HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

2. $\frac{\log b}{\log b+\log a+\log c}+\frac{\log c}{\log c+\log a+\log b}+\frac{\log a}{\log c+\log a+\log b}=1$

3. $1+x>0,\ 1-x>0,\ 1-x^2>0,\ x\neq 0$

4. $(7x-9)^2(3x-4)^2=100$

$\Rightarrow (21x^2-55x+36)^2=100$

$\Rightarrow \quad 21x^2-55x+36=\pm 10$

$21x^2-55x+26=0$

$$x=\frac{55\pm\sqrt{3025-2184}}{42}=\frac{55\pm 29}{42}=2,\ \frac{13}{21}$$

only two real solution

5. Take log on both sides of equation & solve the equation simultaneously.

6. Use $a^{\log_b c}=c^{\log_b a}$

$\Rightarrow 3^{\log_4 5}+4^{\log_5 3}-3^{\log_4 5}-4^{\log_5 3}=0$

7. Let $\log_3 n=x$

$y=5x^2-12x+9$

y is minimum at $x=-\frac{b}{2a}=\frac{12}{10}=\frac{6}{5}$

Here $\quad \log_3 n=\frac{6}{5}$

$\Rightarrow \quad n=3^{6/5}\cong 3.70s$

which is not natural hence minimum occurs at the closest integer

now $\quad 4>3^{6/5}$

$4^5>3^6$

$1024>729$ which is true

8. Using property we get

$$\frac{a^4-(a^2+1)-2a}{a^2-a-1}=\frac{a^4-(a+1)^2}{a^2-a-1}=a^2+a+1$$

11. Let $\quad \log_2 x=y$

$\Rightarrow \quad 1+2y+y^2+y+2y^2+y^3=1$

$\Rightarrow \quad y(y^2+3y+3)=0$

$\Rightarrow \quad y=0 \quad$ or $\quad y^2+3y+3=0$

$\Rightarrow \quad \log_2 x=0 \quad$ or $\quad D<0$ no real solution

$\Rightarrow \quad x=1$

(which is not in domain as x is in the base in one term)

12. $(1+k)^n=\frac{s}{p} \quad \Rightarrow \ n\log(1+k)=\log \quad (s/p)$

$$\Rightarrow \quad n=\frac{\log s/p}{\log(1+k)}$$

13. **Case-I**

when B is a quadratic equation

$D_1=(m+3)^2$ and $D_2=(m-2)^2$

roots of 1st equation are $2,-(m+1)$ set A

roots of 2nd equation are $-1,\ \frac{1}{1-m}$ set B

For exactly there elements in $A\cup B$ two of the roots must be same note that $2\neq -1$

possibilities are

$2=-(m+1) \quad \Rightarrow \quad m=-3$

$2=\frac{1}{1-m} \quad \Rightarrow \quad 2-2m=1$

$\Rightarrow \quad m=1/2$

$-m-1=-1 \quad \Rightarrow \quad m=0$

$-(m+1)=\frac{1}{1-m} \quad \Rightarrow \quad 1-m^2=-1$

$\Rightarrow \quad m=\pm\sqrt{2}$

$\frac{1}{1-m}=-1 \quad \Rightarrow \quad m-1=1$

$\Rightarrow \quad m=2.$

Case-II

Now if $m=1$, then B becomes linear

roots of B as $x=-1$

roots of A are 2 and -2

$\Rightarrow \quad$ 3 elements in common

$\therefore$ all permissible m are $\{-3,\ \frac{1}{2},\ \sqrt{2},-\sqrt{2},2,0,1\}$

15. $\dfrac{\log x}{\log \frac{p}{q}} = \dfrac{\log x}{\log p - \log q} = \dfrac{1}{\dfrac{\log p}{\log x} - \dfrac{\log q}{\log x}}$

$= \dfrac{1}{\log_x p - \log_x q} = \dfrac{1}{\frac{1}{\alpha} - \frac{1}{\beta}} = \dfrac{\alpha\beta}{\beta - \alpha}$

17. $B = \dfrac{12}{3 + \sqrt{5} + \sqrt{8}}$

$B = \left(\dfrac{12(3 + \sqrt{5} - \sqrt{8})}{(3 + \sqrt{5})^2 - 8}\right) = \dfrac{12(3 + \sqrt{5} - \sqrt{8})}{6 + 6\sqrt{5}}$

$= \left(\dfrac{2(3 + \sqrt{5} - 2\sqrt{2})}{1 + \sqrt{5}}\right) = \dfrac{6 + 2\sqrt{5}}{\sqrt{5} + 1} - \dfrac{4\sqrt{2}}{\sqrt{5} + 1}$

$= \dfrac{(\sqrt{5} + 1)^2}{\sqrt{5} + 1} - \dfrac{4\sqrt{2}(\sqrt{5} - 1)}{4}$

$= \sqrt{5} + 1 - \sqrt{10} + \sqrt{2} = A \quad \Rightarrow \quad \log_A B = 1$

20. $x = \left(\dfrac{5}{3}\right)^{-100} \Rightarrow \log_{10} x = -100(\log 5 - \log 3)$

$= -100\left(\log_{10} 10 - \log_{10} 2 - \log_{10} 3\right)$

$= -100(1 - .3010 - .4771)$

$= -22.19 = \overline{23}.81$ hence 0's $= 23 - 1 = 22$

EXERCISE - 2

Part # I : Multiple Choice

1. **(A)** $\log_3 19 . \log_{1/7} 3 . \log_4 \dfrac{1}{7} = \log_3 19 \log_4 3$

$= \log_4 19 > 2$

(B) $\dfrac{1}{5} > \dfrac{1}{23} > \dfrac{1}{25}$

$\log_5 \dfrac{1}{5} > \log_5 \dfrac{1}{23} > \log_5 \dfrac{1}{25}$

(C) $m = 7$ & $n = 7^4$

$\Rightarrow$ $n = m^4$

(D) $\log_{\sqrt{5}} 5^2 = 4$

7. $\sin^2 \beta = \sin\alpha \cos\alpha$

$\dfrac{1 - \cos 2\beta}{2} = \dfrac{\sin 2\alpha}{2}$

$\cos 2\beta = 1 - \sin 2\alpha$ **(A)**

$= 1 - \cos(\pi/2 - 2\alpha) = 2\sin^2\left(\dfrac{\pi}{4} - \alpha\right) \Rightarrow$ **(B)**

$= 2\cos^2\left(\dfrac{\pi}{4} + \alpha\right) \Rightarrow$ **(D)**

10.

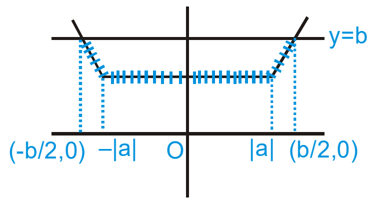
$\log_2 3 > 1, \ \log_{12} 10 < 1 \quad \Rightarrow \log_2 3 > \log_{12} 10$

$\log_6 5 < 1, \ \log_7 8 > 1 \quad \Rightarrow \log_6 5 < \log_7 8$

$\log_3 26 < 3, \log_2 9 > 3 \quad \Rightarrow \log_3 26 < \log_2 9$

$\log_{16} 15 < 1, \log_{10} 11 > 1 \quad \Rightarrow \log_{16} 15 < \log_{10} 11$

11.



13. Let any two distinct odd number be (2n + 3) and (2n + 1) when n ∈ W

Now According to question $(2n + 3)^2 - (2n + 1)^2$

$= (4n^2 + 12n + 9) - (4n^2 + 4n + 1)$

$4n^2 + 12n + 9 - 4n^2 - 4n - 1$

$= 8n + 8 = 8(n + 1)$

Which is always divisible by 4 & 8.

16. $2x^2 + 2x + a + 3$ must be positive hence $D < 0$

i.e. $4 - 8(a + 3) < 0 \quad \Rightarrow \quad 1 - 2a - 6 < 0$

$\Rightarrow \quad -2a < 5 \quad \Rightarrow \quad a > -\dfrac{5}{2}$ ....(i)

Also base of the logarithm

$7 - a > 0$ and $7 - a \neq 1$

$a < 7$ & $a \neq 6$ ....(ii)

from (1) and (2)

$a \in \left(-\dfrac{5}{2}, 6\right) \cup (6, 7)$

$\Rightarrow$ (B), (C) and (D) are correct

17. $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

$z_1\bar{z}_2 + \bar{z}_1 z_2 = 0 \quad \Rightarrow \quad \dfrac{z_1}{z_2} = -\dfrac{\bar{z}_1}{\bar{z}_2}$

$\dfrac{z_1}{z_2} + \overline{\left(\dfrac{z_1}{z_2}\right)} = 0 \quad \Rightarrow \quad \dfrac{z_1}{z_2}$ Pure imaginary

$\text{amp}\left(\dfrac{z_1}{z_2}\right) = \dfrac{\pi}{2}$

19. **(A)** $\log_{10}5(2\log_{10}2 + \log_{10}5) + (\log_{10}2)^2$

$= (\log_{10}2 + \log_{10}5)^2 = (\log_{10}10)^2 = 1$

(B) $\dfrac{\log 4 + \log 3}{2\log 4 + \log 3 - \log 4} = 1$

(C) $-\log_5 \log_3 3^{1/5} = -\log_5 \dfrac{1}{5} = 1$

(D) $\dfrac{1}{6}\log_{\sqrt[3]{4}}\left(\dfrac{4}{3}\right)^3 = \log_{3/4}\dfrac{4}{3} = -1$

20. $[2 - x] + 2[x - 1] \geq 0$

$\Rightarrow \quad 2 + [-x] + 2[x] - 2 \geq 0$

$2[x] + [-x] \geq 0$

Case- I

$x \in I$ then $2I - I \geq 0$

$\Rightarrow \quad I \geq 0$

so $x \in \{0, 1, 2, 3 .....\}$ ... (i)

case II

$x \notin I$ then $I - 1 \geq 0$

$\Rightarrow \quad I \geq 1 \quad \Rightarrow \quad I = 1, 2, 3 .....$

so $x \in (1, 2) \cup (2, 3) \cup (3, 4)$ ....(ii)

from (1) and (2)

$x \in \{0\} \cup [1, \infty)$

21. Let $\log_3 2 = y$

$N = \dfrac{1 + 2y}{(1 + y)^2} + \dfrac{1}{(\log_2 6)^2}$

$= \dfrac{1 + 2y}{(1 + y)^2} + \dfrac{1}{\left(1 + \dfrac{1}{y}\right)^2} = \dfrac{1 + 2y + y^2}{(1 + y)^2} = 1$

$\log_7 6 < 1 < \log_3 \pi$

22. $\log_{x^2} 16 + \log_{2x} 64 = 3$

$\Rightarrow \quad 4\log_{x^2} 2 + 6\log_{2x} 2 = 3$

$\Rightarrow \quad \dfrac{4}{\log_2 x^2} + \dfrac{6}{\log_2 2x} = 3$

$\Rightarrow \quad \dfrac{2}{\log_2 x} + \dfrac{6}{1 + \log_2 x} = 3$

but $\log_2 x = t$

$\therefore \quad \dfrac{2}{t} + \dfrac{6}{1 + t} = 3$

$\Rightarrow \quad 2 + 2t + 6t = 3t + 3t^2$

$\Rightarrow \quad 3t^2 - 5t - 2 = 0$

$\Rightarrow \quad 3t^2 - 5t - 2 = 0$

$\Rightarrow \quad (3t + 1)(t - 2) = 0$

$\Rightarrow \quad t = -\dfrac{1}{3}$, $\quad\quad t = 2$

$\Rightarrow \quad \log_2 x = -\dfrac{1}{3} \quad\quad \log_2 x = 2$

$\Rightarrow \quad x = 2^{-1/3} \quad\quad x = 4$

$= \dfrac{1}{2^{1/3}}$.

24. Only value of x satisfying given equation are 1 & 4.

25. $\left(\dfrac{1}{3}\right)^{\{x\}} > \left(\dfrac{1}{3}\right)^{1/2} \quad \Rightarrow \quad \{x\} < \dfrac{1}{2}$

$\therefore$ solution $x = \pi, \ x = 2 + \dfrac{1}{\sqrt[3]{9}}, \ x = \dfrac{e}{2}$

26. $\log_p \log_p (p)^{1/p^n} = \log_p \dfrac{1}{p^n} = \log_p p^{-n} = -n$

29. $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$

$\Rightarrow \ (\log_3 x)^3 - \frac{9}{2}\log_3 x + 5 = \log_x 3\sqrt{3}$

$\Rightarrow \ (\log_3 x)^2 - \frac{9}{2}\log_3 x + 5 = \frac{3}{2}\log_x 3$

let $\log_3 x = t$

$\Rightarrow \ t^2 - \frac{9}{2}t + 5 = \frac{3}{2t}$

$\Rightarrow \ 2t^3 - 9t^2 + 10t - 3 = 0$

$t = 1$ satisfied

So $2t^3 - 9t^2 + 10t - 3 = 2t^2(t-1) - 7t(t-1) + 3(t-1)$

$= (t-1)\left(2t^2 - 7t + 3\right)$

$= (t-1)(2t-1)(t-3)$

$\Rightarrow \ t = 1 \qquad | \qquad t = \frac{1}{2}$

$t = 3$

$\Rightarrow \ \log_3 x = 1 \qquad | \qquad \log_3 x = \frac{1}{2}$

$\log_3 x = 3$

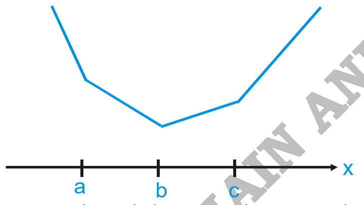
$\Rightarrow \ x = 3 \qquad | \qquad x = 3^{1/2}$

$x = 27.$

Part # II : Assertion & Reason

1. **(A)**

Graph of $y = |x-a| + |x-b| + |x-c|$



We get its minimum value at $x = b$.

So minimum value $|b-a| + |b-c|$

3. **(A)**

Statement 2 is correct and from statement 1

$\Rightarrow \ x^2 - 5x + 6 = 0 \ (\text{for } x \in z)$

$\Rightarrow \ x = \{2, 3\}$

Also, $x^2 - 5x + 6 = -1 \ (\text{for } x \notin z)$

$\Rightarrow \ x^2 - 5x + 7 = 0 \quad \Rightarrow$ no real root

$\Rightarrow$ St. 1 is true.

5. $-\log_{2+|x|}(5 + x^2) = \log_{3+x^2}(15 + \sqrt{x})$

$\because LHS < 0 \text{ and } RHS > 0$

hence no solution.

EXERCISE - 3

Part # I : Matrix Match Type

1. **(A) → (r), (B) → (t), (C) → (q), (D) → (p)**

(A) $\frac{5x + 1 - x^2 - 2x - 1}{(x+1)^2} < 0$

$-x^2 + 3x < 0, \ x \neq -1$

$x(x-3) > 0, \ x \neq -1$

$\therefore \quad x \in (-\infty, -1) \cup (-1, 0) \cup (3, \infty)$

(B) $|x| + |x-3| = \begin{cases} -2x + 3 & : \quad x < 0 \\ 3 & : \quad 0 \le x \le 3 \\ 2x - 3 & : \quad x > 3 \end{cases}$

$\therefore \quad x \in (-\infty, 0) \cup (3, \infty)$

(C) $\frac{1}{|x| - 3} - \frac{1}{2} < 0$

$\Rightarrow \ \frac{2 - |x| + 3}{2(|x| - 3)} < 0$

$(5 - |x|) / (|x| - 3) < 0$

$\Rightarrow \ |x| < 3 \text{ or } |x| > 5$

$\Rightarrow \ x \in (-\infty, -5) \cup (-3, 3) \cup (5, \infty)$

(D) $\frac{x^4}{(x-2)^2} > 0$

$\Rightarrow \ x \in (-\infty, 0) \cup (0, 2) \cup (2, \infty)$

2. **(A)** $2\log_{10}(x-3) = \log_{10}(x^2 - 21)$

$\Rightarrow \ (x-3)^2 = x^2 - 21$

$\Rightarrow \ 6x = 30 \quad \Rightarrow \quad x = 5$

(B) $x^{\log_2 x + 4} = 32$

$\Rightarrow \ (\log_2 x + 4)\log_2 x = \log_2 32$

Let $\log_2 x = y$

$\Rightarrow \ y^2 + 4y - 5 = 0$

$\Rightarrow \ (y+5)(y-1) = 0$

$\log_2 x = -5 \qquad \log_2 x = 1$

$\Rightarrow \ x = \frac{1}{32} \quad \& \quad x = 2$

(C) $5^{\log_{10}x} + \frac{5^{\log_{10}x}}{5} = 3.3^{\log_{10}x} + \frac{3^{\log_{10}x}}{3}$

$\Rightarrow \quad \left(\frac{6}{5}\right)5^{\log_{10}x} = \left(\frac{10}{3}\right)3^{\log_{10}x}$

$\Rightarrow \quad \left(\frac{5}{3}\right)^{\log_{10}x} = \left(\frac{5}{3}\right)^{2}$

$\Rightarrow \quad \log_{10} x = 2 \quad \Rightarrow \quad x = 100$

(D) $9.9^{\log_3 x} - 3.3^{\log_3 x} - 210 = 0$

$\Rightarrow \quad 9x^2 - 3x - 210 = 0$

$\Rightarrow \quad 3x^2 - x - 70 = 0$

$\Rightarrow \quad 3x^2 - 15x + 14x - 70 = 0$

$\Rightarrow \quad x = 5 \;;\; x = \frac{-14}{3}$ (Reject)

3. **(A)** $\rightarrow$ (p,r,s), **(B)** $\rightarrow$ (r,s), **(C)** $\rightarrow$ (t), **(D)** $\rightarrow$ (p,r,s)

(A) $(3 - x) > 3\sqrt{1 - x^2}$

Case-I **(i)** $3 - x \geq 0 \quad \Rightarrow \quad x \leq 3$

(ii) $\sqrt{1 - x^2} \geq 0 \quad \Rightarrow \quad x \in [-1, 1]$

(iii) $9 + x^2 - 6x > 9 - 9x^2$

$10x^2 - 6x > 0$

$x(5x - 3) > 0$

$\Rightarrow \quad x \in (-\infty, 0) \cup \left(\frac{3}{5}, \infty\right)$

$\therefore \quad x \in [-1, 0) \cup \left(\frac{3}{5}, 1\right]$

Case-II **(i)** $3 - x < 0$

– ve > + ve not possible $\quad \therefore$

by case-I & II $\quad x \in [-1, 0) \cup \left(\frac{3}{5}, 1\right]$

(B) $-\sqrt{x+2} < -x \Leftrightarrow x < \sqrt{x+2}$

case-I **(i)** $x \geq 0$

(ii) $x + 2 > 0$

(iii) $x^2 < x + 2$

so $\quad x \in [0, 2)$

case-II **(i)** $x < 0$

(ii) $x + 2 \geq 0$

(iii) –ve < + ve

so $\quad x \in [-2, 0)$

by case-I & II $x \in [-2, 2)$

(C) $\log_5 (x - 3) + \frac{1}{2}\log_5 3 < \frac{1}{2}\log_5(2x^2 - 6x + 7)$

$3(x - 3)^2 < (2x^2 - 6x + 7)$

$\Rightarrow \quad x \in (2, 10)$

$\because \quad x > 3$

so $\quad x \in (3, 10)$

(D) $7^{x+2} - \frac{1}{7} . 7^{x+1} - 14.7^{x-1} + 2.7^x = 48$

Let $\quad 7^x = t$

$49t - t - 2t + 2t = 48 \quad \therefore \quad t = 1$

Part # II : Comprehension

Comprehension 1

1. **(A)** $|x^3 - x| + |2 - x| = (x^3 - x) - (2 - x)$

$\therefore \quad x^3 - x \geq 0$ and $2 - x \leq 0$

$x^3 - x \geq 0$ and $x \geq 2$

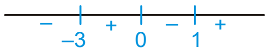
$x(x^2 - 1) \geq 0$ and $x \geq 2$

$\therefore \quad x \in [2, \infty)$

2. **(D)**

$(x^2 - x)(x + 3) \leq 0$

$x(x - 1)(x + 3) \leq 0$



$x \in (-\infty, -3] \cup [0, 1]$

3. **(C)**

$| f(x) - g(x) | = | f(x) | + | g(x) |$

obviously $\quad f(x) . g(x) \leq 0$

Comprehension 3

1. **(C)**

Since $2m - n = 3$ has the solution $m = 4$

and $a_5 - (a_1 + a_2 + a_3 + a_4) = 9 - (1 + 3 + 4 + 7) = -6 < 5$

$\therefore$ there are 2 solutions

2. **(A)**

Since $2m - n = 2$ is not possible

but $2m - n + 1 = 2$ has the solution $m = 3$ and $2 < 5$

and $10 - (1 + 3 + 4) = 2 > 1$

$\therefore$ there is no solution

3. **(C)**

Since $2m - n = 2$ has no solution

$2m - n + 1 = 2$ has a solution $m = 3$ and $2 < 5$

and $7 - (1 + 2 + 4) = 0 < 10$

$\therefore$ there are two solutions.

EXERCISE - 4

Subjective Type

1. $\begin{bmatrix} a>0 \ ; & a\neq 1 \\ N>0 \ ; & N\neq 1 \\ b>0 \ ; & b\neq 1/a \end{bmatrix}$

2. 5

3. $\dfrac{\log_a x}{\log_a y}=4$ & $\dfrac{\log_a z}{\log_a y}=7$

$\Rightarrow \dfrac{\log_a x}{4}=\dfrac{\log_a y}{1}=\dfrac{\log_a z}{7}=\lambda$

$\Rightarrow x=a^{4\lambda},\ y=a^{\lambda},\ z=a^{7\lambda}$

Now $\log_a a^{4\lambda}\log_a(a^{4\lambda+\lambda+7\lambda})=48$

$48\lambda^2=48 \Rightarrow \lambda^2=1 \Rightarrow \lambda=\pm 1$

4. $\dfrac{a^4}{b^4}$

5. $a^2+b^2=c^2$ $a>0,\ b>0,\ c>0$

$\log_{c+b} a+\log_{c-b} a=2\log_{c+b} a\log_{c-b} a$

LHS $=\dfrac{1}{\log_a(c+b)}+\dfrac{1}{\log_a(c-b)}$

$=\dfrac{\log_a(c^2-b^2)}{\log_a(c+b)\log_a(c-b)}=\dfrac{\log_a a^2}{\log_a(c+b)\log_a(c-b)}$

$=\dfrac{2}{\log_a(c+b)\log_a(c-b)}=2\log_{c+b} a\log_{c-b} a$

6. $x=2, y=1$

7. $x=50\log_{10} x \Rightarrow 10^x=x^{50}$

$\Rightarrow 10^{2x}=x^{50\times 2} \Rightarrow 100^x=x^{100}$

$\Rightarrow x=100$

8. $x=2$ or 81

9. $\log 4+\log 3+\log 3^{1/2x}=\log\left(3^{1/x}+27\right)$

$\Rightarrow 12.3^{\frac{1}{2x}}=3^{1/x}+27$

Let $3^{1/2x}=y$ $\Rightarrow y^2-12y+27=0$

$\Rightarrow y=9$ or $y=3 \Rightarrow x=\dfrac{1}{4}$ and $x=\dfrac{1}{2}$

but $x\in N \geq 2$ have no solution

10. $x=10/3, y=20/3$ & $x=-10, y=20$

11. $\log_{100}|x+y|=\dfrac{1}{2}$ and

$\log_{10} y-\log_{10}|x|=\log_{100} 4=\log_{10} 2$

$\Rightarrow |x+y|=10$ & $\dfrac{y}{|x|}=2 \Rightarrow y=2|x|$

when $x>0$; $y=2x$

$\Rightarrow x=\dfrac{10}{3}$ & $y=\dfrac{20}{3}$

when $x<0; y=-2x$

$\Rightarrow |-x|=10$

$\Rightarrow x=-10\quad y=20$

12. (i) $\dfrac{(1+i)x-2i}{3+i}+\dfrac{(2-3i)y+i}{3-i}=i$

$\dfrac{(3-i)(x+ix-2i)+(3+i)(2y-3iy+i)}{(3+i)(3-i)}=i$

$3x+3ix-6i-ix+x-2+6y-9iy+3i+2iy+3y-1=10i$

$4x+9y-3+i(2x-7y-13)=10i$

$4x+9y-3=0$ ...(i)

$2x-7y-13=0$ ...(ii)

$23y+23=0$

$y=-1$

$4x-9-3=0$

$4x=12$

$x=3.$

13. $k_1p+7=296$

$k_2p+11=436$

$k_3p+15=542$

$k_1p=289=(17)(17)$

$k_2p=425=(17)(25)$

$k_3p=527=(17)(31)$

Hence p is 17

14. **(i)** $x\in\left[-2,-\dfrac{3}{2}\right)$

(ii) $(\log_2 5,\infty)$

(iii) $(0,10^{-1}]\cup[10^2,\infty)$

(iv) $(-\infty,-5)\cup(-5,-1)\cup(3,\infty)$

(v) $(-\infty,-1)\cup(1,\infty)$

(vi) $x\in(-2,-1)\cup(-1,0)\cup(0,1)\cup(2,\infty)$

15. **(i)** $(-\infty, 1) \cup (5, \infty)$

(ii) $[-1, (\sqrt{5}-1)/2)$ **(iii)** $x \in [3, \infty)$

(iv) $x \in \left[\frac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7+\sqrt{21}}{2}\right]$ **(v)** $x = 2$

(vi) $\left(0, \frac{1}{4}\right] \cup [1, 4)$

16. **(i)** $|x| + 2 = 3$

$\Rightarrow$ $|x| = 1$

$\Rightarrow$ $x = \pm 1$

(ii) $|x| - 2x + 5 = 0$

case (i)

$x < 0$

$-x - 2x + 5 = 0$

$\Rightarrow$ $x = \frac{5}{3}$ (not possible) ($\because$ $x < 0$)

case (ii)

$x \geq 0$

$x - 2x + 5 = 0$

$\therefore$ $x = 5$

(iii) $x|x| = 4$

case (i)

$x < 0$

$\therefore$ $-x^2 = 4$ (no solution)

case (ii)

$x \geq 0$

$x^2 = 4$

$x = \pm 2$

$\therefore$ $x = 2$

(iv) $\left|\,|x-1| - 2\,\right| = 1$

(vi) $|x-3| + 2|x+1| = 4$

case (i)

$x < -1$

$-x + 3 - 2x - 2 = 4$

$-3x = 3$

$x = -1$

case (ii)

$-1 \leq x < 3$

$-x + 3 + 2x + 2 = 4$

$x = -1$

case (iii)

$x \geq 3$

$x - 3 + 2x + 2 = 4$

$3x = 5$

$x = \frac{5}{3}$ (not possible)

$\therefore$ $x = -1$

(vii) $||x-1| - 2| = |x-3|$

by using property

$||a| - |b|| = |a - b|$ $\Rightarrow$ $a \cdot b \geq 0$

$2(x-1) \geq 0$ $\Rightarrow$ $x \geq 1$

$\Rightarrow$ $x \in [1, \infty)$

17. $x = 9, \frac{1}{9}$

18. $x = 3$ or -3

20. $\left\{\frac{1}{4}\right\}$

21. $(x^2 + y^2)$

22. $x \in \phi$

23. $x \in \left[\frac{1}{2}, 5\right]$

24. **(i)** $x \in (-\infty, 1] \cup [5, \infty)$

(ii) $x = 5$ or $x = -1$

(iii) $x \in R - \{3\}$

(iv) $x \in [0, 6]$

25. 2

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

6. (A)

$\left(x^2-5x+5\right)^{x^2+4x-60}=\left(x^2-5x+5\right)^0$

$\Rightarrow\ x^2+4x-60=0\quad [a^x=a^y\ \Rightarrow\ x=y \text{ if } a\neq 1,0,-1]$

$x=-10,6\ \ \&\ \ \text{base } x^2-5x+5=0\ \text{ or }\ 1\ \text{ or }\ -1$

If $x^2-5x+5=0$

But it will not satisfy original equation

$x^2-5x+5=1$ | $x^2-5x+5=-1$

$\therefore\ x=4,1$ | $\therefore\ x=2,3$

$x=3$ does not satisfy equ.

Part # II : IIT-JEE ADVANCED

12. (4)

Let $\sqrt{4-\frac{1}{3\sqrt2}\sqrt{4-\frac{1}{3\sqrt2}\sqrt{4-\frac{1}{3\sqrt2}\ldots}}}=y$

So, $4-\frac{1}{3\sqrt2}y=y^2\qquad (y>0)$

$\Rightarrow\ y^2+\frac{1}{3\sqrt2}y-4=0\quad\Rightarrow\ y=\frac{8}{3\sqrt2}$

so, the required value is $6+\log_{3/2}\left(\frac{1}{3\sqrt2}\times\frac{8}{3\sqrt2}\right)$

$=6+\log_{3/2}\frac{4}{9}=6-2=4.$

13. (ABC)

$\log_2 3^x=(x-1)\log_2 4=2(x-1)$

$\Rightarrow\ x=\log_2 3=2x-2$

$\Rightarrow\ x=\frac{2}{2-\log_2 3}$

Rearranging, we get

$x=\frac{2}{2-\frac{1}{\log_3 2}}=\frac{2\log_3 2}{2\log_3 2-1}$

Rearranging again,

$x=\frac{\log_3 4}{\log_3 4-1}=\frac{\frac{1}{\log_4 3}}{\frac{1}{\log_4 3}-1}=\frac{1}{1-\log_4 3}$

MOCK TEST

1. (C)

$\log_2 15\ \log_{1/6} 2\ \log_3 1/6=\frac{\log 15}{\log 2}\times\frac{\log 2}{\log 1/6}\times\frac{\log 1/6}{\log 3}$

$=\frac{\log(3\times5)}{\log 3}=1+\log_3 5>2\qquad(\text{but }<3)$

2. (C)

(i) $\log_{\frac{1}{3}}(x^2+x+1)>-1$

$\Rightarrow\ x^2+x+1<3$

$\Rightarrow\ x^2+x-2<0$

$\Rightarrow\ (x+2)(x-1)<0\ \Rightarrow\ x\in(-2,1)$ ....(i)

and (ii) $x^2+x+1>0\ \Rightarrow\ x\in R$ ....(ii)

by (i) & (ii) $x\in(-2,1)$

3. (D)

$|x^2-9|+|x^2-4|=5$

$|x^2-9|+|x^2-4|\ =|(x^2-9)-(x^2-4)|$

$\Rightarrow\ (x^2-9)(x^2-4)\le 0$

$\{\because\ |a|+|b|=|a-b|\Leftrightarrow a.b\le 0\}$

$\Rightarrow\ x\in[-3,-2]\cup[2,3]$

4. (A)

$|a|+|b|=|a-b|\quad\Rightarrow\ a.b\le 0$

$(x^2-5x+7)(x^2-5x-14)\le 0$

$(x-7)(x+2)\le 0\quad\Rightarrow\ x\in[-2,7]$

5. (A)

$S_1:\ e^{y\,\ell n 7-x\,\ell n 11}=e^{\ell n\frac{7^y}{11^x}}=\frac{7^y}{11^x}=\frac{7^{\sqrt{\log_7 11}}}{11^{\sqrt{\log_{11} 7}}}$

$=\frac{7^{\frac{\log_7 11}{\sqrt{\log_7 11}}}}{11^{\sqrt{\log_{11}7}}}=\frac{11^{\sqrt{\log_{11}7}}}{11^{\sqrt{\log_{11}7}}}=1$

$S_2:\ \log_x 3>\log_x 2\quad\Rightarrow\quad x>1$

$S_3:\ |x-2|=[-\pi]$

$|x-2|=-4$ no solution

S_4 : $\log_{25}(2 + \tan^2\theta) = 0.5$

$\Rightarrow \frac{1}{2}\log_5(2 + \tan^2\theta) = \frac{1}{2} \quad \Rightarrow \quad 2 + \tan^2\theta = 5$

$\Rightarrow \tan^2\theta = 3 \quad \Rightarrow \quad \tan\theta = \pm\sqrt{3}$

$\Rightarrow$ θ may take values $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$

6. (C)

Since x = 0 is one of the solution so the product will be zero.

7. (B)

Case I

$[x] - 2x = 4 \quad$ ....(i)

$\Rightarrow [x] - 2([x] + \{x\}) = 4$

$\Rightarrow [x] + 2\{x\} + 4 = 0 \quad$ ....(ii)

$\because 0 \le 2\{x\} < 2$

$\therefore 0 \le -[x] - 4 < 2$

$\Rightarrow -6 < [x] \le -4 \quad \Rightarrow \quad [x] = -4, -5$

$\therefore$ from (i) we get $x = -4, \frac{-9}{2}$

Case II

$[x] - 2x = -4 \quad$ ....(iii)

$\Rightarrow [x] = 2x - 4$

$\Rightarrow [x] = 2([x] + \{x\}) - 4$

$\Rightarrow 2\{x\} = 4 - [x] \quad$ ....(iv)

$\therefore 0 \le 2\{x\} < 2$

$\Rightarrow 0 \le 4 - [x] < 2$

$\Rightarrow 2 < [x] \le 4 \quad \therefore \quad [x] = 3, 4$

$\therefore$ from (iii) we get $x = 4, \frac{7}{2}$

$\therefore$ Number of solutions of $|[x] - 2x| = 4$ are 4.

8. (A)

$5\{x\} = x + [x] \quad$ ....(i)

$[x] - \{x\} = \frac{1}{2} \quad$ ....(ii)

$\because 0 \le \{x\} < 1$

$\Rightarrow 0 \le [x] - \frac{1}{2} < 1 \quad$ (by (ii))

$\Rightarrow [x] = 1 \quad \therefore \quad \{x\} = \frac{1}{2}$

$\therefore$ from (i) we get $\frac{5}{2} = x + 1$

$\therefore x = \frac{3}{2}$, (one value)

9. (D)

$0 \le \log_e[2x] \le 1$

$1 \le [2x] \le e$

$\Rightarrow [2x] = 1, 2$

$\Rightarrow 1 \le 2x < 3$

$\therefore \frac{1}{2} \le x < \frac{3}{2}$

10. (A)

$$N = \frac{(3^4)^{\log_9 5} + 3^{3\log_3\sqrt{6}}}{409}[7^{\log_7 25} - (5^3)^{\log_{5^2} 6}]$$

$$\Rightarrow N = \frac{3^{\log_3 25} + 3^{\log_3 \sqrt{6}^3}}{409}[25 - 6\sqrt{6}]$$

$$N = \frac{(25 + 6\sqrt{6})(25 - 6\sqrt{6})}{409}$$

$N = 1$

$\log_2 N = \log_2 1 = 0$

11. (A, D)

$\ell n(x + z) + \ell n(x - 2y + z) = 2\,\ell n(x - z)$

$\ell n(x + z)(x - 2y + z) = \ell n(x - z)^2$

$x^2 - 2xy + 2zx - 2yz + z^2 = x^2 + z^2 - 2zx$

$\Rightarrow y = \frac{2xz}{z + x}$ or $\frac{x}{z} = \frac{x - y}{y - z}$

12. (C,D)

$\log_{10}5 \,.\, \log_{10}20 + (\log_{10}2)^2$

$= \log_{10}5\,(1 + \log_{10}2) + (\log_{10}2)^2 = \log_{10}5 + \log_{10}2\,[\log_{10}5 + \log_{10}2]$

$= 1$

13. (B)

Here $x \ne 0$

Case I when $x \ge -2$

$\frac{|x + 2| - x}{x} < 2 \quad \Rightarrow \quad \frac{2}{x} < 2 \quad \Rightarrow \quad \frac{1}{x} < 1$

$\Rightarrow (x - 1)/x > 0$

$x \in [-2, 0) \cup (1, \infty) \quad$ ....(i)

Case II when $x < -2$

$$\frac{|x+2|-x}{x} < 2 \Rightarrow \frac{-2-2x}{x} < 2$$

$$\Rightarrow \frac{1+x}{x} + 1 > 0$$

$\Rightarrow (1+2x)/x > 0 \quad \Rightarrow x \in (-\infty, -2)$ ....**(ii)**

$\therefore$ from **(i)** and **(ii)** we get $x \in (-\infty, 0) \cup (1, \infty)$

14. (B, D)

(A) $\log_3 \log_{27} \log_4 64 = \log_3 \log_{27} 3 = \log_3 \left(\frac{1}{3}\right) = -1$

(B) $2 \log_{18} (\sqrt{2}+\sqrt{8}) = 2 . \log_{(3\sqrt{2})^2} \left(\sqrt{2}+2\sqrt{2}\right) = 2 .$

$\log_{(3\sqrt{2})^2} \left(3\sqrt{2}\right) = \frac{2}{2} = 1$

(C) $\log_2 \left(\sqrt{10} \times \frac{2}{\sqrt{5}}\right) = \log_2 2\sqrt{2} = \frac{3}{2}$

(D) $-\log_{\sqrt{2}-1} (\sqrt{2}+1) = \log_{\sqrt{2}-1} \left(\sqrt{2}+1\right)^{-1}$

$= \log_{\sqrt{2}-1} (\sqrt{2}-1) = 1$

15. (A, B)

$\because$ AM $\geq$ GM

$$\frac{x+y}{2} \geq \sqrt{xy}$$

$$\Rightarrow \left(\frac{2-z}{2}\right) \geq \sqrt{xy}$$

$$\frac{2-z}{2} \geq \sqrt{\frac{z^2+4}{2}}$$

$$\frac{4+z^2-4z}{4} \geq \frac{4+z^2}{2}$$

$(z+2)^2 \leq 0 \quad \therefore \quad z = -2$

$x + y = 4$ and $xy = 4$

$x - y = 0$

$\therefore x = 2, y = 2$ and $z = -2$

only one real solution.

16. (D) Statement I is false

$\because$ Sum of the length of any two sides of a triangle is greater than length of third side

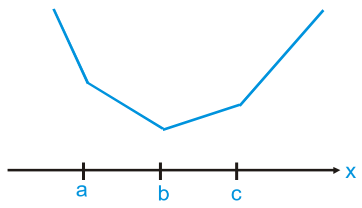
Statement II is true

$\because a^2 + c^2 - b^2 < 0$

then $\cos B < 0 \quad \Rightarrow$ B is obtuse

17. (A)

Graph of $y = |x-a| + |x-b| + |x-c|$



We get its minimum value at $x = b$.

So minimum value $|b-a| + |b-c|$

18. (C)

The result can be easily understood with the help of nature of graph of $y = \log_a x$

19. (A)

Statement 2 is correct and from statement 1

$\Rightarrow x^2 - 5x + 6 = 0$ (for $x \in z$)

$\Rightarrow x = \{2, 3\}$

Also, $x^2 - 5x + 6 = -1$ (for $x \notin z$)

$\Rightarrow x^2 - 5x + 7 = 0$

$\Rightarrow$ no real root $\quad \Rightarrow$ St. 1 is true.

20. (A)

$\log_a b < 0$

$\Rightarrow$ either $\{0 < a < 1$ and $b > 1\}$

or $\{a > 1$ and $b < 1\}$

$\Rightarrow$ 1 lies between the roots

$\therefore a + \alpha + \beta < 0$ and so $\alpha + \beta < 0$

$\therefore$ both the statements are true. Statement-2 is a correct explanation for Statement-1.

21. (A) - (r), (B) - (t), (C) - (p,q,r), (D) - (p,q)

(A) $\log_{\sin x} (\log_3 (\log_{0.2} x)) < 0$

$\Rightarrow \log_3 (\log_{0.2} x) > 1$

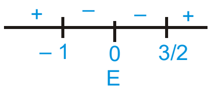
$\Rightarrow \log_{0.2} x > 3$

$\Rightarrow 0 < x < (0.2)^3$

$\Rightarrow 0 < x < \frac{1}{125}$ (which also satisfy $0 < \sin x < 1$)

(B) $\dfrac{(e^x-1)(2x-3)(x^2+x+2)}{(\sin x-2)\,x(x+1)} \le 0$

$$\frac{(e^x-1)(x-3/2)}{x(x+1)} \ge 0$$



$\Rightarrow \quad x \in (-\infty,-1) \cup [3/2, \infty)$

(C) $|2-|[x]-1|| \le 2 \quad \Rightarrow \quad ||[x]-1|-2| \le 2$

$\Rightarrow \quad 0 \le |[x]-1| \le 4 \quad \Rightarrow \quad -3 \le [x] \le 5$

$\Rightarrow \quad x \in [-3, 6)$

(D) $|\sin^{-1}(3x-4x^3)| \le \dfrac{\pi}{2}$

$\Rightarrow \quad -\dfrac{\pi}{2} \le \sin^{-1}(3x-4x^3) \le \dfrac{\pi}{2}$

$\Rightarrow \quad -1 \le 3x-4x^3 \le 1 \quad \Rightarrow \quad -1 \le x \le 1$

22. **(A)** $\to$ (p,r,s), **(B)** $\to$ (r,s), **(C)** $\to$ (t), **(D)** $\to$ (p,r,s)

(A) $(3-x) > 3\sqrt{1-x^2}$

Case-I **(i)** $3-x \ge 0 \quad \Rightarrow \quad x \le 3$

(ii) $\sqrt{1-x^2} \ge 0 \quad \Rightarrow \quad x \in [-1, 1]$

(iii) $9+x^2-6x > 9-9x^2$

$10x^2-6x > 0$

$x(5x-3) > 0 \quad \Rightarrow \quad x \in (-\infty, 0) \cup \left(\dfrac{3}{5}, \infty\right)$

$\therefore \quad x \in [-1, 0) \cup \left(\dfrac{3}{5}, 1\right]$

Case-II **(i)** $3-x < 0$

– ve > + ve not possible

$\therefore$ by case-I & II $\quad x \in [-1, 0) \cup \left(\dfrac{3}{5}, 1\right]$

(B) $-\sqrt{x+2} < -x \quad \Leftrightarrow \quad x < \sqrt{x+2}$

Case-I **(i)** $x \ge 0$

(ii) $x+2 > 0$

(iii) $x^2 < x+2$

so $\quad x \in [0, 2)$

Case-II **(i)** $x < 0$

(ii) $x+2 \ge 0$

(iii) –ve < + ve

so $x \in [-2, 0)$

by case-I & II $x \in [-2, 2)$

(C) $\log_5(x-3) + \dfrac{1}{2}\log_5 3 < \dfrac{1}{2}\log_5(2x^2-6x+7)$

$3(x-3)^2 < (2x^2-6x+7) \quad \Rightarrow \quad x \in (2, 10)$

$\because \quad x > 3$

so $\quad x \in (3, 10)$

(D) $7^{x+2} - \dfrac{1}{7}\cdot 7^{x+1} - 14.7^{x-1} + 2.7^x = 48$

Let $7^x = t$

$49t - t - 2t + 2t = 48 \quad \therefore \quad t = 1$ so $\quad x = 0$

23.

1. (A) $|x^3-x| + |2-x| = (x^3-x) - (2-x)$

$\therefore \quad x^3 - x \ge 0$ and $2 - x \le 0$

$x^3 - x \ge 0$ and $x \ge 2$

$x(x^2-1) \ge 0$ and $x \ge 2$

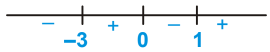
$\therefore \quad x \in [2, \infty)$

2. (D)

$(x^2-x)(x+3) \le 0$

$x(x-1)(x+3) \le 0$

$x \in (-\infty, -3] \cup [0, 1]$



3. (C)

$|f(x)-g(x)| = |f(x)| + |g(x)|$

obviously $\quad f(x)\,.\,g(x) \le 0$

24.

1. This equation is equivalent to the system

$$\begin{cases} 2x > 0 \\ (2x)^2 = 7x - 2 - 2x^2 \end{cases} \Rightarrow \begin{cases} x > 0 \\ 6x^2 - 7x + 2 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x > 0 \\ (x-1/2)(x-2/3) = 0 \end{cases} \Rightarrow \begin{cases} x = \dfrac{1}{2} \\ x = \dfrac{2}{3} \end{cases}$$

$\therefore \quad$ Number of solutions = 2.

2. This equation is equivalent to the system

$$\begin{cases} 4x-15>0 \\ 2x=(4x-15)^2 \end{cases}$$

$$\Rightarrow \begin{cases} x>\dfrac{15}{4} \\ 2x=16x^2-120x+225 \end{cases}$$

$$\Rightarrow \begin{cases} x>\dfrac{15}{4} \\ 16x^2-122x+225=0 \end{cases}$$

$\therefore$ $x=\dfrac{9}{2}$

$\therefore$ Number of solutions = 1.

3 This equation is equivalent to the system

$$\begin{cases} 3x^2+x-2>0 \\ 3x^2+x-2=(3x-2)^3 \end{cases}$$

$$\Rightarrow \begin{cases} (x-2/3)(x+1)>0 \\ (x-2/3)(9x^2-13x+3)=0 \end{cases}$$

$$\Rightarrow \begin{cases} x<2/3 \text{ and } x<-1 \\ x=2/3,\ x=\dfrac{13\pm\sqrt{61}}{18} \end{cases}$$

$\therefore$ No root.

25.

1. (C)

Since $2m-n=3$ has the solution $m=4$

and $a_5-(a_1+a_2+a_3+a_4)=9-(1+3+4+7)=-6<5$

$\therefore$ there are 2 solutions

2. (A)

Since $2m-n=2$ is not possible

but $2m-n+1=2$ has the solution $m=3$ and $2<5$

and $10-(1+3+4)=2>1$

$\therefore$ there is no solution

3. (C)

Since $2m-n=2$ has no solution

$2m-n+1=2$ has a solution $m=3$ and $2<5$

and $7-(1+2+4)=0<10$

$\therefore$ there are two solutions.

26.

4.

Case – I

$0<x^2<1 \quad \Rightarrow \quad x\in(-1,1)-\{0\}$ ....(i)

$|x-1|<1 \quad \Rightarrow \quad -1<x-1<1$

$\Rightarrow \quad 0<x<2$ ....(ii)

from (i) and (ii), we get $x\in(0,1)$

Case – II

$x^2>1$

$\Rightarrow \quad x\in(-\infty,-1)\cup(1,\infty)$ ....(iii)

$|x-1|>1$

$\Rightarrow \quad x-1>1$ or $x-1<-1$

$\Rightarrow \quad x>2$ or $x<0$ ....(iv)

from (iii) and (iv), we get $x\in(-\infty,-1)\cup(2,\infty)$

$\therefore \quad x\in(-\infty,-1)\cup(0,1)\cup(2,\infty)$

inequality is not defined for

$x=-1,0,1,2$

$\therefore$ sum of their absolute values

$=|-1|+|0|+|1|+|2|=4.$

27.

2.

$|x^2-3x-1|<|3x^2+2x+1|+|2x^2+5x+2|,\ x^2-3x-1\neq0$

$\Leftrightarrow |(3x^2+2x+1)-(2x^2+5x+2)|<|3x^2+2x+1|+|2x^2+5x+2|,$

$x^2-3x-1\neq0$

The inequality holds if and only if

$(3x^2+2x+1)(2x^2+5x+2)>0$

i.e. $2x^2+5x+2>0$

i.e. $(2x+1)(x+2)>0$

i.e. $x\in(-\infty,-2)\cup(-1/2,\infty)$

$\Rightarrow \quad a=2$ and $b=\dfrac{1}{2}$

$\therefore \quad a+\log ab=2$

28. $\underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots p\sqrt[p]{p}}}}}_{2008\text{ times}}=(p)^{p^{\frac{1}{2008}}}=p^{p^{-2008}}$

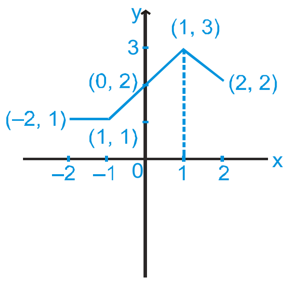
$\log_p\log_p\underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots p\sqrt[p]{p}}}}}_{2008\text{ times}}=\log_p\log_p\left(p^{p^{-2008}}\right)$

$=\log_p(p^{-2008})=-2008,$

$\therefore \quad \lambda=2008$

29. (3)

Drawing the graph of $y = f(x)$



Clearly the range of $y = f(x)$ is $[1, 3]$

when $-2 \le x \le -1$, $\{f(x)\} = 0$

when $-1 \le x \le 0$, $\{f(x)\}$ will have the value $\frac{1}{2}$ for one value of x.

when $0 \le x \le 1$, $\{f(x)\}$ will have the value $\frac{1}{2}$ for one value of x.

when $1 \le x \le 2$, $\{f(x)\}$ will have the value $\frac{1}{2}$ for one value of x.

Hence the total number of values of x for which $\{f(x)\} = \frac{1}{2}$ are 3

30. (1)

$$\sqrt{\left[x + \left[\frac{x}{2}\right]\right]} + \left[\sqrt{\{x\}} + \left[\frac{x}{3}\right]\right] = 3$$

$$\Rightarrow \sqrt{[x] + \left[\frac{x}{2}\right]} + \left[\sqrt{\{x\}}\right] + \left[\frac{x}{3}\right] = 3$$

$$\Rightarrow \sqrt{[x] + \left[\frac{x}{2}\right]} + \left[\frac{x}{3}\right] = 3$$

Case-1 $\sqrt{[x] + \left[\frac{x}{2}\right]} = 3$ and $\left[\frac{x}{3}\right] = 0$

i.e. $[x] + \left[\frac{x}{2}\right] = 9$ and $0 \le x < 3$

$\because$ $0 \le x < 3 \Rightarrow [x] = 0, 1$ or 2 ; $\left[\frac{x}{2}\right] = 0$ or 1

$\therefore$ There is no solution in this case.

Case-2 $\sqrt{[x] + \left[\frac{x}{2}\right]} = 2$ and $\left[\frac{x}{3}\right] = 1$

i.e. $[x] + \left[\frac{x}{2}\right] = 4$ and $3 \le x < 6$

$\because$ $3 \le x < 6 \Rightarrow [x] = 3, 4$ or 5 ; $\left[\frac{x}{2}\right] = 1$ or 2

$\therefore$ $[x] + \left[\frac{x}{2}\right] = 4$ has a solution for $[x] = 3$ and $\left[\frac{x}{2}\right] = 1$ and $\left[\frac{x}{3}\right] = 1$.

i.e. $3 \le x < 4, 2 \le x < 4$ and $3 \le x < 6$

$\therefore$ $[3, 4)$ are solutions.

Case-3 $\sqrt{[x] + \left[\frac{x}{2}\right]} = 1$ and $\left[\frac{x}{3}\right] = 2$

i.e. $[x] + \left[\frac{x}{2}\right] = 1$ and $6 \le x < 9$

$\because$ $6 \le x < 9$

$\Rightarrow [x] = 6, 7$ or 8 ; $\left[\frac{x}{2}\right] = 3$ or 4

$\therefore$ $[x] + \left[\frac{x}{2}\right] = 1$ and $\left[\frac{x}{3}\right] = 2$ is not possible.

Case-4 $\sqrt{[x] + \left[\frac{x}{2}\right]} = 0$ and $\left[\frac{x}{3}\right] = 3$

i.e. $[x] + \left[\frac{x}{2}\right] = 0$ and $9 \le x < 12$

$\because$ $9 \le x < 12$

$\Rightarrow [x] = 9, 10$ or 11 ; $\left[\frac{x}{2}\right] = 4$ or 5

$\therefore$ $[x] + \left[\frac{x}{2}\right] = 0$ has no solution.

Hence the solution set is $[3, 4)$.