Trigonometry

TRIGONOMETRY

Introduction:

The word ‘ trigonometry’ is derived from the Greek roots .. ‘tri’ means ‘three’ ; ‘gonia’ means ‘an angle’ ; and metron’ means ‘ meaning ‘ measure’. Thus ‘trigonometry’ means three angle measure. It is analytical study of a three angled geometric figure -- namely the triangle.

Hipparchus (140BC) , a Greek mathematician established the relationshipss between the sides and angles of a triangle. Greek trigonomentry was further developed by Hindu Mathematicians. They replaced the chords used by the Greeks by half chords of circles with given radii i.e ., the equivalent of our sine functions. The earliest such tables are in siddantas (systems of Astronomy) of the 4th and 5th centuries A.D. Like numbers, modern trigonometry migreted from the Hindus to Europe via Arabs.

The study of trigometry is of great importance in several fields - for example : in Surverying, Astronomy, Navigation and Engineering. In recent times, trigonometry is widely applied in many branches of Science and Engineering such as Seismology, design of electrical circuits, estimating the heights of tides in ocean etc.,

Definition of Sinθ , cosθ, tanθ and their reciprocals (for 0 < θ < $\frac{\pi}{2}$)

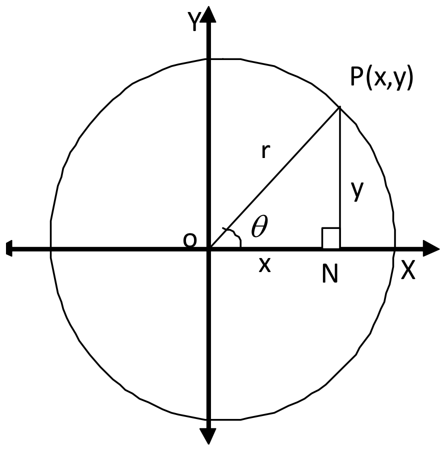
We shall now learn the relations existing between the sides and angles of a right angled triangle.

Consider the coordinate axes OX and OY in a plane. With O as the centre, construct a circle of raduis r. Let the terminal side of an acute angle θ intersect the circle at P (x,y) as shown in figure . Draw a perpendicular PN from P to OX. Then ON is the projection of OP on OX

From figure note that ON = x ,NP = y and OP = r

$\angle PON = \theta$, $\angle PNO = 90^0$

Hence Δ PNO is a right angled triangle.



The following six ratios of the sides relate the angle θ as

sin $\theta = \frac{NP}{OP} = \frac{\text{side opposite to } \theta}{hypotenuse} = \frac{y}{r}$

Cos $\theta = \frac{NP}{OP} = \frac{side\,adjacent\,to\,\theta}{hypotenuse} = \frac{x}{r}$

tan $\theta = \frac{NP}{ON} = \frac{side\,opposite\,to\,\theta}{side\,adjacent\,to\,\theta} = \frac{y}{x}$

cosec $\theta = \frac{OP}{NP} = \frac{Hypotenuse}{side\,opposite\,to\,\theta} = \frac{r}{y}$

sec $\theta = \frac{OP}{NP} = \frac{Hypotenuse}{side\,opposite\,to\,\theta} = \frac{r}{x}$

cot $\theta = \frac{ON}{NP} = \frac{side\,adjacent\,to\,\theta}{side\,opposite\,to\,\theta} = \frac{x}{y}$

NOTE:

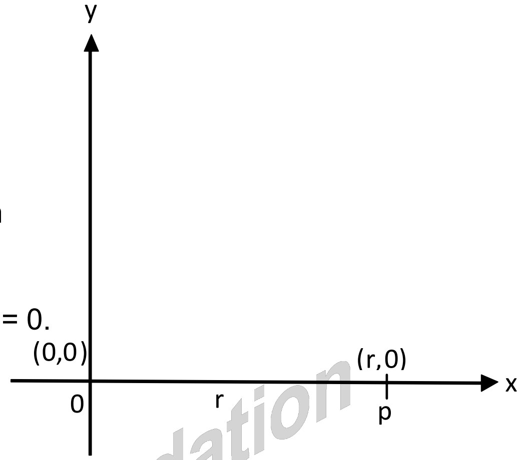
1. For $a \neq 0$, we know that $\frac{1}{a}$ is the reciprocal (multiplicative inverse) of a, since $a.\frac{1}{a} = 1$ (multiplicative identity). Since sinθ X cosecθ = $\frac{y}{r}$ X $\frac{r}{y}$ =1, cosecθ is the reciprocal of sinθ and vice - versa. Similarly the two ratios secθ and cotθ are the reciprocals of cosθ and tanθ respectively.

   Hence cosecθ = $\frac{1}{\sin\theta}$; secθ = $\frac{1}{\cos\theta}$ and cotθ = $\frac{1}{\tan\theta}$
2. Since the six trigonometrical ratios discussed above represent the ratios of sides of a right angled triangle, they are all real numbers.
3. The values of the six trigonometrical ratios discussed above are indepedent of the length of the radius vector $\overline{OP}$. They are dependent only on the magnitude or measure of the angle.
4. The six trigonometrical ratios are defined with respect to a certain angle θ . Hence sine, cosine, tangent, etc., by themselves do not have any meaning. They are meaningful only when they associated with an angle like θ .

Behaviour of trigonometric functions sinθ,cosθ and tanθ as θ changes from 0^0. 90^0 i.e ., 0^0 i.e., $\leq \theta \leq 90^0$

Case (i): For θ = 90^0

Let OX be the initial position of the rotating line. Take any point P on this line at a distance r units form the origin. Then the coordinates of the point Pare (r, θ). From Fig note that OP makes an angle θ=0^0 with the X - axis. This means that x = r and y = 0. Then by definition.



Sinθ=sin0^0 = $\frac{y}{r} = \frac{0}{r} = 0$

Cosθ=cos0^0=$\frac{x}{r} = \frac{r}{r} = 1$; tanθ = tan0^0 = $\frac{y}{x} = \frac{0}{r} = 0$

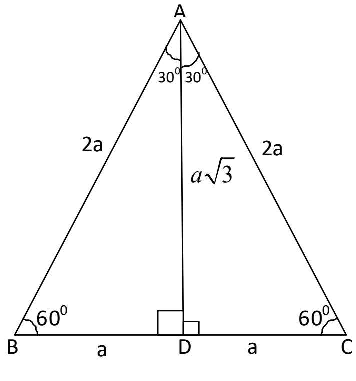
cosecθ = csec0^0 = $\frac{r}{y} = \frac{r}{0}$ (This ratio is not defined)

secθ = sec0^0 = $\frac{r}{x} = \frac{r}{r} = 1$

cotθ = cot 0^0 = $\frac{x}{y} = \frac{r}{0}$(This ratio is also not defined)

Case(ii) : For θ = 30^0 or $\frac{\mu^c}{6}$ and θ = 60^0 or $\frac{\mu^c}{3}$

To obtain the values of trigonometrical ratios of θ = 30^0 and 60^0, consider the quailateral triangle ABC of side 2a. Draw the perpendicular AD from the vertex A to BC . Then AD bisects side BC and the angle A



$\therefore$ BD = DC = a; $\angle BAD = \angle CAD = 30^0$

We know that the height of an equilateral triangle with side 2a is equal to $a\sqrt{3}$. Then from the right angled Δ ADB.

Sin30^0 = $\frac{opp.Side}{Hyp} = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

Cos30^0 = $\frac{adj.side}{Hyp} = \frac{DA}{AB} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$

tan30^0 = $\frac{opp.side}{adj.Side} = \frac{BD}{AD} = \frac{a}{a\sqrt{3}} = \frac{2}{\sqrt{3}}$

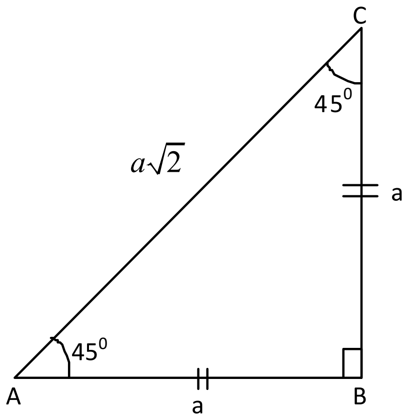
cosec30^0 = $\frac{AB}{BD} = 2$; sec30^0 $\frac{AB}{AD} = \frac{2}{\sqrt{3}}$ and , cot 30^0 = $\frac{AD}{BD} = \sqrt{3}$

sin 60^0 = $\frac{AD}{AB} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$; cos$60^0$ $\frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$;

tan60^0 = $\frac{AD}{BD} = \frac{a\sqrt{3}}{a} = \sqrt{3}$; cosec 60^0 = $\frac{AB}{AD} = \frac{2}{\sqrt{3}}$

sec 60^0 = $\frac{AB}{BD} = 2$; cot 60^0 = $\frac{BD}{AD} = \frac{1}{\sqrt{3}}$

Case (iii): For θ = 45^0 or $\frac{\pi^c}{4}$



To obtain the values of the six trigonometrical ratios corresponding to θ = 45^0. Consider the Δ ABC with AB=BC=a and $\angle B = 90^0$. Then it follows that $\angle A = \angle C = 45^0$.

From pythogorous theorem.

$AB^2 + BC^2 = AC^2$ and

AC = $\sqrt{AB^2 + BC^2} = a\sqrt{2}$

Then sin45^0 = $\frac{AB}{AC}$ = $\frac{a}{a\sqrt{2}}$ = $\frac{1}{\sqrt{2}}$;

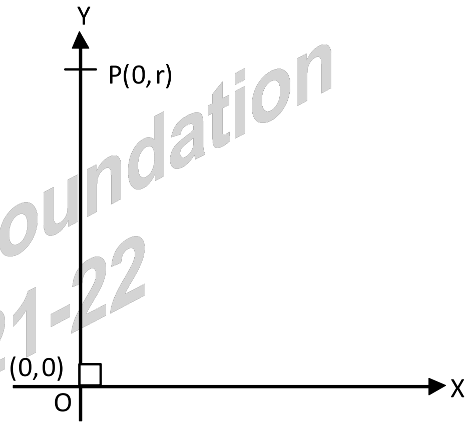
cos45^0 $\frac{AB}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$; $\tan 45^0 = \frac{BC}{AB} = \frac{a}{a} = 1$

cosec45^0 = $\frac{AC}{BC} = \frac{a\sqrt{2}}{a} = \sqrt{2}; \sec 45^0 = \frac{AC}{AB} = \frac{a\sqrt{2}}{a} = \sqrt{2}; \cot 45^0 = \frac{AB}{BC} = \frac{a}{a}$ =1

Note: Observe here that instead of considering $\angle A$ = θ= 45^0, if we consider $\angle C$ = θ= 45^0, we get same values for all the six ratios considered above.

Case(iv): For θ = 90^0 or $\frac{\pi^c}{2}$

Let the rotating line OP now make an angle θ = 90^0 with the x - axis OX. Then OP will be along the Y - axis (infact OP overlaps on y - axis) as shown in the figure . Since P lies on Y - axis and OP = r, coordinates of P are (0,r). Observe here that as P moves towards Y-axis and lies exactly on Y-axis, y=r and x=0.



SinceOY=OP=r, sin90^0 = $\frac{y}{r} = \frac{r}{r} = 1$

cos 90^0 = $\frac{x}{r} = \frac{0}{r} = 0$; tan$90^0$ = $\frac{y}{x} = \frac{y}{0}$ (undefined)

Cosec90^0 = $\frac{r}{y} = \frac{r}{r} = 1$

sec90^0 = $\frac{r}{x} = \frac{r}{0}$ (undefined)

cot 90^0= $\frac{x}{y} = \frac{0}{y} = 0$

f (-θ) for all the values of 'θ'

sin (-θ) = -sinθ; cosec (-θ) = -cosecθ

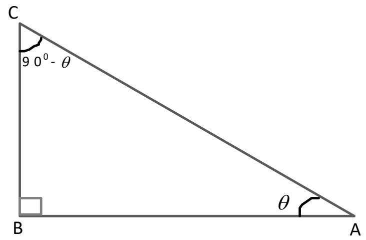
cos (-θ) = cosθ; sec (-θ)= secθ

tan (-θ) = -tanθ; cot(-θ) = -cotθ

sign will change for sin, cosec, tan and cot values. There is no change in cos and sec values.

*** Trigonometric function of complementary angles:**

1. Consider a right angled triangle ABC, right angled at B. Let $\angle A = \theta$ then $\angle C = (90 - \theta)$ From the diagram.



Sin $\theta = \frac{BC}{AC}$ sin(90- θ) = $\frac{AB}{AC}$

cos $\theta = \frac{AB}{AC}$ cos(90- θ) = $\frac{BC}{AC}$

tan θ = $\frac{BC}{AB}$ tan(90- θ) = $\frac{AB}{BC}$

cosec θ = $\frac{AC}{BC}$ cosec(90- θ) $\frac{AC}{AB}$

sec θ = $\frac{AC}{AB}$ sec (90- θ) = $\frac{AC}{BC}$

cot θ = $\frac{AB}{BC}$ cot(90- θ) = $\frac{BC}{AB}$

From the above results,

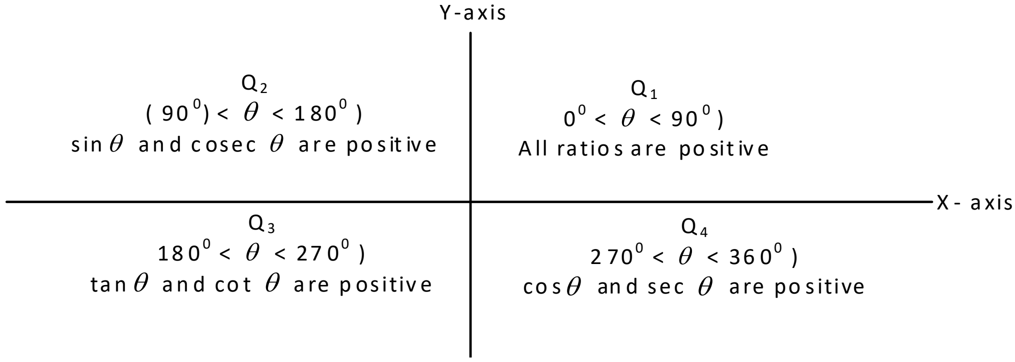
sin(90- θ) = cos θ, cosec(90- θ) = sec θ

cos(90- θ) = sin θ, sec (90- θ) = cosec θ

tan(90- θ) = cot θ, cot(90- θ) = tan θ

2. Sign of Trigonometric Ratios.

If 'θ' lies in Q_1, Q_2, Q_3, Q_4 quadrants, then the sign of trigonometric ratios are as follows:



Note:

1. $0^0, 90^0. 180^0, 270^0, 360^0$. .... are called quadrant angles.
2. With the phrase **A**ll **S**ilver **T**ea **C**ups we can remember the sign of trigometric ratios

To find the values of trigonometric functions of any angle;

Quadrant	Q_1	Q_2		Q_3		Q_4	
Tr.Ratios	$90^0 - \theta$	$90^0 + \theta$	$180^0 - \theta$	$180^0 + \theta$	$270^0 - \theta$	$270^0 + \theta$	$360^0 - \theta$
Sin	$+\cos\theta$	$+\cos\theta$	$+\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$
Cos	$+\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$+\sin\theta$	$+\cos\theta$
Tan	$+\cot\theta$	$-\cot\theta$	$-\tan\theta$	$+\tan\theta$	$+\cot\theta$	$-\cot\theta$	$-\tan\theta$
Cot	$+\tan\theta$	$-\tan\theta$	$-\cot\theta$	$+\cot\theta$	$+\tan\theta$	$-\tan\theta$	$-\cot\theta$
Sec	$+\text{cosec}\theta$	$-\text{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\theta$	$+\text{cosec}\theta$	$+\sec\theta$
Cosec	$+\sec\theta$	$+\sec\theta$	$+\text{cosec}\theta$	$-\text{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\theta$

Note:

1. For 0^0 $0^0 \pm \theta, 180^0 \pm \theta, 360^0 \pm \theta,$ there is no change in the trigonometric ratios.
2. $90^0 \pm \theta, 270^0 \pm \theta,$ the change in the trigonometric ratios is as follows:

   $\sin \leftrightarrow \cos, \tan \leftrightarrow \cot, \sec \leftrightarrow \cos ec$
3. Whether we get + or - sign in the answer, it should be decided by considering the quadrant in which the angle ($n.360^0 \pm \theta$) lies.

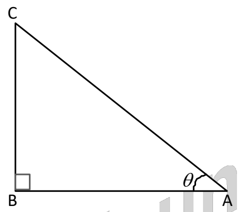
TRIGONOMETRY IDENTITIES

Introduction: The equations which are satisfied by all valuesof 'θ' are called trigometric identities. We shall establish some basic trigonometric identities and use them to draw some useful results.

Identity 1: $\boxed{\sin^2\theta + \cos^2\theta = 1}$ $\forall \theta \in R$

Consider a right angled triangle ABC, with $\angle B = 90^0$ and $\angle CAB = \theta$

From diagram, $\sin\theta = \frac{BC}{AC}, Cos\theta = \frac{AB}{AC}$



From pythagoras therom, $AB^2 + BC^2 = AC^2$
Dividing both sides with AC^2, we get

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

$$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1$$

$$(COS\theta)^2 + (SIN\theta)^2 = 1$$

Identity II:

From the first identity $(Cos\theta)^2 + (Sin\theta)^2 = 1$

Dividing both sides with $\cos^2\theta$,

we get

$$\frac{Sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\left(\frac{Sin\theta}{\cos\theta}\right)^2 + 1 = \left(\frac{1}{\cos\theta}\right)^2$$

$$(\tan\theta)^2 + 1 = (\sec\theta)^2$$

$\tan^2\theta + 1 = \sec^2\theta$ or $\sec^2\theta - \tan^2\theta = 1$

Identity III:

From the first identify $\sin^2\theta+\cos^2\theta=1$

Dividing both sides with '$\sin^2\theta$'

we get $$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$1+\cot^2\theta=\operatorname{cosec}^2\theta$$

$$\operatorname{cosec}^2\theta-\cot^2\theta=1$$

USUAL FORMULAE FROM IDENTITIES:

1. $\sin^2\theta+\cos^2\theta=1$

$\sin^2\theta=1-\cos^2\theta$ $\quad Cos^2\theta=1-\sin^2\theta$

$\sin\theta=\pm\sqrt{1-\cos^2\theta}$ $\quad Cos\theta=\pm\sqrt{1-\sin^2\theta}$

2. $\sec^2\theta-\tan^2\theta=1$

$\sec^2\theta=1-\tan^2\theta$ $\quad \tan^2\theta=\sec^2\theta-1$

$\sec\theta=\pm\sqrt{1-\tan^2\theta}$ $\quad \tan\theta=\pm\sqrt{\sec^2\theta-1}$

$(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=1$

$$\sec\theta+\tan\theta=\frac{1}{\sec\theta-\tan\theta}$$

3. $\operatorname{cosec}^2\theta-\cot^2\theta=1$

$1+\cot^2\theta=\operatorname{cosec}^2\theta$ $\quad \cot^2\theta=\operatorname{cosec}^2\theta-1$

$\operatorname{cosec}\theta=\pm\sqrt{1+\cot^2\theta}$ $\quad \cot\theta=\pm\sqrt{\operatorname{cosec}^2\theta-1}$

$(\operatorname{cosec}\theta+\cot\theta)(\operatorname{cosec}\theta-\cot\theta)=1$

$$(\operatorname{cosec}\theta+\cot\theta)=\frac{1}{(\operatorname{cosec}\theta-\cot\theta)}$$

Radian:- A radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

Circular system of angles

1 revolution = 2π radians = 360^0

π radians = 180^0

1 degree (1^0) = $\frac{\pi}{180}$ radians = 0.0172 radians

1 radian $(1^c) = \frac{180}{\pi}$ degrees = $57^0\, 17^{'}\, 44.8^{''}$

Sexagesimal system of angles			Centesimal system of angles		
1rt angle	=	90 degrees (90^0)	1rt angle	=	100 grades (100^g)
1 degree	=	60 minutes (60′)	1 grade	=	100 minutes (100′)
1 minute	=	60 seconds (60′′)	1 minute	=	100 seconds (100′′)

Degrees	0^0	30^0	45^0	60^0	90^0	120^0	135^0	150^0	180^0	210^0	225^0	240^0	270^0	300^0	315^0	330^0	360^0
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

Some standard angles:- Note:- n . d $\rightarrow$ not defined

T.Ratio	0^0	30^0	45^0	60^0	90^0	180^0	270^0	360^0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	– 1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	– 1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	n.d	0	n.d	0
$\operatorname{cosec}\theta$	n.d	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	n.d	– 1	n.d
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	n.d	– 1	n.d	1
$\cot\theta$	n.d	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	n.d	0	n.d

FUNDAMENTAL IDENTITIES:-

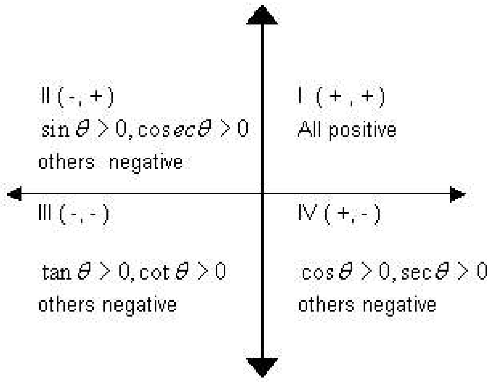
i) $\tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}, \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}, \cot\theta = \frac{1}{\tan\theta}$

ii) $\sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^2\theta = 1 - \cos^2\theta, \cos^2\theta = 1 - \sin^2\theta$

iii) $1 + \tan^2\theta = \sec^2\theta \rightarrow \sec^2\theta - \tan^2\theta = 1, (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

iv) $1 + \cot^2\theta = \operatorname{cosec}^2\theta \rightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1, (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

The signs of Trigonometric functions:-



Problems:

1. In a right angled triangle $\triangle ABC\ \angle B = 90^0$ and $\tan A = \frac{5}{12}$ then

find (i) cosA (ii) cosecA - cotA

2. Find the value of

(i) $\sin^2 30^0 + \cos^2 30^0$ (ii) $\sec^2 45^0 - \tan^2 45^0$ (iii) $co\sec^2 60^0 - \cot^2 60^0$

3. A Pole 10 mts along rests slantly againt a vertical wall AB making 60^0 with the hori zontal line (Ground). Find how far is the foot of the pole from the wall.

4 . In a $\triangle ABC$, $\underline{|A} = 60^0$ $\angle B = 30^0$ show that

(i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

(ii) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

5. If $A = 30^0$ then show that

(i) $\sin 2A = 2\sin A\cos A$

(ii) $\cos 2A = \cos^2 A - \sin^2 A$

(iii) $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

6. In a , if sin A= $\frac{9}{15}$ then find (i) cosA (ii) tanA (iii) $\cos ec^2 A - \cot^2 A$

7. If tan(A-B) = $\frac{1}{\sqrt{3}}$, sin B= $\frac{1}{\sqrt{2}}$ find B in circular measure

8. find the value of 32 $\cot^2 \frac{\pi}{4} - 8\sec^2 \frac{\pi}{3} + 8\cot^3 \frac{\pi}{6}$

9. An equilateral triangleis inscribed in a circle of radius 6 cms. Find its sides.

10 . Show that $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$

11. If sec θ= $\frac{m+n}{2\sqrt{mn}}$ then find the value of sin θ

MCQs:

1. $\sec A + \tan A = 3 \Rightarrow \sec A =$

1) $\frac{10}{3}$ 2) $\frac{5}{3}$ 3) $\frac{2}{3}$ 4) $\frac{4}{3}$

2. $\sec\theta - \tan\theta = 3 \Rightarrow \theta$ lies in the quadrant

1) I 2) II 3) III 4) IV

3. $\sin\theta = \frac{2ab}{a^2+b^2} \Rightarrow \sec\theta + \tan\theta =$

1) $\frac{a-b}{a+b}$ 2) $\frac{a+b}{a-b}$ 3) $\frac{ab}{a^2+b^2}$ 4) $\frac{ab}{a+b}$

4. $\tan 20^0 + \tan 40^0 + \tan 60^0 + ... + \tan 180^0 =$

1) 0 2) 1 3) 2 4) 3

5. $3[\sin x - \cos x]^4 + 6[\sin x + \cos x]^2 + 4[\sin^6 x + \cos^6 x] =$

1) 3 2) 6 3) 4 4) 13

6. $\cos(40^0 + \theta) + \cos(120^0 + \theta) + \cos(220^0 + \theta) + \cos(300^0 + \theta) =$

1) 3 2) 2 3) 1 4) 0

7. $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}, \sin\alpha - \cos\alpha = k\sin\theta$ then k =

1) 1 2) $\sqrt{3}$ 3) $\sqrt{2}$ 4) 1/2

8. A, B, C are the angles of a triangle ABC$\Rightarrow$ $\cos\left(\frac{3A+2B+C}{2}\right) + \cos\left(\frac{A-C}{2}\right) =$

1) 0 2) 1 3) cosA 4) cosC

9. $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} =$

1) $\dfrac{1-\cos\theta}{\sin\theta}$ 2) $\dfrac{\sin\theta}{1+\cos\theta}$ 3) $\dfrac{1+\cos\theta}{\sin\theta}$ 4) $\dfrac{1-\sin\theta}{\cos\theta}$

10. $\dfrac{\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}}{\cos^2\frac{\pi}{18} + \cos^2\frac{\pi}{9} + \cos^2\frac{7\pi}{18} + \cos^2\frac{4\pi}{9}} =$

1) 1 2) 2 3) 3 4) 4

11. $1+\cos\frac{\pi}{7}+\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{5\pi}{7}+\cos\frac{6\pi}{7}=$

1) 0 2) 1 3) 2 4) 3

12. $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\sec\alpha + \cos\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha \Rightarrow k =$

1) 9 2) 7 3) 5 4) 3

13. If θ is not in 4th quadrant

$\tan\theta = -4/3 \Rightarrow 5\sin\theta + 10\cos\theta + 9\sec\theta + 16\operatorname{cosec}\theta + 4\cot\theta =$

1) −1 2) 2/5 3) 4/5 4) 0

14. If $\frac{\pi}{2} < \theta < \pi$ and $\sin\theta = \frac{3}{5}$, then the ascending order of $\sin\theta, \cos\theta, \tan\theta$ is

1) $\cos\theta, \sin\theta, \tan\theta$ 2) $\cos\theta, \tan\theta, \sin\theta$ 3) $\sin\theta, \cos\theta, \tan\theta$ 4) $\sin\theta, \tan\theta, \cos\theta$

15. If θ lies in the first quadrant and $5\tan\theta = 4$, then $\left[\dfrac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta}\right] =$

1) $\dfrac{5}{14}$ 2) $\dfrac{3}{14}$ 3) $\dfrac{1}{14}$ 4) 0

16. $\dfrac{\sin(-660^0)\tan(1050^0)\sec(-420^0)}{\cos(225^0)\operatorname{cosec}(315^0)\cos(510^0)} =$

1) $\dfrac{\sqrt{3}}{4}$ 2) $\dfrac{\sqrt{3}}{2}$ 3) $\dfrac{2}{\sqrt{3}}$ 4) $\dfrac{4}{\sqrt{3}}$

17. $\tan 25^0 = p \Rightarrow \dfrac{\tan 245^0 + \tan 335^0}{\tan 205^0 - \tan 115^0} =$

1) $\dfrac{p^2-1}{p^2+1}$ 2) $\dfrac{p^2+1}{p^2-1}$ 3) $\dfrac{1-p^2}{1+p^2}$ 4) $\dfrac{1+p^2}{1-p^2}$

18. $a_n = \cos^n\alpha + \sin^n\alpha \Rightarrow 2a_6 - 3a_4 =$

1) 0 2) −1 3) 3 4) 4

19. $\sin\theta_1+\sin\theta_2+\sin\theta_3=3\Rightarrow\ \cos\theta_1+\cos\theta_2+\cos\theta_3=$

1) 0 2) 1 3) 2 4) 3

20. $5\sin x+4\cos x=3\Rightarrow 4\sin x-5\cos x=$

1) 4 2) $4\sqrt{2}$ 3) $3\sqrt{2}$ 4) $\sqrt{2}$

21. $8\sin^2 x+3\cos^2 x=5\Rightarrow \cot x=$

1) $\pm\frac{1}{\sqrt{2}}$ 2) $\pm\frac{1}{\sqrt{3}}$ 3) $\pm\sqrt{\frac{3}{2}}$ 4) $\pm\sqrt{\frac{2}{3}}$

22. If $\tan(\alpha+\beta)=\sqrt{3},\tan(\alpha-\beta)=1$ then $\tan 6\beta=$

1) -1 2) 0 3) 1 4) 2

23. If ABCD is a cyclic quadrilateral then

$\cos(180^0+A)+\cos(180^0-B)+\cos(180^0-C)-\sin(90^0-D)=$

1) -1 2) 0 3) 1 4) 2

24. If $\cos\theta=k(0<k<1)$ and θ does not lie in the first quadrant, then $\tan\theta=$

1) $-\frac{\sqrt{1-k^2}}{k}$ 2) $\frac{k}{\sqrt{1-k^2}}$ 3) $-\frac{k}{\sqrt{1+k^2}}$ 4) $\frac{\sqrt{1-k^2}}{k}$

25. If $a\sec\theta+b\tan\theta=c$ then $(a\tan\theta+b\sec\theta)^2=$

1) $a^2+b^2+c^2$ 2) $-a^2+b^2+c^2$ 3) $a^2-b^2+c^2$ 4) $a^2+b^2-c^2$

26. $\cos^2 1^0+\cos^2 2^0+\cos^2 3^0+....+\cos^2 90^0=$

1) 0 2) 1 3) 45 4) $\frac{89}{2}$

27. If $\theta=\frac{11\pi}{6}$, then $\cos\theta+\sin\theta=$

1) $\frac{\sqrt{3}+1}{\sqrt{2}}$ 2) $\frac{\sqrt{3}-1}{\sqrt{2}}$ 3) $\frac{\sqrt{3}-1}{2}$ 4) $\frac{\sqrt{3}+1}{2}$

28. $\tan\theta=\frac{1}{\sqrt{5}},0<\theta<90^0\Rightarrow\frac{\cos ec^2\theta-\sec^2\theta}{\cos ec^2\theta+\sec^2\theta}$

1) $\frac{2}{3}$ 2) $\frac{3}{2}$ 3) $\frac{1}{3}$ 4) $\frac{5}{6}$

29. $(1+\sin A)(1+\sin B)(1+\sin C)=(1-\sin A)(1-\sin B)(1-\sin C)=k\Rightarrow k=$

1) $+\sin A\sin B\sin C$ 2) $\pm\cos A\cos B\cos C$

3) $\pm\sec A\sec B\sec C$ 4) $\pm$ cosec A cosecB cosecC

30. $\tan^2\theta+\sec\theta=5\Rightarrow\sec\theta=$

1) 3 2) 2 3) 1 4) -1

31. $\log\left(\tan 18^0\right)+\log\left(\tan 36^0\right)+\log\left(\tan 54^0\right)+\log\left(\tan 72^0\right)=$

1) $\log 4$ 2) $\log 3$ 3) $\log 2$ 4) log1

32. The value of $\cot^2\alpha\left(\frac{\sec\alpha-1}{1+\sin\alpha}\right)+\sec^2\alpha\left(\frac{\sin\alpha-1}{1+\sec\alpha}\right)$ is

1) 0 2) 1 3) 2 4) -2

33. $\sec A-\tan A=4\Rightarrow\tan A=$

1) $\frac{15}{8}$ 2) $-\frac{15}{8}$ 3) $\frac{17}{8}$ 4) $-\frac{17}{8}$

34. $\sec A-\tan A=5\Rightarrow\sin A=$

1) $\frac{6}{13}$ 2) $-\frac{6}{13}$ 3) $\frac{12}{13}$ 4) $-\frac{12}{13}$

35. $\operatorname{cosec}A-\cot A=5\Rightarrow\operatorname{cosec}A=$

1) $\frac{5}{13}$ 2) $\frac{13}{5}$ 3) $-\frac{5}{13}$ 4) $-\frac{13}{5}$

36. $\operatorname{cosec}A-\cot A=6\Rightarrow\cot A=$

1) $\frac{12}{35}$ 2) $-\frac{12}{35}$ 3) $\frac{35}{12}$ 4) $-\frac{35}{12}$

37. $\operatorname{cosec}A+\cot A=\frac{2}{3}\Rightarrow\cos A=$

1) $\frac{5}{13}$ 2) $\frac{13}{5}$ 3) $-\frac{5}{13}$ 4) $-\frac{13}{5}$

38. $\operatorname{cosec}\theta-\cot\theta=5\Rightarrow\theta$ lies in the quadrant

1 I 2) II 3) III 4) IV

39. $\sin\alpha=\frac{2xy}{x^2+y^2}\Rightarrow\sec\alpha-\tan\alpha=$

1) $\frac{x-y}{x+y}$ 2) $\frac{x+y}{x-y}$ 3) $\frac{xy}{x^2+y^2}$ 4) $\frac{xy}{x+y}$

40. $\sin x=\frac{2pq}{p^2+q^2}\Rightarrow\operatorname{cosec}x+\cot x=$

1) $\frac{p}{q}$ 2) $\frac{q}{p}$ 3) $\frac{2p}{q}$ 4) $\frac{2q}{p}$