^2}-\cancel{ac}+\cancel{ac}-\cancel{a^2}}$

$\Rightarrow x^0 = 1.$

Ans: C

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(22) $\left(\frac{5}{2}\right)^{-5} \left(\frac{2}{3}\right)^{-5} = \left(\frac{5}{2}\right)^{-5} \left(\frac{3}{2}\right)^{5}$

$\Rightarrow \left(\frac{2}{5}\right)^{5} \cdot \left(\frac{3}{2}\right)^{5} = \frac{\cancel{2^5}}{5^5} \cdot \frac{3^5}{\cancel{2^5}} = \frac{3^5}{5^5}$

Ans: No correct option.

INTEGER TYPE

(23) $5^{2x-1} = 25^{x+2}$

$5^{2x-1} = 5^{2(x+2)}$

$5^{2x-1} = 5^{2x+4}.$

Bases are equal power should be equal

$2x-1 = 2x+4$

$2x-2x = 4+1$

$0 = 5.$

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$4^{x-2} = 8^{2x+1}$

$2^{2(x-2)} = 2^{3(2x+1)}$

$\Rightarrow 2^{2x-4} = 2^{6x+3}$

Bases are equal

power should be equal

$2x-4 = 6x+3$

$-4-3 = 6x-2x$

$-7 = 4x$

$x = \frac{-7}{4}$.

25

$2^{3x} = 16 \cdot 2^{x}$

$\Rightarrow 2^{3x} = 2^{4} \cdot 2^{x}$

$\Rightarrow 2^{3x} = 2^{x+4}$

B.e.p.s.e

$3x = x+4$

$3x - x = 4$

$2x = 4$

$x = \cancel{4}^{2}/\cancel{2}_{1}$

$x = 2$

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$3^{x+2} = 18$

$\frac{3^{x}}{3^{2}} = 18$

$3^{x} = 18 \times 3^{2}$

$3^{x} = 18 \times 9$

$3^{x} = 162$.

27

$8^{x-1} = 2^{x+3}$

$2^{3(x-1)} = 2^{x+3}$

$2^{3x-3} = 2^{x+3}$

Bases are equal

power should be equal

$3x-3 = x+3$

$3x - x = 6$

$2x = 6$

$x = 6/2$

$x = 3$

Matrix matching type:

(28)

(a) $5^{-5} = \frac{1}{5^5}$ Ans: t

(b) $6^{-6} \times 6^{6} = 6^{-6+6} = 6^{0} = 1$ Ans: q

(c) $x^3 = 7^3$; $x = 7$ Ans: s

(d) $2^{123} = 2^{1\times2\times3} = 2^6$ Ans:

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(29)

(a) $\frac{2^{10}}{2^{-10}} = 2^{10} \cdot 2^{10} \Rightarrow 2^{10+10} \rightarrow 2^{20}$ Ans: s

(b) $6^{18} = (2\times3)^{18} = 2^{18} \times 3^{18}$ Ans: u

(c) $2^x = 16$, $2^x = 2^4$; $x = 4$ Ans: t

(d) $5^0 - 6^0 = 1 - 1 = 0$ Ans: q

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LEARNER's TASK

Conceptual understanding Questions (CUQ's)

① $\left(\frac{-2}{5}\right)^{-3} = \left(\frac{-5}{2}\right)^{3} = \frac{-5^3}{2^3} = \frac{-125}{8}$ Ans: B

② $(-n^2)^2 \times ((-n)^2)^3 = n^{2\times 2} \times -n^{2\times 3}$

$= n^4 \times -n^6 \Rightarrow -n^{4+6} \Rightarrow -n^{10}$. Ans: A

③ $x = (a^m)^n \; ; \; y = (a^n)^m$

$x = a^{mn} \; ; \; y = a^{nm}$ Ans: A

$\therefore x = y$

④ $\left(\frac{x^m}{x^n}\right)^p \left(\frac{x^n}{x^p}\right)^m \left(\frac{x^p}{x^m}\right)^n$

$\Rightarrow \frac{x^{mp}}{x^{np}} \cdot \frac{x^{nm}}{x^{mp}} \cdot \frac{x^{np}}{x^{mn}}$

$\Rightarrow x^{mp} \cdot x^{-np} \cdot x^{nm} \cdot x^{-mp} \cdot x^{np} \cdot x^{-mn}$

$\Rightarrow x^{\cancel{mp} - \cancel{np} + nm - \cancel{mp} + \cancel{np} - \cancel{mn}}$

$\Rightarrow x^0 = 1$

Ans: B

(5) $(x^{m+n})^{m-n} \cdot (x^{n+p})^{n-p} \cdot (x^{n+m})^{n-m}$

$\Rightarrow x^{(m+n)(m-n)} \cdot x^{(n+p)(n-p)} \cdot x^{(n+m)(n-m)}$

$\Rightarrow x^{m^2-\cancel{nm}+\cancel{nm}-n^2} \cdot x^{n^2-\cancel{np}+\cancel{np}-p^2} \cdot x^{n^2+\cancel{nm}+\cancel{mn}-m^2}$

$\Rightarrow x^{m^2-n^2} \cdot x^{n^2-p^2} \cdot x^{n^2-m^2}$

$\Rightarrow x^{\cancel{m^2}-\cancel{n^2}+\cancel{n^2}-p^2+n^2-\cancel{m^2}}$

$\Rightarrow x^{n^2-p^2}$ Ans: A

(6) $20 \times (x-y)^4$

$\Rightarrow 20(x-4)^4$ Ans: A

(7) $2010 \times 2010 \times 2010 \times \ldots \times 2010$ (2010 times)

$\Rightarrow (2010)^{2010}$. Ans: B

(8) $(7^0-3^0) \times 6^0$

$\Rightarrow (1-1) \times 1$ Ans: D

$= 0 \times 1 = 0$

(9) $(-1)^{2014} = 1$ Ans: A

$\Rightarrow$ 2014 is a even power so, the negative number will be converted into positive.

(10)

$a=5,\ b=3$

$a^3-b^5 \Rightarrow 5^3-3^5$

$= 125-243=-118$

Ans: D

(11) $\frac{a^{10}}{a^{12}} = a^{10}\cdot a^{-12}$

$= a^{10-12} = a^{-2}$ Ans: C

(12) $(4^3)^2-4^8 = 4^9-4^8 = 262144-65536$

$=196608$

$= 3\times 2^{16}$ Ans: B

(13)

$8^x \times 2^2 = 2^6$

$\Rightarrow 2^{3x}\times 2^2 = 2^6$

$2^{3x+2} = 2^6.$

Bases are equal power should be equal

$3x+2=6$

$3x=6-2$

$3x=4$

$x=4/3$

Ans: B

(14) 96326000000

$\Rightarrow 9.6326\times 10^{10}.$ Ans: B

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JEE MAINS LEVEL QUESTIONS

(1) $\frac{1}{2}\times(3^x - 3^{x-1}) = 81$

$\rightarrow \frac{1}{2}3^x - \frac{1}{2}(3^{x-1}) = 81$

$$\frac{3^x}{2} - \frac{3^{x-1}}{2} = 81$$

$$\frac{3^x}{2} - \frac{\frac{3^x}{3}}{2} = 81 \Rightarrow \frac{3^x}{2} - \frac{3^x}{6} = 81$$

$$\Rightarrow \frac{3\times3^x - 3^x}{6} = 81$$

$$\frac{\cancel{2}^{1}.3^x}{\cancel{6}_{3}} = 81 \Rightarrow 3^x = 81\times3 \Rightarrow 3^x = 243$$

$\Rightarrow 3^x = 3^5$ $\quad \therefore \boxed{x = 5}$

Ans: D

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(2) $0.000019 = \frac{19}{1000000} = \frac{19}{10^6} = \frac{1.9}{10^5}$

$$= \frac{1.9}{10^5} = 1.9\times10^{-5}$$

Ans: A

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(3) $(x^{-1}+y^{-1})^{-1} = \left(\frac{1}{x}+\frac{1}{y}\right)^{-1}$

$$\Rightarrow \left(\frac{x+y}{xy}\right)^{-1} = \frac{xy}{x+y}$$

Ans: C

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(4) $\left[\frac{x^{2-3n} \cdot x^{4+3n}}{x^3}\right]^3$

$\Rightarrow \left[\frac{x^{2-\cancel{3n}+4+\cancel{3n}}}{x^3}\right]^3 = \left[\frac{x^6}{x^3}\right]^3$

$\Rightarrow \frac{x^{18}}{x^9} \Rightarrow x^{18} \cdot x^{-9} \Rightarrow x^{18-9} \Rightarrow x^9$

Ans: D

(5) $3^{x-1} = 9$

$\frac{3^x}{3} = 9 \Rightarrow 3^x = 9 \times 3$

$3^x = 3^3$

$\boxed{x=3}$

$4^{y+2} = 64$

$\frac{4^y}{4^{-2}} = 64$

$4^y = 64 \times 4^{-2}$

$4^y = \frac{64}{4^2} = \frac{\cancel{64}^{\,4}}{\cancel{16}_{\,1}}$

$4^y = 4^1$

$\therefore \boxed{y=1}$

$\therefore \frac{x}{y} = \frac{3}{1} = 3$

Ans: B

(6) $\left\{\left(\frac{3}{2}\right)^{-1} \div \left(\frac{-2}{5}\right)^{-1}\right\}$

$\Rightarrow \left(\frac{2}{3}\right) \div \left(\frac{-5}{2}\right)$

$\Rightarrow \frac{2}{3} \times \frac{-2}{5} = \frac{-4}{15}$

Ans: D

⑦ $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$

let $x^{a-b} = y$, $x^{b-a} = \frac{1}{y}$

$\frac{1}{1+y} + \frac{1}{1+\frac{1}{y}} \Rightarrow \frac{1}{1+y} + \frac{y}{y+1}$

$\Rightarrow \frac{1+y}{1+y} = 1$ Ans: C

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⑧ $\frac{5x-3}{19x+2} = \frac{3}{5}$

$5(5x-3) = 3(19x+2)$

$\Rightarrow 25x-15 = 57x+6$

$\Rightarrow 57x-25x = -15-6$

$\Rightarrow 32x = -21$

$x = -21/32$ Ans: A

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⑨ $x+y = 5$, $x^2+y^2 = 10$, $xy = ?$

$x+y=5$ (Squaring on Both sides)

$(x+y)^2 = 5^2$

$x^2+2xy+y^2 = 25$

$x^2+y^2+2xy = 25$

$10+2xy = 25$

$2xy = 25-10$

$xy = \frac{15}{2}$

$xy = 7\frac{1}{2}$

Ans: A

(10) $(50.5)^2-(49.5)^2 \Rightarrow a^2-b^2=(a+b)(a-b)$

$\Rightarrow (50.5+49.5)(50.5-49.5)$

$\Rightarrow (100)(1)=100$ Ans: B

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(12) $2^{0.64} \times 2^{0.36}$

$\Rightarrow 2^{0.64+0.36} = 2^1 = 2$ Ans: B

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(13) $a^3 \times b^2 = a^5 \times b$

$\Rightarrow \frac{a^3 x}{a^5} = \frac{b}{b^2}$ $\frac{a^3 \times b^3}{}$

$\Rightarrow a^{3-5} = b^{1-2}$

$\Rightarrow a^{-2} = b^{-1}$

$= \frac{1}{a^2} = \frac{1}{b} \Rightarrow b = a^2$

Substitute b value in above equation

$\Rightarrow \frac{a^3}{a^5} \times (a^2)^2$

$\Rightarrow \frac{a^3}{a^5} \times a^4 \Rightarrow a^7 \cdot a^{-5}$

$= a^2.$

Ans: A

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(14) $x=2,\ y=3$; $\left[\frac{1}{x^x}+\frac{1}{y^y}\right]$

$= \left[\frac{1}{2^2}+\frac{1}{3^3}\right] = \left[\frac{1}{4}+\frac{1}{27}\right]$

$= \frac{27+4}{108} = \frac{31}{108}$ Ans: B

---

(15) $\left[\dfrac{x^4 - x^7}{x^3}\right] = \dfrac{x^4}{x^3} - \dfrac{x^7}{x^3}$

$\Rightarrow x^4 \cdot x^{-3} - x^7 \cdot x^{-3}$

$\Rightarrow x^{4-3} - x^{7-3} \Rightarrow x^1 - x^4$

$\therefore x - x^4$ Ans: No correct option

(16) $3^4 \times x = 729$

$81 \times x = 729 \Rightarrow x = \dfrac{\cancel{729}^{\,\cancel{81}\,9}}{\cancel{81}_{\,\cancel{9}\,1}} \Rightarrow x = 9$

$x = 9 \Rightarrow x = 3^2.$ Ans: No correct option

(17) $\left(\dfrac{x^m}{x^n}\right)^p \left(\dfrac{x^n}{x^p}\right)^m \left(\dfrac{x^p}{x^m}\right)^n$

$\Rightarrow \left(\dfrac{x^{mp}}{x^{np}}\right)\left(\dfrac{x^{nm}}{x^{mp}}\right)\left(\dfrac{x^{np}}{x^{mn}}\right) \Rightarrow x^{mp} \cdot x^{-np} \cdot x^{nm} \cdot x^{-mp} \cdot x^{np} \cdot x^{-mn}$

$\Rightarrow x^{\cancel{mp} - \cancel{np} + \cancel{nm} - \cancel{mp} + \cancel{np} - \cancel{mn}}$

$\Rightarrow x^0 = 1$ Ans: D

(18) $3^{2x-1} = 27$

$3^{2x-1} = 3^3$

Bases are equal power should be equal

$2x - 1 = 3$

$2x = 3 + 1$

$x = \cancel{4}^{2} / \cancel{2}_{1}$

$x = 2$ Ans: C

(19) $(2x^2)^3 \times (3x)^2$

$\Rightarrow 2^3 \times (x^2)^3 \times (3)^2 \times (x)^2$

$\Rightarrow 8 \times x^6 \times 9 \times x^2$

$= 72 \times x^8$

Ans: No correct option

(20) $\frac{a^3 . b^{-2}}{a^{-1} . b^4} = \frac{a^3 \times a^1}{b^4 \times b^2} = \frac{a^{3+1}}{b^{4+2}} = \frac{a^4}{b^6}$ (or) $a^4 . b^{-6}$

Ans: No correct option.

(21) $\left(\frac{p^2 q^{-3}}{r^2}\right)^{-1} . \left(\frac{r^3}{pq}\right)^2$

$\Rightarrow \left(\frac{p^2}{q^3 r^2}\right)^{-1} . \frac{r^6}{p^2 q^2} \Rightarrow \left(\frac{q^3 r^2}{p^2}\right) \times \frac{r^6}{p^2 q^2}$

$\Rightarrow \frac{q^3 q^{-2} . r^2 . r^6}{p^2 . p^2} = \frac{q^{3-2} \, r^{2+6}}{p^{2+2}} = \frac{q^1 r^8}{p^4}$

$\Rightarrow \frac{q^1 r^8}{p^4} = q^1 r^8 p^{-4}$ Ans: No correct option.

(22) $(ab)^2 = \frac{c}{d3}$

Multiply both side with ab.

$(ab)^2 \times ab = ab \times \frac{c}{d3}$ Ans: No correct option.

$(ab)^3 = \frac{abc}{d3}$

ADVANCED LEVEL QUESTIONS

MULTI CORRECT ANSWER TYPE

① $m^n \times m^{-2n} \times m^{4n} = m^p$

$m^{n-2n+4n} = m^p \Rightarrow m^p = m^{3n}$

Bases are equal, power also equal

$3n = p$ Ans: C

② $\left(\frac{x^3}{y^2}\right)^{-2} \cdot \left(\frac{y^5}{x^{-1}}\right)^3$

$\Rightarrow \left(\frac{y^2}{x^3}\right)^2 \cdot \left(\frac{y^5}{\frac{1}{x}}\right)^3$

$\Rightarrow \left(\frac{y^2}{x^3}\right)^2 (y^5 . x)^3$

$\frac{y^4}{x^6} \times y^{15} x^3$

$\Rightarrow y^{15+4} x^{3-6}$

$\Rightarrow y^{19} x^{-3}$

Ans: No correct optio

③ $8^{2/3} = (2^{\cancel{3}})^{2/\cancel{3}} = 2^2 = 4$ Ans: A, B and C.

④ $(2^3)^3 \times (2^7)^2$

$= 2^9 \times 2^{14}$ (or) $2^{9+14} = 2^{23}$

Ans: A, B and D

(5) $$\frac{9^n \times 3^2 \times \left(3^{-n/2}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{\left(9^n \times 3^2 \times 3^n - 3^{3n}\right)}{3^{3m} \times 8} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 8} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 8} = \frac{1}{27} \Rightarrow \frac{3^{(3n+2)} - 3^{3n}}{3^{3m} \times 8} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{3n} \times \left(3^{2n} - 1\right)}{3^{3m} \times 8} = \frac{1}{27}$$

$$3n = 3m - 3$$

$$3n - 3m = -3$$

$$\frac{-3n}{3} + \frac{3m}{3} = \frac{3}{3}$$

$$m - n = 1$$

Ans: A

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REASON AND ASSERTION TYPE

(6) The assertion states that $9^{3/2} = \sqrt{9^3}$, which is true because $9^{3/2} = (9^3)^{1/2} = \sqrt{9^3}$.

The reason given is also true and correctly explains the assertion. The exponent $3/2$ can be written as $(3)^{\frac{1}{2}}$ wich means taking the square root of 9 raised to the power of 3. This is exactly what the reason states.

Ans: A

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(7) The assertion states that $7^3 \div (7^2)^2 = 7^{-1}$ which is true because,

$$7^3 \div (7^2)^2 = \frac{7^3}{7^4} \Rightarrow 7^{3-4} = 7^{-1}$$

The reason given also true, and correctly explains the assertion when dividing two exponential expressions with the same base, we subtract their exponents, which is exactly what was done in the calculation above. the fact that second term is raised to the power of 2 i.e $(7^2)^2$ is correctly handled by applying the power rule $(a^m)^n = a^{mn}$.

Ans: A

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(8) The assertion states that $7^{-4} \times 5^3 = \frac{5^3}{2^4}$
which is false. because

$$7^{-4} \times 5^3 = \frac{1}{7^4} \times 5^3 \Rightarrow \frac{5^3}{7^4}.$$

The reason given is true, but it does not correctly explain the assertion. the product of exponential expressions with different bases can inded be simplified by combining the terms, but in this case, the assertion is incorrect

The correct result is $\frac{5^3}{7^4}$. not $\frac{5^3}{2^4}$.

Ans: D

STATEMENT TYPE:

(9) Statement-1: is true. $(2^3)^2 = 2^{(3\times2)} = 2^6 = 64$.

$3^2 \times 3^3 = 2^5$ is true.

$3^2 \times 3^3 = 3^{2+3} = 3^5$.

Statement-II: $(a^m)^n = a^{mn}$ is true

$a^m \times a^n = a^{m+n}$ is true.

Ans: A

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(10) Statement-1: $a^m = a^n$ $(a \neq 1)$ then $m=n$ is true because, If the bases are the same a, and the values are equal, then the exponents m and n must also be equal. This is a fundamental property of exponential equations.

Statement-II: If $8^{x+2} = 2^{4x-3}$, then the value of x is 9 is true because

$8^{x+2} = 2^{3(x+2)} = 2^{3x+6}$

$2^{3x+6} = 2^{4x-3}$

$3x+6 = 4x-3$

$6+3 = 4x-3x$

$\boxed{9 = x}$

Ans: A

(11) Statement-1: $x^y = y^x$, then $\left(\frac{x}{y}\right)^{x/y} = x^{(x/y - 1)}$ is true because

$x^y = y^x \Rightarrow x = y^{x/y} \Rightarrow x = (y^x)^{1/y}$

$\left(\frac{x}{y}\right)^{x/y} = \left(\frac{(y^x)^{1/y}}{y}\right)^{x/y} = \left(\frac{(y^x)^{1/y}}{(y^x)^{1/x}}\right)^{x/y}$

$\Rightarrow (y^x)^{(1/y - 1/x)^{x/y}} \Rightarrow x^{(x/y - 1)}$

Statement-II: $2^x = 4^y \Rightarrow 2^x = (2^2)^y \Rightarrow x = 2y$

$4^y = 8^z \Rightarrow (2^2)^y = (2^3)^z \Rightarrow 2y = 3z$

$xyz = 288 \Rightarrow 2y \times y\,(3z/2) = 288 \Rightarrow y^2 z = 96,\ z = 2y/3$

$y^2 (2y/3) = 96 \Rightarrow y^3 = 144 \Rightarrow y = 6$; Substitute $y = 6$

$\frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z} = \frac{11}{8y} = \frac{11}{48} = \frac{11}{96}$ Ans: A