6. SQUARE, SQUARE ROOTS AND CUBE, CUBE ROOTS ①

Class: VIII. MATHEMATICS

INTEGRATED†. SOLUTIONS

2025/26

TEACHING TASK

JEE MAINS LEVEL

01. We know, $32 = 2^5$

If we multiply with 2, it becomes

$32 \times 2 = 2^5 \times 2 = 2^6 = (2^3)^2 = 8^2$

Ans: B

02. $28\sqrt{x} + 1426 = \frac{3}{4} \times 2872$

$= 2154$

$\therefore 28\sqrt{x} = 728$

$\Rightarrow \sqrt{x} = 26$

$\Rightarrow x = 676$

Ans: B

03. $\therefore 4320 = 2^5 \times 3^3 \times 5$

$$\begin{array}{r|l} 2 & 4320 \\ 2 & 2160 \\ 2 & 1080 \\ 2 & 540 \\ 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ & 5 \end{array}$$

If we multiply with 2×5^2 it becomes perfect cube

i.e 50

Ans: C

04 Divisor = 3

quotient = $\frac{4}{3} \times 3 = 4$

Remainder = 17

let the no. be x

We know

Divident = Divisor x quotient + Remainder

$= 3 \times 4 + 17$

$= 29$

Ans: —

05 We know

$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

$\therefore 1^3 + 2^3 + 3^3 + \cdots + 10^3 = \frac{10^2(10+1)^2}{4} = 3025$

$\therefore \sqrt{1^3 + 2^3 + 3^3 + \cdots + 10^3} = \sqrt{3025} = 55$ Ans: B

06 Let $\sqrt[3]{x} = a$

$\Rightarrow a + \frac{48}{a} = 16$

$\Rightarrow a^2 - 16a + 48 = 0$

$\Rightarrow (a-4)(a-12) = 0$

$\Rightarrow a = 4$ or $a = 12$

$\Rightarrow \sqrt[3]{x} = 4$ | $\sqrt[3]{x} = 12$

$\Rightarrow x = 4^3$ | $x = (12)^3$

$\Rightarrow x = 64$ | $= 1728$

Ans: D

07. $\sqrt[3]{1+3+5+7+\cdots 53}$ (3)

$1+3+5+7+\cdots 53 \longrightarrow$ Arithmetic Series

$a+(n-1)d = 53$

$\Rightarrow 1+(n-1)\cdot 2 = 53$

$\Rightarrow n = 27.$

$\therefore S_n = \frac{n}{2}[a+l]$

$\therefore S_{27} = \frac{27}{2}[1+53] = 27\times 27 = 3^6$

$\therefore \sqrt[3]{1+3+5+7+\cdots+53} = \sqrt[3]{3^6} = 3^2 = 9$

Ans. D

08 $a^b = 2^9 = (2^3)^3 = 8^3$

$\therefore a = 8,\quad b = 3$

$\sqrt[b]{a} = \sqrt[3]{8} = 2$

$$\begin{array}{r|l} 2 & 512 \\ 2 & 256 \\ 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

Ans: A

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09. $\left(5-\sqrt{y^2-25}\right)^2 = \cancel{25} - 2\cdot 5\cdot\sqrt{y^2-25} + (y^2 - \cancel{25})$

$= y^2 - 10\sqrt{y^2-25}$ (OR)

$= y^2 - \sqrt{(10)^2\cdot y^2 - 10^2\cdot 25}$

$= y^2 - \sqrt{(10y)^2 - (50)^2}$

Ans: C, D

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10. $\sqrt[3]{0.125} + \sqrt[3]{0.729}$

$= \sqrt[3]{(0.5)^3} + \sqrt[3]{(0.9)^3}$

$= 0.5 + 0.9$

$= 1.4$

option (B): $\sqrt{\frac{49}{25}} = \frac{7}{5} = 1.4$ Ans.: A, B

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11. Statement I: $1^2 + 2^2 + 3^2 = 1 + 4 + 9 = 14$ (True)

Statement II: Let the three consecutive numbers be $n-1, n, n+1$

$(n-1)^2 + n^2 + (n+1)^2 = 3n^2 + 2$ (True)

Ans.: A

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12. We know $\sqrt{1752} \approx 41.84$ (approx)

$\therefore 41^2 = 1681$; $42^2 = 1764$

Now, $1764 - 1752 = \boxed{12}$

$\therefore$ 12 should be added to 1752 to make it a perfect square

Ans: C

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13. $\sqrt{1300} \approx 36.05$ (approx)

We know $36^2 = 1296$

Now, $1300 - 1296 = 4$

$\therefore$ 4 should be subtracted from 1300 to make it a perfect square Ans: B

14.	We know $882 = 2 \times 3^2 \times 7^2$ clearly to make 882, a perfect square we have to multiply with 2 Ans: C
15	Let $a=1$ 1, 3, 4 → not prime numbers $a=2$ 2, 4, 6 → not prime No. $a=3$ 3, 5, 7 → prime No.. Ans: 1
16	a) 121 (r) b) 12321 (p) c) 1234321 (q) d) 225 (t) Ans. r, p, q, t

Learner's Task (CUQ's)

01	8	Ans. C
02	$\frac{5}{3}$	Ans.. B
03	-2	Ans: A
04	$\sqrt[3]{512} = 8$	Ans: A
05	$\sqrt[3]{-343} = -7$	Ans: C
$\boxed{06}$	$32 = 2^5$, clearly 2 should be multiplied $\boxed{\text{Ans: B, C}}$	

07. $\sqrt[3]{1331} = 11$ Ans: B

08 Conceptual Ans: A

09 $\sqrt[3]{2}, \sqrt[2]{3}, \sqrt[3]{36}$

Inorder to multiply we have to make the orders same.

L.C.M $\{3, 2, 3\} = 6.$

$\sqrt[3\times 2]{2^2} = \sqrt[6]{4}$ | $\sqrt[2]{3} = \sqrt[2\times 3]{3^3} = \sqrt[6]{27}$

$\sqrt[3]{36} = \sqrt[3\times 2]{(36)^2} = \sqrt[6]{1296}$

$\therefore$ Answer $= \sqrt[6]{4} \times \sqrt[6]{27} \times \sqrt[6]{1296}$

$= \sqrt[6]{4 \times 27 \times 1296}$

$= \sqrt[6]{2^2 \times 3^3 \times 2^4 \times 3^4}$

$= \sqrt[6]{2^6 \times 3^6 \times 3} = 2 \times 3 \times \sqrt[6]{3}$

$= 6 \times \sqrt[6]{3}$ $\boxed{\text{Ans: —}}$

10. A) Factors of $28 = \{1, 2, 4, 7, 14, 28\}$

Sum of the factors $= 1 + 2 + 4 + 7 + 14 + 28$

$= 56 = 2 \times 28$

B) Factors of $6 = \{1, 2, 3, 6\}$

sum of the factors $= 1 + 2 + 3 + 6 = 12 = 2 \times 6$

$\therefore$ Both 28, 6 are perfect numbers

Ans: D

01 $3(x-2)^2 = 507$

$\Rightarrow (x-2)^2 = 169$

$\Rightarrow x-2 = \pm 13$

$\Rightarrow x = \pm 13 + 2$

$\Rightarrow x = 15$ or -11

Ans: C

02 $\sqrt{\dfrac{32.4}{x}} = 2 \Rightarrow \dfrac{32.4}{x} = 4 \Rightarrow x = 8.1$

Ans: B

03 $\sqrt{(272)^2 - (128)^2}$

$= \sqrt{(272+128)(272-128)}$

$= \sqrt{400 \times 144} = 20 \times 12 = 240$

Ans: B

04 $3a = 4b = 6c$

$3a = 4b \Rightarrow \dfrac{a}{b} = \dfrac{4}{3} \Rightarrow a:b = 4:3$

$4b = 6c \Rightarrow \dfrac{b}{c} = \dfrac{6}{4} = \dfrac{3}{2} \Rightarrow b:c = 3:2$

$\therefore a:b:c = 4:3:2$

Now, $a+b+c = 27\sqrt{29}$

$\Rightarrow 4x+3x+2x = 27\sqrt{29} \Rightarrow x = 3\sqrt{29}$

$\sqrt{a^2+b^2+c^2} = \sqrt{16x^2+9x^2+4x^2} = \sqrt{29x^2} = \sqrt{29}\,x$

$\therefore \sqrt{29}\,x = \cancel{3\sqrt{29}} = \sqrt{29} \cdot 3\sqrt{29}$

$= 87$

Ans: C

8

05 $x = \sqrt{2\sqrt{2\sqrt{2\cdots}}}$

$\Rightarrow x = \sqrt{2x}$

$\Rightarrow x^2 = 2x$

$\Rightarrow x = 2$

Similarly $y = 3$

I $\rightarrow$ $x + y \leq 5$

$2 + 3 \leq 5$

$5 \leq 5$ (True)

II $\rightarrow$ $x^2 + y^2 = 6xy$

$\Rightarrow 4 + 9 = 6.2.3$

$\Rightarrow 13 = 36$ (False)

III $\rightarrow$ $x^2 . y^2 = 6xy$

$= 4 \times 9 = 6.2 \times 3$

$\Rightarrow 36 = 36$ (True)

Ans: C

06 $\sqrt{x} + \sqrt{y} = \sqrt{1980} = \sqrt{36 \times 55} = 6\sqrt{55}$

Let $\sqrt{x} = p\sqrt{55}$, $\sqrt{y} = q\sqrt{55}$

$\therefore p\sqrt{55} + q\sqrt{55} = 6\sqrt{55}$

$\Rightarrow p + q = 6$

$\therefore (p, q) \in (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)$

$\boxed{\text{Ans: C}}$

(~~$p^2 + q^2$~~) No. of pairs $= 5$

07

07. Given $x^2+y^2+z^2=6$

$$\left(\frac{x}{y}+\frac{y}{x}\right)^2+\left(\frac{y}{z}+\frac{z}{y}\right)^2+\left(\frac{x}{z}+\frac{y}{z}\right)^2=165$$

$$\Rightarrow \frac{x^2}{y^2}+\frac{y^2}{x^2}+2+\frac{y^2}{z^2}+\frac{z^2}{y^2}+2+\frac{x^2}{z^2}+\frac{y^2}{z^2}+2=165$$

$$\Rightarrow \frac{x^2+z^2}{y^2}+\frac{y^2+z^2}{x^2}+\frac{x^2+y^2}{z^2}=159$$

$$\Rightarrow \frac{6-y^2}{y^2}+\frac{6-x^2}{x^2}+\frac{6-z^2}{z^2}=159$$

$$=6\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)=159+3$$

$$\Rightarrow \frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=27$$

Ans: C

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08 Conceptual Ans: A, B

09 Conceptual Ans: B, C

10 Statement I:

$1^2+2^2+3^2+\ldots+5^2=55$ (False)

Statement II: Conceptual (True) Ans.- D

11 $1+3+5+\ldots99=$

$a+(n-1)d=99$

$1+(n-1)\cdot 2=99$

$n=50$

SUM $=(50)^2=2500$

Ans: C

12 $S = 3 + 9 + 15 + 21 + \cdots + 63$

$= 3(1 + 3 + 5 + 7 + \cdots + 21)$ A.P.

~~here~~ $a + (n-1)d = 21$

$\Rightarrow 1 + (n-1)\cdot 2 = 21$

$\Rightarrow n = 11$

$\therefore$ Sum $= 3 \times 11^2 = 3 \times 121 = 363$

Ans. A

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13 $S = 2 + 4 + 6 + \cdots + 400$ A.P.

$a + (n-1)d = 400$

$\Rightarrow 2 + (n-1)\cdot 2 = 400$

$\Rightarrow n = 200$

Sum $= \frac{n}{2}(a + l) = \frac{200}{2}(2 + 400)$

$= 40200$

Ans. D

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14 Let the number be a

Given $\sqrt[3]{\frac{a}{25}} = x \Rightarrow \frac{a}{25} = x^3 \Rightarrow a = 25x^3$

$\sqrt[3]{5a} = y \Rightarrow 5a = y^3 \Rightarrow a = \frac{y^3}{5}$

$\therefore 25x^3 = \frac{y^3}{5} \Rightarrow 5x = y$

Given $x + y = 36 \Rightarrow x + 5x = 36 \Rightarrow x = 6$

$\therefore a = 25 \times 6^3 = 5400$

$\therefore 5395 + a = 5400$

$\Rightarrow a = 5$

Ans. 5

(11)

15 a) $(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$

$= x^2+y^2+z^2+2xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ (t)

b) $x+y+z=0 \Rightarrow x^3+y^3+z^3 = 3xyz$ (q)

c) $x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{1/3} = 0$

$\Rightarrow \left(x^{\frac{1}{3}}\right)^3+\left(y^{\frac{1}{3}}\right)^3+\left(z^{\frac{1}{3}}\right)^3 = 3\cdot x^{\frac{1}{3}}\cdot y^{\frac{1}{3}}\cdot z^{1/3}$

$\Rightarrow x+y+z = 3(xyz)^{\frac{1}{3}}$

$\Rightarrow (x+y+z)^3 = 27xyz$ (or)

$= 27\sqrt[3]{x^3y^3z^3}$ (p)

d) $x+y=z \Rightarrow x+y+(-z)=0$

$x^3+y^3+(-z)^3 = 3(x)(y)(-z)$

$\Rightarrow x^3+y^3-z^3 = -3xyz$ (r)

Ans. t, q, p, r

$\Rightarrow$ THE END $\in$