WS-15 for 8th class

Itask

(1) Given $\vec{P} = 2\hat{i}+3\hat{j}-4\hat{k}$

$\vec{Q} = 5\hat{i}+2\hat{j}+4\hat{k}$

Angle between $\vec{P}$ and $\vec{Q}$

$\cos\theta = \dfrac{\vec{P}\cdot\vec{Q}}{|\vec{P}||\vec{Q}|}$

$\vec{P}\cdot\vec{Q} = (2\hat{i}+3\hat{j}-4\hat{k})\cdot(5\hat{i}+2\hat{j}+4\hat{k})$

$= 2(5)+3(2)-4(4)$

$\Rightarrow 10+6-16 = 0$

$\cos\theta = \dfrac{0}{|\vec{P}||\vec{Q}|} = 0$

$\theta = 90^\circ$

(2) $\vec{F} = 5\hat{i}-3\hat{j}+2\hat{k}$

$\vec{r_1} = 2\hat{i}+7\hat{j}+4\hat{k}$

$\vec{r_2} = 5\hat{i}+2\hat{j}+8\hat{k}$

$\vec{S} = \vec{r_2}-\vec{r_1}$

$\Rightarrow (5\hat{i}+2\hat{j}+8\hat{k})-(2\hat{i}+7\hat{j}+4\hat{k})$

$\vec{S} = 3\hat{i}-5\hat{j}+4\hat{k}$

$W = \vec{F}\cdot\vec{S}$

$= (5\hat{i}-3\hat{j}+2\hat{k})\cdot(3\hat{i}-5\hat{j}+4\hat{k})$

$= 5(3)+(-3)(-5)+2(4)$

$\Rightarrow 15+15+8$

$= 38\,J$

(3) $\vec{A} = \overset{x}{1}\hat{i}+\overset{y}{2}\hat{j}$, $|\vec{B}| = 3\sqrt{5}$

$|\vec{A}| = \sqrt{x^2+y^2} = \sqrt{1^2+2^2} = \sqrt{5}$

The vector $\perp^{er}$ to $\vec{A}$ is

$\vec{B} = |\vec{B}| \times \dfrac{\vec{A}}{|\vec{A}|}$

$= 3\sqrt{5} \times \dfrac{(\hat{i}+2\hat{j})}{\sqrt{5}}$

$\vec{B} = 3(\hat{i}+2\hat{j})$

$\vec{B} = 3\hat{i}+6\hat{j}$

(4) Given $\vec{P} = a\hat{i}+a\hat{j}+3\hat{k}$

$\vec{Q} = a\hat{i}-2\hat{j}-\hat{k}$

Given $\vec{P}$ and $\vec{Q}$ are Perpendicular

$\Rightarrow \vec{P}\cdot\vec{Q} = 0$

$\Rightarrow (a\hat{i}+a\hat{j}+3\hat{k})\cdot(a\hat{i}-2\hat{j}-\hat{k}) = 0$

$\Rightarrow a\cdot a+a(-2)+3(-1) = 0$

$\Rightarrow a^2-2a-3 = 0$

$\Rightarrow a^2-3a+a-3 = 0$

$\Rightarrow a(a-3)+1(a-3) = 0$

$\Rightarrow (a-3)(a+1) = 0$

$\Rightarrow a-3 = 0$ (or) $a+1 = 0$

$\Rightarrow a = 3$ (or) $a = -1$.

(5) Given $F = 2\hat{i}+3\hat{j}+2\hat{k}$ N

$\vec{S} = 3\hat{i}+4\hat{j}+5\hat{k}$ m

$t = 4$ sec.

Power $= \dfrac{W}{t} = \dfrac{\vec{F}\cdot\vec{S}}{t}$

$= \dfrac{(2\hat{i}+3\hat{j}+2\hat{k})\cdot(3\hat{i}+4\hat{j}+5\hat{k})}{4}$

$= \dfrac{2\times3+3\times4+2\times5}{4}$

$= \dfrac{6+12+10}{4}$

$= \dfrac{28}{4} = 7W$

(6) Given $\vec{A} = \overset{x}{2}\hat{i}+\overset{y}{3}\hat{j}$

$\vec{B} = 2\hat{j}+3\hat{k}$

$|\vec{A}| = \sqrt{x^2+y^2} = \sqrt{2^2+3^2}$

$= \sqrt{13}$.

$\vec{A}\cdot\vec{B} = (2\hat{i}+3\hat{j})\cdot(2\hat{j}+3\hat{k})$

$\Rightarrow 3(2) = 6$

Component of $\vec{B}$ along $\vec{A}$ is $B\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}|}$

$B\cos\theta = \dfrac{6}{\sqrt{13}}$

(7) Given

$A\cos\theta = \sqrt{3}\,B\cos\theta$

$\Rightarrow A = \sqrt{3}\,B$

$\Rightarrow \dfrac{A}{B} = \dfrac{\sqrt{3}}{1}$

(8) Given $\vec{F} = 8\hat{i}+4\hat{j}$ N

$\vec{S} = 3\hat{i}-3\hat{j}$ m

$P = 0.6W$

$P = \dfrac{W}{t} = \dfrac{\vec{F}\cdot\vec{S}}{t}$

$\Rightarrow 0.6 = \dfrac{(8\hat{i}+4\hat{j})\cdot(3\hat{i}-3\hat{j})}{t}$

$\Rightarrow 0.6 = \dfrac{8(3)+4(-3)}{t}$

$\Rightarrow t = \dfrac{24-12}{0.6} = \dfrac{120}{6} = 20$ sec

(9) Given $\vec{A}+\vec{B} = \vec{R}$

$\vec{B} = \vec{R}-\vec{A}$ and Given $A = -R$.

$\Rightarrow \vec{B} = \vec{R}-(-\vec{R})$

$\vec{B} = 2\vec{R}$.

$2\vec{A}+\vec{B} \perp^{er}$ $\vec{B}$.

$\Rightarrow (2\vec{A}+\vec{B})\cdot\vec{B} = 0$

$\Rightarrow 2\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{B} = 0$

$\Rightarrow 2\vec{A}(\vec{R}-\vec{A})+\cancel{B^2} B^2 = 0$

$\Rightarrow 2\vec{A}\cdot(\vec{R}-\vec{A})+(\vec{R}-\vec{A})^2 = 0$

$\Rightarrow \cancel{2\vec{A}\vec{R}}-2\vec{A}\cdot\vec{A}+R^2+A^2-\cancel{2RA} = 0$

$\Rightarrow -2A^2+R^2+A^2 = 0$

$\Rightarrow R^2 = A^2 \Rightarrow R = \pm A$.

(10) Given $\vec{v} = 2\hat{i} + c\hat{j}$.

$\vec{a} = 3\hat{i} + 4\hat{j}$

Given $\vec{v}$ ⊥er to $\vec{a}$

$\Rightarrow \vec{v} \cdot \vec{a} = 0$

$\Rightarrow (2\hat{i} + c\hat{j}) \cdot (3\hat{i} + 4\hat{j}) = 0$

$\Rightarrow 2(3) + c(4) = 0$

$\Rightarrow 4c = -6$

$\Rightarrow c = -\frac{3}{2} = -1.5$

(11) Given $\vec{a} = m\vec{b} + \vec{c}$

By multiplying with $\vec{b}$ on both sides.

$\Rightarrow \vec{a} \cdot \vec{b} = (m\vec{b} + \vec{c}) \cdot \vec{b}$

$\Rightarrow \vec{a} \cdot \vec{b} = m\vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b}$

$\Rightarrow \vec{a} \cdot \vec{b} = mb^2 + \vec{c} \cdot \vec{b}$

$\Rightarrow \vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b} = m b^2$

$\Rightarrow m = \frac{\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b}}{b^2}$

L Task

(1) Given $\vec{a} = \overset{x}{2}\hat{i} + \overset{y}{1}\hat{j} - \overset{z}{1}\hat{k}$

$\vec{B} = \underset{x}{1}\hat{i} - \underset{3}{1}\hat{k}$

$|\vec{a}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{2^2 + 1^2 + (-1)^2}$

$= \sqrt{6}$

$|\vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\vec{a} \cdot \vec{b} = (2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{k})$

$\Rightarrow 2 + (-1)(-1) = 2 + 1$

$= 3$

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{3}{\sqrt{6}\sqrt{2}}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30°$

(2) let $\vec{A} = 3\hat{i} + 4\hat{j}$

$\vec{B} = \underset{x}{1}\hat{i} + \underset{y}{1}\hat{j}$

$|\vec{B}| = \sqrt{x^2 + y^2}$

$= \sqrt{1^2 + 1^2} = \sqrt{2}$

$\vec{A} \cdot \vec{B} = (3\hat{i} + 4\hat{j}) \cdot (\hat{i} + \hat{j})$

$= 3 + 4$

$= 7$

component of $\vec{A}$ along $\vec{B}$

$A\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$

$= \frac{7}{\sqrt{2}}$

(3) $|\vec{a}_1 + \vec{a}_2| = \sqrt{3}$

$\Rightarrow \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\theta} = \sqrt{3}$

$\Rightarrow 1^2 + 1^2 + 2 \times 1 \times 1 \cos\theta = 3$

$\Rightarrow 2 + 2\cos\theta = 3$

$\Rightarrow 2\cos\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\theta = 60°$

$(a_1 - a_2) \cdot (2a_1 + a_2) =$

$\Rightarrow 2a_1 \cdot a_1 + a_1 \cdot a_2 - 2a_1 \cdot a_2 + a_2 \cdot a_2$

$\Rightarrow 2a_1^2 - a_1a_2\cos\theta - a_2^2$

$\Rightarrow 2(1)^2 - 1 \times 1 \cos 60 - 1^2$

$= 2 - \frac{1}{2} - 1 = 1 - \frac{1}{2} = \frac{1}{2}$

(4) $|\vec{a}| = |\vec{b}| = 1$

$(\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 4\vec{b}) = 0$

$\Rightarrow 5\vec{a} \cdot \vec{a} - 4\vec{a} \cdot \vec{b} + 2 \times 5\vec{b} \cdot \vec{a} - 2 \times 4(\vec{b} \cdot \vec{b}) = 0$

$\Rightarrow 5a^2 - 4ab\cos\theta + 10ab\cos\theta - 8b^2 = 0$

$\Rightarrow 5(1)^2 + 6ab\cos\theta - 8(1)^2 = 0$

$\Rightarrow 5 + 6\cos\theta - 8 = 0$

$\Rightarrow 6\cos\theta = 3$

$\Rightarrow \cos\theta = \frac{3}{6} = \frac{1}{2}$

$\theta = 60°$

(5) $\vec{A} = 5\hat{i} + 7\hat{j} - 3\hat{k}$

$\vec{B} = 2\hat{i} + 2\hat{j} - c\hat{k}$

$\vec{A}$ is ⊥er to $\vec{B}$

(ie) $\vec{A} \cdot \vec{B} = 0$

$\Rightarrow (5\hat{i} + 7\hat{j} - 3\hat{k}) \cdot (2\hat{i} + 2\hat{j} - c\hat{k}) = 0$

$\Rightarrow 5 \times 2 + 7 \times 2 + 3c = 0$

$\Rightarrow 10 + 14 + 3c = 0$

$\Rightarrow 24 + 3c = 0$

$\Rightarrow 3c = -24$

$\Rightarrow c = -8.$

(6) let $\vec{F} = 3\hat{i} + 4\hat{k}$ N

$\vec{s} = 2\hat{i} + 4\hat{j}$ m

$t = 3$ sec

$P = \frac{\omega}{t} = \frac{\vec{F} \cdot \vec{s}}{t}$

$P = \frac{(3\hat{i} + 4\hat{k}) \cdot (2\hat{i} + 4\hat{j})}{3}$

$P = \frac{(3 \times 2) + (4 \times 4)}{3}$

$P = \frac{6 + 16}{3} = \frac{22}{3} = 7.3\omega$

(7) let $\vec{A} = 2\hat{i} + 5\hat{k}$

$\vec{B} = 3\hat{j} + 4\hat{k}$

$\vec{A} \cdot \vec{B} = (2\hat{i} + 5\hat{k}) \cdot (3\hat{j} + 4\hat{k})$

$\Rightarrow 5 \times 4 =$

$= 20$

(10) $|\vec{A}| = 3$

$|\vec{B}| = 4$

$\vec{A} \cdot \vec{B} = 6$

$\Rightarrow AB\cos\theta = 6$

$\Rightarrow \cancel{3} \times 4 \cos\theta = \cancel{6}\, 2$

$\Rightarrow 4\cos\theta = 2$

$\Rightarrow \cos\theta = \frac{2}{4}$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\theta = 60^\circ$

(1) G

$\vec{r}_1$

$\vec{r}_2 =$

$\vec{r} =$

$= C$

$\vec{r} =$

$\cdot(3\hat{j}+5\hat{k})$

(8) Given $(3\hat{i}-2\hat{j}+2\hat{k})\cdot(2\hat{i}-x\hat{j}+3\hat{k})=12$

$\Rightarrow 3\times 2+(-2)(-x)+2\times 3=-12$

$\Rightarrow 6+2x+6=-12$

$\Rightarrow 2x+12=-12$

$\Rightarrow 2x=-12-12$

$\Rightarrow 2x=-24$

$\Rightarrow x=-12.$

th class

(9) $\vec{A}=6\hat{i}+5\hat{j}$

$\vec{B}=4\hat{i}-3\hat{j}$

$\vec{A}+\vec{B}=6\hat{i}+5\hat{j}+4\hat{i}-3\hat{j}$

$=10\hat{i}+2\hat{j}$

$\vec{A}-\vec{B}=6\hat{i}+5\hat{j}-4\hat{i}+3\hat{j}$

$=2\hat{i}+8\hat{j}.$

$(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B})=(10\hat{i}+2\hat{j})\cdot(2\hat{i}+8\hat{j})$

$=10\times 2+2\times 8$

$=20+16=36$