

Task

①

Given $\vec{p} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{q} = 5\hat{i} + 2\hat{j} + 4\hat{k}$

$$\vec{p} \cdot \vec{q} = (2\hat{i} + 3\hat{j} - 4\hat{k}) \cdot (5\hat{i} + 2\hat{j} + 4\hat{k})$$

$$= 2 \cdot 5 + 3 \cdot 2 - 4 \cdot 4 = 10 + 6 - 16 = 0$$

$$\cos \theta = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}| |\vec{q}|} = \frac{0}{|\vec{p}| |\vec{q}|} = 0 \Rightarrow \theta = 90^\circ \rightarrow D$$

②

Given $\vec{F} = 5\hat{i} - 3\hat{j} + 2\hat{k}$; $r_1 = 2\hat{i} + 7\hat{j} + 4\hat{k}$; $r_2 = 7\hat{j} + 5\hat{i} + 8\hat{k}$

Displacement $\vec{s} = r_2 - r_1 \Rightarrow 5\hat{i} + 2\hat{j} + 8\hat{k} - 2\hat{i} - 7\hat{j} - 4\hat{k}$
 $= 3\hat{i} - 5\hat{j} + 4\hat{k}$

$\therefore$ work done $w = \vec{F} \cdot \vec{s} = (5\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (3\hat{i} - 5\hat{j} + 4\hat{k})$
 $= 5 \cdot 3 + (-3)(-5) + 2(4)$
 $= 15 + 15 + 8 = 38 \text{ units} \rightarrow A$

③

Given $\vec{A} = \hat{i} + 2\hat{j}$; $|\vec{B}| = 3\sqrt{5}$

$\vec{C}$ is a vector which is $\perp$ to $\vec{A}$ and $|\vec{C}| = |\vec{B}|$

let $\vec{C} = x\hat{i} + y\hat{j}$ $\Rightarrow \vec{C} \cdot \vec{A} = 0 \Rightarrow (x\hat{i} + y\hat{j}) \cdot (\hat{i} + 2\hat{j}) = 0$
 $\Rightarrow x + 2y = 0 \rightarrow (1) \Rightarrow x = -2y$

also $= \sqrt{x^2 + y^2} = 3\sqrt{5} \Rightarrow x^2 + y^2 = 45 \rightarrow (2)$

$\Rightarrow (-2y)^2 + y^2 = 45 \Rightarrow 5y^2 = 45$
 $y = \sqrt{9} = 3$

$\therefore x = -6$

$\therefore \vec{C} = -6\hat{i} + 3\hat{j} \rightarrow B$

(4)

Given $\vec{p} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{q} = a\hat{i} - 2\hat{j} - \hat{k}$

$$\therefore \vec{p} \cdot \vec{q} = 0$$

$$\Rightarrow (a\hat{i} + a\hat{j} + 3\hat{k}) \cdot (a\hat{i} - 2\hat{j} - \hat{k}) = 0$$

$$\Rightarrow a^2 - 2a - 3 = 0 \Rightarrow a^2 - 3a + a - 3 = 0$$

$$\Rightarrow a(a-3) + 1(a-3) = 0$$

$$\Rightarrow (a-3) = 0 \text{ (or) } (a+1) = 0$$

$$\Rightarrow a = 3 \text{ (or) } -1 \rightarrow D$$

(5)

Given

$$F = 2\hat{i} + 3\hat{j} + 2\hat{k} \text{ N} ; \vec{S} = 3\hat{i} + 4\hat{j} + 5\hat{k} \text{ m} ; t = 4 \text{ sec}$$

$$\text{work} = \vec{F} \cdot \vec{S} = (2\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (3\hat{i} + 4\hat{j} + 5\hat{k})$$

$$= 2 \times 3 + 3 \times 4 + 2 \times 5$$

$$\Rightarrow 6 + 12 + 10 = 28$$

$$\text{Power} = \frac{W}{t} = \frac{28}{4} = 7 \text{ W} \rightarrow C$$

(6)

Given $\vec{A} = 2\hat{i} + 3\hat{j}$ & $\vec{B} = 2\hat{i} + 3\hat{j}$

component of $\vec{B}$ along $\vec{A} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|}$

$$= \frac{(2\hat{i} + 3\hat{j}) \cdot (2\hat{i} + 3\hat{j})}{\sqrt{2^2 + 3^2}} = \frac{2 \times 2 + 3 \times 3 + 3 \times 0}{\sqrt{4 + 9}}$$

$$\Rightarrow \frac{13}{\sqrt{13}} \rightarrow C$$

(7)

Given that component of $\vec{A}$ along $\vec{B} = \sqrt{3}$ component of $\vec{B}$ along $\vec{A}$

$$A \cos \theta = \sqrt{3} B \cos \theta$$

$$\Rightarrow A = \sqrt{3} B \Rightarrow \frac{A}{B} = \frac{\sqrt{3}}{1} = A$$

(8)

Given $\vec{F} = 8\hat{i} + 4\hat{j}$ N; $\vec{S} = 3\hat{i} - 3\hat{j}$ m; Power = 0.6 W

$$\text{Work} = \vec{F} \cdot \vec{S} = (8\hat{i} + 4\hat{j}) \cdot (3\hat{i} - 3\hat{j})$$

$$= 8 \times 3 + 4(-3) = 24 - 12 = 12$$

$$\text{Power} = \frac{W}{t} \Rightarrow t = \frac{W}{P} = \frac{12}{0.6} = \frac{120}{6} = 20 \text{ s} \rightarrow A$$

(9)

Given $\vec{A} + \vec{B} = \vec{R} \rightarrow (1)$ $2\vec{A} + \vec{B}$ is perpendicular to $\vec{B}$

$$\Rightarrow (2\vec{A} + \vec{B}) \cdot \vec{B} = 0 \Rightarrow 2\vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{B} = 0$$

$$\Rightarrow 2AB = -\vec{B} \cdot \vec{B}$$

$$\Rightarrow 2AB = -B^2 \rightarrow (2)$$

$$\text{From } \vec{A} + \vec{B} = \vec{R}$$

$$\Rightarrow (\vec{A} + \vec{B}) \cdot \vec{B} = \vec{R} \cdot \vec{B} \quad (or)$$

$$\Rightarrow \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{B} = \vec{R} \cdot \vec{B}$$

$$\Rightarrow \vec{A} \cdot \vec{B} - 2AB = \vec{R} \cdot \vec{B}$$

$$\Rightarrow -2AB = \vec{R} \cdot \vec{B}$$

$$\Rightarrow -A \times (-2A) = \vec{R} (2A)$$

$$\Rightarrow 2A^2 = 2AR$$

$$\Rightarrow R = A$$

$$\text{From (1)} (\vec{A} + \vec{B})^2 = R^2$$

$$\Rightarrow A^2 + B^2 + 2AB = R^2$$

$$\Rightarrow A^2 - 2AB + 2AB = R^2$$

$$= R^2 = A^2 \quad [\text{From (2)}]$$

$$\Rightarrow \underline{R = A}$$

$$\rightarrow A$$

(10)

Given $\vec{v} = 2\hat{i} + c\hat{j}$ and $\vec{a} = 3\hat{i} + 4\hat{j}$.

Given velocity and acceleration are perpendicular

$$\Rightarrow \vec{v} \cdot \vec{a} = 0$$

$$\Rightarrow (2\hat{i} + c\hat{j}) \cdot (3\hat{i} + 4\hat{j}) = 0$$

$$\Rightarrow 2 \times 3 + c \times 4 = 0$$

$$\Rightarrow 6 + 4c = 0 \Rightarrow 4c = -6 \Rightarrow c = -\frac{3}{2} = -1.5 \rightarrow c$$

Advanced level

(1)

Given $\vec{a} = m\vec{b} + \vec{c}$.

on doing dot product with $\vec{b}$ on both sides

$$\vec{a} \cdot \vec{b} = (m\vec{b} + \vec{c}) \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = m\vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b} = m b^2$$

$$\Rightarrow m = \frac{\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b}}{b^2} \rightarrow A$$

(3)

$$\textcircled{c} \sqrt{\vec{A} \cdot \vec{A}} = \sqrt{AA \cos 0} = \sqrt{A^2 \cos 0} = \sqrt{A^2} = A \quad [\text{Both } \vec{A} \text{ and } \vec{A} \text{ are Parallel}]$$

(4)

$$\vec{a} = m\vec{b}$$

on doing dot product with $\vec{b}$ on both side

$$\Rightarrow \vec{a} \cdot \vec{b} = m\vec{b} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = m b^2 \Rightarrow m = \frac{\vec{a} \cdot \vec{b}}{b^2} \quad (A, B, C, D)$$

(4)

Given $\vec{A} = 5\hat{i} + 6\hat{j} + 3\hat{k}$ and $\vec{B} = 6\hat{i} - 2\hat{j} - 6\hat{k}$

$$\vec{A} \cdot \vec{B} = [5\hat{i} + 6\hat{j} + 3\hat{k}] \cdot [6\hat{i} - 2\hat{j} - 6\hat{k}]$$

$$= 5 \cdot 6 + 6(-2) + 3(-6) = 30 - 12 - 18 = 0.$$

$$\vec{B} \cdot \vec{A} = 0 \rightarrow A, B, O.$$

(5)

Given $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C}$

$$\Rightarrow AB \cos \theta_1 = BC \cos \theta_2 \quad \text{if } \theta_1 = \theta_2 \text{ then } A = C$$

Other wise $\vec{A}$ is not equal to $\vec{C}$.

$\rightarrow D$

(6)

Given $\theta = 60^\circ$

$$\vec{a} \cdot \vec{b} = ab \cos \theta = ab \cos 60^\circ = \frac{ab}{2} \rightarrow c$$

(7)

Component of $\vec{b}$ along $\vec{a} = b \cos \theta = \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|}$

$$= b \frac{\vec{a}}{|\vec{a}|} = b \hat{a} \rightarrow B$$

(8)

Given $\vec{A} = \hat{i} + y\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$

$\vec{A} \perp \vec{B}$ to $\vec{B} \quad (\vec{A} \cdot \vec{B}) = 0 \Rightarrow (\hat{i} + y\hat{j} + 2\hat{k}) \cdot (2\hat{i} + 4\hat{j} - 3\hat{k}) = 0$

$$\Rightarrow 2 + 4y - 6 = 0$$

$$\Rightarrow 4y - 4 = 0 \Rightarrow 4y = 4 \Rightarrow y = 1$$



8

Given $\vec{P} = \hat{i} + 2\hat{j} - 4\hat{k}$ $\vec{Q} = \hat{i} + 2\hat{j} - \hat{k}$

$$\vec{P} + \vec{Q} = \hat{i} + 2\hat{j} - 4\hat{k} + \hat{i} + 2\hat{j} - \hat{k} = 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\vec{P} - \vec{Q} = \hat{i} + 2\hat{j} - 4\hat{k} - \hat{i} - 2\hat{j} + \hat{k} = -3\hat{k}$$

$$(\vec{P} + \vec{Q}) \cdot (\vec{P} - \vec{Q}) = (2\hat{i} + 4\hat{j} - 5\hat{k}) \cdot (-3\hat{k})$$

$$= (-5)(-3) = 15$$

20

(a)

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

$$\Rightarrow A^2 + B^2 + 2AB \cos \theta = A^2 + B^2 - 2AB \cos \theta$$

$$\Rightarrow 2AB \cos \theta = -2AB \cos \theta \Rightarrow 4AB \cos \theta = 0$$

$$\Rightarrow \cos \theta = 0$$

$$\theta = 90^\circ$$

(b)

$$|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$$

$$\Rightarrow AB \sin \theta = AB \cos \theta \Rightarrow \sin \theta = \cos \theta \Rightarrow \theta = 45^\circ$$

(c)

$$\vec{A} \cdot \vec{B} = \frac{AB}{2}$$

$$\Rightarrow AB \cos \theta = \frac{AB}{2} \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

(d)

$$|\vec{A} \times \vec{B}| = \frac{AB}{2}$$

$$\Rightarrow AB \sin \theta = \frac{AB}{2} \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Task

Ques

(4)

Given

$$\vec{A} + \vec{B} = \vec{C}$$

on doing dot product with $\vec{B}$ on both sides

$$(\vec{A} + \vec{B}) \cdot \vec{B} = \vec{C} \cdot \vec{B}$$

$$\Rightarrow \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{B} = \vec{C} \cdot \vec{B}$$

$$\Rightarrow \vec{B}^2 = \vec{C} \cdot \vec{B} - \vec{A} \cdot \vec{B}$$

$$\Rightarrow \vec{B} = \sqrt{\vec{C} \cdot \vec{B} - \vec{A} \cdot \vec{B}}$$

(5)

Given $\vec{A} \cdot \vec{B} = 0$ i.e. $\vec{A}$ is perpendicular to $\vec{B}$

$\vec{A} \cdot \vec{C} = 0$ i.e. $\vec{A}$ is perpendicular to $\vec{C}$

(i.e.) $\vec{A}$ must lie along the direction that is perpendicular to the plane formed by $\vec{B}$ and $\vec{C}$.

$$\text{so } \vec{A} = \vec{B} \times \vec{C}$$

(6)

we know

$$\text{torque } \vec{\tau} = \vec{r} \times \vec{F} \rightarrow (1)$$

$$\therefore \vec{\tau} \cdot \vec{\tau} = \vec{\tau} \cdot (\vec{r} \times \vec{F}) \Rightarrow \vec{\tau} \cdot \vec{\tau} = 0$$

The dot product of a vector ($\vec{r}$) with a cross product of itself and another vector ($\vec{F}$) will be zero $\vec{\tau} \cdot \vec{\tau} = 0$

Again from (1)

$$\vec{F} \cdot \vec{\tau} = \vec{F} \cdot (\vec{r} \times \vec{F}) \Rightarrow \vec{F} \cdot \vec{\tau} = 0 \rightarrow (2)$$

(By using same above rule for eqn (2))

(8)

Let the given vectors be $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{j} + \hat{k}$

$$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j}) \cdot (\hat{j} + \hat{k}) = \hat{j} \cdot \hat{j} = 1$$

$$|\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2} \quad ; \quad |\vec{B}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{1}{\sqrt{2} \sqrt{2}} = \frac{1}{2} \Rightarrow \theta = 60^\circ \rightarrow C$$

(9)

Given $\vec{A} = (4, -2, 5)$ $\vec{B} = (3, 1, -2)$

$$\vec{A} \cdot \vec{B} = (4 \cdot 3) + (-2 \cdot 1) + (5 \cdot -2) = 12 - 2 - 10 = 0$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{0}{|\vec{A}| |\vec{B}|} = 0 \Rightarrow \theta = 90^\circ \rightarrow C$$

(10)

Given $\vec{A} = 2\hat{i} + 3\hat{j}$

Angle made by the vector with y-axis

$$\tan \theta = \left(\frac{A_x}{A_y} \right) = \frac{2}{3}$$

$$\theta = \tan^{-1}\left(\frac{2}{3}\right) \rightarrow B$$

①

Given $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - \hat{k}$

$$\vec{A} \cdot \vec{B} = (2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{k}) = 2 \times 1 + (-1)(-1) = 2 + 1 = 3$$

$$|\vec{A}| = \sqrt{2^2 + 1^2 + (-1)^2} \quad ; \quad |\vec{B}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$= \sqrt{6}$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{3}{\sqrt{6} \sqrt{2}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = 30^\circ \rightarrow A$$

②

Given $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$?

Component of $\vec{A}$ along $\vec{B} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = \frac{(3\hat{i} + 4\hat{j}) \cdot (\hat{i} + \hat{j})}{\sqrt{1^2 + 1^2}}$

$$= \frac{3+4}{\sqrt{2}} = \frac{7}{\sqrt{2}} \rightarrow B$$

③

Given $|\vec{a}_1| = |\vec{a}_2| = 1$

also $|\vec{a}_1 + \vec{a}_2| = \sqrt{3} \quad \therefore (\vec{a}_1 - \vec{a}_2) \cdot (2\vec{a}_1 + \vec{a}_2) = 2\vec{a}_1 \cdot \vec{a}_1 + \vec{a}_1 \cdot \vec{a}_2 - 2\vec{a}_2 \cdot \vec{a}_1 - \vec{a}_2 \cdot \vec{a}_2$

$$\Rightarrow \sqrt{a_1^2 + a_2^2 + 2\vec{a}_1 \cdot \vec{a}_2} = \sqrt{3}$$

$$\Rightarrow 1^2 + 1^2 + 2\vec{a}_1 \cdot \vec{a}_2 = (\sqrt{3})^2$$

$$\Rightarrow 2 + 2\vec{a}_1 \cdot \vec{a}_2 = 3$$

$$\Rightarrow 2\vec{a}_1 \cdot \vec{a}_2 = 1$$

$$\Rightarrow 2 \times 1 \times 1 + \frac{1 \times 1}{2} = 1 - 1$$

$$\Rightarrow \frac{4+1}{2} - 2$$

$$\Rightarrow \frac{5}{2} - 2$$

$$\Rightarrow \frac{1}{2} \rightarrow C$$

(4)

Given that

$$(5\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 4\vec{b}) = 0 \quad (1)$$

$$\Rightarrow 5|\vec{a}|^2 - 8|\vec{b}|^2 - 4\vec{a} \cdot \vec{b} + 10\vec{b} \cdot \vec{a} = 0$$

$$\Rightarrow -3|\vec{a}|^2 + 6\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow 6|\vec{a}||\vec{b}|\cos\theta = 3$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ \rightarrow D$$

(5)

Given

$$\vec{A} = 5\hat{i} + 7\hat{j} - 3\hat{k} \text{ and } \vec{B} = 2\hat{i} + 2\hat{j} - c\hat{k}$$

 $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

$$\vec{A} \cdot \vec{B} = 0$$

$$\Rightarrow (5\hat{i} + 7\hat{j} - 3\hat{k}) \cdot (2\hat{i} + 2\hat{j} - c\hat{k}) = 0$$

$$\Rightarrow 5 \times 2 + 7 \times 2 - 3(-c) = 0$$

$$\Rightarrow 10 + 14 + 3c = 0 \Rightarrow 3c = -24$$

$$\Rightarrow c = -8 \rightarrow B$$

(6)

Given

Force

$$\vec{F} = 3\hat{i} + 4\hat{k} \text{ N. displacement } \vec{s} = 2\hat{i} + 4\hat{j} \text{ m}$$

Time = 1 sec.

$$\text{Power} = \frac{W}{t} = \frac{\vec{F} \cdot \vec{s}}{t} = \frac{(3\hat{i} + 4\hat{k}) \cdot (2\hat{i} + 4\hat{j})}{1 \text{ s}}$$

$$P = \frac{3 \cdot 2 + 4 \cdot 4}{1} = \frac{6 + 16}{1} = \frac{22}{1}$$

$$P = 22 \text{ W}$$

(6)

(7)

$$\text{let } \vec{A} = 2\hat{i} + 5\hat{k} \quad \& \quad \vec{B} = 3\hat{j} + 4\hat{k}$$

$$\text{Given } \vec{A} \cdot \vec{B} = (2\hat{i} + 5\hat{k}) \cdot (3\hat{j} + 4\hat{k})$$

$$\Rightarrow 2 \times 5 \times 4 = 20 \rightarrow A$$

(8)

Given

$$(3\hat{i} - 2\hat{j} + 2\hat{k}) \cdot (2\hat{i} - x\hat{j} + 3\hat{k}) = -12$$

$$\Rightarrow 3 \times 2 + (-2)(-x) + 2 \times 3 = -12$$

$$\Rightarrow 6 + 2x + 6 = -12$$

$$\Rightarrow 2x = -12 - 12 \Rightarrow 2x = -24$$

$$\Rightarrow x = -12 \rightarrow D$$

(9)

$$\text{If } \vec{A} = 6\hat{i} + 5\hat{j} \text{ and } \vec{B} = 4\hat{i} - 3\hat{j}$$

$$\vec{A} + \vec{B} = 6\hat{i} + 5\hat{j} + 4\hat{i} - 3\hat{j} \Rightarrow 10\hat{i} + 2\hat{j}$$

$$\vec{A} - \vec{B} = 6\hat{i} + 5\hat{j} - 4\hat{i} + 3\hat{j} \Rightarrow 2\hat{i} + 8\hat{j}$$

$$(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = (10\hat{i} + 2\hat{j}) \cdot (2\hat{i} + 8\hat{j})$$

$$\Rightarrow 10 \times 2 + 2 \times 8$$

$$\Rightarrow 20 + 16 = 36 \rightarrow B$$

(10)

$$\text{let } |\vec{A}| = 3 \quad ; \quad |\vec{B}| = 4 \text{ units} \quad \vec{A} \cdot \vec{B} = 6$$

$$\Rightarrow \vec{A} \cdot \vec{B} = 6$$

$$\Rightarrow \cos \theta = \frac{2}{4} = \frac{1}{2}$$

$$\Rightarrow AB \cos \theta = 6$$

$$\Rightarrow \theta = 60^\circ \rightarrow A$$

$$\Rightarrow 3 \times 4 \times \cos \theta = 6$$

$$\Rightarrow 4 \cos \theta = 2$$

(12)

let $\vec{A} = 6\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} - \hat{k}$

$$\vec{A} \cdot \vec{B} = (6\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - 2\hat{j} - \hat{k})$$

$$\Rightarrow 6 \times 2 + 3 \times (-2) + 2 \times (-1) = 12 - 6 - 2 = 4$$

$$|\vec{A}| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{49} = 7$$

$$|\vec{B}| = \sqrt{2^2 + (-2)^2 + (-1)^2} = \sqrt{9} = 3$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{4}{7 \times 3} = \frac{4}{21}$$

$$\theta = \cos^{-1}\left(\frac{4}{21}\right) \rightarrow c$$

(13)

$$P \cdot Q = PQ \Rightarrow P \cdot Q = PQ$$

$$\Rightarrow PQ \cos \theta = PQ \Rightarrow \cos \theta = 1$$

$$\Rightarrow \theta = 0^\circ$$

$$\vec{P} \cdot \vec{Q} = \frac{\sqrt{3}}{2} PQ$$

$$\Rightarrow PQ \cos \theta = \frac{\sqrt{3}}{2} PQ \Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$$

(14), (15)

Given $\vec{F} = 5\hat{i} - 3\hat{j} + 2\hat{k}$; $\vec{r}_1 = 2\hat{i} + 7\hat{j} + 4\hat{k}$; $\vec{r}_2 = 5\hat{i} + 2\hat{j} + 8\hat{k}$

displacement $\vec{s} = \vec{r}_2 - \vec{r}_1 = 5\hat{i} + 2\hat{j} + 8\hat{k} - 2\hat{i} - 7\hat{j} - 4\hat{k}$
 $= 3\hat{i} - 5\hat{j} + 4\hat{k} \rightarrow c$

work done = $\vec{F} \cdot \vec{s} \Rightarrow (5\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (3\hat{i} - 5\hat{j} + 4\hat{k})$

$$\Rightarrow 5 \times 3 + (-3) \times (-5) + 2 \times 4$$

$$\Rightarrow 15 + 15 + 8$$

$$\Rightarrow 38 \rightarrow c$$

(16)

Given

$$(3\hat{i} + 4\hat{j} - 7\hat{k}) \cdot (2\hat{i} + 4\hat{j} + p\hat{k}) = 8$$

$$\Rightarrow 3 \cdot 2 + 4 \cdot 4 - 7p = 8$$

$$\Rightarrow 6 + 16 - 7p = 8 \Rightarrow 7p = 22 - 8$$

$$\Rightarrow 7p = 14 \Rightarrow p = 2.$$

(17)

Given vector is $\sqrt{3}\hat{i} + \hat{j}$; $\theta = \frac{\pi}{k}$

Angle made by the vector with x-axis

$$\tan \theta = \frac{y\text{-comp}}{x\text{-comp}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^\circ = \frac{\pi}{6}$$

$$\Rightarrow \frac{\pi}{k} = \frac{\pi}{6} \Rightarrow k = 6$$

(18)

Angle between two vectors is 90° means the vectors are Perpendicular to each other. Their dot Product is 0.

$$\Rightarrow (1\hat{i} - \hat{j} - 3\hat{k}) \cdot (-2\hat{i} - c\hat{j} + \hat{k}) = 0$$

$$\Rightarrow (1 \times -2) + (-1)(-c) + (-3)(1) = 0$$

$$\Rightarrow -2 + c - 3 = 0$$

$$\Rightarrow c = 5$$

(19)

$$\text{If } \vec{A} \cdot \vec{B} = \frac{AB}{2}$$

$$\Rightarrow AB \cos \theta = \frac{AB}{2} \Rightarrow \theta = 60^\circ$$

$$\Rightarrow \cos \theta = \frac{1}{2}$$