

class: IX

TRIGONOMETRIC
EQUATIONS-II

WORKSHEET-6

TEACHING TASK

①

Q1. We know,

$$\tan x + \tan(120^\circ + x) + \tan(x - 120^\circ) = 3 \tan 3x$$

$$\therefore 3 \tan 3x = 0$$

$$\Rightarrow \tan 3x = 0$$

$$\Rightarrow 3x = n\pi, \quad n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{n\pi}{3}, \quad \forall n \in \mathbb{Z}$$

Ans: A

Q2. We know

$$\tan x \cdot \tan(120^\circ + x) \cdot \tan(120^\circ - x) = \tan 3x$$

$$\therefore \tan 3x = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan 3x = \tan \frac{\pi}{6}$$

$$\Rightarrow 3x = n\pi + \frac{\pi}{6}$$

$$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{18}$$

$$\Rightarrow x = (6n+1)\frac{\pi}{18}, \quad n \in \mathbb{Z}$$

Ans: A

03. We know, $\cos^3 \alpha + \cos^3(120^\circ + \alpha) + \cos^3(120^\circ - \alpha)$
 $= \frac{3}{4} \cos 3\alpha$ (2)

$$\therefore \frac{3}{4} \cos 3\alpha = \frac{3\sqrt{3}}{4}$$

$$\Rightarrow \cos 3\alpha = \sqrt{3}$$

Since $-1 \leq \cos 3\alpha \leq 1$ *have*
 This equation does not *have* any solution.
 Ans: A

04. We know $\cos 0 \cdot \cos(120^\circ + 0) \cdot \cos(120^\circ - 0) = \frac{1}{4} \cos 30$

~~Given~~
 $\Rightarrow 4 \cos 0 \cdot \cos(120^\circ + 0) \cdot \cos(120^\circ - 0) = \cos 30$

$$\therefore \cos 30 = \frac{1}{2}$$

$$\Rightarrow \cos 30 = \cos \frac{\pi}{3}$$

$$\Rightarrow 30 = 2n\pi \pm \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}, \quad n \in \mathbb{Z}$$

Ans: C

05. Given $x^2 + 4 + 3 \sin(ax+b) - 2x = 0$

$$\Rightarrow x^2 + 4 - 2x = -3 \sin(ax+b)$$

$$\Rightarrow (x-1)^2 + 3 = -3 \sin(ax+b)$$

(3)

clearly $(x-1)^2 + 3 \geq 3$

$$\therefore -3 \sin(ax+b) \geq -3$$

$$\Rightarrow \sin(ax+b) \leq -1$$

$$\Rightarrow \sin(ax+b) = -1$$

$$\Rightarrow ax+b = \frac{3\pi}{2} \text{ or } \frac{7\pi}{2}$$

But given $(a, b) \in [0, 2\pi]$

$$\therefore \text{let } x=1 \quad a+b = \frac{3\pi}{2}$$

Ans: B

Q6. Given $\cos x + \cos y = 1 \rightarrow \textcircled{1}$

$$\cos x \cdot \cos y = \frac{1}{4}$$

$$\Rightarrow \cos x (1 - \cos x) = \frac{1}{4}$$

$$\Rightarrow 4 \cos x - 4 \cos^2 x = 1$$

$$\Rightarrow 4 \cos^2 x - 4 \cos x + 1 = 0$$

$$\Rightarrow (2 \cos x - 1)^2 = 0$$

$$\Rightarrow 2 \cos x - 1 = 0$$

④

$$\Rightarrow \cos x = \frac{1}{2}$$

$$\cos x + \cos y = 1$$

Now From (1),

$$\Rightarrow \frac{1}{2} + \cos y = 1$$

$$\Rightarrow \cos y = \frac{1}{2}$$

Consider $\cos x = \frac{1}{2}$

$$\Rightarrow \cos x = \cos \frac{\pi}{3}$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

$$\Rightarrow \text{Similarly } y = 2m\pi \pm \frac{\pi}{3}, m \in \mathbb{Z}$$

Ans: B

Q7. Given $\tan A, \tan B$ are the roots of

$$x^2 - bx + c = 0.$$

We have

$$\tan A + \tan B = b$$

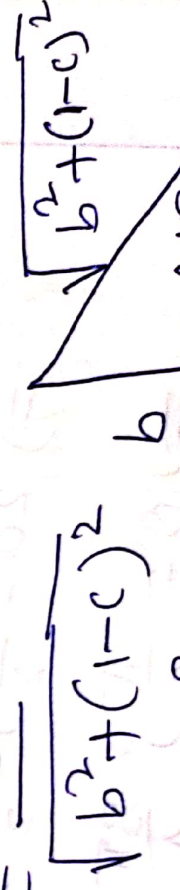
$$\tan A \cdot \tan B = c$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\text{Now } \tan(A+B) = \frac{b}{1-c}$$

$$= \frac{b}{1-c}$$

$$\therefore \sin(A+B) = \frac{b}{\sqrt{b^2 + (1-c)^2}}$$



$$\Rightarrow \sin^2(A+B) = \frac{b^2}{b^2 + (1-c)^2}$$

Ans: D

85

Q8 Given $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$

$$\Rightarrow 2 \cos\left(\frac{6\theta+4\theta}{2}\right) \cdot \cos\left(\frac{6\theta-4\theta}{2}\right) + 2\cos^2\theta = 0$$

$$\Rightarrow 2\cos 5\theta \cdot \cos \theta + 2\cos^2\theta = 0$$

$$\Rightarrow 2\cos\theta [\cos 5\theta + \cos \theta] = 0$$

$$\Rightarrow 2\cos\theta \cdot \cos 3\theta \cdot \cos 2\theta = 0$$

$$\Rightarrow \cos\theta = 0 \quad \text{or} \quad \cos 3\theta = 0 \quad \text{or} \quad \cos 2\theta = 0.$$

Consider

$$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

$$\text{Consider, } \cos 3\theta = 0 \Rightarrow 3\theta = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow 3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

$$\Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \dots$$

$$\text{Consider } \cos 2\theta = 0 \Rightarrow 2\theta = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \dots$$

6

$$\text{Since } 0 \leq \theta \leq \pi$$

$$\theta \in \left\{ \frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

Ans: B

09. Given $y = \frac{\cos^2 \theta - 1}{\cos^2 \theta + \cos \theta}$

$$\Rightarrow y = \frac{(\cos \theta + 1)(\cos \theta - 1)}{\cos \theta (\cos \theta + 1)}$$

$$\Rightarrow y = \frac{\cos \theta - 1}{\cos \theta} \quad \text{Since } \cos \theta + 1 \neq 0$$

$$\Rightarrow y = 1 - \sec \theta$$

$$\text{We know } \sec \theta \leq -1 \quad \text{and} \quad \sec \theta \geq 1$$

$$\Rightarrow 1 - \sec \theta \geq 2 \quad \text{and} \quad 1 - \sec \theta \leq 0$$

$$\Rightarrow y \geq 2 \quad \text{and} \quad y \leq 0$$

Ans: B

10. ~~Consider~~ Given $e^{-\frac{1}{\sqrt{2}}} (e^{\sin x} + e^{\cos x}) = 2$

$$\text{Let } x = \frac{\pi}{4}$$

$$\text{LHS} = e^{-\frac{1}{\sqrt{2}}} \left(e^{\sin \frac{\pi}{4}} + e^{\cos \frac{\pi}{4}} \right)$$

7

$$= e^{-\frac{1}{\sqrt{2}}} (e^{\frac{1}{\sqrt{2}}} + e^{\frac{1}{\sqrt{2}}})$$

$$= e^{-\frac{1}{\sqrt{2}}} (2 \cdot e^{\frac{1}{\sqrt{2}}})$$

$$= 2 \cdot e^0$$

$$= 2 = \text{RHS}$$

Hence $x = (4m+1) \frac{\pi}{4}$, Ans: B

11. Given $\sqrt{2} \tan^2 x - \sqrt{10} \tan x + \sqrt{2} = 0$

$$\Rightarrow \tan x = \frac{\sqrt{10} \pm \sqrt{10-8}}{2\sqrt{2}}$$

$$= \frac{\sqrt{10} \pm \sqrt{2}}{2\sqrt{2}}$$

$$= \frac{\sqrt{2} \cdot \sqrt{5} \pm \sqrt{2}}{2\sqrt{2}}$$

$$= \frac{\sqrt{5} \pm 1}{2}$$

8

12. Given $\sin \theta = \cos \phi$

$$\Rightarrow \sin \theta = \sin\left(\frac{\pi}{2} - \phi\right)$$

$$\Rightarrow \theta = n\pi + (-1)^n \left(\frac{\pi}{2} - \phi\right)$$

$$\text{Let } n = 0 \Rightarrow \theta = \frac{\pi}{2} - \phi$$

$$\Rightarrow \theta + \phi = \frac{\pi}{2}$$

$$\text{Let } n = 1 \Rightarrow \theta = \pi - \left(\frac{\pi}{2} - \phi\right)$$

$$\Rightarrow \theta = \frac{\pi}{2} + \phi$$

$$\Rightarrow \theta - \phi = \frac{\pi}{2}$$

$$\therefore \theta \pm \phi = \frac{\pi}{2}$$

$$\text{Let } n = 2 \Rightarrow \theta = 2\pi + \frac{\pi}{2} - \phi$$

$$\Rightarrow \theta = \frac{5\pi}{2} - \phi \Rightarrow \theta + \phi = \frac{5\pi}{2}$$

$$\text{Given } \frac{1}{\pi} \left(\theta \pm \pi - \frac{\pi}{2} \right) = \frac{1}{\pi} \left(\frac{\pi}{2} - \frac{\pi}{2} \right) = 0$$

$$\text{Also } \frac{1}{\pi} \left(\frac{5\pi}{2} - \frac{\pi}{2} \right) = \frac{1}{\pi} \times 2\pi = 2$$

$\therefore$ The possible values are 0 or 2

Ans: A, C

9

13. Given $\sin x + \cos x = \sqrt{y + \frac{1}{y}}$

We know $A.M \geq G.M$

$$\Rightarrow \frac{y + \frac{1}{y}}{2} \geq \sqrt{y \times \frac{1}{y}}$$

$$\Rightarrow y + \frac{1}{y} \geq 2$$

$$\Rightarrow \sqrt{y + \frac{1}{y}} \geq \sqrt{2}$$

i.e. The minimum value of $\sqrt{y + \frac{1}{y}}$ is $\sqrt{2}$

$$\text{When } x = \frac{\pi}{4}, \quad \sin \frac{\pi}{4} + \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

$$\text{Also, when } y = 1, \quad \sqrt{y + \frac{1}{y}} = \sqrt{2}$$

$\therefore$ Ans: A, C

14. Statement I: Given $\cos x - 2 \cdot 3^{\cos x} + 1 = 0$

$$\Rightarrow \left(\frac{\cos x}{3} \right)^2 - 2 \cdot \frac{\cos x}{3} + 1 = 0.$$

$$\text{Let } \frac{\cos x}{3} = t$$

10

$$\Rightarrow t^2 - 2t + 1 = 0$$

$$\Rightarrow (t-1)^2 = 0$$

$$\Rightarrow t-1=0$$

$$\Rightarrow t=1$$

$$\Rightarrow \cos x = \frac{0}{3}$$

$$\Rightarrow \cos x = 0$$

$$\Rightarrow x = (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}$$

Statement I is TRUE.

Statement II: Given $\sqrt{3} \sin x + \cos x = 4$

$$\Rightarrow \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = 2$$

$$\Rightarrow \sin x \cdot \cos \frac{\pi}{6} + \cos x \cdot \sin \frac{\pi}{6} = 2$$

$$\Rightarrow \sin \left(x + \frac{\pi}{6} \right) = 2$$

which is not possible, since the maximum value of $\sin \theta$ is 1.

$\therefore$ The given equation has no solutions.

Statement II is Not TRUE

Ans: C

Comprehension-I

11

Given $7\cos\theta + 4\cos\theta - 1 = 0$

$$\Rightarrow \cos\theta = \frac{-4 \pm \sqrt{16 + 28}}{14}$$

$$\Rightarrow \cos\theta = \frac{-4 \pm \sqrt{44}}{14}$$

$$\Rightarrow \cos\theta = \frac{-4 \pm 2\sqrt{11}}{14}$$

$$\Rightarrow \cos\theta = \frac{-2 \pm \sqrt{11}}{7}$$

Now $\cos 2\theta = 2\cos^2\theta - 1$

$$= 2 \left(\frac{-2 \pm \sqrt{11}}{7} \right)^2 - 1$$

$$= \frac{2}{49} (4 + 11 \pm 4\sqrt{11}) - 1$$

$$= \frac{30 \pm 8\sqrt{11} - 49}{49}$$

$$= \frac{8\sqrt{11} - 19}{49} = \frac{-19 \pm 8\sqrt{11}}{49} \rightarrow \textcircled{1}$$

Given $p\cos 2\theta + q\cos\theta + r = 0$

$$\Rightarrow \cos 2\theta = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p} \rightarrow \textcircled{2}$$

12

Comparing ① and ②

→ ③

$$\text{We, get } 2p = 49 \Rightarrow p = \frac{49}{2}$$

→ ④

$$-q = -19 \Rightarrow q = 19$$

$$\sqrt{q^2 - 4px} = 8\sqrt{11}$$

$$\Rightarrow q^2 - 4px = 704$$

$$\Rightarrow 19^2 - 4 \cdot \frac{49}{2} \cdot x = 704$$

$$\Rightarrow x = -\frac{343}{98}$$

$$\Rightarrow x = -\frac{7}{2} \rightarrow \text{⑤}$$

$$\text{Given } p \cos^2 \theta + q \cos 2\theta + x = 0$$

$$\Rightarrow \frac{49}{2} \cos^2 \theta + 19 \cos 2\theta - \frac{7}{2} = 0$$

$$\Rightarrow 49 \cos^2 \theta + 38 \cos 2\theta - 7 = 0$$

$$\therefore p = 49, q = 38, x = -7$$

Ans: B

15. $p = 49$

16. $q = 38$

17. $x = -7$

Ans: A

Comprehension - III

Trigonometric (13)
class equations
after continuation

$$\text{Given } 21 \cos^3 y + 32 \cos y \cdot \sin^2 y = 14 \rightarrow \textcircled{1}$$

$$21 \sin^3 y + 32 \cos^2 y \cdot \sin y = 13 \rightarrow \textcircled{2}$$

Now, $\textcircled{1} + \textcircled{2}$

$$21 (\cos^3 y + 3 \cos y \sin^2 y + 3 \cos^2 y \sin y + \sin^3 y) = 27 \rightarrow \textcircled{3}$$
$$= 27$$

$$\Rightarrow 21 (\cos y + \sin y)^3 = 27 \rightarrow \textcircled{3}$$

$$\text{Now, } \textcircled{1} - \textcircled{2} \quad 3 = 1 \rightarrow \textcircled{4}$$

$$\Rightarrow 21 (\cos y - \sin y)^3 = 1$$
$$\Rightarrow \frac{\textcircled{3}}{\textcircled{4}} \Rightarrow \left(\frac{\cos y + \sin y}{\cos y - \sin y} \right)^3 = 27$$

$$\Rightarrow \frac{\cos y + \sin y}{\cos y - \sin y} = 3$$

$$\Rightarrow \cos y + \sin y = 3 \cos y - 3 \sin y$$

$$\Rightarrow 4 \sin y = 2 \cos y$$

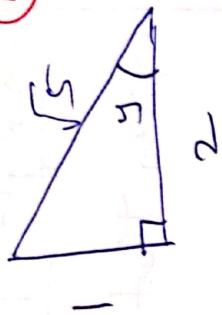
$$\Rightarrow \tan y = \frac{1}{2} \Rightarrow$$

$$\Rightarrow \tan y = \frac{1}{2}$$

14

$$\sin y = \frac{1}{\sqrt{5}}$$

$$\text{and } \cos y = \frac{2}{\sqrt{5}}$$



~~Ques~~

21. Given $x \cos^3 y + 3x \cdot \cos y \cdot \sin^2 y = 14$

$$\Rightarrow x \left(\frac{2}{\sqrt{5}} \right)^3 + 3x \cdot \left(\frac{2}{\sqrt{5}} \right) \left(\frac{1}{\sqrt{5}} \right)^2 = 14$$

$$\Rightarrow \frac{8x}{5\sqrt{5}} + \frac{6x}{5\sqrt{5}} = 14$$

$$\Rightarrow 14x = 70\sqrt{5}$$

$$\Rightarrow x = 5\sqrt{5}$$

Ans: A

22. The number of values of $y \in [0, 6\pi]$ are 6.

Ans: D

23. Given $\sin^2 y + 2 \cos^2 y$

$$= \left(\frac{1}{\sqrt{5}} \right)^2 + 2 \left(\frac{2}{\sqrt{5}} \right)^2$$

$$= \frac{1}{5} + \frac{8}{5} = \frac{9}{5}$$

Ans: B

Integer Type

(15)

24 Given that $a \sin x + \cos 2x = 2a - 7$

$$\Rightarrow a \sin x + 1 - 2 \sin^2 x = 2a - 7$$

$$\Rightarrow 2 \sin^2 x - a \sin x + 2a - 8 = 0$$

$$\Rightarrow \sin x = \frac{a \pm \sqrt{a^2 - 4 \cdot 2 \cdot (2a - 8)}}{4}$$

$$= \frac{a \pm \sqrt{a^2 - 16a + 64}}{4}$$

$$= \frac{a \pm \sqrt{(a-8)^2}}{4}$$

$$= \frac{a \pm (a-8)}{4}$$

$$\therefore \sin x = \frac{a+a-8}{4} \quad \text{or} \quad \sin x = \frac{a-(a-8)}{4}$$

$$\Rightarrow \sin x = \frac{2a-8}{4}$$

$$\Rightarrow \sin x = \frac{8}{4} = 2$$

$$\Rightarrow \sin x = \frac{a-4}{2}$$

which is not possible

16

We know, $-1 \leq \sin x \leq 1$

$$\Rightarrow -1 \leq \frac{a-4}{2} \leq 1$$

$$\Rightarrow -2 \leq a-4 \leq 2$$

$$\Rightarrow 2 \leq a \leq 6$$

$\therefore$ The maximum integral value of a is 6

25 a) $4 \sin^2 x + \sin^2 x = 3$

$$\Rightarrow 4 \sin^2 x + 4 \sin^2 x \cdot \cos^2 x = 3$$

$$\Rightarrow \frac{4 \sin^2 x}{\cos^2 x} + \frac{4 \sin^2 x \cdot \cos^2 x}{\cos^2 x} = \frac{3}{\cos^2 x}$$

$$\Rightarrow 4 \tan^2 x + 4 \sin^2 x = 3 \sec^2 x$$

$$\Rightarrow 4 \tan^2 x + 4 (1 - \cos^2 x) = 3 \sec^2 x$$

$$\Rightarrow 4 \sin^2 x + 4 \sin^2 x (1 - \sin^2 x) = 3$$

$$\Rightarrow 4 \sin^2 x + 4 \sin^2 x - 4 \sin^4 x = 3$$

$$\Rightarrow 8 \sin^2 x - 4 \sin^4 x = 3$$

$$\Rightarrow 4 \sin^4 x - 8 \sin^2 x + 3 = 0$$

$$\Rightarrow \frac{4 \sin^4 x - 8 \sin^2 x + 3 = 0}{4 \sin^4 x - 2 \sin^2 x - 6 \sin^2 x + 3 = 0}$$

$$\Rightarrow 2 \sin^2 x (2 \sin^2 x - 1) - 3 (2 \sin^2 x - 1) = 0$$

(17)

$$\Rightarrow (2\sin^2 x - 1)(2\sin^2 x - 3) = 0$$

$$\Rightarrow 2\sin^2 x - 1 = 0 \quad | \quad 2\sin^2 x - 3 = 0$$

$$\Rightarrow \sin x = \pm \frac{1}{\sqrt{2}} \quad \Rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \pm \frac{\pi}{4} \quad x = \pm \frac{\pi}{3}$$

$$\therefore \tan \frac{\pi}{4} = 1$$

$$\therefore \tan x = 1$$

$$\Rightarrow x = \tan^{-1}(1)$$

b) Given $4\cos^2 x + 8\cos^2 x = 7$

$$\text{Let } x = \frac{\pi}{6}$$

$$\text{Now, LHS} = 4\cos^2\left(\frac{\pi}{6}\right) + 8\cos^2\left(\frac{\pi}{6}\right)$$

$$= 4\cos^2\frac{\pi}{6} + 8\cos^2\frac{\pi}{6}$$

$$= 4\left(\frac{1}{2}\right)^2 + 8\left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 1 + 6$$

$$= 7 = \text{RHS}$$

$$\therefore \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\therefore \tan x = \frac{1}{\sqrt{3}} \Rightarrow x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\begin{aligned}
 c) \quad & 3(1 + \sin x) = 1 + \cos^2 x \\
 \Rightarrow & 3(1 + \sin x) = 2 \cos^2 x \\
 \Rightarrow & 3(1 + \sin x) = 2(1 - \sin^2 x) \\
 \Rightarrow & 2 \sin^2 x + 3 \sin x + 1 = 0 \\
 \Rightarrow & (2 \sin x + 1)(\sin x + 1) = 0 \\
 \Rightarrow & \sin x = -\frac{1}{2} \quad \text{or} \quad \sin x = -1 \\
 \Rightarrow & \sin x = -1 \Rightarrow x = \sin^{-1}(-1)
 \end{aligned}$$

d) Given $4x^4 + x^6 + \sin^2 x = 0$
 $x = 0$, satisfies the above equation

Ans: a-p, b-q, c-r, d-s

e) Given $7 \cos^2 x + \sin x \cdot \cos x - 3 = 0$
 Dividing with $\cos^2 x$ on either side

$$\begin{aligned}
 \Rightarrow & 7 + \tan x - 3 \sec^2 x = 0 \\
 \Rightarrow & 7 + \tan x - 3(1 + \tan^2 x) = 0 \\
 \Rightarrow & 7 + \tan x - 3 - 3 \tan^2 x = 0 \\
 \Rightarrow & 3 \tan^2 x - \tan x - 4 = 0 \\
 \Rightarrow & (3 \tan x - 4)(\tan x + 1) = 0
 \end{aligned}$$

(19)

$$\Rightarrow \tan x = \frac{4}{3} \quad \text{or} \quad \tan x = -1$$

b) Given $2 \sin^2\left(\frac{\pi}{2} \cdot \cos^2 x\right) = 1 - \cos(\pi \sin 2x)$

$$\Rightarrow 2 \sin^2\left(\frac{\pi}{2} \cdot \cos^2 x\right) = 2 \sin^2\left(\frac{\pi}{2} \cdot \sin 2x\right)$$

$$\Rightarrow \frac{\pi}{2} \cdot \cos^2 x = \frac{\pi}{2} \cdot \sin 2x$$

$$\Rightarrow \cos^2 x = 2 \sin x \cos x$$

$$\Rightarrow \cos x (\cos x - 2 \sin x) = 0$$

$$\Rightarrow \cos x = 0 \quad \text{or} \quad \tan x = \frac{1}{2}$$

c) Given $6 \sec^2 x - 11 \tan x - 2 = 0$

$$\Rightarrow 6(1 + \tan^2 x) - 11 \tan x - 2 = 0$$

$$\Rightarrow 6 \tan^2 x - 11 \tan x + 4 = 0$$

$$\Rightarrow (2 \tan x - 1)(3 \tan x - 4) = 0$$

$$\Rightarrow \tan x = \frac{1}{2} \quad \text{or} \quad \tan x = \frac{4}{3}$$

d) ~~$2 \cos x - \sin x = \cos^2 x - 4 \sin x$~~

~~$\text{Let } x = \frac{\pi}{3}$~~

$$\text{Given } 2 \cos x - \sin x = \cos^2 x - 4 \sin x$$

20

$$\Rightarrow 2 \cos x - \cos^2 x = -3 \sin x$$

$$\Rightarrow 2 - \cos x = -3 \tan x$$

$$\Rightarrow 2 + 3 \tan x = \cos x$$

$$\Rightarrow \frac{1}{2 + 3 \tan x} = \sec x$$

$$\Rightarrow \frac{1}{(2 + 3 \tan x)^2} = \sec^2 x$$

$$\Rightarrow \frac{1}{(2 + 3 \tan x)^2} = 1 + \tan^2 x$$

$$\Rightarrow (1 + \tan^2 x) (2 + 3 \tan x)^2 = 1$$

For any value of x , the above equation does not satisfy.

21) 27a) Given $\sqrt{p} \cos x - 2 \sin x = \sqrt{2} + \sqrt{2-p} \rightarrow \textcircled{1}$

We know, the maximum value of

$$\sqrt{p} \cos x - 2 \sin x = c + \sqrt{a^2 + b^2}$$

$$= 0 + \sqrt{4+p}$$

Also, minimum value = $c - \sqrt{a^2 + b^2}$

$$= -\sqrt{4+p}$$

$$\therefore |\sqrt{p} \cos x - 2 \sin x| \leq \sqrt{4+p} \rightarrow \textcircled{2}$$

also equation $\textcircled{1}$ has solutions if

$$p \geq 0 \text{ and } 2-p \geq 0 \text{ i.e. } p \leq 2$$

Now from $\textcircled{2}$

$$|\sqrt{2} + \sqrt{2-p}| \leq \sqrt{4+p}$$

$$\Rightarrow (\sqrt{2} + \sqrt{2-p})^2 \leq 4+p$$

$$\Rightarrow p^2 + 2p - 4 \geq 0$$

$$\Rightarrow (p+1)^2 \geq 5$$

$$\Rightarrow |p+1| \geq \sqrt{5}$$

22

$$\text{i.e. } p+1 \geq \sqrt{5} \quad \text{or} \quad p+1 \leq -\sqrt{5}$$
$$\Rightarrow p \geq \sqrt{5}-1 \quad \text{or} \quad p \leq -1-\sqrt{5}$$

We have $p \geq 0$, $p \leq 2$, $p \geq -1+\sqrt{5}$

and $p \leq -1-\sqrt{5}$.

The common solution of the above inequality is

$$p \in (-1+\sqrt{5}, 2).$$

b) See the solution of Q.No. 24.

c) Given $a \sin \frac{x}{2} = \sin x + \sin \frac{3x}{2}$

d) Given $\sin^4 x + \cos^4 x + \sin 2x + a = 0$

$$\Rightarrow (\sin^2 x)^2 + (\cos^2 x)^2 + \sin 2x + a = 0$$

$$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cdot \cos^2 x + \sin 2x + a = 0$$

$$\Rightarrow 1 - \frac{1}{2}(\sin 2x)^2 + \sin 2x + a = 0$$

Let $\sin 2x = t$

$$\Rightarrow 1 - \frac{t^2}{2} + t + a = 0$$

$$\Rightarrow 2 - t^2 + 2t + 2a = 0$$

$$\Rightarrow t^2 - 2t - 2 - 2a = 0$$

$$\Rightarrow t = \frac{2 \pm \sqrt{4 + 4(2+2a)}}{2}$$

$$\Rightarrow t = \frac{2 \pm 2\sqrt{3+2a}}{2}$$

$$\Rightarrow t = 1 \pm \sqrt{3+2a}$$

24

$$\Rightarrow \sin 2\alpha = 1 - \sqrt{3+2a}$$

~~Since~~

~~$-1 \leq \sin 2\alpha \leq 1$~~

$$\text{Since } -1 \leq \sin 2\alpha \leq 1$$

$$\Rightarrow -1 \leq 1 - \sqrt{3+2a} \leq 1$$

$$\Rightarrow -2 \leq -\sqrt{3+2a} \leq 0$$

$$\Rightarrow 2 \geq \sqrt{3+2a} \geq 0$$

$$\Rightarrow 0 \leq \sqrt{3+2a} \leq 2$$

$$\Rightarrow 0 \leq 3+2a \leq 4$$

$$\Rightarrow -3 \leq 2a \leq 1$$

$$\Rightarrow -\frac{3}{2} \leq a \leq \frac{1}{2}$$

$$\therefore a \in \left[-\frac{3}{2}, \frac{1}{2}\right]$$

25

LEARNERS TASK

01. Given that $4 \cos^2 \theta = 3$

$$\Rightarrow \cos^2 \theta = \frac{3}{4}$$

$$\Rightarrow \cos \theta = \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \cos \theta = \cos \frac{\pi}{6}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$$

$$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Ans: A

02. $3 \tan \theta = \cot \theta$

$$\Rightarrow 3 \tan^2 \theta = 1$$

$$\Rightarrow \tan^2 \theta = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\Rightarrow \tan \theta = \tan \frac{\pi}{6}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$$

$$\therefore \theta = \pm 30^\circ$$

Ans: A

26

Q1 Given $\cos m\theta = \sin n\theta$

$$\Rightarrow \cos m\theta = \cos\left(\frac{\pi}{2} - n\theta\right)$$

$$\Rightarrow m\theta = 2k\pi \pm \frac{\pi}{2} - n\theta, k \in \mathbb{Z}$$

$$\Rightarrow m\theta + n\theta = 2k\pi \pm \frac{\pi}{2}$$

$$\Rightarrow (m+n)\theta = 2k\pi \pm \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{2k\pi \pm \frac{\pi}{2}}{m+n}$$

Ans: C

Q2 Given $\tan m\theta = \cot n\theta$

$$\begin{aligned} \Rightarrow \tan m\theta &= \tan\left(\frac{\pi}{2} - n\theta\right) \\ \Rightarrow m\theta &= k\pi + \frac{\pi}{2} - n\theta \\ \Rightarrow m\theta + n\theta &= k\pi + \frac{\pi}{2} \\ \Rightarrow (m+n)\theta &= k\pi + \frac{\pi}{2} \end{aligned}$$

$$\Rightarrow \tan m\theta = \tan\left(\frac{\pi}{2} - n\theta\right)$$

$$\Rightarrow m\theta = k\pi + \frac{\pi}{2} - n\theta, k \in \mathbb{Z}$$

$$\Rightarrow (m+n)\theta = k\pi + \frac{\pi}{2}$$

$$\Rightarrow (m+n)\theta = (2k+1)\frac{\pi}{2}$$

(27)

Ans: B

$$\Rightarrow \theta = \frac{(2k+1)\pi}{2(m+n)}$$

05 Given $\sqrt{3} \sin \theta = \cos \theta$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan \theta = \tan \frac{\pi}{6}$$

$$\Rightarrow \theta = n\pi + \frac{\pi}{6}, \quad n \in \mathbb{Z}$$

06 ~~Given~~ $\sin 2x = 4 \cos x$

$$\Rightarrow 2 \sin x \cos x = 4 \cos x$$

$$\Rightarrow \sin x \cos x - 2 \cos x = 0$$

$$\Rightarrow \cos x (\sin x - 2) = 0$$

$$\Rightarrow \cos x = 0 \quad \text{or} \quad \sin x = 2$$

which is NOT possible

$$\therefore x = (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}$$

Ans: A

28
Q7. Given $\tan\left(\frac{\pi}{2} \sin \theta\right) = \cot\left(\frac{\pi}{2} \cos \theta\right)$

$$\Rightarrow \tan\left(\frac{\pi}{2} \sin \theta\right) = \tan\left(\pm \frac{\pi}{2} - \frac{\pi}{2} \cos \theta\right)$$

$$\Rightarrow \frac{\pi}{2} \sin \theta = \pm \frac{\pi}{2} - \frac{\pi}{2} \cos \theta$$

$$\Rightarrow \sin \theta = \pm 1 - \cos \theta$$

$$\Rightarrow \sin \theta + \cos \theta = \pm 1$$

$$\Rightarrow \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \pm \frac{1}{\sqrt{2}} \quad \text{Ans: B}$$

Q8. Given $\tan(\pi \cos x) = \cot(\pi \sin x)$

$$\Rightarrow \tan(\pi \cos x) = \tan\left(\pm \frac{\pi}{2} - \pi \sin x\right)$$

$$\Rightarrow \pi \cos x = \pm \frac{\pi}{2} - \pi \sin x$$

$$\Rightarrow \cos x + \sin x = \pm \frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = \pm \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \pm \frac{1}{2\sqrt{2}} \quad \text{Ans: C}$$

(28)

9. Given that $\tan(\cot x) = \cot(\tan x)$

$$\Rightarrow \tan(\cot x) = \tan\left(\frac{\pi}{2} - \tan x\right)$$

$$\Rightarrow \cot x = n\pi + \frac{\pi}{2} - \tan x, n \in \mathbb{Z}$$

$$\Rightarrow \tan x + \cot x = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \frac{1}{\sin x \cdot \cos x} = \frac{2n\pi + \pi}{2}$$

$$\Rightarrow \frac{1}{\sin 2x} = \frac{2n\pi + \pi}{4}$$

$$\Rightarrow \sin 2x = \frac{4}{(2n+1)\pi}$$

Ans. B

$$10. \sin\left(\frac{\pi}{4} \cot \theta\right) = \cos\left(\frac{\pi}{4} \tan \theta\right)$$

$$\Rightarrow \cos\left(\frac{\pi}{2} - \frac{\pi}{4} \cot \theta\right) = \cos\left(\frac{\pi}{4} \tan \theta\right)$$

$$\text{Let } \theta = \frac{\pi}{4}$$

$$\text{LHS} = \sin\left(\frac{\pi}{4} \cot \frac{\pi}{4}\right)$$

30

$$= \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\text{RHS} = \cos\left(\frac{\pi}{4} + \tan\frac{\pi}{4}\right)$$

$$= \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

Ans: B

$$\text{Hence } \theta = n\pi + \frac{\pi}{4}$$

JEE MAINS LEVEL QUESTIONS

Q1. Given $\frac{\tan 2x - \tan x}{1 + \tan 2x \cdot \tan x} = 1$

$$\Rightarrow \tan(2x - x) = 1$$

$$\Rightarrow \tan x = \tan \frac{\pi}{4}$$

$$\Rightarrow x = n\pi + \frac{\pi}{4}$$

Ans: C

Q2. Given $\sin x \cdot \sin(60^\circ + x) \cdot \sin(60^\circ - x) = \frac{1}{8}$

$$\text{We know } \sin x \cdot \sin(60^\circ + x) \cdot \sin(60^\circ - x) = \frac{1}{4} \sin 3x$$

$$\therefore \frac{1}{4} \sin 3x = \frac{1}{8}$$

$$\Rightarrow \sin 3x = \frac{1}{2}$$

$$\Rightarrow \sin 3x = \sin \frac{\pi}{6}$$

31

$$\Rightarrow 3x = n\pi + (-1)^n \cdot \frac{\pi}{6}, \quad n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{n\pi}{3} + (-1)^n \cdot \frac{\pi}{18}$$

Ans: B

Q4. Given $\cot^2 \theta - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) \cot \theta + 1 = 0$

~~Let~~ $\theta = \frac{\pi}{6}$

$$\text{LHS} = \cot^2 \theta - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) \cot \theta + 1$$

$$= \cot^2 \frac{\pi}{6} - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) \cot \frac{\pi}{6} + 1$$

$$= (\sqrt{3})^2 - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) (\sqrt{3}) + 1$$

$$= 3 - 3 - 1 + 1$$

$$= 0 = \text{RHS}$$

Hence, the general solution, $\theta = n\pi + \frac{\pi}{6}$, $n \in \mathbb{Z}$

Q4 Given $\sin \theta \cdot \sin(60^\circ + \theta) \cdot \sin(60^\circ - \theta) = \frac{1}{4}$

We know, $\sin \theta \cdot \sin(60^\circ + \theta) \cdot \sin(60^\circ - \theta) = \frac{1}{4} \sin 3\theta$

$$\therefore \frac{1}{4} \sin 3\theta = \frac{1}{4}$$

32

$$\Rightarrow \sin 3\theta = 1$$

$$\Rightarrow \sin 3\theta = \sin \frac{\pi}{2}$$

$$\Rightarrow 3\theta = n\pi + (-1)^n \cdot \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{n\pi}{3} + (-1)^n \cdot \frac{\pi}{6}, \quad n \in \mathbb{Z}$$

Ans: B

Q5

$$\sin x + \cos x = \sqrt{2} \cos x$$

$$\Rightarrow \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \cos x$$

$$\Rightarrow \cos \left(x - \frac{\pi}{4} \right) = \cos x$$

$$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm x, \quad n \in \mathbb{Z}$$

$$\Rightarrow x = 2n\pi \pm x + \frac{\pi}{4}$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{4} \pm x \quad \text{Ans: C}$$

Q6

$$\text{Given } \sin \left(x + \frac{\pi}{4} \right) = \sin 2x$$

$$\Rightarrow x + \frac{\pi}{4} = n\pi + (-1)^n \cdot 2x, \quad n \in \mathbb{Z}$$

$$\Rightarrow x - (-1)^n 2x = n\pi - \frac{\pi}{4}$$

$$\Rightarrow x(1 - (-1)^n \cdot 2) = n\pi - \frac{\pi}{4}$$

$$\alpha = \frac{n\pi - \frac{\pi}{4}}{1 - (-1)^n \cdot 2}$$

$$, n \in \mathbb{Z}$$

Ans: ①

Q7. Given that $81 \sin^2 \alpha + 81 \cos^2 \alpha = 30$

$$\Rightarrow 81 \sin^2 \alpha + 81 = 30$$

$$\Rightarrow 81 \sin^2 \alpha + \frac{81}{\sin^2 \alpha} = 30$$

$$\Rightarrow t + \frac{81}{t} = 30, \text{ where } 81 \sin^2 \alpha = t$$

$$\Rightarrow t^2 - 30t + 81 = 0$$

$$\Rightarrow (t-27)(t-3) = 0$$

$$\Rightarrow t = 27$$

$$\Rightarrow 81 \sin^2 \alpha = 27$$

$$\Rightarrow 4 \sin^2 \alpha = 3$$

$$\Rightarrow \sin^2 \alpha = \frac{3}{4}$$

$$\Rightarrow \sin \alpha = \frac{\sqrt{3}}{2}$$

$$t = 3$$

$$\Rightarrow 81 \sin^2 \alpha = 3$$

$$\Rightarrow 4 \sin^2 \alpha = 3$$

$$\Rightarrow 4 \sin^2 \alpha = 1$$

$$\Rightarrow \sin^2 \alpha = \frac{1}{4}$$

$$\Rightarrow \sin \alpha = \frac{1}{2}$$

$$\Rightarrow x = \frac{\pi}{6} \propto \frac{\pi}{3} \quad \text{Since } 0 \leq x \leq \frac{\pi}{2}$$

34

Ans: A

Given $\sec^2(a+2)x + (a^2-1) = 0$

$$\Rightarrow \sec^2(a+2)x - 1 + a^2 = 0$$

$$\Rightarrow \tan^2(a+2)x + a^2 = 0$$

$$\Rightarrow \tan^2(a+2)x = 0 \quad \text{and} \quad a^2 = 0$$

$$\Rightarrow a = 0$$

$$\Rightarrow \tan 2x = 0$$

$$\Rightarrow 2x = 0, \pi, -\pi, \text{ Since } -\pi < x < \pi$$

$$\Rightarrow x = 0, \frac{\pi}{2}, -\frac{\pi}{2}$$

So, the required ordered pairs are

$$(0, 0), (0, \frac{\pi}{2}), (0, -\frac{\pi}{2})$$

$\therefore$ 3 possible ordered pairs Ans: C

Given $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$

Divide with $\cos^2 \theta$ on either side

$$\Rightarrow \sec^2 \theta + \tan^2 \theta = 3 \tan \theta$$

$$\Rightarrow (1 + \tan^2 \theta) + \tan^2 \theta = 3 \tan \theta$$

35

$$\Rightarrow 2 \tan^2 \theta - 3 \tan \theta + 1 = 0$$

$$\Rightarrow (2 \tan \theta - 1)(\tan \theta - 1) = 0$$

$$\Rightarrow \tan \theta = \frac{1}{2} \text{ or } \tan \theta = 1$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta = \frac{\pi}{4}$$

Ans: B

10. Given $2 \cos x + \cos 2x = 3$

one of the possible case is

Let $2 \cos x = 2$ ~~$\cos 2x = 1$~~

$$\Rightarrow \cos x = 1$$

$$\Rightarrow \cos x = \cos 0$$

$$\Rightarrow x = 2n\pi, n \in \mathbb{Z}$$

~~If $x \in \mathbb{Q}$, the above equation has infinitely~~

Let $\cos 2x = 1$

~~If $x = 0$ i.e. $x \in \mathbb{Q}$, is the only solution for both the equations~~

~~If $x \in \mathbb{Q}$, there are infinitely many~~

Solutions

and, if $\pi \notin Q$, then $x=0$ is the only one solution satisfying both the equations.

Hence, π is an irrational number
Ans: C

11. Given $\sin^4 x + \cos^4 x = 1$

$$\Rightarrow \sin^4 x = 1 - \cos^4 x$$

$$\Rightarrow \sin^4 x = (1 + \cos^2 x)(1 - \cos^2 x)$$

$$\Rightarrow \sin^4 x = (1 + \cos^2 x) \cdot \sin^2 x$$

$$\Rightarrow \sin^4 x - \sin^2 x (1 + \cos^2 x) = 0$$

$$\Rightarrow \sin^2 x (\sin^2 x - 1 - \cos^2 x) = 0$$

$$\Rightarrow \sin^2 x = 0 \quad \left| \quad \sin^2 x - \cos^2 x - 1 = 0 \right.$$

$$\Rightarrow \sin x = 0 \quad \left| \quad \cos^2 x - \sin^2 x = -1 \right.$$

$$\Rightarrow x = n\pi \quad \left| \quad \cos 2x = -1 \right.$$

$$\Rightarrow \cos 2x = \cos\left(\frac{\pi}{2}\right)$$

$$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{2}$$

Ans A, D

$$\Rightarrow x = n\pi \pm \frac{\pi}{2}$$

(37)

12. Statement I: Given $\sqrt{3} \tan \theta - 1 = 0$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan \theta = \tan \frac{\pi}{6}$$

$$\Rightarrow \theta = n\pi + \frac{\pi}{6}, n \in \mathbb{Z}$$

Statement I is true

Statement II: Given $\tan x = k$

$$\Rightarrow x = n\pi + \alpha, n \in \mathbb{Z}$$

$\therefore$ Statement II is true

Ans: A

13. Statement I: Given $\sin \theta = \frac{1}{2}$ and $\tan \theta = \frac{1}{\sqrt{3}}$

$$\Rightarrow \sin \theta = \sin \frac{\pi}{6} \text{ and } \tan \theta = \tan \frac{\pi}{6}$$

$$\text{General Solution} = \left\{ 2n\pi + \frac{\pi}{6}, n \in \mathbb{Z} \right\}$$

Statement I is TRUE

38

Statement II: Given $\cos \theta = -\frac{1}{\sqrt{2}}$, $\tan \theta = -1$

Now, $\cos \theta = \cos \frac{3\pi}{4}$ and $\tan \theta = \tan \frac{3\pi}{4}$

$\therefore$ The G.S. = $\{2n\pi + \frac{3\pi}{4}, n \in \mathbb{Z}\}$

$\therefore$ Statement II is TRUE Ans: A

14. Given $2\sin^2 \theta + 3\sin \theta + 1 = 0$

$$\Rightarrow (2\sin \theta + 1)(\sin \theta + 1) = 0$$

$$\Rightarrow 2\sin \theta + 1 = 0 \quad \text{or} \quad \sin \theta + 1 = 0$$

$$\therefore \sin \theta = -\frac{1}{2}$$

$$\sin \theta = -1$$

$$\Rightarrow \sin \theta = \sin \left(-\frac{\pi}{2}\right)$$

$$\Rightarrow \theta = n\pi + (-1)^n \left(-\frac{\pi}{2}\right)$$

$$\Rightarrow \theta = n\pi + (-1)^n \left(-\frac{\pi}{6}\right)$$

$$\Rightarrow \theta = \frac{3\pi}{2}$$

$$\Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}$$

Total No. of solutions = 4.

15. a) Given $\cos x = -\frac{1}{2}$

~~Given~~

$$\text{Now } \cos \frac{8\pi}{3} = \cos \left(2\pi + \frac{2\pi}{3} \right)$$

$$= \cos \frac{2\pi}{3}$$

$$= \cos \left(\pi - \frac{\pi}{3} \right)$$

$$= -\cos \frac{\pi}{3}$$

$$= -\frac{1}{2}$$

$$\therefore x = \frac{8\pi}{3}$$

b) Given $\sin x = \frac{\sqrt{3}}{2}$

$$\text{Now } \sin \frac{8\pi}{3} = \sin \left(2\pi + \frac{2\pi}{3} \right)$$

$$= \sin \frac{2\pi}{3}$$

$$= \sin \left(\pi - \frac{\pi}{3} \right)$$

$$= \sin \frac{\pi}{6}$$

$$= \frac{\sqrt{3}}{2}$$

$$\text{Also, } \sin \frac{7\pi}{3} = \sin \left(2\pi + \frac{\pi}{3} \right)$$

$$= \sin \frac{\pi}{3}$$

$$= \frac{\sqrt{3}}{2}$$

40

hence $x = \frac{8\pi}{3}$ and $\frac{7\pi}{3}$

c) Given $\tan x = \frac{1}{\sqrt{3}}$

Now, $\tan \frac{19\pi}{6} = \tan \left(2\pi + \frac{7\pi}{6} \right)$

$= \tan \frac{7\pi}{6}$

$= \tan \left(\pi + \frac{\pi}{6} \right)$

$= \tan \frac{\pi}{6}$

$= \frac{1}{\sqrt{3}}$

hence, $x = \frac{19\pi}{6}$

d) Given $\cot x = -1$

$\Rightarrow \tan x = -1$

Now, ~~$\tan \frac{11\pi}{4} = \tan \left(2\pi + \frac{3\pi}{4} \right)$~~

~~$= \tan \frac{3\pi}{4}$~~

~~$= \tan \left(\frac{\pi}{2} + \frac{\pi}{4} \right)$~~

Now, $\tan \frac{11\pi}{4} = \tan \left(2\pi + \frac{3\pi}{4} \right)$

$= \tan \frac{3\pi}{4}$

(41)

$$= \tan\left(\pi - \frac{\pi}{4}\right)$$

$$= -\tan\frac{\pi}{4}$$

$$= -1$$

$$\text{hence } x = \frac{11\pi}{4}$$

Ans: $a-x$, $b-(p,x)$, $c-q$, $d-s$

ADDITIONAL PRACTICE QUESTIONS FOR STUDENTS

Q1. Given $K = \cos 20^\circ$

$$\begin{aligned}\text{Now, } \cos x &= 2K^2 - 1 \\ &= 2\cos^2 20^\circ - 1 \\ &= \cos 2 \times 20^\circ \\ &= \cos 40^\circ\end{aligned}$$

$$\begin{aligned}\therefore \cos x &= \cos 40^\circ & \cos x &= \cos (36^\circ - 40^\circ) \\ \Rightarrow x &= 40^\circ & \Rightarrow \cos x &= \cos 32^\circ \\ & & \Rightarrow x &= 32^\circ\end{aligned}$$

Ans: C

Q3. Given that $\tan\left(\theta + \frac{\pi}{4}\right) = 3 \cdot \tan 3\theta$.

$$\begin{aligned}
 & \Rightarrow \frac{\tan \theta + \tan \frac{\pi}{4}}{1 - \tan \theta \cdot \tan \frac{\pi}{4}} = 3 \left(\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right) \\
 & \Rightarrow \frac{\tan \theta + 1}{1 - \tan \theta} = 3 \left(\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right) \\
 & \Rightarrow (1 + \tan \theta)(1 - 3 \tan^2 \theta) = 9 \tan \theta - 3 \tan^3 \theta \\
 & \quad (1 - \tan \theta)
 \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow 1 - 3 \tan^2 \theta + \tan \theta - 3 \tan^3 \theta \\
 & \quad = 9 \tan \theta - 9 \tan^2 \theta - 3 \tan^3 \theta + 3 \tan^3 \theta \\
 & \Rightarrow 3 \tan^4 \theta - 6 \tan^2 \theta + 8 \tan \theta - 1 = 0.
 \end{aligned}$$

Sum of the roots = $\tan \alpha + \tan \beta + \tan \delta +$

$$\tan \delta = \frac{0}{3} = 0.$$

Given $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$

Given this equation has real roots

$\therefore$ Discriminant ≥ 0

$$\Rightarrow \cos^2 p - 4(\cos p - 1)\sin p \geq 0$$

$$\Rightarrow \cos^2 p - 4 \cos p \sin p + 4 \sin p \geq 0$$

$$\Rightarrow \cos^2 p - 4 \cos p \sin p + 4 \sin^2 p - 4 \sin^2 p + 4 \sin p \geq 0 \quad (43)$$

$$\Rightarrow (\cos p - 2 \sin p)^2 + 4 \sin p (1 - \sin p) \geq 0$$

here $(\cos p - 2 \sin p)^2 \geq 0$ always.

$\therefore 4 \sin p (1 - \sin p)$ should be positive

here $\sin p$ is positive in $[0, \pi]$

$$\text{also, } 0 \leq 1 - \sin p \leq 2$$

also, $(1 - \sin p)$ is also positive in $[0, \pi]$

$\therefore$

Hence, p belongs to $(0, \pi)$

Ans: D

4.

$$\text{Given } \sin^2 x - \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 + \cos^2 x$$

The maximum value of $\sin^2 x$ is

1.

also, $\cos^2 x$ is always positive and

must be equal to 0.

$$\therefore \cos^2 x = 0$$

$$\Rightarrow \cos x = 0$$

$$\Rightarrow x = (2n+1) \frac{\pi}{2} \Rightarrow x = n\pi + \frac{\pi}{2}$$

Ans: C

Given $\cos^4 x - (a+2)\cos^2 x - (a+3) = 0$ 44

Let $\cos^2 x = t$

$$\Rightarrow t^2 - (a+2)t - (a+3) = 0$$

$$\Rightarrow t^2 - ((a+3)-1)t - (a+3) = 0$$

$$\Rightarrow t^2 + t - (a+3)t - (a+3) = 0$$

$$\Rightarrow t(t+1) - (a+3)(t+1) = 0$$

$$\Rightarrow (t+1)(t - (a+3)) = 0$$

$$\Rightarrow t+1=0 \quad \text{or} \quad t - (a+3) = 0$$

$$\Rightarrow t = a+3$$

$$\Rightarrow \cos^2 x + 1 = 0$$

$$\Rightarrow \cos^2 x = -1$$

which is not

possible

$$\Rightarrow \cos^2 x = a+3$$

We know

$$0 \leq \cos^2 x \leq 1$$

$$\Rightarrow 0 \leq a+3 \leq 1$$

$$\Rightarrow -3 \leq a \leq -2$$

$$\therefore a \in [-3, -2]$$

Ans: A

06.

Given $\sin x - \sqrt{3} \cos x = \frac{4m-6}{4-m}$

45

$$\Rightarrow 2 \left(\frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x \right) = \frac{4m-6}{4-m}$$

$$\Rightarrow \sin x \cos \frac{\pi}{3} - \cos x \cdot \sin \frac{\pi}{6} = \frac{2m-3}{4-m}$$

$$\Rightarrow \sin \left(x - \frac{\pi}{3} \right) = \frac{2m-3}{4-m}$$

We know $-1 \leq \sin \left(x - \frac{\pi}{3} \right) \leq 1$

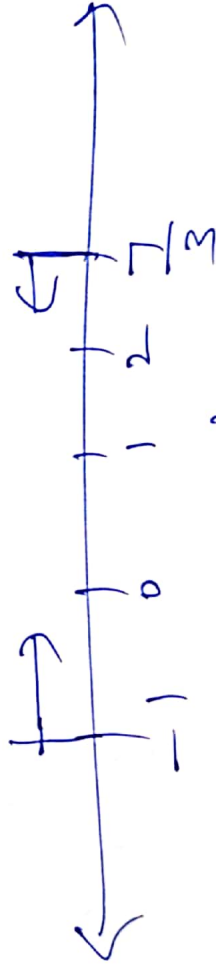
$$\Rightarrow -1 \leq \frac{2m-3}{4-m} \leq 1$$

$$\Rightarrow -1 \leq \frac{2m-3}{4-m} \quad \text{and} \quad \frac{2m-3}{4-m} \leq 1$$

$$\Rightarrow -4+m \leq 2m-3 \quad \text{and} \quad 2m-3 \leq 4-m$$

$$\Rightarrow m \geq -1 \quad \Rightarrow 3m \leq 7$$

$$\Rightarrow m \leq -1 \quad \Rightarrow m \leq \frac{7}{3}$$



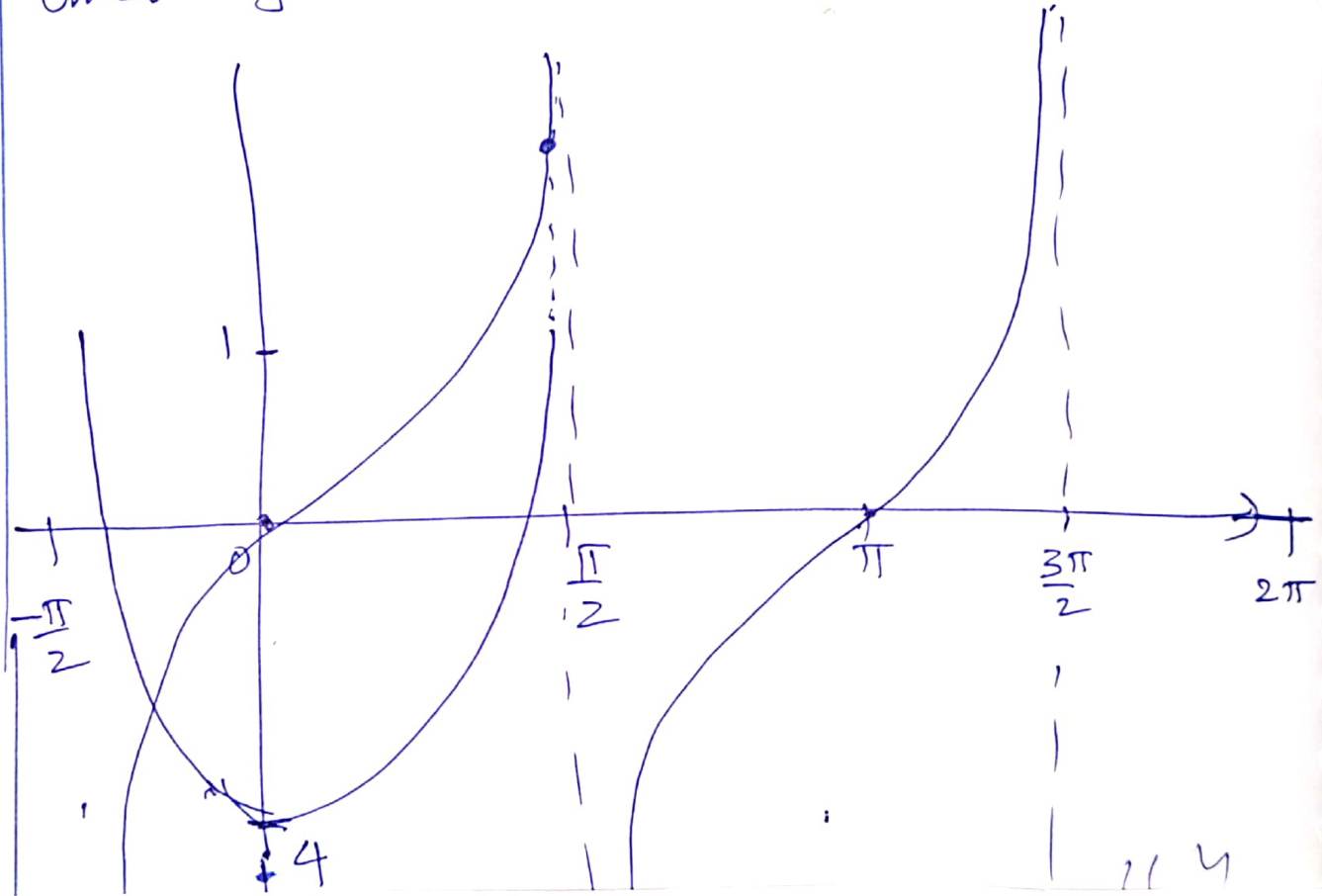
$$\therefore m \in \{-1, 0, 1, 2\}$$

No. of integral values of m are 4

67 Given $3\sec\theta - 5 = 4\tan\theta$

46

Let us draw the graphs of $y = 3\sec\theta - 5$
and $y = 4\tan\theta$ in $[0, \pi]$



clearly Both the graphs intersect at 47

2 points between $[0, \pi]$

Since, both the functions are periodic functions, in $[0, 4\pi]$, they intersect

in 8 points.
Hence, they have 8 solutions in $[0, 4\pi]$

so ~~THE~~ END $\Leftarrow$ $\therefore$ $\boxed{8}$ ✓

Comprehension-II

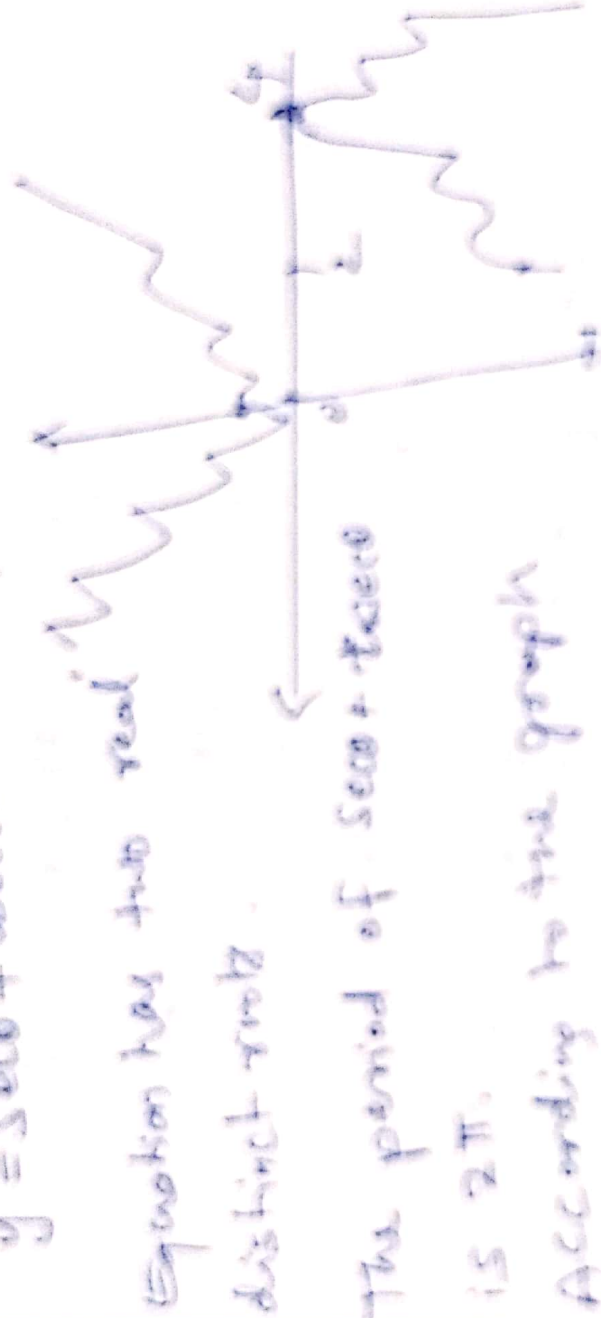
48

Given $\sec \theta + \csc \theta = a$,

Given $\theta \in (0, 2\pi) - \left\{ \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\}$

Now, let us draw the graphs of

$$y = \sec \theta + \csc \theta \quad \text{and} \quad y = a$$



Equation has two real distinct roots.

The period of $\sec \theta + \csc \theta$ is 2π .

According to the graph

$$f(x) \rightarrow \infty, \text{ so in}$$

$(0, \frac{\pi}{2})$ $f(x)$ is increasing.

At $\frac{\pi}{4}$ it has least value $2\sqrt{2}$, so in the given interval $|a| < 2\sqrt{2}$.

From the figure, we can say that

$f(x) = 0$ has two distinct roots if

$y = a$ cuts the graph $y = f(x)$ between

$$y = 2\sqrt{2} \quad \text{and} \quad y = -2\sqrt{2}$$

$$\text{i.e. } |a| < 2\sqrt{2}$$