PROPERTIES OF TRIANGLES-II
WORK SHEET-8
TEACHING TASK

1. Given $\frac{\Delta^2}{a^2+b^2+c^2}\left\{\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}+\frac{1}{r^2}\right\}$

$$= \frac{\Delta^2}{a^2+b^2+c^2}\left\{\frac{(s-a)^2}{\Delta^2}+\frac{(s-b)^2}{\Delta^2}+\frac{(s-c)^2}{\Delta^2}+\frac{s^2}{\Delta^2}\right\}$$

$$= \frac{\Delta^2}{a^2+b^2+c^2}\left\{\frac{(s-a)^2+(s-b)^2+(s-c)^2+s^2}{\Delta^2}\right\}$$

$$= \frac{\Delta^2}{a^2+b^2+c^2}\left\{\frac{4s^2-2s(a+b+c)+a^2+b^2+c^2}{\Delta^2}\right\}$$

$$= \frac{1}{a^2+b^2+c^2}\left\{\frac{4s^2-2s(2s)+a^2+b^2+c^2}{1}\right\}$$

$$= \frac{a^2+b^2+c^2}{a^2+b^2+c^2}$$

= 1

Ans : D

2. Given $\frac{s-a}{\Delta}=\frac{1}{8} \Rightarrow \frac{1}{r_1}=\frac{1}{8} \Rightarrow r_1=8$

$$\frac{s-a}{\Delta}=\frac{1}{12} \Rightarrow \frac{1}{r_2}=\frac{1}{12} \Rightarrow r_2=12$$

$$\frac{s-c}{\Delta}=\frac{1}{20} \Rightarrow \frac{1}{r_3}=\frac{1}{20} \Rightarrow r_3=24$$

We know, $\frac{1}{r} = \frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$

$$= \frac{1}{8}+\frac{1}{12}+\frac{1}{24}$$

$$= \frac{3+2+1}{24}$$

$$= \frac{6}{24}$$

$$= \frac{1}{4}$$

$\therefore \quad r = 4$

We have $b^2 = (r_2 - r)(r_3 + r_1)$

$= (12-4)(24+8)$

$= 8 \times 32$

$= 8 \times 8 \times 4$

$\therefore \quad b = 16$

Ans : A

3. Given in $\Delta ABC,\ \dfrac{\sqrt{r.r_1.r_2.r_3}}{2Rr(\sin A + \sin B + \sin C)}$

$$= \frac{\sqrt{\frac{\Delta}{s}.\frac{\Delta}{s-a}.\frac{\Delta}{s-b}.\frac{\Delta}{s-c}}}{2Rr\left(\frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}\right)}$$

$$= \frac{\Delta}{r(a+b+c)}$$

$$= \frac{s}{a+b+c} \qquad \text{since} \quad r = \frac{\Delta}{s}$$

$$= \frac{s}{2s} \qquad \Rightarrow s = \frac{\Delta}{r}$$

$$= \frac{1}{2}$$

Ans : D

4. Given $\left(\dfrac{1}{r} - \dfrac{1}{r_1}\right)\left(\dfrac{1}{r} - \dfrac{1}{r_2}\right)\left(\dfrac{1}{r} - \dfrac{1}{r_3}\right)$

$$= \left(\frac{s}{\Delta} - \frac{s-a}{\Delta}\right)\left(\frac{s}{\Delta} - \frac{s-b}{\Delta}\right)\left(\frac{s}{\Delta} - \frac{s-c}{\Delta}\right)$$

$$= \left(\frac{a}{\Delta}\right)\left(\frac{b}{\Delta}\right)\left(\frac{c}{\Delta}\right)$$

$$= \frac{abc}{\Delta^3}$$

Ans : A

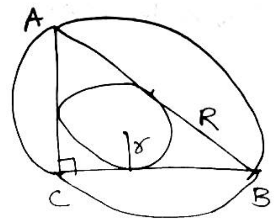
5. Given $\underline{|C} = \dfrac{\pi}{2}$

We have c = 2RsinC

$$\Rightarrow c = 2R\sin\frac{\pi}{2}$$

$\Rightarrow c = 2R$

$\Rightarrow R = \frac{c}{2}$ .... (i)



Also, $r = (s-c).\tan\left(\frac{c}{2}\right)$

$\Rightarrow r = (s-c).\tan\left(\frac{\pi}{4}\right)$

$\Rightarrow r = (s-c)$ .... (ii)

Now, $2(r+R) = 2\left(s - c + \frac{c}{2}\right)$ $\quad [\because \textit{ from } (i),(ii)]$

$= 2\left(s - \frac{c}{2}\right)$

= 2s - c
= a+b+c - c
= a+b

Ans : A,B

6. Given $\frac{a\cos A + b\cos B + c\cos C}{a\sin B + b\sin C + c\sin A} = \frac{a+b+c}{9R}$

$$\Rightarrow \frac{2R\sin A\cos A + 2R\sin B\cos B + 2R\sin C\cos C}{a\left(\frac{b}{2R}\right) + b\left(\frac{c}{2R}\right) + c\left(\frac{a}{2R}\right)} = \frac{a+b+c}{9R}$$

$$\Rightarrow \frac{R(\sin 2A + \sin 2B + \sin 2C)}{ab+bc+ca} = \frac{a+b+c}{9R}$$

$$\Rightarrow \frac{2R^2(4\sin A.\sin B.\sin C)}{ab+bc+ca} = \frac{a+b+c}{9R}$$

$$\Rightarrow \frac{8R^2.\frac{a}{2R}.\frac{b}{2R}.\frac{c}{2R}}{ab+bc+ca} = \frac{a+b+c}{9R}$$

$\Rightarrow (a+b+c)(ab+bc+ca) = 9abc$

Rearangeing the terms we get

$a(b^2+c^2-2bc) + b(c^2+a^2-2ca) + c(a^2+b^2-2ab) = 0$

$\Rightarrow a(b-c)^2 + b(c-a)^2 + c(a-b)^2 = 0$

This is possible only when
a-b = 0 and b-c=0 and c-a = 0

$\Rightarrow a = b = c$

$\therefore \Delta ABC$ is an equilateral triangle

We have area of $\therefore \Delta ABC = \frac{\sqrt{3}}{4}a^2$

$$= \frac{\sqrt{3}}{4}\left(\sqrt{3}R\right)^2$$

$$= \frac{3\sqrt{3}}{4}\ R^2$$

Ans : D

7. Let $A=\pi r^2$, $A_1 = \pi r_1^2$, $A_2 = \pi r_2^2$ and $A_3 = \pi r_3^2$

Now, $\frac{1}{\sqrt{A_1}}+\frac{1}{\sqrt{A_2}}+\frac{1}{\sqrt{A_3}}$

$$= \frac{1}{\sqrt{\pi r_1^2}}+\frac{1}{\sqrt{\pi r_2^2}}+\frac{1}{\sqrt{\pi r_3^2}}$$

$$= \frac{1}{\sqrt{\pi}}\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)$$

$$= \frac{1}{\sqrt{\pi}}\left(\frac{1}{r}\right)$$

$$= \frac{1}{\sqrt{\pi r^2}}$$

$$= \frac{1}{\sqrt{A}}$$

Ans : c

8. Let ΔABC be an equilateral triangle with side 1 unit

We have $P_1=P_2=P_3 = \frac{\sqrt{3}}{2}$

Now, $\frac{\cos A}{P_1}+\frac{\cos B}{P_2}+\frac{\cos C}{P_3}$

$$= \frac{\cos 60^0}{\left(\frac{\sqrt{3}}{2}\right)}+\frac{\cos 60^0}{\left(\frac{\sqrt{3}}{2}\right)}+\frac{\cos 60^0}{\left(\frac{\sqrt{3}}{2}\right)}$$

$$= 3.\frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \sqrt{3}$$

Now, $\frac{1}{R} = \frac{1}{\left(\frac{a}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{a} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Ans : B

9. We have $\Delta = \frac{1}{2} aP_1$

$\Rightarrow P_1 = \frac{2\Delta}{a}$

similarly $P_2 = \frac{2\Delta}{b}$ *and* $P_3 = \frac{2\Delta}{c}$

Now, $P_1.P_2.P_3 = \frac{2\Delta}{a}.\frac{2\Delta}{b}.\frac{2\Delta}{c}$

$$= \frac{8\Delta^3}{abc}$$

$$= \frac{8\Delta}{abc}\left(\frac{abc}{4R}\right)^3$$

$$= \frac{8}{abc}.\frac{(abc)^3}{64.R^3}$$

$$= \frac{(abc)^2}{8.R^3}$$

$$= \frac{2}{R}\left(\frac{abc}{4R}\right)^2$$

$$= \frac{2\Delta^2}{R}$$

10. We have $r.r_1.r_2.r_3 = \Delta^2$

$$\Rightarrow r_1.r_2.r_3 = \frac{\Delta^2}{r}$$

Also, $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

Now, $r_1r_2r_3 - r\left(r_1r_2 + r_2r_3 + r_3r_1\right)$

$= \frac{\Delta^2}{r} - r(s^2)$

$= \frac{r^2s^2}{r} - rs^2 \qquad \text{since } r = \frac{\Delta}{s}$

$= rs^2 - rs^2$

= 0

Ans : A

11. Given $r_1 = 8$, $r_2 = 12$ and $r_3 = 24$

We have $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

$= \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$

$= \frac{1}{4}$

$\therefore\ r = 4$

We have $r.r_1.r_2.r_3 = \Delta^2$

$\Rightarrow 4.8.12.24 = \Delta^2$

$\Rightarrow \Delta = 96$

We have $r = \frac{\Delta}{s}$

$\Rightarrow s = \frac{\Delta}{r}$

$\Rightarrow\ s = \frac{96}{4}$

$\Rightarrow\ s = 24$

Now $r_1 = \frac{\Delta}{s-a}$

$\Rightarrow\ 8 = \frac{96}{24-a}$

$\Rightarrow\ a = 12$

Similarly, b = 16, c = 20

Ans : B,C,D

12. Given $(r_2-r_1)(r_3-r_1) = 2r_2r_3$

$$\Rightarrow \left(\frac{\Delta}{s-b} - \frac{\Delta}{s-a}\right)\left(\frac{\Delta}{s-c} - \frac{\Delta}{s-a}\right) = 2.\frac{\Delta}{s-b}.\frac{\Delta}{s-c}$$

$$\Rightarrow \left(\frac{s-a-s+b}{(s-b)(s-a)}\right)\left(\frac{s-a-s+c}{(s-c)(s-a)}\right) = 2.\frac{1}{(s-b)}.\frac{1}{(s-c)}$$

$$\Rightarrow (b-a)(c-a) = 2(s-a)^2$$

$$\Rightarrow (b-a)(c-a) = \frac{1}{2}(2s-2a)^2$$

$$\Rightarrow 2\left(bc-ba-ac+a^2\right)=b^2+c^2+a^2+2bc-2ca-2ab$$

$$\Rightarrow 2bc-2ba-2ac+2a^2=b^2+c^2+a^2+2bc-2ca-2ab$$

$$\Rightarrow a^2=b^2+c^2$$

$\therefore \Delta ABC$ is a right angled triangle with $\underline{|A}=90^0$

$$\Rightarrow \underline{|B}+\underline{|C}=90^0$$

Ans : B,C

13. **Statement I:**

Given a > b > c

$$\Rightarrow -a<-b<-c$$

$$\Rightarrow s-a<s-b<s-c$$

$$\Rightarrow \frac{1}{s-a}>\frac{1}{s-b}>\frac{1}{s-c}$$

$$\Rightarrow \frac{\Delta}{s-a}>\frac{\Delta}{s-b}>\frac{\Delta}{s-c}$$

$$\Rightarrow r_1>r_2>r_3$$

Hence, Statement I is TRUE

Statement II:

Given a > b > c

consider cosA - cosB

$$=\frac{b^2+c^2-a^2}{2bc}-\frac{c^2+a^2-b^2}{2ca}$$

$$=\frac{ab^2+ac^2-a^3-bc^2-ba^2+b^3}{2abc}$$

$$=\frac{ab(b-a)+c^2(a-b)+b^3-a^3}{2abc}$$

$$=\frac{(a-b)\left(-ab+c^2+a^2+ab+b^2\right)}{2abc}$$

$$=\frac{(a-b)\left(a^2+b^2+c^2\right)}{2abc}>0$$

$$\therefore \cos A-\cos B>0$$

$$\Rightarrow \cos A>\cos B$$

Similarly we can prove cosB > cosC

$$\therefore \cos A>\cos B>\cos C$$

Hence, statement II is TRUE

Ans : A

14. **Statement I :**

With the given measurements a = 3, b = 4, $\underline{|A} = 30^0$, we cam draw one triangle only

Hence, statement I is FALSE

Statement II :

Let ΔABC be an equilateral triangle with side 1 unit

We have a = b = c = 1

A = B = C = 60^0 and

$$R = \frac{1}{\sqrt{3}}$$

Now, $\dfrac{(a+b)\cos C + (b+c)\cos A + (c+a)\cos B}{\sin A + \sin B + \sin C}$

$$= \frac{(1+1)\cos 60^0 + (1+1)\cos 60^0 + (1+1)\cos 60^0}{\sin 60^0 + \sin 60^0 + \sin 60^0}$$

$$= \frac{2\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right)}{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}$$

$$= \frac{3}{\left(\frac{3\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}} \neq R$$

Hence, statement II is FALSE

Ans : B

15. **Statement I :**

Given $r_1 = r_2 = r_3$

$$\Rightarrow \frac{\Delta}{s-a} = \frac{\Delta}{s-b} = \frac{\Delta}{s-c}$$

$$\Rightarrow \frac{1}{s-a} = \frac{1}{s-b} = \frac{1}{s-c}$$

$$\Rightarrow s-a = s-b = s-c$$

$$\Rightarrow a = b = c$$

ΔABC is an equilateral triangle

Hence, Statement is FALSE

Statement II :

Given $rr_1 = r_2r_3$

$$\Rightarrow \left(\frac{\Delta}{s}\right)\left(\frac{\Delta}{s-a}\right) = \left(\frac{\Delta}{s-b}\right)\left(\frac{\Delta}{s-c}\right)$$

$$\Rightarrow s(s-a)=(s-b)(s-c)$$

$$\Rightarrow [s(s-a)]^2 = s(s-a)(s-b)(s-c)$$

$$[s(s-a)]^2 = \Delta^2$$

$$\Rightarrow \left[\frac{\Delta}{s(s-a)}\right]^2 = 1$$

$$\Rightarrow \tan^2\left(\frac{A}{2}\right) = 1$$

$$\Rightarrow \left(\frac{A}{2}\right) = \frac{\pi}{4}$$

$$\Rightarrow A = \frac{\pi}{2}$$

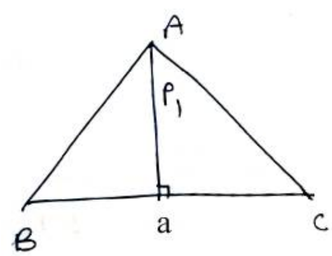
ΔABC is a right angled triangle

Hence, statement II is FALSE

Ans : B

COMPREHENSION-I

We have $\Delta = \frac{1}{2}.a.p_1$



$$\Rightarrow P_1 = \frac{2\Delta}{a}$$

similarly

$$P_2 = \frac{2\Delta}{b} \ and \ p_3 = \frac{2\Delta}{c}$$

16. Given $\frac{1}{P_1}+\frac{1}{P_2}+\frac{1}{P_3} = \frac{1}{2}$

We have $A.M \geq G.M$

$$\Rightarrow \frac{\frac{1}{P_1}+\frac{1}{P_2}+\frac{1}{P_3}}{3} \geq \left(\frac{1}{P_1}.\frac{1}{P_2}.\frac{1}{P_3}\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{\left(\frac{1}{2}\right)}{3} \geq \left(\frac{1}{P_1P_2P_3}\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{1}{6} \geq \left(\frac{1}{P_1P_2P_3}\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{1}{216} \geq \frac{1}{P_1P_2P_3}$$

$$\Rightarrow P_1P_2P_3 \geq 216$$

$\therefore$ The least value of $P_1P_2P_3 = 216$

Ans : D

17. We know $A.M \geq G.M$

$$\Rightarrow \frac{1}{3}\left(\frac{b^2P_1}{c}+\frac{c^2P_2}{a}+\frac{a^2P_3}{b}\right) \geq \left(\frac{b^2P_1}{c}.\frac{c^2P_2}{a}.\frac{a^2P_3}{b}\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{1}{3}\left(\frac{b^2P_1}{c}+\frac{c^2P_2}{a}+\frac{a^2P_3}{b}\right) \geq \left(abc\,P_1P_2P_3\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{1}{3}\left(\frac{b^2P_1}{c}+\frac{c^2P_2}{a}+\frac{a^2P_3}{b}\right) \geq \left(8\Delta^3\right)^{\frac{1}{3}}$$

$$\Rightarrow \frac{1}{3}\left(\frac{b^2P_1}{c}+\frac{c^2P_2}{a}+\frac{a^2P_3}{b}\right) \geq 2\Delta$$

$$\Rightarrow \frac{b^2P_1}{c}+\frac{c^2P_2}{a}+\frac{a^2P_3}{b} \geq 6\Delta$$

$\therefore$ The minimum value = 6Δ

Ans : D

18. Given $P_1P_2P_3$ are in A.P

$\Rightarrow \frac{\Delta}{s-a},\frac{\Delta}{s-b},\frac{\Delta}{s-c}$ are in A.P

$\Rightarrow \frac{1}{s-a},\frac{1}{s-b},\frac{1}{s-c}$ are in A.P

$\Rightarrow$ $s-a$, $s-b$, $s-c$ are in H.P

$\Rightarrow$ $-a$, $-b$, $-c$ are in H.P

$\Rightarrow$ a, b, c, are in H.P

Ans : B

COMPREHENSION-II

Let ΔABC be an equilateral triangle with side 1 unit

we have $a=b=c=1$

$A= B= C= 60^0$

$R= \frac{a}{\sqrt{3}}=\frac{1}{\sqrt{3}}$

$r = \frac{R}{2} = \frac{1}{2\sqrt{3}}$

19. $\frac{a}{\tan A} + \frac{b}{\tan B} + \frac{c}{\tan C}$

$= \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}$

$= \frac{3}{\sqrt{3}}$

$= \sqrt{3}$

Ans : D

20. $\text{Cos}^2 \frac{A}{2} + \text{Cos}^2 \frac{B}{2} + \text{Cos}^2 \frac{C}{2}$

$= \text{Cos}^2 30^0 + \text{Cos}^2 30^0 + \text{Cos}^2 30^0$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$

$= \frac{9}{4}$

Consider $2 + \frac{r}{2R}$

$= 2 + \frac{\left(\frac{1}{2\sqrt{3}}\right)}{2\left(\frac{1}{\sqrt{3}}\right)} = 2 + \frac{1}{4} = \frac{9}{4}$

Ans: C

21. $\text{Sin}^2 \frac{A}{2} + \text{Sin}^2 \frac{B}{2} + \text{Sin}^2 \frac{C}{2}$

$= \text{Sin}^2 30^0 + \text{Sin}^2 30^0 + \text{Sin}^2 30^0$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

$= \frac{3}{4}$

Consider $1-\frac{r}{2R}$

$$=1-\frac{\left(\frac{1}{2\sqrt{3}}\right)}{2\left(\frac{1}{\sqrt{3}}\right)}=1-\frac{1}{4}=\frac{3}{4}$$

Ans: D

Integer answer type

22. Give $2R+r=r_2$

$$2\left(\frac{abc}{4\Delta}\right)+\frac{\Delta}{s}=\frac{\Delta}{s-b}$$

since $\Delta=\frac{abc}{4R}\Rightarrow R=\frac{abc}{4\Delta}$

$$\Rightarrow\frac{abc}{2\Delta}=\frac{\Delta}{s-b}-\frac{\Delta}{s}$$

$$\Rightarrow\frac{abc}{2\Delta}=\frac{\Delta s-\Delta s+\Delta b}{s(s-b)}$$

$$\Rightarrow\frac{abc}{2\Delta}=\frac{\Delta b}{s(s-b)}$$

$$\Rightarrow\frac{ac}{2\Delta}=\frac{\Delta}{s(s-b)}$$

$$\Rightarrow\ \frac{1}{\sin B}=\tan\frac{B}{2}$$

$$\Rightarrow Tan\frac{B}{2}.\sin B=1$$

Clearily $B=90^0$, satisfies the above equation

$\therefore Ans: B=90^0$

Ans : 90^0

23. Given r_1, r_2 and r_3 are in H.P

$$\Rightarrow\frac{1}{r_1},\frac{1}{r_2}\,and\,\frac{1}{r_3}\,are\,in\,A.P$$

$$\Rightarrow\frac{2}{r_2}=\frac{1}{r_1}+\frac{1}{r_3}$$

$$\Rightarrow\frac{2}{r_2}+\frac{1}{r_2}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$$

$$\Rightarrow\frac{3}{r_2}=\frac{1}{r}$$

$$\Rightarrow \frac{r_2}{r} = 3$$

Ans: 3

24. **Matrix matching type**

a) $(r_1 - r)\ (r_2 - r)\ (r_3 - r)$

$$= \left(\frac{\Delta}{s-a} - \frac{\Delta}{s}\right)\left(\frac{\Delta}{s-b} - \frac{\Delta}{s}\right)\left(\frac{\Delta}{s-c} - \frac{\Delta}{s}\right)$$

$$= \Delta^3\left(\frac{1}{s-a} - \frac{1}{s}\right)\left(\frac{1}{s-b} - \frac{1}{s}\right)\left(\frac{1}{s-c} - \frac{1}{s}\right)$$

$$= \Delta^3\left(\frac{a}{s(s-a)}\right)\left(\frac{b}{s(s-b)}\right)\left(\frac{c}{s(s-c)}\right)$$

$$= \frac{\Delta^3 abc}{s(s-a)(s-b)(s-c).s^2}$$

$$= \frac{\Delta^3 abc}{\Delta^2.s^2}$$

$$= \frac{\Delta\, abc}{s^2} \qquad \text{since}\ \Delta = \frac{abc}{4R}$$

$$= \frac{\Delta}{s^2}.4R\Delta$$

$$= 4R\frac{\Delta^2}{s^2}$$

$$= 4R.r^2 \qquad \text{since}\ \ r = \frac{\Delta}{s}$$

b) $(r_1 + r_2)(r_2 + r_3)(r_3 + r_1)$

$$= \left(\frac{\Delta}{s-a} + \frac{\Delta}{s-b}\right)\left(\frac{\Delta}{s-b} + \frac{\Delta}{s-c}\right)\left(\frac{\Delta}{s-c} + \frac{\Delta}{s-a}\right)$$

$$= \Delta^3\left(\frac{1}{s-a} + \frac{1}{s-b}\right)\left(\frac{1}{s-b} + \frac{1}{s-c}\right)\left(\frac{1}{s-c} + \frac{1}{s-a}\right)$$

$$= \Delta^3\left(\frac{2s-a-b}{(s-a)(s-b)}\right)\left(\frac{2s-b-c}{(s-b)(s-c)}\right)\left(\frac{2s-a-c}{(s-c)(s-a)}\right)$$

$$= \left(\frac{abc}{(s-a)^2(s-b)^2(s-c)^2}\right)$$

$$= \frac{\Delta^3 . s^2 . abc}{\Delta^4}$$

$$= \frac{s^2 . abc}{\Delta} \qquad \text{since } \Delta = \frac{abc}{4R}$$

$$= \frac{s^2 . 4R . \Delta}{\Delta}$$

$$= 4Rs^2$$

c) $r_1 + r_2 + r_3 - r$

Now, $r_1 + r_2 = 4R \sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} + 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

$$= 4R\cos\frac{C}{2}\left(\sin\frac{A}{2}\cos\frac{B}{2} + \cos\frac{A}{2}\sin\frac{B}{2}\right)$$

$$= 4R\cos\frac{C}{2}.\sin\left(\frac{A}{2} + \frac{B}{2}\right)$$

$$= 4R\cos\frac{C}{2}.\cos\frac{C}{2}$$

$$= 4R\cos^2\frac{C}{2}$$

similarly $r_3 - r = 4R\sin^2\frac{C}{2}$

$$\therefore r_1 + r_2 + r_3 - r = 4R\cos^2\frac{C}{2} + 4R\sin^2\frac{C}{2}$$

$$= 4R\left(\cos^2\frac{C}{2} + \sin^2\frac{C}{2}\right)$$

$$= 4R$$

d) $r + r_3 + r_1 - r_2$

Now, $r - r_2 = 4R\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2} - 4R\cos\frac{A}{2}.\sin\frac{B}{2}.\cos\frac{C}{2}$

$$= 4R\sin\frac{B}{2}\left(\sin\frac{A}{2}.\sin\frac{C}{2} - \cos\frac{A}{2}.\cos\frac{C}{2}\right)$$

$$= -4R\sin\frac{B}{2}.\cos\left(\frac{A}{2} + \frac{C}{2}\right)$$

$$= -4R\sin\frac{B}{2}.\sin\frac{B}{2}$$

$$= -4R\sin^2\frac{B}{2}$$

similarly $r_1 + r_3 = 4R\cos^2\frac{B}{2}$

$$\therefore r + r_3 + r_1 - r_2 = 4R\cos^2\frac{B}{2} - 4R\sin^2\frac{B}{2}$$

$$= 4R\left(\cos^2\frac{B}{2} - \sin^2\frac{B}{2}\right)$$

$$= 4R\cos B$$

Ans: $A - q, B - r, C - p, D - t$

LEARNERS TASK

1. Given $a = 13$, b = 14, c = 15

Now, s $= \dfrac{a+b+c}{2}$

$$= \frac{13+14+15}{2}$$

$= 21$

Now, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21\text{x}8\text{x}7\text{x}6}$$

$= 84$

We know, r $= \dfrac{\Delta}{s}$

$$= \frac{84}{21}$$

$= 4$

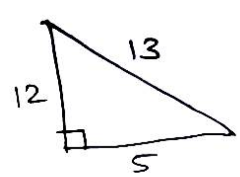
Ans : A

2. Given $a = 5$, b = 12, c = 13

Now, s $= \dfrac{a+b+c}{2}$

$$= \frac{5+12+13}{2}$$

$= 15$



$$\Delta = \frac{1}{2}.12.5 = 30$$

Now, $r_1 = \frac{\Delta}{s-a}$

$= \frac{30}{15-5} = \frac{30}{10} = 3$

Ans: B

3. Given $\Delta = 48, s = 16,$ and $c = 4$

Now, $r_3 = \frac{\Delta}{s-c}$ 16.26pg .4th que

$= \frac{48}{16-4}$

$= \frac{48}{12} = 4$

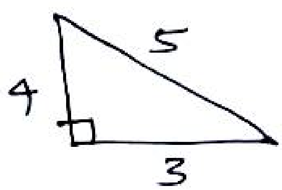
Ans: C

4. Given, a = 4, b = 5, c = 3

$s = \frac{a+b+c}{2}$

$= \frac{3+4+5}{2}$

$= 6$



Now, $\Delta = \frac{1}{2}.4.3 = 6$

Now, $r = \frac{\Delta}{s}$

$= \frac{6}{6}$

$= 1$

Ans: A

5. Given s = 15, b = 7, $\Delta = 24$

Now, $r_2 = \frac{\Delta}{s-b}$

$= \frac{24}{15-7}$

$= \frac{24}{8}$

$= 3$

Ans: D

6. Given, $r_1 = 8$, $r_2 = 12$, $r_3 = 24$

we know $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

$$= \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$$

$$= \frac{3+2+1}{24}$$

$$= \frac{6}{24} \quad = \frac{1}{4}$$

$$\therefore r = 4$$

We know $\Delta^2 = r r_1 r_2 r_3$

$$= 4.8.12.24$$

$$\therefore \Delta = 96$$

We know, $r = \frac{\Delta}{s}$

$$\Rightarrow s = \frac{\Delta}{r}$$

$$= \frac{96}{4} = 24$$

We know, $r_1 = \frac{\Delta}{s-a}$

$$\Rightarrow 8 = \frac{96}{24-a}$$

$$\Rightarrow 24 - a = 12$$

$$\Rightarrow a = 12$$

Ans : B

7. We know $\Delta^2 = r.r_1.r_2.r_3$

$$\Rightarrow = r_1.r_2.r_3 = \frac{\Delta^2}{r}$$

$$= \frac{r^2 s^2}{r} \ \text{since} \ r = \frac{\Delta}{s}$$

$= r.s^2$

Ans : C

8. Given $a=30$, b=24, c=18

Now, $s=\frac{a+b+c}{2}$

$=\frac{30+24+18}{2}$

= 36

Now, $\frac{1}{r_1}:\frac{1}{r_2}:\frac{1}{r_3}=\frac{s-a}{\Delta}:\frac{s-b}{\Delta}:\frac{s-c}{\Delta}$

= s-a : s-b : s-c
= 6 : 12 : 18
= 1 : 2 : 3

Ans : A

9. Given $\Delta=8,\ s=1.5=\frac{3}{2}$

We know, $r=\frac{\Delta}{s}$

$=\frac{8}{\left(\frac{3}{2}\right)}=\frac{16}{3}$

Ans : D

10. Given $r=2,\ r_1=4,\ r_2=6,\ r_3=12$

We know, $\Delta^2=r.r_1.r_2.r_3$

$\Rightarrow\ \Delta^2=2.4.6.12$

$\Rightarrow\ \Delta=24$

Ans : B

JEE MAINS LEVEL QUESTIONS

1. We know, $r=(s-b)\tan\frac{B}{2}$

Ans : B

2. We know, $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r}$

Ans : B

3. $\left(r_1-r\right)\left(r_2-r\right)\left(r_3-r\right)=4Rr^2$

Ans : B

4. $\sqrt{\frac{rr_1}{r_2r_3}} = \sqrt{\frac{4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}.4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}{4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}.4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}}}$

$= \tan\frac{A}{2}$

Ans : B

5. Given $r = \frac{R}{6}$ and r_1=7r

We have $r_1 - r = 4R\sin^2\frac{A}{2}$

$\Rightarrow\ 7r - r = 4R\sin^2\frac{A}{2}$

$\Rightarrow\ 6r = 4R\sin^2\frac{A}{2}$

$\Rightarrow\ 6\left(\frac{R}{6}\right) = 4R\sin^2\frac{A}{2}$

$\Rightarrow\ \sin^2\frac{A}{2} = \frac{1}{4}$

$\Rightarrow\ \sin\frac{A}{2} = \frac{1}{2}$

$\Rightarrow\ \frac{A}{2} = \frac{\pi}{6}$

$\Rightarrow\ A = \frac{\pi}{3}$

Ans : D

6. Given $r\left(\cot\frac{B}{2} + \cot\frac{C}{2}\right)$

$= r\left(\frac{s(s-b)}{\Delta} + \frac{s(s-c)}{\Delta}\right)$

$= \frac{r}{\Delta}\left(2s^2 - s(b+c)\right)$

$= \frac{1}{s}\left(2s^2 - s(2s-a)\right)$ since $r = \frac{\Delta}{s}$

$= \frac{1}{s}(sa)$

$= a$

Ans : A

7. Given $r^2.\cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}$

$$= r^2.\frac{s(s-a)}{\Delta}.\frac{s(s-b)}{\Delta}.\frac{s(s-c)}{\Delta}$$

$$= \frac{r^2}{\Delta^3}.\ s(s-a)(s-b)(s-c).s^2$$

$$= \frac{r^2}{\Delta^3}.\ \Delta^2.s^2$$

$$= (rs)^2.\ \Delta$$

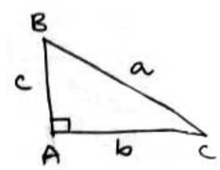
$$= \Delta^2.\ \Delta \qquad \text{since} \quad r = \frac{\Delta}{s}$$

$$= \Delta^3$$

8. Since right angle is at A
We have $a^2 = b^2+c^2$

Also, $R = \frac{a}{2},\ \Delta = \frac{1}{2}bc$

$$r_2 + r_3 = \frac{\Delta}{s-b} + \frac{\Delta}{s-c}$$



$$= \Delta\left(\frac{2s-b-c}{(s-b)(s-c)}\right)$$

$$= \frac{a\Delta}{(s-b)(s-c)} = \left(\frac{abc}{2(s-b)(s-c)}\right)$$

Now, $\frac{R}{r_2+r_3} = \frac{\left(\frac{a}{2}\right)}{\left(\frac{abc}{2(s-b)(s-c)}\right)}$

$$= \frac{(s-b)(s-c)}{bc}$$

$$= \frac{\left(\frac{a+b+c}{2}-b\right)\left(\frac{a+b+c}{2}-c\right)}{bc}$$

$$= \frac{(a-b+c)\,(a+b-c)}{4bc}$$

$$= \frac{a^2 + ab - ac - ba - b^2 + bc + ac + bc - c^2}{4bc}$$

$$= \frac{2bc}{4bc} \qquad \text{since} \quad a^2 = b^2 + c^2$$

$$= \frac{1}{2}$$

$$\therefore \ \cos^{-1}\left(\frac{R}{r_2 + r_3}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^0$$

Ans : B

9. Given $r_1 + r = r_2 + r_3$

$$\Rightarrow \frac{\Delta}{s-a} + \frac{\Delta}{s} = \frac{\Delta}{s-b} + \frac{\Delta}{s-c}$$

$$\Rightarrow \frac{1}{s-a} + \frac{1}{s} = \frac{1}{s-b} + \frac{1}{s-c}$$

$$\Rightarrow \frac{2s-a}{s(s-a)} = \frac{2s-b-c}{(s-b)(s-c)}$$

$$\Rightarrow \frac{2s-a}{s(s-a)} = \frac{a}{(s-b)(s-c)}$$

$$\Rightarrow \frac{2s-a}{a} = \frac{s(s-a)}{(s-b)(s-c)}$$

$$\Rightarrow 2\left(\frac{s}{a}\right) - 1 = Tan^2\left(\frac{A}{2}\right)$$

$$\Rightarrow 2\left(\frac{s}{a}\right) - 1 = Tan^2 30^0$$

$$\Rightarrow 2\left(\frac{s}{a}\right) - 1 = \frac{1}{3}$$

$$\Rightarrow \frac{s}{a} = \frac{2}{3}$$

Ans : A

10. Given $r_1 = 2r_2 = 3r_3$

$$\Rightarrow \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c}$$

$$\Rightarrow \frac{1}{s-a} = \frac{2}{s-b} = \frac{3}{s-c}$$

Let $\frac{1}{s-a} = \frac{2}{s-b} = \frac{3}{s-c} = \frac{1}{K}$

$\Rightarrow$ s-a = K, s-b = 2K, s-c = 3K

Adding these equations, we get 3s-(a+b+c) = 6K

$\Rightarrow$ 3s-2s = 6K

$\Rightarrow$ s = 6K

Now

$s-a = K$	$s-b = 2K$	$s-c = 3K$
$\Rightarrow 6K-a = K$	$\Rightarrow 6K-b = 2K$	$\Rightarrow 6K-c = 3K$
$\Rightarrow a = 5K$	$\Rightarrow b = 4K$	$\Rightarrow c = 3K$

Now, $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$

$= \frac{5K}{4K}+\frac{4K}{3K}+\frac{3K}{5K}$

$= \frac{5}{4}+\frac{4}{3}+\frac{3}{5}$

$= \frac{191}{60}$

Ans : D

ADVANCED LEVEL QUESTIONS

11. In ΔABC

Inradius $r = \frac{\Delta}{s}$

$= 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Ans : A,B

12. We have $r_1 = s\tan\frac{A}{2}$

$= (s-c)\cot\frac{B}{2}$

$= (s-b)\cot\frac{C}{2}$

Ans : A,B,C

Statement Type:

13. **Statement I:** Let a = 2, b = 3

We know, Area of triangle $\Delta = \frac{1}{2}ab\sin C$

$\Delta = \frac{1}{2} \text{x } 2 \text{ x } 3 \text{ x } \sin C$

= 3 sin C

We know $-1 \leq \sin C \leq 1$

$\Rightarrow -3 \leq 3\sin C \leq 3$

$\Rightarrow -3 \leq \Delta \leq 3$

$\therefore$ The area can not exleed 3

Hence, statement I is TRUE

Statement II :

Area of $\Delta ABC = \frac{1}{2} ab \sin C$

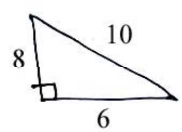
Hence, statement II is TRUE

Ans : A

Comprehension I:

Given $a=6$, b = 10, c = 8

We know $s=\frac{a+b+c}{2}$



$\Rightarrow s=\frac{6+10+8}{2}$

$\Rightarrow s=12$

Also $\Delta=\frac{1}{2}.8.6 = 24$

14. $r = \frac{\Delta}{s}$

$= \frac{24}{12} = 2$

Ans : B

15. $r_1 = \frac{\Delta}{s-a}$

$= \frac{24}{12-6} = \frac{24}{6} = 4$

Ans : A

16. $r_3 = \frac{\Delta}{s-c}$

$= \frac{24}{12-8} = \frac{24}{4} = 6$

Ans : C

Comprehension-II:

Given in ΔABC, r is the inradius and r_1, r_2, r_3 are the radii of excircles

17. $r\left(\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}\right)$

$=r\left(\frac{s(s-a)}{\Delta}+\frac{s(s-b)}{\Delta}+\frac{s(s-c)}{\Delta}\right)$

$=\frac{r}{\Delta}\left(3s^2-s(a+b+c)\right)$

$=\frac{r}{\Delta}\left(3s^2-2s^2\right)$

$=\frac{rs^2}{\Delta}$

$=\frac{rs^2}{rs}$ since $r=\frac{\Delta}{s}$

= s

Ans : A

18. $r_1\cot\frac{A}{2}+r_2\cot\frac{B}{2}+r_3\cot\frac{C}{2}$

$= \Sigma r_1\cot\frac{A}{2}$

$= \Sigma\frac{\Delta}{s-a}.\frac{s(s-a)}{\Delta}$

$= \Sigma s$

= s + s + s

= 3s

Ans : C

19. Conceptual

Integer answer type questions

20. In an equilateral triangle, the orthocentre and incentre coincides. Hence, the distance between them is zero

Ans : 0

21. In an equilateral triangle

We have $a=\sqrt{3}R$ and

$r_1=\frac{3}{2}R$

Now $\frac{a}{r_1} = \frac{\sqrt{3}R}{\frac{3}{2}R} = \frac{2\sqrt{3}}{3}$

$\therefore$ The denominator = 3

Ans : 3

22. **Matrix matching type :**

a) Given $\Delta = 400\sqrt{3}$

$s = 20\left(2+\sqrt{3}\right)$

We know, $r = \frac{\Delta}{s}$

$\Rightarrow r = \frac{400\sqrt{3}}{20\left(2+\sqrt{3}\right)} \text{x} \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$\Rightarrow r = 20\sqrt{3}\left(2-\sqrt{3}\right)$

$\Rightarrow r = 20\left(2\sqrt{3}-3\right)$

b) Given $\Delta = 400\sqrt{3}$ and

$s-a = 20\sqrt{3}$

Now, $r_1 = \frac{\Delta}{s-a}$

$= \frac{400\sqrt{3}}{20\sqrt{3}}$

= 20

c) Given $\Delta = 400\sqrt{3}$ and a+b-c = 40$\left(2-\sqrt{3}\right)$

$\Rightarrow$ 2s-2c = 40 $\left(2-\sqrt{3}\right)$

$\Rightarrow$ s-c = 20$\left(2-\sqrt{3}\right)$

Now, $r_3 = \frac{\Delta}{s-c}$

$= \frac{400\sqrt{3}}{20\left(2-\sqrt{3}\right)} \text{x} \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= 20\sqrt{3}\left(2+\sqrt{3}\right)$

d) Given $\Delta = 400\sqrt{3}$ and $abc = 64000\sqrt{3}$

We know $\Delta = \dfrac{abc}{4R}$

$$\Rightarrow R = \frac{abc}{4\Delta}$$

$$\Rightarrow R = \frac{64000\sqrt{3}}{4\text{x}400\sqrt{3}}$$

$$\Rightarrow R = 40$$

Ans : A-q, B-s, C-r, D-p

ADDITIONAL QUESTIONS

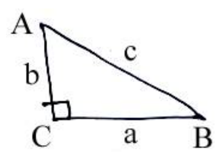
1. Given a+b+c = 70 .... (i)

r = 6

Now, 2s = a+b+c

= 70

$\therefore$ s = 35



Also, $\Delta = \dfrac{1}{2}ab$

$\Rightarrow rs = \dfrac{1}{2}ab$ since $r = \dfrac{\Delta}{s}$

$\Rightarrow 6.35 = \dfrac{1}{2}ab$

$\Rightarrow ab = 420$ .... (ii)

We have, $a^2+b^2 = c^2$

$\Rightarrow (a+b)^2 - 2ab = c^2$

$\Rightarrow (70-c)^2 - 2(420) = c^2$ [from (i), (ii)]

upon solving, we get c = 29

Now, $|a-b| = \sqrt{(a+b)^2 - 4ab}$

$= \sqrt{(70-c)^2 - 4ab}$

$= \sqrt{(70-29)^2 - 4.(420)}$

$= \sqrt{1681-1680}$

= 1

Ans : A

2. We know that $r_1 + r_2 + r_3 - r = 4R$

Also, $r_1 + r_2 = 4R\cos^2\frac{C}{2}$

$$\therefore\ 4R\cos^2\frac{C}{2} + r_3 - r = 4R$$

$$\Rightarrow\ r_3 - r = 4R - 4R\cos^2\frac{C}{2}$$

$$\Rightarrow\ r_3 - r = 4R\left(1 - \cos^2\frac{C}{2}\right)$$

$$\Rightarrow\ r_3 - r = 4R\sin^2\frac{C}{2}$$

Given $r_3 = r_1 + r_2 + r$

$$\Rightarrow\ r_3 - r = r_1 + r_2$$

$$\Rightarrow\ 4R\sin^2\frac{C}{2} = 4R\cos^2\frac{C}{2}$$

$$\Rightarrow\ Tan^2\frac{C}{2} = 1$$

$$\Rightarrow\ Tan\frac{C}{2} = 1$$

$$\Rightarrow\ \frac{C}{2} = \frac{\pi}{4}$$

$$\Rightarrow\ \underline{|C} = \frac{\pi}{2}$$

$$\therefore\ \underline{|A} + \underline{|B} = \frac{\pi}{2}$$

Ans : C

3. Let ΔABC be an equilateral triangle with side 1unit we have

$$R = \frac{a}{\sqrt{3}}$$

$$\Rightarrow R = \frac{1}{\sqrt{3}}$$

$$r_1 = r_2 = r_3 = \frac{3}{2}R$$

$$\Rightarrow\ r_1 = r_2 = r_3 = \frac{\sqrt{3}}{2}$$

Also $r = \frac{R}{2}$

$$\Rightarrow\ r = \frac{1}{2\sqrt{3}}$$

Now, $r_1^2 + r_2^2 + r_3^2 + r^2$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2\sqrt{3}}\right)^2$$

$$= \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{1}{12}$$

$$= \frac{28}{12} = \frac{7}{3}$$

Now, $16R^2-(a^2+b^2+c^2)$

$$= 16\left(\frac{1}{\sqrt{3}}\right)^2 - \left(1^2 + 1^2 + 1^2\right)$$

$$= \frac{16}{3} - 3$$

$$= \frac{7}{3}$$

Ans : A

4. Given $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$

$$= \sin^2 30^0 + \sin^2 30^0 + \sin^2 30^0$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4}$$

Now, $1 - \frac{r}{2R} = 1 - \frac{\left(\frac{1}{2\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)}$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

Ans : D

5. We know cosA+cosB+cosC = $1+\frac{r}{R}$

Given 4r = 3R

$$\Rightarrow \frac{r}{R} = \frac{3}{4}$$

$$\therefore \cos A+\cos B+\cos C = 1+\frac{3}{4}$$

$$= \frac{7}{4}$$

Ans : D

6. Let ΔABC be an equilateral triangle with side 1 unit

We have a = b = c = 1

A = B = C = 60^0

$P_1 = P_2 = P_3 = \frac{\sqrt{3}}{2}$ and

$\Delta = \frac{\sqrt{3}}{4}$

Now, $\frac{1}{P_1}+\frac{1}{P_2}-\frac{1}{P_3} = \frac{2}{\sqrt{3}}+\frac{2}{\sqrt{3}}-\frac{2}{\sqrt{3}}$

$$= \frac{2}{\sqrt{3}}$$

Consider $\frac{2ab\cos^2\frac{c}{2}}{\Delta(a+b+c)}$

$$= \frac{2.1.1.\cos^2 30^0}{\frac{\sqrt{3}}{4}.(1+1+1)}$$

$$= \frac{2.\frac{3}{4}}{\left(\frac{3\sqrt{3}}{4}\right)} = \frac{2}{\sqrt{3}}$$

Ans : A

7. Let ΔABC be an equilateral triangle with side 1 unit

We have a = b = c = 1

$R = \frac{a}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$r_1 = r_2 = r_3 = \frac{3}{2}R = \frac{\sqrt{3}}{2}$

Also area $\Delta = \frac{\sqrt{3}}{4}$

Now, $r_1 r_2 \sqrt{\dfrac{4R - r_1 - r_2}{r_1 + r_2}}$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \sqrt{\frac{\frac{4}{\sqrt{3}} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}}$$

$$= \frac{\sqrt{3}}{4} = \Delta$$

Ans : C

8. Given $\dfrac{r_1 + r_2}{1 + \cos C}$

$$= \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \cos 60^0}$$

$$= \frac{\sqrt{3}}{1 + \frac{1}{2}} = \frac{2}{\sqrt{3}}$$

Consider $\dfrac{abc}{2\Delta} = \dfrac{1.1.1}{2\left(\frac{\sqrt{3}}{4}\right)} = \dfrac{2}{\sqrt{3}}$

Ans : C

9. Conceptual