WS-13 For 9th class

T task (Moment of inertia and theorem)

(1) $I = 20\ kgm^2$

$I = \frac{mR^2}{2}$

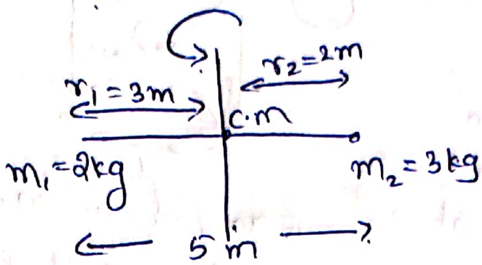
$\Rightarrow I = Mk^2$

$\Rightarrow \cancel{20}^{4} = 5k^2$

$\Rightarrow k^2 = 4$

$\Rightarrow k = 2m$

(2)



$r_1 = \frac{m_2 d}{m_1 + m_2} = \frac{3 \times 5}{2+3} = \frac{3 \times \cancel{5}}{\cancel{5}}$

$r_1 = 3m$

$r_2 = d - r_1 = 5 - 3 = 2m.$

$\therefore I = m_1 r_1^2 + m_2 r_2^2$

$= 2 \times (3)^2 + 3(2)^2$

$\Rightarrow 2 \times 9 + 3 \times 4$

$\Rightarrow 18 + 12$

$= 30\ kgm^2$

(3)

$I_{cm} = MR^2$

Acc to parallel axis theorem

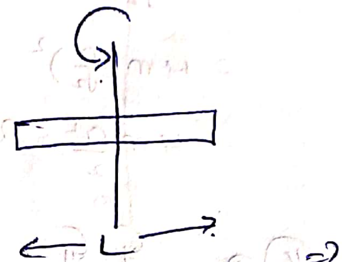
$I = I_{cm} + mr^2$

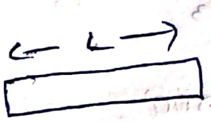
'r' is $\perp^{er}$ distance b/w two axis

$\therefore I = MR^2 + MR^2$

$I = 2MR^2.$

(4)





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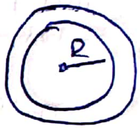


$I_1 = \frac{ML^2}{12}$

$I_1 = \frac{ML^2}{4} \times 3$

$\Rightarrow I_1 = I_2 \frac{\pi^2}{3}$

$\Rightarrow \frac{I_1}{I_2} = \frac{\pi^2}{3}$

$L = 2\pi R$

$R = \frac{L}{2\pi}$

$I_2 = MR^2$

$= M\left[\frac{L}{2\pi}\right]^2$

$\Rightarrow \frac{ML^2}{4\pi^2}$

$I_2 \pi^2 = \frac{ML^2}{4}$

(5)

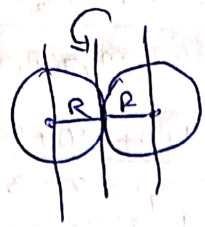
$\Rightarrow MR^2 = 2I.$

$I = \frac{MR^2}{2} \Rightarrow$ M of Inertia about diametrical axis of ring

$\therefore$ Moment of inertia about an axis passing through centre is

$I_{cm} = MR^2 = 2I$

(6)



moment of inertia of 1st sphere about tangential axis

$I_1 = I_{c.m} + mr^2$

$I_1 = \frac{2}{5}MR^2 + MR^2$

$I_1 = \frac{7}{5}MR^2.$

moment of inertia of 2nd sphere about tangential axis

$I_2 = I_{c.m} + mr^2$

$= \frac{2}{5}mR^2 + MR^2$

$= \frac{7}{5}mR^2$

~~Moment of~~

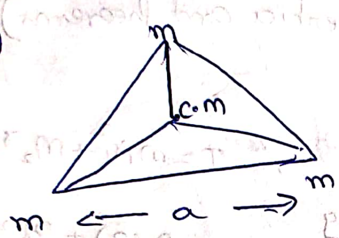
for Total system

$\therefore I = I_1 + I_2$

$= \frac{7}{5}MR^2 + \frac{7}{5}MR^2$

$= \frac{14}{5}MR^2.$

(7)



Three particles are at a distance of $\frac{a}{\sqrt{3}}$ from com

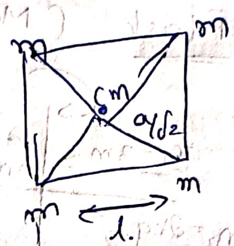
$\therefore r_1 = r_2 = r_3 = \frac{a}{\sqrt{3}}$

$\therefore I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$

$= m\left(\frac{a}{\sqrt{3}}\right)^2 + m\left(\frac{a}{\sqrt{3}}\right)^2 + m\left(\frac{a}{\sqrt{3}}\right)^2$

$\Rightarrow 3m\frac{a^2}{3} = ma^2$

(8)



Four particles are at a distance of $\frac{a}{\sqrt{2}}$ from com

$r_1 = r_2 = r_3 = r_4 = \frac{a}{\sqrt{2}}$

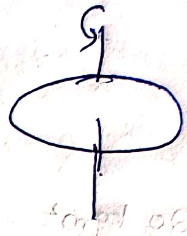
$\therefore I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + m_4r_4^2$

$= m\left(\frac{a}{\sqrt{2}}\right)^2 + m\left(\frac{a}{\sqrt{2}}\right)^2 + m\left(\frac{a}{\sqrt{2}}\right)^2 + m\left(\frac{a}{\sqrt{2}}\right)^2$

$= 4m\left(\frac{a}{\sqrt{2}}\right)^2$

$= 4m\frac{a^2}{2} = 2ma^2$

(9)



D_1, D_2 are densities

$D = \frac{M}{V} = \frac{M}{A \times t}$

$\Rightarrow D \propto \frac{M}{A} \Rightarrow D \propto \frac{1}{R^2}$

moment of inertia of a disc about an axis passing through centres

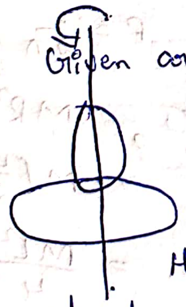
$I = \frac{MR^2}{2}.$

$I \propto R^2$

$\Rightarrow I \propto \frac{1}{D}$

$\Rightarrow \frac{I_1}{I_2} = \frac{D_2}{D_1}$

(10)



Given arrangement

$I = I_1 + I_2.$

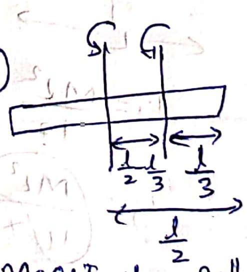
where I_1, I_2 are the M.O.I of two discs about diametrical axis.

$\Rightarrow I_1 = \frac{MR^2}{4};\ I_2 = \frac{MR^2}{4}.$

$\therefore I = \frac{MR^2}{4} + \frac{MR^2}{4}$

$I = \frac{MR^2}{2}.$

(16) (P)



M.O.I of a Rod of length L about an axis passing through centre

$I_{cm} = \frac{ML^2}{12}$

$r = \frac{l}{2} - \frac{l}{3} = \frac{l}{6}$

$I = I_{cm} + mr^2$

$= \frac{ML^2}{12} + m\left(\frac{L}{6}\right)^2$

$I = \frac{ML^2}{12} + \frac{ML^2}{36}$

$\Rightarrow I = \frac{ML^2}{9}.$

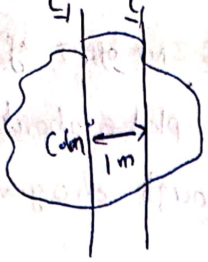
(ii)

$I = Mk^2$

$\Rightarrow Mk^2 = \frac{ML^2}{9}$

$\Rightarrow k = \frac{L}{3}$

(17)



M.O.I of a body about an axis through C.G is $I_{cm} = 10\ kg\,m^2$.

The distance between two parallel axis 1m.

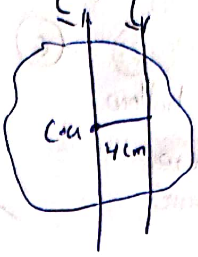
$I = I_{c.m} + mr^2$

$= 10 + 2(1)^2$

$= 10 + 2$

$I = 12\ kg\,m^2$

(18)



$k' = ?$ According to Parallel axis theorem $I = I_{c.m} + Mr^2$

$\Rightarrow Mk^2 = Mk'^2 + Mr^2$

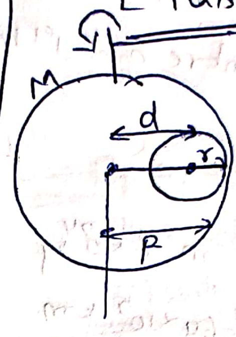
$\Rightarrow k^2 = k'^2 + r^2$

$\Rightarrow 25 = k'^2 + 16 \Rightarrow k'^2 = 25 - 16$

$\Rightarrow k'^2 = 9$

$\Rightarrow k' = 3\,cm$

L Task



$d = R - r$.

Let $\frac{m}{2}$ be mass of removed disc

r be it's radius

Since $M \propto R^2 \Rightarrow \frac{m_1}{m_2} = \frac{r_1^2}{r_2^2} \Rightarrow \frac{M}{\frac{M}{2}} = \frac{R^2}{r^2}$

$\Rightarrow 2 = \frac{R^2}{r^2} \Rightarrow r^2 = \frac{R^2}{2} \Rightarrow r = \frac{R}{\sqrt{2}}$

$\therefore d = R - \frac{R}{\sqrt{2}} = R\left[1 - \frac{1}{\sqrt{2}}\right]$

The M.o.I about the central axis perpendicular to it's plane is

$I' = I_{c.m} + md^2$

$= \frac{\frac{M}{2} r^2}{2} + \frac{M}{2}\underbrace{\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]^2}_{<<1}R^2$. Neglect

$\approx \frac{M}{4}\left[\frac{R}{\sqrt{2}}\right]^2$

$I' = \frac{MR^2}{8}$

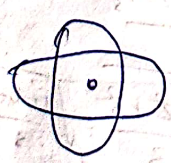
(2) M.O.I of disc about an axis passing through centre is $I = \frac{MR^2}{2}$

$\therefore$ M.O.I about tangential axis $I_{tangent} = \frac{5}{4}MR^2$

$= \frac{5}{2}\frac{MR^2}{2}$

$I_{tangent} = \frac{5}{2}I$

(3) Given that two rings are arranged



M.O.I of ring of the system about the central axis $\perp$er to it's plane would be.

$I' = MR^2$

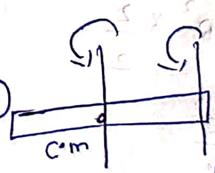
$\Rightarrow \boxed{I' = \frac{2}{3}I}$

M.O.I of the system $I = I_1 + I_2$

$= \frac{MR^2}{2} + MR^2$

$I = \frac{3}{2}MR^2 = \frac{3}{2}I'$

(4)



M.O.I of thin rod about an axis passing through one end $\perp$er to it's length L is

$I = \frac{ML^2}{3}$

About central axis

$I' = \frac{ML^2}{12} = \frac{ML^2}{4\times3}$

$I' = \frac{1}{4}\frac{ML^2}{3} = \frac{1}{4}I$

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⑤ Given that

M.O.I of solid sphere = M.O.I of hollow sphere.

$\Rightarrow I_S = I_H$

$\Rightarrow \frac{2}{5} MR_S^2 = \frac{2}{3} MR_H^2$

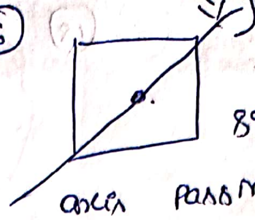
$\Rightarrow \frac{1}{5} R_S^2 = \frac{1}{3} R_H^2$

$\Rightarrow \frac{1}{5} \left(\frac{D_S}{2}\right)^2 = \frac{1}{3} \left(\frac{D_H}{2}\right)^2$

$\Rightarrow \frac{D_S^2}{5} = \frac{D_H^2}{3}$

$\rightarrow \frac{D_S^2}{D_H^2} = \frac{5}{3}$

$\Rightarrow \frac{D_S}{D_H} = \sqrt{\frac{5}{3}}$

⑥ 

The M.of.I of thin square plate about an axis passing through diagonal is

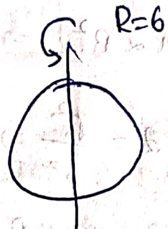
$I = \frac{ML^2}{12}$.

M.O.I about an axis passing through it's centre and $\perp^{er}$ to plane $I' = \frac{ML^2}{6} = 2\frac{ML^2}{2\times 6}$

$I' = 2\frac{ML^2}{12} \Rightarrow I' = 2I$

⑦ Given $m = 6\times10^{24}$ kg

$R = 64\times10^5$ km



Here Earth is in the form of sphere

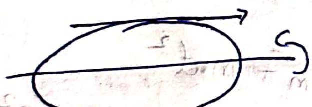
The M.O.I about an axis passing through pole's (ie) centre and $\perp^{er}$ to plane

$I = \frac{2}{5}MR^2 \Rightarrow \frac{2}{5} 6\times10^{24} \times (64\times10^5)^2$

$\Rightarrow \frac{6}{5} \times 2\times10^{24} \times (2^6\times10^5)^2$

$\Rightarrow 1.2 \times 2\times10^{24} \times 2^{12} \times 10^{10}$

$\Rightarrow 1.2 \times 2^{13} \times 10^{34}$

⑧ 

M.O.I about diametrical axis is $I = \frac{MR^2}{4}$

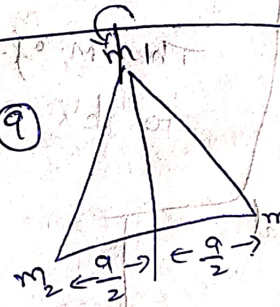
About tangential axis

$I_t = \frac{3}{2}MR^2$

$I_t = \frac{3}{2}MR^2 \times \frac{2}{2}$

$\Rightarrow 6\frac{MR^2}{4}$

$I_t = 6I$.

⑨ 

m_1 is located on axis of rotation so $r_1 = 0$

$r_2 = r_3 = \frac{a}{2}$ ∴ M.O.I of the system

$I = I_1 + I_2 + I_3 = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$

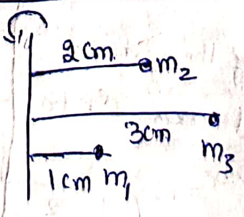
$= m_1(0)^2 + m_2\left(\frac{a}{2}\right)^2 + m_3\left(\frac{a}{2}\right)^2$

$I = 0 + m_2\frac{a^2}{4} + m_3\frac{a^2}{4} \Rightarrow (m_2+m_3)\frac{a^2}{4}$

⑩ 

M.O.I of Rod about an axis passing through centre is $I = \frac{ML^2}{12}$

$\therefore k^2 = \frac{L^2}{12} \Rightarrow k = \frac{L}{2\sqrt{3}}$

⑮ 

$m_1 = 1g; m_2 = 2g; m_3 = 3g$

$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$

$I = 1\times1^2 + 2\times2^2 + 3\times3^2$

$= 1 + 2\times4 + 3\times9 = 1+8+27 = 36 \text{ gm cm}^2$

$k = \sqrt{\frac{r_1^2+r_2^2+r_3^2}{3}}$

$= \sqrt{\frac{1^2+2^2+3^2}{3}} = \sqrt{\frac{1+4+9}{3}}$

$k = \sqrt{\frac{14}{3}} = \sqrt{4.66}$

$k = 2.45$ cm

(17) M. of I of ring about it's diameter is $I = \frac{MR^2}{2}$.

$\Rightarrow 2I = MR^2$

$\therefore$ M.O.I about an axis passing through it's centre and $\perp^{er}$ to plane is

$I' = MR^2 = 2I$.

(18) The M.O.I of fly wheen about ~~an~~ ~~an axis~~ central axis $I = 16$ kg m²

Given $M = 4$ kg. $\Rightarrow Mk^2 = 16$

$\Rightarrow \cancel{4} k^2 = \cancel{16}\,4$

$\Rightarrow k^2 = 4$

$k = 2$ cm