TRIGONOMETRIC IDENTITIES
Teaching Task

1. **Ans : C**

Given $\sec\theta = \frac{m+m}{2\sqrt{mn}}$

$$\Rightarrow \cos\theta = \frac{2\sqrt{mn}}{m+n}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} \qquad since\ 0<\theta\le\frac{\pi}{2}$$

$$= \sqrt{1-\left(\frac{2\sqrt{mn}}{m+n}\right)^2}$$

$$= \sqrt{\frac{(m+n)^2-4mn}{(m+n)^2}} = \sqrt{\left(\frac{m-n}{m+n}\right)^2} = \frac{m-n}{m+n}$$

2. **Ans:B**

Given $\frac{(1+\tan^2 A).\cot A}{\operatorname{cosec}^2 A}$

$$= \frac{\sec^2 A.\cos A}{\operatorname{cosec}^2 A.\sin A}$$

$$= \frac{\left(\frac{1}{\cos^2 A}\times\cos A\right)}{\left(\frac{1}{\sin^2 A}\times\sin A\right)}$$

$$= \frac{\sin A}{\cos A}$$

=tanA

3. **Ans :A**

$\sec^2A.\operatorname{cosec}^2A-(\tan^2A+\cot^2A)$

$$= \frac{1}{\cos^2 A}.\frac{1}{\sin^2 A} - \left(\sec^2 A - 1 + \operatorname{cosec}^2 A - 1\right)$$

$$= \frac{\sin^2 A+\cos^2 A}{\cos^2 A.\sin^2 A} - \sec^2 A - \operatorname{cosec}^2 A + 2$$

$=\sec^2A+\operatorname{cosec}^2A-\sec^2A-\operatorname{cosec}^2A+2$

$=2$

4. **Ans : C**

$\left(\cos^2\theta - 1\right).\cos ec^2\theta$

$= -\sin^2\theta.\cos ec^2\theta = -1$

5. **Ans:A**

$\left(\sec\theta + \cos\theta\right)\left(\sec\theta - \cos\theta\right)$

$= \sec^2\theta - \cos^2\theta$

$= (1 + \tan^2\theta) - (1 - \sin^2\theta)$

$= 1 + \tan^2\theta - 1 + \sin^2\theta$

$= \tan^2\theta + \sin^2\theta$

6. **Ans : A**

$\left(1 + \frac{1}{\tan^2 A}\right)\left(1 + \frac{1}{\cot^2 A}\right)$

$=(1+\cos^2A)(1+\tan^2A)$

$=\text{cosec}^2A \cdot \sec^2A$

7. **Ans :** D

$\sin^2A.\cos^2B-\cos^2A.\sin^2B$

$=(1-\cos^2A)\cos^2B-\cos^2A(1-\cos^2B)$

$=\cos^2B-\cos^2A.\cos^2B-\cos^2A+\cos^2A.\cos^2B$

$=\cos^2B-\cos^2A$

8. **Ans : C**

Given $\frac{\tan\theta}{(1+\tan^2\theta)^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$

$= \frac{\tan\theta}{(\sec^2\theta)^2} + \frac{\cot\theta}{(\cos ec^2\theta)^2}$

$= \frac{\tan\theta}{\sec^4\theta} + \frac{\cot\theta}{\cos ec^4\theta}$

$= \frac{\sin\theta}{\cos\theta.\sec^4\theta} + \frac{\cos\theta}{\sin\theta.\cos ec^4\theta}$

$= \frac{\sin\theta}{\sec^3\theta} + \frac{\cos\theta}{\cos ec^3\theta}$

$= \sin\theta.\cos^3\theta + \cos\theta.\sin^3\theta$

$= \sin\theta c\cos\theta\left(\cos^2\theta + \sin^2\theta\right)$

$= \sin\theta.\cos\theta$

9. **Ans :**

given $\sec\theta+\tan\theta=p \quad \rightarrow(1)$

$\Rightarrow \sec\theta-\tan\theta=\frac{1}{p} \quad \rightarrow(2)$

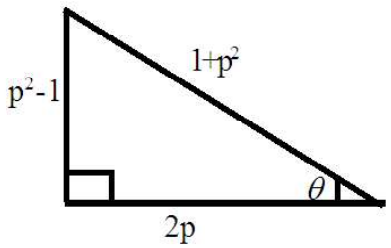
$(1)+(2)\Rightarrow 2\sec\theta=p+\frac{1}{p}$

$\Rightarrow 2\sec\theta=\frac{p^2+1}{p}$

$\Rightarrow \sec\theta=\frac{p^2+1}{2p}$

$\Rightarrow \cos\theta=\frac{2p}{1+p^2}$

$\therefore \sin\theta=\frac{p^2-1}{p^2+1}$



10. Ans : D

$\frac{\cos ec^2\alpha}{1+\cot^2\alpha}+\frac{\tan^2\alpha}{1+\tan^2\alpha}+\cos^2\alpha$

put $\alpha=\frac{\pi}{4}$

$=\frac{\left(\sqrt{2}\right)^2}{1+(1)^2}+\frac{(1)^2}{1+(1)^2}+\left(\frac{1}{\sqrt{2}}\right)^2$

$=\frac{2}{2}+\frac{1}{2}+\frac{1}{2}$

=1+1

=2

11. **Ans : D**

Given $\sin^6\theta+\cos^6\theta+\sin^2\theta.\cos^2\theta=a\sin^4\theta+b\cos^4\theta$

put $\theta=0^0$

$\sin^6 0^0+\cos^6 0^0+\sin^2 0^0.\cos^2 0^0=a.\sin^4 0^0 b.\cos^4 0^0$

0+1+0=a(0)+b(1)

$\therefore$ b=1

put $\theta=\frac{\pi}{2}$

$\sin^6\frac{\pi}{2}+\cos^6\frac{\pi}{2}+\sin^2\frac{\pi}{2}.\cos^2\frac{\pi}{2}=a.\sin^4\frac{\pi}{2}+b.\cos^4\frac{\pi}{2}$

1+0+0=a(1)+b(0)

$\therefore a=1$

now a+2b=1+2(1)=3

12. **Ans : A,C**

Given $\sin^2x.\cos^2y - \cos^2x.\sin^2y$

$= \sin^2x(1-\sin^2y)-(1-\sin^2x).\sin^2y$

$= \sin^2x -\sin^2x.\sin^2y -\sin^2y+\sin^2x.\sin^2y$

$=\sin^2x=\sin^2y$

again $\sin^2x.\cos^2y-\cos^2x.\sin^2y$

$= (1-\cos^2x).\cos^2y-\cos^2x(1-\cos^2y)$

$= \cos^2 y - \cos^2 x.\cos^2 y - \cos^2 x + \cos^2 x.\cos^2 y$

$=\cos^2y-\cos^2x$

13. **Ans : A,C**

given $\sin(\alpha+\beta)=1$ | $\sin(\alpha-\beta)=\frac{1}{2}$

$\Rightarrow \alpha+\beta=90^0$ | $\Rightarrow \alpha-\beta=30^0$

solving we get $\alpha = 60^0, \beta = 30^o$

14. **Ans : B**

$a\sin^2\alpha + b\cos^2\alpha = C$

divide with $\cos^2\theta$

$\Rightarrow a\tan^2\theta + b = c\sec^2\theta$

$\Rightarrow a\tan^2\theta + b = c\left(1+\tan^2\theta\right)$

$\Rightarrow a\tan^2\theta + b = C + C\tan^2\theta$

$\Rightarrow (a-c)\tan^2\theta = c-b$

$\Rightarrow \tan^2\theta = \frac{c-b}{a-c}$

15. **Statement - I**

given $(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma) = k$

$= (1+\cos\alpha)(1-\cos\alpha)(1+\cos\beta)(1-\cos\beta)(1+\cos\gamma)(1-\cos\gamma) = k$

$\Rightarrow (1-\cos^2\alpha)(1-\cos^2\beta)(1-\cos^2\gamma) = k$

$\Rightarrow \sin^2\alpha.\sin^2\beta.\sin^2\gamma = k$

Statement - II

$LHS = \frac{\sin\theta}{\cos\theta-1}$

$= \frac{\sin\theta}{-(1-\cos\theta)} \times \frac{1+\cos\theta}{1+\cos\theta}$

$$= \frac{-\sin\theta(1+\cos\theta)}{1-\cos^2\theta} = \frac{-\sin\theta(1+\cos\theta)}{\sin^2\theta}$$

$$= \frac{-(1+\cos\theta)}{\sin\theta} = RHS$$

statement II is correct

ans : A

16. Statement I : given $A+B=\frac{\pi}{2}$

$$\Rightarrow A = \frac{\pi}{2} - B$$

$$\Rightarrow \sin A = \sin\left(\frac{\pi}{2} - B\right)$$

$$\Rightarrow \sin A = \cos B$$

$$\Rightarrow \sin^2 A = \cos^2 B$$

$$\Rightarrow \sin^2 A = 1 - \sin^2 B$$

$$\Rightarrow \sin^2 A + \sin^2 B = 1$$

Statement I is correct

Statement - II

Given $\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} = 3$

$$LHS = \sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}$$

$$= \left(\sin^2\frac{\pi}{18} + \sin^2\frac{4\pi}{18}\right) + \left(\sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18}\right)$$

$=1+1$ $\qquad \sin ce\ A+B=\frac{\pi}{2} \Rightarrow \sin^2 A + \sin^2 B = 1$

$=2$

$\therefore$ statement II is wrong

17. **Ans : A**

Statement I : $1+\cot^2 A = \operatorname{cosec}^2 A$

$\Rightarrow \operatorname{cosec}^2 A - \cot^2 A = 1$, which is true

Statement II :

$$LHS = \frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A}$$

$=(\operatorname{cosec} A + \cot A) - \operatorname{cosec} A = \cot A$

$$RHS = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$$

=cosecA -(cosecA-cotA)=cotA

statement - II is true

$\therefore$

18. **Ans: D**

Given sinA+cosA= $\sqrt{2}$

$\Rightarrow$ $\sin^2$A+$\cos^2$A+2sinAcosA=2

$\Rightarrow$ 1+2sinAcosA=2

$\Rightarrow \sin A.\cos A = \frac{1}{2}$

19. **Ans : A**

$$\sec A.\cos ecA = \frac{1}{\cos A.\sin A} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

20. **Ans : A**

since sinA+cosA= $\sqrt{2}$

$$\Rightarrow A = \frac{\pi}{4}$$

now $\sin^2$A-$\cos^2$A= $\sin^2\frac{\pi}{4} - \cos^2\frac{\pi}{4} = 0$

21. **Ans:D**

Given $\sin\theta \, and \, \cos\theta$ are the roots of the equation ax^2 -bx+c=0

we have $\sin\theta + \cos\theta = \frac{b}{a}$

$$\sin\theta + .\cos\theta = \frac{c}{a}$$

22. **Ans : A**

$$\sin^4\theta + \cos^4\theta$$

$$= \left(\sin^2\theta\right)^2 + \left(\cos^2\theta\right)^2$$

$$= \left(\sin^2\theta + \cos^2\theta\right) - 2.\sin^2\theta.\cos^2\theta$$

$$= (1)^2 - 2(\sin\theta\cos\theta)^2$$

$$= 1 - 2\left(\frac{c}{a}\right)^2$$

$$= \frac{a^2 - 2c^2}{a^2}$$

23. **Ans : a-s, b-q,c-q,d-p**

a) $2\sec^2\theta + 2\tan^2\theta - 7$

$= 2\sec^2\theta + 2(\sec^2\theta - 1) - 7$

$= 4\sec^2\theta - 9$

$= 4\left(\frac{3}{2}\right)^2 - 9 = 0$

$\sin ce \quad \cos\theta = \frac{2}{3}$

b) $\frac{\tan 55^0}{\cot 55^0} + \cot 1^0.\cot 2^0.....\cot 90^0$

$= \frac{\tan(90^0 - 35^0)}{\cot 35^0} + \cot 1^0.\cot 2^0.....0$

$= \frac{\cot 35^0}{\cot 35^0} + 0$

$= 1 + 0 = 1$

c) $x.\tan 45^0.\cos 60^0 = \sin 60^0.\cot 60^0$

$\Rightarrow x.(1).\cos 60^0 = \sin 60^0.\frac{\cos 60^0}{\sin 60^0}$

$\Rightarrow x = 1$

d) given $\sec^2\theta = 3$

$\Rightarrow 1 - \tan^2\theta = 3$

$\Rightarrow \tan\theta = \sqrt{2}$

$\Rightarrow \cot\theta = \frac{1}{\sqrt{2}}$

now $\frac{\tan^2\theta - \cos ec^2\theta}{\tan^2\theta + \cos ec^2\theta}$

$= \frac{\tan^2\theta - (1 + \cot^2\theta)}{\tan^2\theta + (1 + \cot^2\theta)}$

$= \frac{(\sqrt{2})^2 - \left(1 + \left(\frac{1}{\sqrt{2}}\right)^2\right)}{\left(\sqrt{2}\right)^2 + \left(1 + \left(\frac{1}{\sqrt{2}}\right)^2\right)} = \frac{1}{7}$]

24. **Ans : a-5,b-3,c-1,d-5**

a) given $\alpha = n\pi + \theta$

now $\sin\alpha = \sin(n\pi+\theta) = (-1)^n.\sin\theta$

b) Given $\alpha = n\pi + \theta$

now $\cos\alpha = \cos(n\pi+\theta) = (-1)^n.\cos\theta$

c)given $\alpha = (2n+1)\frac{\pi}{2} - \theta$

now $\tan\alpha = \tan\left[(2n+1)\frac{\pi}{2} - \theta\right] = \cot\theta$

d)given $\alpha = (2n+1)\frac{\pi}{2} + \theta$

$\cos\alpha = \cos\left[(2n+1)\frac{\pi}{2} + \theta\right] = (-1)^n \sin\theta$

Learns Task

1. **Ans : B**

$\sin\theta = +\sqrt{1-\cos^2\theta}$,

2. **Ans A**

$\cos ec\theta + \cot\theta = \frac{1}{\cos ec\theta - \cot\theta}$

3. **Ans:D**

$\cot^2\theta - \cos ec^2\theta = -(\cos ec^2\theta - \cot^2\theta) = -1$

4. **Ans : D**

$\cot\theta = \pm\sqrt{\cos ec^2 - 1}$

5. **Ans** : B

$\frac{\sec\theta - 1}{\tan\theta} \times \frac{\sec\theta+1}{\sec\theta+1}$

$= \frac{\sec^2\theta - 1}{\tan\theta(\sec\theta+1)}$

$= \frac{\tan^2\theta}{\tan\theta(\sec\theta+1)} = \frac{\tan\theta}{\sec\theta+1}$

6. **Ans : B**

$$\frac{\cos ec\theta+1}{\cot\theta}\times\frac{\cos ec\theta-1}{\cos ec\theta-1}$$

$$=\frac{\cos ec^2\theta-1}{\cot\theta(\cos ec\theta-1)}=\frac{\cot^2\theta}{\cot\theta(\cos ec\theta-1)}=\frac{\cot\theta}{\cos ec\theta-1}$$

7. **Ans :D**

Given

$$4\sin\theta-3\cos\theta=0$$
$$\Rightarrow 4\sin\theta=3\cos\theta$$
$$\Rightarrow\frac{\sin\theta}{\cos\theta}=\frac{3}{4}$$

$$\tan\theta=\frac{3}{4}$$
$$\Rightarrow\sin\theta=\frac{3}{5}$$
$$\Rightarrow\cos\theta=\frac{4}{5}$$

now $\sin^2\theta-\cos^2\theta$

$$=\left(\frac{3}{5}\right)^2-\left(\frac{4}{5}\right)^2$$
$$=\frac{9}{25}-\frac{16}{25}=\frac{-7}{25}$$

8. **Ans : B**

given $\theta=45^0$

$$\tan^2\theta+\frac{1}{\tan^2\theta}$$
$$=\tan^2 45^0+\frac{1}{\tan^2 45^0}=1+\frac{1}{1}=2$$

9. **Ans :A**

given

$$\cos\theta=\sec\theta$$
$$\Rightarrow\cos\theta=\frac{1}{\cos\theta}$$
$$\Rightarrow\cos^2\theta=1$$

$$\sin\theta=\cos ec\theta$$
$$\Rightarrow\sin\theta=\frac{1}{\sin\theta}$$
$$\Rightarrow\sin^2\theta=1$$

Now $\sin^2\theta-\cos\theta=1-1=0$

10. **Ans :**

given $(1-\sin^2\theta)\sec^2\theta$

$=\sec^2\theta-\sin^2\theta.\sec^2\theta$

$=\sec^2\theta-\tan^2\theta=1$

Jee Main Level Questions

1. **Ans :B**

given $\frac{1}{1+\tan^2\theta}+\frac{1}{1+\cot^2\theta}$

$=\frac{1}{\sec^2\theta}+\frac{1}{\operatorname{cosec}^2\theta}$

$=\cos^2\theta+\sin^2\theta=1$

2. **Ans :D**

given $\operatorname{cosec}\theta=\sqrt{n+1}$

$\Rightarrow \sin\theta=\frac{1}{\sqrt{n+1}}$

we know $\cos\theta=\sqrt{1-\sin^2\theta}$

$\sqrt{1-\frac{1}{n+1}}=\sqrt{\frac{n}{n+1}}$

3. **Ans : B**

Given $1-\frac{\sin^2 x}{1+\cos x}+\frac{1+\cos x}{\sin x}-\frac{\sin x}{1-\cos x}$

$=1-\frac{(1-\cos^2 x)}{(1+\cos x)}+\frac{(1-\cos^2 x)-\sin^2 x}{\sin x(1-\cos x)}$

$=1-\frac{(1+\cos x)(1-\cos x)}{(1+\cos x)}+\frac{\sin^2 x-\sin^2 x}{\sin x(1-\cos x)}$

$=1-(1-\cos x)+0$

$=\cos x$

4. **Ans : B**

Given $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

Let $\theta=\frac{\pi}{4}$

$$= \frac{1-\cos\frac{\pi}{4}+\sin\frac{\pi}{4}}{1+\cos\frac{\pi}{4}-\sin\frac{\pi}{4}}$$

$$= \frac{1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}} = 1+\frac{2}{\sqrt{2}} = 1+\sqrt{2}$$

$$\frac{1+\sin\theta}{\cos\theta} = \frac{1+\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}}$$

$$= \frac{1+\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)} = 1+\sqrt{2}$$

5. given $\sin^4\theta+\cos^4\theta = l+3+k.\sin^2\theta.\cos^2\theta$

let $\theta = \frac{\pi}{4}$

$$\sin^4\frac{\pi}{4}+\cos^4\frac{\pi}{4} = l+3+k.\sin^2\frac{\pi}{4}.\cos^2\frac{\pi}{4}$$

$$\Rightarrow \frac{1}{4}+\frac{1}{4} = l+3+\frac{k}{4}$$

$$\Rightarrow \frac{1}{2} = l+3+\frac{k}{4}$$

$$\Rightarrow 2 = 4l+12+k$$

$$\Rightarrow 4l+k = -10$$

6. **Ans:B**

given $\frac{1}{\cos ec\theta-\cot\theta}+\frac{1}{\cos ec\theta+\cot\theta}$

$$= \frac{\cos ec\theta+\cot\theta+\cos ec\theta-\cot\theta}{(\cos ec\theta-\cot\theta)(\cos ec\theta+\cot\theta)}$$

$$= \frac{2\cos\sec\theta}{\cos ec^2\theta-\cot^2\theta}$$

$$= 2\cos ec\theta$$

7. **Ans :C**

given $\sec^4A - \sec^2A$

$=\sec^2A(\sec^2A-1)$

$=\left(1+\tan^2\theta\right)\left(\tan^2\theta\right)$

$=\tan^2\theta+\tan^4\theta$

8. Given $(\sin A+\sec A)^2+(\cos A+\operatorname{cosec} A)^2$

Let $A=45^0$

$(\sin 45^0+\sec 45^0)^2+(\cos 45^0+\operatorname{cosec} 45^0)^2$

$=\left(\frac{1}{\sqrt{2}}+\sqrt{2}\right)^2+\left(\frac{1}{\sqrt{2}}+\sqrt{2}\right)^2$

$=\left(\frac{3}{\sqrt{2}}\right)^2+\left(\frac{3}{\sqrt{2}}\right)^2=9$

Option B : $(1+\sec A.\operatorname{cosec} A)^2$

$=(1+\sec 45^0.\operatorname{cosec} 45^0)^2$

$=\left(1+\sqrt{2}.\sqrt{2}\right)^2=9$

9. **Ans : C**

$(\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)=K \rightarrow$ (1)

$(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)=k \rightarrow$ (2)

(1) x (2)

$(\sec^2A-\tan^2A)(\sec^2B-\tan^2B)(\sec^2C-\tan^2C)=k^2$

$\Rightarrow k^2=1$

$\Rightarrow K=\pm 1$

10. Given $\frac{1}{\sec A+\tan A}-\frac{1}{\cos A}$

Let $A=30^0$

$=\frac{1}{\sec 30^0+\tan 30^0}-\frac{1}{\cos 30^0}$

$=\frac{1}{\left(\frac{2}{\sqrt{3}}\right)+\frac{1}{\sqrt{3}}}-\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}$

$=\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}=\frac{-1}{\sqrt{3}}$

$-\tan A=-\tan 30^0=-\frac{1}{\sqrt{3}}$

Advanced Level Question

11. **AnsL B,C,D**

$\sec\theta=\sqrt{1+\tan^2\theta}$

$= \frac{1}{\cos\theta}$

$= \frac{1}{\sqrt{1-\sin^2\theta}}$

12. **Ans: A,C**

Given $(\sec\theta + \tan\theta)(1-\sin\theta)$

$Let \theta = 30^0$

$= (\sec 30^0 + \tan 30^0)(1-\sin 30^0)$

$= \left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)\left(1-\frac{1}{2}\right)$

$= \left(\sqrt{3}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

Option A: $\cos 30^0 = \frac{\sqrt{3}}{2}$

Option B: $\frac{1}{\sec 30^0} = \frac{\sqrt{3}}{2}$

13. **Statement I**

Given $a = \cos\theta, b = \sin\theta$

$Let \theta = 45^0, a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{2}}$

Now $(a-b)^4 = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)^4 = 0$

$1 + 4a^2b^2 - 4ab = 1 + 4.\frac{1}{2}.\frac{1}{2} - 4.\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}$

$= 1 + 1 - 2 = 0$

Statement I is correct

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4(\sin^6\theta + \cos^6\theta)$

$Let \theta = 0^0$

$= 3(\sin 0^0 - \cos 0^0)^4 + 6(\sin 0^0 + \cos 0^0)^2 + 4(\sin^6 0^0 + \cos^6 0^0)$

$=3(0-1)^4+6(0+1)^2+4(0^6+1^6)$

$= 3+6+4 = 13$

Statement II is correct

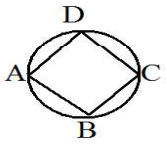
14. given A,B,C,D are the angle of a cyclic quadrilaterla

we have $A+C=180^0$

$B+D= 180^0$

We know that sinA+sinB +sinC+sinD=0

sinA+sinB = -(sinC+sinD)



15. **Ans:C**
We know that A+B+C+D=360^0
$\Rightarrow$A+B=360^0-(C+D)

$$\Rightarrow \frac{A+B}{4} = 90^0 - \left(\frac{C+D}{4}\right)$$

$$\Rightarrow \tan\left(\frac{A+B}{4}\right) = \cot\left(\frac{C+D}{4}\right)$$

16. **Ans: A**
We know A+B+C+D=360^0

$$\Rightarrow \frac{A+B}{2} = 180^0 - \left(\frac{C+D}{2}\right)$$

$$\cos\left(\frac{A+B}{2}\right) = -\cos\left(\frac{C+D}{2}\right)$$

$$\Rightarrow \cos\left(\frac{A+B}{2}\right) + \cos\left(\frac{C+D}{2}\right) = 0$$

17. given $\sin(A+B) = \frac{\sqrt{3}}{2}$ | $\cos(A-B) = \frac{\sqrt{3}}{2}$

$\Rightarrow A+B=60^0$ | $\Rightarrow A-B = 30^0$

on solving, we get A=45^0, B=15^0
sin^{2}A+cos2A=sin90^0 +cos30^0

$$= 1 + \frac{\sqrt{3}}{2}$$

18. **Ans: C**
sin^{2}A+cos^{2}3B = sin^{2}45^0 =cos^{2}45^0

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

19. **Ans: D**
tan^{2}2A+tan^{2}B=tan^{2}90^0 +tan^{2}15^0
= not defined

20. **Ans: 1**
tan1^0 . tan2^0. tan3^0 .....tan89^0
=(tan1^0.tan89^0) (tan2^0.tan88^0)..... tan45^0
=(tan1^0.cot1^0) (tan2^0).....1
= 1 x 1 x1 .....x1=1

21. **Ans: 0**

$\cos1^0.\cos2^0.\cos3^0.....\cos180^0$

$=\cos1^0.\cos2^0.\cos3^0.....\cos90^0.....\cos180^0$

$=\cos1^0.\cos2^0.\cos3^0..... 0 .....\cos180^0 = 0$

22. **Ans:** 1

$$Given\ 3(\cos^4\alpha+\sin^4\alpha)-2(\sin^6\alpha+\cos^6\alpha)$$

$$let\alpha=0^0$$

$$=3(\cos^4 0^0+\sin^4 0^0)-2(\sin^6 0^0+\cos^6 0^0)$$

$$=3(1+0)-2(0+1)$$

$$=1$$

23. **Ans: 1**

Given $\cos A+\cos^2 A=1$

$\Rightarrow \cos A = 1-\cos^2 A$

$\Rightarrow \cos A = \sin^2 A$

now $\sin^2 A = \sin^2 A(1+\sin^2 A)$

$= \cos A(1+\cos A)$

$=\cos A+\cos^2 A$

$=1$

24. a) $\sin^4 A- \cos^4 A$

$=(\sin^2 A)^2 - (\cos^2 A)^2$

$= (\sin^2 A +\cos^2 A) (\sin^2 A - \cos^2 A)$

$= \sin^2 A - \cos^2 A$

$= (1- \cos^2 A) - \cos^2 A$

$= 1- 2\cos^2_A$

again $= \sin^2 A - \cos^2 A$

$= \sin^2 A - (1-\sin^2 A) = 2\sin^2 A-1$

b) $1-\dfrac{\cos^2 A}{1+\sin A}$

$$=1-\frac{1-\sin^2 A}{1+\sin A}$$

$$=1-\frac{(1+siA)(1-\sin A)}{(1+\sin A)}$$

$= 1-(1-\sin A) = \sin A$

c) $\tan^2 A+\tan^4 A$

$= \tan^2 A(1+\tan^2 A)$

$(\sec^2 A-1) (\sec^2 A)$

$= \sec^4 A - \sec^2 A$

Ans: a-r,s; b-q,c-p

25. a) $\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}}$

$$\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}\times\frac{1-\sin\alpha}{1-\sin\alpha}}=\sqrt{\frac{(1-\sin\alpha)^2}{1-\sin^2\alpha}}$$

$$=\sqrt{\frac{(1-\sin\alpha)^2}{\cos^2\alpha}}$$

$$=\sqrt{\left(\frac{1-\sin\alpha}{\cos\alpha}\right)^2}$$

$$=\frac{1-\sin\alpha}{\cos\alpha}$$

$$=\frac{1}{\cos\alpha}-\frac{\sin\alpha}{\cos\alpha}$$

$$=\sec\alpha-\tan\alpha$$

b) $\sqrt{\frac{1-\sin\alpha}{1-\sin\alpha}}$

$$\sqrt{\frac{(1-\sin\alpha)(1+\sin\alpha)}{(1-\sin\alpha)(1-\sin\alpha)}}=\sqrt{\frac{(1+\sin\alpha)^2}{1-\sin^2\alpha}}$$

$$=\sqrt{\frac{(1+\sin\alpha)^2}{\cos^2\alpha}}=\sqrt{\left(\frac{1+\sin\alpha}{\cos\alpha}\right)^2}=\frac{1+\sin\alpha}{\cos\alpha}$$

$$=\frac{1}{\cos\alpha}+\frac{\sin\alpha}{\cos\alpha}=\sec\alpha+\tan\alpha$$

c) $\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}\times\frac{1+\cos\alpha}{1+\cos\alpha}}=\sqrt{\frac{(1+\cos\alpha)^2}{1-\cos^2\alpha}}$

$$=\sqrt{\frac{(1+\cos\alpha)^2}{\sin^2\alpha}}=\sqrt{\left(\frac{1+\cos\alpha}{\sin\alpha}\right)^2}=\frac{1+\cos\alpha}{\sin\alpha}$$

$$=\frac{1}{\sin\alpha}+\frac{\cos\alpha}{\sin\alpha}=\cos ec\alpha+\cot\alpha$$

d) $\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}\times\frac{1-\cos\alpha}{1-\cos\alpha}}=\sqrt{\frac{(1-\cos\alpha)^2}{1-\cos^2\alpha}}$

$$= \sqrt{\frac{(1-\cos\alpha)^2}{\sin^2\alpha}} = \sqrt{\left(\frac{1-\cos\alpha}{\sin\alpha}\right)^2} = \frac{1-\cos\alpha}{\sin\alpha}$$

$$= \frac{1}{\sin\alpha} - \frac{\cos\alpha}{\sin\alpha} = \operatorname{cosec}\alpha - \cot\alpha$$

Ans: a-s, b-p, c-t, d-q

Additional Practice Questions for students

1. **Ans: D**

$$\cos^2\theta + \frac{1}{1+\cot^2\theta}$$

$$= \cos^2\theta + \frac{1}{\operatorname{cosec}^2\theta} = \cos^2\theta + \sin^2\theta = 1$$

2. **Ans;D**

$$\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)$$

$$= \sec\theta - \sec\theta.\sin\theta)(\sec\theta+\tan\theta)$$

$$= (\sec\theta - \tan\theta)(\sec\theta+\tan\theta)$$

$$= \sec^2\theta - \tan^2\theta = 1$$

3. **Ans:A**

$$\sin^2\theta + \frac{1}{1+\tan^2\theta}$$

$$= \sin^2\theta + \frac{1}{\sec^2\theta} = \sin^2\theta + \cos^2\theta = 1$$

4. given

$$\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$$

$Let\ \theta = 30^0$

$$= \frac{1+\sin 30^0}{\cos 30^0} + \frac{\cos 30^0}{1+\sin 30^0}$$

$$= \frac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}} + \frac{\left(\frac{\sqrt{3}}{2}\right)}{1+\frac{1}{2}}$$

$$= \frac{3}{\sqrt{3}} + \frac{\sqrt{3}}{3}$$

$$= \frac{4\sqrt{3}}{3}$$

$$= 2\left(\frac{2}{\sqrt{3}}\right)$$

$$= 2(\sec 30^0) = 2\sec\theta$$

Ans; B

5. Ans: C

$$\left(\sec^2\theta-1\right)\left(\cos ec^2\theta-1\right)$$

$$Let\theta=45^0$$

$$=(\sec^2 45^0-1)(\cos ec^2 45^0-1)$$

$$=(2-1)(2-1)=1x1=1$$

6. **Ans:C**

$$\left(1+\cot\theta-\cos ec\theta\right)(1+\tan\theta+\sec\theta)$$

$$Let\theta=45^0$$

$$=(1+\cot 45^0-\cos ec45^0)(1+\tan 45^0+\sec 45^0)$$

$$=(1+1-\sqrt{2})(1+1+\sqrt{2})$$

$$=\left(2-\sqrt{2}\right)(2+\sqrt{2})$$

$$=(2)^2-(\sqrt{2})^2=4-2=2$$

7. $$\cot^2\alpha\left(\frac{\sec\alpha-1}{1+\sin\alpha}\right)+\sec^2\alpha\left(\frac{\sin\alpha-1}{1+\sec\alpha}\right)$$

$$Let\alpha=45^0$$

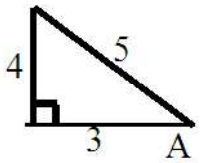
$$=\cot^2 45^0\left(\frac{\sec 45^0-1}{1+\sin 45^0}\right)+\sec^2 45^0\left(\frac{\sin 45^0-1}{1+\sec 45^0}\right)$$

$$=1\left(\frac{\sqrt{2}-1}{1+\frac{1}{\sqrt{2}}}\right)+2\left(\frac{\frac{1}{\sqrt{2}}-1}{1+\sqrt{2}}\right)$$

$$=\frac{2-\sqrt{2}}{\sqrt{2}+1}+\frac{\sqrt{2}-2}{\sqrt{2}+1}$$

$$=\frac{2-\sqrt{2}+\sqrt{2}-2}{1+\sqrt{2}}=\frac{0}{1+\sqrt{2}}=0$$

8. given in $\Delta ABC, \tan A=\frac{-4}{3}$ $\quad A\in Q_2$



$$\sec A=-\frac{5}{3}$$

$$\cos ecA=\frac{5}{4}$$

now, $x^2-\left(-\frac{5}{3}+\frac{5}{9}\right)x+\left(-\frac{5}{3}\right)\left(\frac{5}{4}\right)=0$

$\Rightarrow x^2 - \left(-\frac{5}{12}\right)x - \frac{25}{12} = 0$

$\Rightarrow 12x^2+5x-25=0$

Ans: A

$\sec^4 A\left(1-\sin^4 A\right) - 2\tan^2 A$

$Let A = 45^0$

$= \left(\sqrt{2}\right)^4\left(1-\left(\frac{1}{\sqrt{2}}\right)^4\right) - 2(1)^2$

9. $= 4\left(1-\frac{1}{4}\right) - 2$

$= 3 - 2$

$= 1$

Ans: B

THE END