8^{TH} CLASS

Compound Angles

Teaching Task

1. Given $A+B+C=\pi$

$\Rightarrow A+B=\pi-C$

$\Rightarrow Tan(A+B)=Tan(\pi-c)$

$\Rightarrow \dfrac{TanA+TanB}{1-TanA\cdot TanB}=-TanC$

$\Rightarrow TanA+TanB=-TanC+TanA\cdot TanB\cdot TanC$

$\Rightarrow TanA+TanB+TanC=TanA\cdot TanB\cdot TanC$

$\Rightarrow \sum TanA=\pi TanA$

Ans: A

2. Given $\dfrac{\tan 225^0-\cot 81^0\cdot\cot 69^0}{\cot 261^0+\tan 21^0}$

$=\dfrac{\tan\left(180^0+45^0\right)-\cot 81^0\cdot\cot 6^0}{\cot\left(180^0+81^0\right)+\tan\left(90^0-69^0\right)}$

$=\dfrac{\tan 45^0-\cot 81^0\cdot\cot 69^0}{\cot 81^0+\cot 69^0}$

$=\dfrac{1-\cot 81^0\cdot\cot 69^0}{\cot 81^0+\cot 69^0}$

$=-\cot\left(81^0+69^0\right)$

$=-\cot\left(150^0\right)$

$=-\cot\left(180^0-30^0\right)$

$=\cot 30^0=\sqrt{3}$

Ans: C

3. Given $\dfrac{\left(1+\tan 13^0\right)\left(1+\tan 32^0\right)}{\left(1+\tan 12^0\right)\left(1+\tan 33^0\right)}$

We know $13^0+32^0=45^0$

$\Rightarrow \tan\left(13^0+32^0\right)=\tan 45^0$

$\Rightarrow \dfrac{\tan 13^0+\tan 32^0}{1-\tan 13^0\cdot\tan 32^0}=1$

$\Rightarrow \tan 13^0+\tan 32^0=1-\tan 13^0\cdot\tan 32^0$

$\Rightarrow \tan 13^0+\tan 32^0+\tan 13^0\cdot\tan 32^0=1$

$\Rightarrow 1+\tan 13^0+\tan 32^0+\tan 13^0\cdot\tan 32^0=2$

$\Rightarrow \left(1+\tan 13^0\right)\left(1+\tan 32^0\right)=2$

Similarly, we can prove

$$\left(1+\tan 12^0\right)\left(1+\tan 33^0\right)=2$$

$$Hence, \frac{\left(1+\tan 13^0\right)\left(1+\tan 32^0\right)}{\left(1+\tan 12^0\right)\left(1+\tan 33^0\right)}=\frac{2}{2}=1$$

ANS : A

4. Given A + B = 300^0

$$\Rightarrow \cot\left(A+B\right)=\cot 300^0$$

$$\Rightarrow \frac{\cot A\cdot\cot B-1}{\cot A+\cot B}=\cot\left(360^0-60^0\right)$$

$$\Rightarrow \frac{\cot A\cdot\cot B-1}{\cot A+\cot B}=-\frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\cot A\cdot\cot B-1}{\cot A+\cot B}=-\frac{\sqrt{3}}{3}$$

$$\Rightarrow 3\cot A\cdot\cot B-3=-\sqrt{3}\cot A-\sqrt{3}\cot B$$

$$\Rightarrow \sqrt{3}\cot A+\sqrt{3}\cot B+3\cot A\cdot\cot B=3$$

$$\Rightarrow 1+\sqrt{3}\cot A+\sqrt{3}\cot B+3\cot A\cdot\cot B=4$$

$$\Rightarrow \left(1+\sqrt{3}\cot A\right)\left(1+\sqrt{3}\cot B\right)=4$$

ANS: A

5. Given A + B + C = 360^0

Let A = B = C = 120^0

Given

$$\cot\frac{A}{4}+\cot\frac{B}{4}+\cot\frac{C}{4}=K.\cot\frac{A}{4}\cdot\cot\frac{B}{4}\cdot\cot\frac{C}{4}$$

$$\Rightarrow \cot 30^0+\cot 30^0+\cot 30^0=K.\cot 30^0\cdot\cot 30^0\cdot\cot 30^0$$

$$\Rightarrow 3\times\cot 30^0=K.\cot^3 30^0$$

$$\Rightarrow 3\times\sqrt{3}=K\cdot\left(\sqrt{3}\right)^3$$

$$\Rightarrow 3\times\sqrt{3}=K\cdot 3\sqrt{3}$$

$$\Rightarrow K=1$$

ANS: D

6. Given tan20^0 = p

Now, $$\frac{\tan 160^0-\tan 110^0}{1+\tan 160^0\cdot\tan 110^0}=\tan\left(160^0-110^0\right)$$

$$=\tan 50^0$$

$$=\tan\left(90^0-40^0\right)$$

$$=\cot 40^0$$

$$=\frac{1}{\tan 40^0}$$

$$= \frac{1}{\tan 2\cdot 20^0}$$
$$= \frac{1-\tan^2 20^0}{2\tan 20^0}$$
$$= \frac{1-p^2}{2p}$$

ANS: B

7. Given A + B = 225^0

$$\Rightarrow \cot(A+B) = \cot 225^o$$
$$\Rightarrow \frac{\cot A\cdot\cot B-1}{\cot A+\cot B} = 1$$
$$\Rightarrow \cot A\cdot\cot B-1 = \cot A+\cot B$$
$$\Rightarrow \cot A\cdot\cot B = 1+\cot A+\cot B$$
$$\Rightarrow 2\cot A\cdot\cot B = 1+\cot A+\cot B+\cot A\cdot\cot B$$
$$\Rightarrow 2\cot A\cdot\cot B = (1+\cot A)(1+\cot B)$$
$$\Rightarrow \frac{\cot A}{1+\cot A}\cdot\frac{\cot B}{1+\cot B} = \frac{1}{2}$$

ANS: B

8. We have A + B + C = 180^0

$$\Rightarrow A+B = 180^0 - C$$
$$\Rightarrow \sin(A+B) = \sin(180^0 - C)$$
$$\Rightarrow \sin A\cos B+\cos A\sin B = \sin C$$
$$\Rightarrow \frac{3}{5}\cdot\frac{12}{3}+\frac{4}{5}\cdot\frac{5}{13} = \sin C$$

Since $\sin A = \frac{3}{5}$	$\sin B = \frac{5}{13}$
$\Rightarrow \cos A = \frac{4}{5}$	$\Rightarrow \cos B = \frac{12}{13}$

$$\Rightarrow \frac{36}{65}+\frac{20}{65} = \sin C$$
$$\Rightarrow \frac{56}{65} = \sin C$$

9. Given $\sum\left(\frac{\sin(A+B)\cdot\sin(A-B)}{\sin^2 A\cdot\sin^2 B}\right)$

$$= \sum\left(\frac{\sin^2 A-\sin^2 B}{\sin^2 A\cdot\sin^2 B}\right)$$
$$= \sum\left(\frac{\sin^2 A}{\sin^2 A\cdot\sin^2 B}-\frac{\sin^2 B}{\sin^2 A\cdot\sin^2 B}\right)$$
$$= \sum\left(\cos ec^2 B-\cos ec^2 A\right)$$
$$= \cos ec^2 B-\cos ec^2 A+\cos ec^2 C-\cos ec^2 B+\cos ec^2 A-\cos ec^2 C = 0$$

ANS: B

10. Given $\frac{\cos A}{\cos B} = n$ $\qquad$ $\frac{\sin A}{\sin B} = m$

$\Rightarrow \cos A = n\cos B$ $\qquad$ $\Rightarrow \sin A = m\sin B$

$\Rightarrow \cos^2 A = n^2\cos^2 B$ $\qquad$ $\Rightarrow \sin^2 A = m^2\sin^2 B$

$\therefore \cos^2 A + \sin^2 A = n^2\cos^2 B + m^2\sin^2 B$

$\Rightarrow 1 = n^2\left(1-\sin^2 B\right) + m^2\sin^2 B$

$\Rightarrow 1 = n^2 + \left(m^2 - n^2\right)\sin^2 B$

$\Rightarrow \left(m^2 - n^2\right)\sin^2 B = 1 - n^2$

11. We have $P + Q + R = \pi$

Given $\underline{|R} = \frac{1}{2}$

$$\therefore P + Q = \frac{\pi}{2}$$

$$\Rightarrow \frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4}$$

$$\Rightarrow \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{\tan\frac{P}{2} + \tan\frac{Q}{2}}{1 - \tan\frac{P}{2}\cdot\tan\frac{Q}{2}} = 1$$

$$\Rightarrow \frac{-\frac{b}{a}}{1 - \frac{c}{a}} = 1 \qquad \text{since } \tan\frac{P}{2} + \tan\frac{Q}{2} = -\frac{b}{a}$$

$$\Rightarrow \frac{b}{c-a} = 1 \qquad \tan\frac{P}{2}\cdot\tan\frac{Q}{2} = \frac{c}{a}$$

$$\Rightarrow a + b = c$$

ANS: A

12. Given $\sin^2 52\frac{1}{2}^0 - \sin^2 22\frac{1}{2}^0$

$$= \sin\left(52\frac{1}{2}^0 + 22\frac{1}{2}^0\right)\cdot\sin\left(52\frac{1}{2}^0 - 22\frac{1}{2}^0\right)$$

$$= \sin 75^0 \cdot \sin 30^0$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{4\sqrt{2}} = \frac{\sqrt{3}+1}{4\sqrt{2}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{2}{4\sqrt{2}\left(\sqrt{3}-1\right)} = \frac{1}{2\sqrt{2}\left(\sqrt{3}-1\right)}$$

ANS: A, D

13. Given $\sin(\theta+\alpha)=\cos(\theta+\alpha)$

$$\Rightarrow \tan(\theta+\alpha)=1$$

$$\Rightarrow \frac{\tan\theta+\tan\alpha}{1-\tan\theta\cdot\tan\alpha}=1$$

$$\Rightarrow \tan\theta+\tan\alpha=1-\tan\theta\cdot\tan\alpha$$

$$\Rightarrow \tan\alpha+\tan\theta\cdot\tan\alpha=1-\tan\theta$$

$$\Rightarrow \tan\alpha(1+\tan\theta)=1-\tan\theta$$

$$\Rightarrow \tan\alpha=\frac{1-\tan\theta}{1+\tan\theta}$$

ANS: B

14. Statement -I

Given $\sin\alpha=\frac{a}{\sqrt{1+a^2}}$ | $\cos\beta=\frac{b}{\sqrt{1+b^2}}$

$\Rightarrow \tan\alpha=a$ | $\tan\beta=b$

Now, $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}$

$$=\frac{a+b}{1-ab}$$

Statement - I is FALSE

Statement II

Given $\tan(A+B)=m$ and $\tan(A-B)=n$

Now, $\tan 2B=\tan((A+B)-(A-B))$

$$=\frac{\tan(A+B)-\tan(A-B)}{1+\tan(A+B)\cdot\tan(A-B)}$$

$$=\frac{m-n}{1+mn}$$

$\therefore$ Statement II is FALSE.

15. Statement I:

Given $\sin\alpha=\frac{12}{13}$

$$\Rightarrow \cos\alpha=\frac{5}{13}$$

Since $\alpha\in Q_1$

Given $\cos\beta=-\frac{3}{5}$

$$\Rightarrow \sin\beta=-\frac{4}{5}$$

Since $\beta\in Q_3$

Now, $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

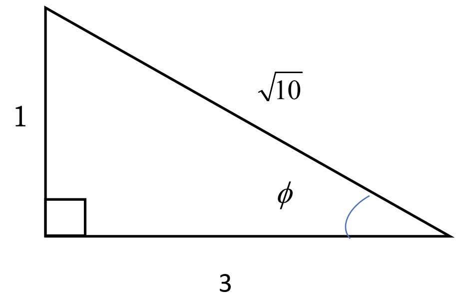
$$=\frac{12}{13}\cdot\left(\frac{-3}{5}\right)+\frac{5}{13}\cdot\left(\frac{-4}{5}\right)$$

$$=\frac{-36}{65}-\frac{20}{65}=-\frac{56}{65}$$

$\therefore$ Statement I is FALSE.

Statement II : Given $\tan\theta=\frac{1}{7}$

And $\sin\phi=\frac{1}{\sqrt{10}}$



$$\Rightarrow \tan\phi=\frac{1}{3}$$

Now, $\tan 2\phi=\frac{2\tan\phi}{1-\tan^2\phi}$

$$=\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}=\frac{\left(\frac{2}{3}\right)}{\left(\frac{8}{9}\right)}=\frac{3}{4}$$

Now, $\tan(\theta+2\phi)=\frac{\tan\theta+\tan 2\phi}{1-\tan\theta\cdot\tan 2\phi}$

$$=\frac{\left(\frac{1}{7}\right)+\left(\frac{3}{4}\right)}{1-\left(\frac{1}{7}\right)\left(\frac{3}{4}\right)}=\frac{25}{25}=1=\tan 45^0$$

$$\therefore \theta+2\phi=45^0$$

$\therefore$ Statement II is TRUE.

16. Statement I

Given $m\cdot\cos(\theta+\alpha)=n\cdot\cos(\theta-\alpha)$

$$\Rightarrow \frac{m}{n}=\frac{\cos(\theta-\alpha)}{\cos(\theta+\alpha)}$$

$$\Rightarrow \frac{m+n}{m-n}=\frac{\cos(\theta-\alpha)+\cos(\theta+\alpha)}{\cos(\theta-\alpha)-\cos(\theta+\alpha)}$$

$$\Rightarrow \frac{m+n}{m-n}=\frac{2\cdot\cos\theta\cdot\cos\alpha}{2\sin\theta\cdot\sin\alpha}$$

$$\Rightarrow \frac{m+n}{m-n}=\cot\theta\cdot\cot\alpha$$

Statement I is FALSE.

Statement II

Given $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\frac{a+b}{a-b}$

$$\Rightarrow \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)}=\frac{(a+b)+(a-b)}{(a+b)-(a-b)}$$

$$\Rightarrow \frac{2\sin\alpha\cdot\cos\beta}{2\cos\alpha\cdot\sin\beta}=\frac{2a}{2b}$$

$$\Rightarrow \frac{\tan\alpha}{\tan\beta}=\frac{a}{b}$$

$$\Rightarrow \tan\alpha\cdot\cot\beta=\frac{a}{b}$$

Statement II is TRUE.

ANS: D

17. $\cos^2\left(\frac{\pi}{44}+x\right)-\sin^2\left(\frac{\pi}{4}-x\right)$

$$=\cos\left(\frac{\pi}{4}+x+\frac{\pi}{4}-x\right)\cdot\cos\left(\frac{\pi}{4}+x-\frac{\pi}{4}+x\right)$$

$$=\cos\frac{\pi}{2}\cdot\cos 2x=0\cdot\cos 2x=0$$

ANS: D

18. $\cos^2\left(\frac{\pi}{6}+\theta\right)-\cos^2\left(\frac{\pi}{6}-\theta\right)$

$$=\sin\left(\frac{\pi}{6}-\theta+\frac{\pi}{6}+\theta\right)\cdot\sin\left(\frac{\pi}{6}-\theta-\frac{\pi}{6}-\theta\right)$$

$$=\sin\left(\frac{\pi}{3}\right)\cdot\sin(-2\theta)$$

$$=\frac{-\sqrt{3}}{2}\cdot\sin 2\theta$$

ANS: B

19. $\cos(A+B)\cdot\cos(A-B)+\sin(A+B)\cdot\sin(A-B)$

$$=\cos^2 A-\sin^2 B+\sin^2 A-\sin^2 B$$
$$=\cos^2 A+\sin^2 A-2\sin^2 B$$
$$=1-2\sin^2 B$$
$$=1-2\left(1-\cos^2 B\right)$$
$$=1-2+2\cos^2 B$$
$$=2\cos^2 B-1$$

ANS: C

20. $\tan 2A=\tan\left((A+B)+(A-B)\right)$

$$=\frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\cdot\tan(A-B)}=\frac{p+q}{1-pq}$$

ANS: C

21. $\cot 2B = \cot\left(\left(A+B\right)-\left(A-B\right)\right)$

$$= \frac{\cot(A+B)\cdot\cot(A-B)+1}{\cot(A-B)-\cot(A+B)}$$

$$= \frac{\left(\frac{1}{p}\right)\left(\frac{1}{q}\right)+1}{\left(\frac{1}{q}\right)-\left(\frac{1}{p}\right)}$$

$$= \frac{1+pq}{p-q}$$

ANS: C

22. $\tan 2A\cdot\cot 2B + \tan 2B\cdot\cot 2A$

$$= \left(\frac{p+q}{1-pq}\right)\cdot\left(\frac{1+pq}{p-q}\right)+\left(\frac{p-q}{1+pq}\right)\cdot\left(\frac{1-pq}{p+q}\right)$$

ANS: D

23. Given $\tan A = \frac{n}{n+1}$ and $\tan B = \frac{1}{2n+1}$

Now, $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\cdot\tan B}$

$$= \frac{\frac{n}{n+1}+\frac{1}{2n+1}}{1-\left(\frac{n}{n+1}\right)\left(\frac{1}{2n+1}\right)} = \frac{2n^2+n+n+1}{2n^2+2n+1}$$

$$= \frac{2n^2+2n+1}{2n^2+2n+1} = 1$$

$\therefore A+B = 45^0$

24. a) Given A + B + C = 180^0

Let A = B = C = 60^0

$$\sum \frac{\cot A + \cot B}{\tan A + \tan B}$$

$$= \sum \frac{\cot 60^0 + \cot 60^0}{\tan 60^0 + \tan 60^0}$$

$$= 3\times\frac{\left(\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}+\sqrt{3}} = 1$$

b) In $\triangle ABC$, we have A + B + C = 180^0

Let A = B = C = 60^0

$$\sum \frac{\cos(B-C)}{\sin B\cdot\sin C}$$

$$=\sum\frac{\cos\left(60^0-60^0\right)}{\sin 60^0\cdot\sin 60^0}=3\times\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}=3\times\frac{4}{3}=4$$

c) In $\triangle ABC$, we have A + B + C = 180^0

Let A = B = C = 60^0

$$\sum\frac{\sin\left(A-B\right)}{\cos A\cdot\cos B}$$

$$=\sum\frac{\sin\left(60^0-60^0\right)}{\cos 60^0\cdot\cos 60^0}=\sum\frac{\sin 0^0}{\cos 60^0\cdot\cos 60^0}=0$$

d) Given A + B + C = 90^0

Let A = B= C = 30^0

Now, $\frac{\cot A+\cot B+\cot C}{\cot A\cdot\cot B\cdot\cot C}$

$$=\frac{\cot 30^0+\cot 30^0+\cot 30^0}{\cot 30^0\cdot\cot 30^0\cdot\cot 30^0}$$

$$=\frac{\sqrt{3}+\sqrt{3}+\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}\cdot\sqrt{3}}=\frac{3\sqrt{3}}{3\sqrt{3}}=1$$

25. a) Given $\frac{\sin\theta}{\sin^2\left(\frac{\pi}{8}+\frac{\theta}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{\theta}{2}\right)}$

$$=\frac{\sin\theta}{\sin\left(\frac{\pi}{2}+\frac{\theta}{2}+\frac{\pi}{8}-\frac{\theta}{2}\right)\cdot\sin\left(\frac{\pi}{8}+\frac{\theta}{2}-\frac{\pi}{8}+\frac{\theta}{2}\right)}$$

$$=\frac{\sin\theta}{\sin\frac{\pi}{4}\cdot\sin\theta}=\sqrt{2}$$

b) Given $\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)=a\rightarrow$ (1)

we know $\tan\left(\frac{\pi}{4}+\theta\right)\cdot\tan\left(\frac{\pi}{4}-\theta\right)$

$$=\frac{\tan^2\frac{\pi}{4}-\tan^2\theta}{1-\tan^2\frac{\pi}{4}\cdot\tan^2\theta}=\frac{1-\tan^2\theta}{1-\tan^2\theta}=1$$

Now, squaring on both sides of (1) gives

$$\tan^2\left(\frac{\pi}{4}+\theta\right)+\tan^2\left(\frac{\pi}{4}-\theta\right)+2\tan\left(\frac{\pi}{4}+\theta\right)\cdot\tan\left(\frac{\pi}{4}-\theta\right)=a^2$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4}+\theta\right)+\tan^2\left(\frac{\pi}{4}-\theta\right)+2=a^2$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4}+\theta\right)+\tan^2\left(\frac{\pi}{4}-\theta\right)=a^2-2$$

c) Given $2\tan A+\cot A=\tan B$

$$\Rightarrow 2\tan A+\frac{1}{\tan A}=\tan B$$

$$\Rightarrow 2\tan^2 A+1=\tan A\cdot\tan B$$

$$\Rightarrow 2\tan^2 A-\tan A\cdot\tan B+1=0 \rightarrow \textcircled{1}$$

Now, $\cot A+2\tan(A-B)$

$$=\frac{1}{\tan A}+2\left(\frac{\tan A-\tan B}{1+\tan A\cdot\tan B}\right)$$

$$=\frac{1+\tan A\cdot\tan B+2\tan^2 A-2\tan A\cdot\tan B}{\tan A(1+\tan A\cdot\tan B)}$$

$$=\frac{2\tan^2 A-\tan A\cdot\tan B+1}{\tan A(1+\tan A\cdot\tan B)}$$

$$=\frac{0}{\tan A(1+\tan A\cdot\tan B)}=0 \qquad \text{(from } \textcircled{1}\text{)}$$

d) Given $A=35^0, B=15^0$ and $C=40^0$

we have $A+B+C=90^0$

$$\Rightarrow A+B=90^0-C$$

$$\Rightarrow \tan(A+B)=\tan(90^0-C)$$

$$\Rightarrow \tan(A+B)=\cot C$$

$$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\cdot\tan B}=\frac{1}{\tan C}$$

$$\Rightarrow \tan A\cdot\tan C+\tan B\cdot\tan C+\tan A\cdot\tan B=1$$

$$\Rightarrow \sum\tan A\cdot\tan B=1$$

LEARNER'S TASK

1. $\sin 40^0\cdot\cos 50^0+\cos 40^0\cdot\sin 50^0$

$$= \sin\left(40^0 + 50^0\right) = \sin 90^0 = 1$$

ANS: A

2. $\cos 70^0 \cdot \cos 20^0 - \sin 70^0 \cdot \sin 20^0$

$= \cos\left(70^0 + 20^0\right) = \cos 90^0 = 0$

ANS: B

3. $\dfrac{\tan 20^0 + \tan 25^0}{1 - \tan 20^0 \cdot \tan 25^0} = \tan\left(20^0 + 25^0\right)$

$= \tan 45^0 = 1$

ANS: A

4. Given $\tan A = \dfrac{1}{2}, \tan B = \dfrac{1}{3}$

Now, $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$$\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \frac{5}{5} = 1 = \tan\frac{\pi}{4}$$

ANS: C

5. $\cos(A+B) - \cos(A-B)$

$$= (\cos A\cos B - \sin A\sin B) - (\cos A\cos B + \sin A\sin B)$$
$$= \cos A\cos B - \sin A\sin B - \cos A\cos B - \sin A\sin B$$
$$= -2\sin A \cdot \sin B$$

ANS: B

6. $\tan\left(\dfrac{\pi}{4} + 45^0\right) = \tan 90^0 = \infty$

ANS: D

7. $\cos(A+B)\cdot\cos(B-A)$

$= (\cos A\cos B - \sin A\sin B)(\cos B\cos A + \sin A\sin B)$

$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$= \left(1-\sin^2 A\right)\cos^2 B - \sin^2 A\left(1-\cos^2 B\right)$

$= \cos^2 B - \sin^2 A\cdot\cos^2 B - \sin^2 A + \sin^2 A\cdot\cos^2 B$

$= \cos^2 B - \sin^2 A$

ANS: A

$\cos^2 45^0 - \sin^2 15^0$

8. $= \cos\left(45^0 + 15^0\right)\cdot\cos\left(45^0 - 15^0\right)$

$= \cos 60^0 \cdot \cos 30^0 = \dfrac{1}{2}\times\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4}$

ANS: C

9. $\cos\left(120^0 + A\right) + \cos\left(120^0 - A\right)$

$$= 2\cos 120^0 \cdot \cos A$$
$$= 2\cos\left(180^0 - 60^0\right)\cdot \cos A$$
$$= -2\cos 60^0 \cdot \cos A$$
$$= -2\left(\frac{1}{2}\right)\cdot \cos A = -\cos A$$

ANS: B

10. $\sin(B+C)\cdot\sin(C-B)$

$$= -\sin(B+C)\cdot\sin(B-C)$$
$$= -\left(\sin^2 B - \sin^2 C\right) = \sin^2 C - \sin^2 B$$

ANS: B

JEE MAINS LEVEL

1. Given $\sin 75^0 \cdot \cos 75^0$

$$= \frac{1}{2}\left(2\sin 75^0 \cos 75^0\right)$$
$$= \frac{1}{2}\left(\sin 2\times 75^0\right) = \frac{1}{2}\sin 150^0$$
$$= \frac{1}{2}\sin\left(180^0 - 30^0\right) = \frac{1}{2}\sin 30^0$$
$$= \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$$

ANS: C

2. Given $\tan^2 75^0 + \cot 75^0$

$$= \left(2+\sqrt{3}\right)^2 + \left(2-\sqrt{3}\right)^2$$
$$= 2\left(2^2 + \left(\sqrt{3}\right)^2\right)$$
$$= 2(4+3) = 14$$

ANS: A

3. Given $\cos A = \frac{3}{5}$ | $\sin B = \frac{5}{13}$

$\Rightarrow \tan A = \frac{4}{3}$ | $\tan B = \frac{5}{12}$

Now, $\dfrac{\tan A + \tan B}{1 - \tan A\cdot\tan B}$

$$= \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\cdot\frac{5}{12}} = \frac{63}{16}$$

ANS: D

4. Given $A+B=45^0$

$$\Rightarrow \tan(A+B)=\tan 45^0$$

$$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\cdot\tan B}=1$$

$$\Rightarrow \tan A+\tan B=1-\tan A\cdot\tan B$$

$$\Rightarrow \tan A+\tan B+\tan A\cdot\tan B=1$$

$$\Rightarrow 1+\tan A+\tan B+\tan A\cdot\tan B=2$$

$$\Rightarrow (1+\tan A)(1+\tan B)=2$$

ANS: B

5. We know $55^0-35^0=20^0$

$$\Rightarrow \tan\left(55^0-35^0\right)=\tan 20^0$$

$$\Rightarrow \frac{\tan 55^0-\tan 35^0}{1+\tan 55\cdot\tan 35}=\tan 20$$

$$\Rightarrow \frac{\tan 55^0-\tan 35^0}{1+\tan 55\cdot\tan\left(90^0-55^0\right)}=\tan 20^0$$

$$\Rightarrow \frac{\tan 55^0-\tan 35^0}{1+\tan 55^0\cdot\cot 55^0}=\tan 20^0$$

$$\Rightarrow \frac{\tan 55^0-\tan 35^0}{1+1}=\tan 20^0$$

$$\Rightarrow \tan 55^0-\tan 35^0=2\tan 20^0$$

$$\Rightarrow \tan 35^0+2\tan 20^0=\tan 55^0=\tan x^0$$

$$\therefore x^0=55^0$$

ANS: C

6. $\cos^2 48^0-\sin^2 12^0$

$$=\cos\left(48^0+12^0\right)\cdot\cos\left(48^0-12^0\right)$$

$$=\cos 60^0\cdot\cos 36^0$$

$$=\frac{1}{2}\cdot\frac{\sqrt{5}+1}{4}=\frac{\sqrt{5}+1}{8}$$

ANS: A

7. Given $x=\tan A-\tan B$

$$\Rightarrow x=\frac{\sin A}{\cos A}-\frac{\sin B}{\cos B}$$

$$\Rightarrow x=\frac{\sin A\cos B-\cos A\sin B}{\cos A\cdot\cos B}$$

$$\Rightarrow x=\frac{\sin(A-B)}{\cos A\cdot\cos B}$$

Given $y=\cot B-\cot A$

$$\Rightarrow y=\frac{\cos B}{\sin B}-\frac{\cos A}{\sin A}$$

$$\Rightarrow y=\frac{\sin A\cos B-\cos A\sin B}{\sin B\cdot\sin A}$$

$$\Rightarrow y=\frac{\sin\left(A-B\right)}{\sin B\cdot\sin A}$$

Now, $\frac{1}{x}+\frac{1}{y}=\frac{\cos A\cdot\cos B}{\sin\left(A-B\right)}+\frac{\sin B\cdot\sin A}{\sin\left(A-B\right)}$

$$=\frac{\cos A\cdot\cos B+\sin A\cdot\sin B}{\sin\left(A-B\right)}$$

$$=\frac{\cos\left(A-B\right)}{\sin\left(A-B\right)}=\cot\left(A-B\right)$$

ANS: A

8. Given $\tan\left(A-B\right)=\frac{7}{24}$

$$\Rightarrow\frac{\tan A-\tan B}{1+\tan A\cdot\tan B}=\frac{7}{24}$$

$$\Rightarrow\frac{\frac{4}{3}-\tan B}{1+\frac{4}{3}\cdot\tan B}=\frac{7}{24}$$

$$\Rightarrow\frac{4-3\tan B}{3+4\tan B}=\frac{7}{24}$$

$$\Rightarrow 96-72\tan B=21+28\tan B$$

$$\Rightarrow 100\tan B=75$$

$$\Rightarrow\tan B=\frac{3}{4}$$

Now, $\tan\left(A+B\right)=\frac{\tan A+\tan B}{1-\tan A\cdot\tan B}$

$$=\frac{\frac{4}{3}+\frac{3}{4}}{1-\frac{4}{3}\cdot\frac{3}{4}}=\frac{\left(\frac{25}{12}\right)}{0}$$

= Not defined

$$\therefore A+B=\frac{\pi}{2}$$

ANS: D

9. Given $\tan A = 1, \tan B = 2, \tan C = 3$

$$\tan(A+B+C) = \frac{(\tan A + \tan B + \tan C) - (\tan A \cdot \tan B \cdot \tan C)}{1 + (\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A)}$$

$$= \frac{(1+2+3)-(1\cdot 2\cdot 3)}{1-(1.2+2.3+3.1)} = \frac{0}{1-11} = \frac{0}{-10} = 0$$

$\therefore A+B+C = \pi$

ANS: B

10. Given $A+B+C = 180^0$

$$\Rightarrow \frac{A}{2}+\frac{B}{2}+\frac{C}{2} = 90^0$$

$$\Rightarrow \frac{A}{2}+\frac{B}{2} = 90^0 - \frac{C}{2}$$

$$\Rightarrow \tan\left(\frac{A}{2}+\frac{B}{2}\right) = \tan\left(90^0 - \frac{C}{2}\right)$$

$$\Rightarrow \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\cdot\tan\frac{B}{2}} = \cot\frac{C}{2}$$

$$\Rightarrow \frac{\frac{5}{6}+\frac{20}{37}}{1-\frac{5}{6}\cdot\frac{20}{37}} = \cot\frac{C}{2}$$

$$\Rightarrow \cot\frac{C}{2} = \frac{305}{122}$$

$$\therefore \tan\frac{C}{2} = \frac{122}{305}$$

ANS: A

11. Given $A+B = \frac{\pi}{4}$

$$\Rightarrow \tan(A+B) = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{\tan A + \tan B}{1-\tan A\cdot\tan B} = 1$$

$$\Rightarrow \tan A + \tan B = 1 - \tan A\cdot\tan B$$

$$\Rightarrow \tan A + \tan B + \tan A\cdot\tan B = 1$$

$$\Rightarrow 1 + \tan A + \tan B + \tan A\cdot\tan B = 2$$

$$\Rightarrow (1+\tan A)(1+\tan B) = 2$$

Also, $\left(1+\frac{1}{\cot A}\right)\left(1+\frac{1}{\cot B}\right) = 2$

$$\Rightarrow (1+\cot A)(1+\cot B) = 2\cot A\cdot\cot B$$

ANS: A, C

12. Given $\frac{\cos 9^0 + \sin 9^0}{\cos 9^0 - \sin 9^0}$

$$= \frac{1+\frac{\sin 9^0}{\cos 9^0}}{1-\frac{\sin 9^0}{\cos 9^0}}$$

$$= \frac{1+\tan 9^0}{1-\tan 9^0}$$

$$= \frac{\tan 45^0 + \tan 9^0}{1-\tan 45^0 \cdot \tan 9^0} = \tan\left(45^0 + 9^0\right)$$

$$= \tan 54^0 = \tan\left(90^0 - 36^0\right) = \cot 36^0$$

ANS: A, C

13. Statement I : Given $\tan A = x \cdot \tan B$

$$\Rightarrow \frac{\tan A}{\tan B} = \frac{x}{1}$$

By componendo and dividendo

$$\Rightarrow \frac{\tan A - \tan B}{\tan A + \tan B} = \frac{x-1}{x+1}$$

$$\Rightarrow \frac{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}} = \frac{x-1}{x+1}$$

$$\Rightarrow \frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B} = \frac{x-1}{x+1}$$

$$\Rightarrow \frac{\sin\left(A-B\right)}{\sin\left(A+B\right)} = \frac{x-1}{x+1}$$

Hence, Statement I is TRUE.

Statement II : Given $\frac{a}{b} = \frac{c}{d}$

By componendo and dividendo

$$\Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Hence, statement II is TRUE.

ANS: A

14. Given $A + B = 45^0$

$$\Rightarrow \cot(A+B)=\cot 45^0$$

$$\Rightarrow \frac{\cot A\cdot\cot B-1}{\cot A+\cot B}=1$$

$$\Rightarrow \cot A\cdot\cot B-1=\cot A+\cot B$$

$$\Rightarrow \cot A\cdot\cot B-\cot A-\cot B=1$$

$$\Rightarrow 1-\cot A-\cot B+\cot A\cdot\cot B=2$$

$$\Rightarrow (\cot A-1)(\cot B-1)=2$$

ANS: C

15. Let $A=22\frac{1}{2}^0$

$$\Rightarrow 2A=45^0$$

$$\Rightarrow \tan 2A=\tan 45^0$$

$$\Rightarrow \frac{2\tan A}{1-\tan^2 A}=1$$

$$\Rightarrow 1-\tan^2 A=2\tan A$$

$$\Rightarrow \tan^2 A+2\tan A-1=0$$

$$\Rightarrow \tan A=\frac{-2\pm\sqrt{4+4}}{2}$$

$$\Rightarrow \tan A=\frac{-2\pm 2\sqrt{2}}{2}$$

$$\Rightarrow \tan A=-1\pm\sqrt{2}$$

$\Rightarrow \tan A=-1+\sqrt{2}$ Since $22\frac{1}{2}^0\in Q_1$

$$\therefore \tan 22\frac{1}{2}^0=\sqrt{2}-1$$

ANS: C

16. Let $A=67\frac{1}{2}^0$

$\Rightarrow 2A = 135^0$

$\Rightarrow Tan2A = Tan135^0$

$\Rightarrow Tan2A = Tan\left(180^0 - 45^0\right)$

$\Rightarrow Tan2A = -Tan45^0$

$\Rightarrow \dfrac{2TanA}{1-Tan^2A} = -1$

$\Rightarrow 2TanA = -1 + Tan^2A$

$\Rightarrow Tan^2A - 2TanA - 1 = 0$

$\Rightarrow TanA = \dfrac{2 \pm \sqrt{4+4}}{2}$

$\Rightarrow TanA = 1 \pm \sqrt{2}$ Since $67\frac{1}{2}^0 \in Q_1$

$\therefore Tan67\frac{1}{2}^0 = \sqrt{2} + 1$

ANS: B

17. We Know $80^0 - 10^0 = 70^0$

$\Rightarrow Tan\left(80^0 - 10^0\right) = Tan70^0$

$\Rightarrow \dfrac{Tan80^0 - Tan10^0}{1 + Tan80^0 \cdot Tan10^0} = Tan70^0$

$\Rightarrow \dfrac{Tan80^0 - Tan10^0}{1 + Tan80^0 \cdot \cot 80^0} = Tan70^0$

$\Rightarrow \dfrac{Tan80^0 - Tan10^0}{1+1} = Tan70^0$

$\Rightarrow \dfrac{Tan80^0 - Tan10^0}{Tan70^0} = 2$

ANS: 2

18. In $\triangle ABC$, We have A + B + C = 180^0

Let A = B = C = 60^0

Now, $\tan\frac{A}{2} \cdot \tan\frac{B}{2} + \tan\frac{B}{2} \cdot \tan\frac{C}{2} + \tan\frac{C}{2} \cdot \tan\frac{A}{2}$

$= Tan30^0 \cdot Tan30^0 + Tan30^0 \cdot Tan30^0 + Tan30^0 \cdot Tan30^0$

$= 3\left(Tan30^0 \cdot Tan30^0\right) = 3\left(\frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}\right) = 1$

ANS: 1

19. a) $\cos\theta + \cos\left(120^0 + \theta\right) + \cos\left(120^0 - \theta\right)$

$= \cos\theta + 2\cos 120^0 \cdot \cos\theta$

$= \cos\theta + 2\cos\left(180^0 - 60^0\right) \cdot \cos\theta$

$= \cos\theta + 2\left(-\cos 60^0\right) \cdot \cos\theta$

$= \cos\theta - 2\frac{1}{2}\cos\theta = 0$

$b) \cos\theta + \cos\left(240^0 + \theta\right) + \cos\left(240^0 - \theta\right)$

$= \cos\theta + 2\cos 240^0 \cdot \cos\theta$

$= \cos\theta + 2\cos\left(180^0 + 60^0\right) \cdot \cos\theta$

$= \cos\theta - 2\cos 60^0 \cdot \cos\theta$

$= \cos\theta - 2\left(\frac{1}{2}\right)\cos\theta = 0$

$c) \cos\theta - \cos\left(60^0 + \theta\right) - \cos\left(60^0 - \theta\right)$

$= \cos\theta - \left[\cos\left(60^0 + \theta\right) + \cos\left(60^0 - \theta\right)\right]$

$= \cos\theta - \left[2\cos 60^0 \cdot \cos\theta\right]$

$= \cos\theta - \left[2\left(\frac{1}{2}\right) \cdot \cos\theta\right] = \cos\theta - \cos\theta = 0$

$d) \sin\theta + \sin\left(120^0 + \theta\right) + \sin\left(\theta - 60^0\right)$

$= \sin\theta + \sin 120^0 \cdot \cos\theta + \cos 120^0 \cdot \sin\theta + \sin\theta \cdot \cos 60^0 - \cos\theta \cdot \sin 60^0$

$= \sin\theta + \frac{\sqrt{3}}{2} \cdot \cos\theta - \frac{1}{2}\sin\theta + \sin\theta\left(\frac{1}{2}\right) - \cos\theta\left(\frac{\sqrt{3}}{2}\right)$

$= \sin\theta$

ANS: $a - t, b - t, c - t, d - s$

20. a) We know, $20^0 + 40^0 = 60^0$

$\Rightarrow \tan\left(20^0 + 40^0\right) = \tan 60^0$

$\Rightarrow \frac{\tan 20^0 + \tan 40^0}{1 - \tan 20^0 \cdot \tan 40^0} = \sqrt{3}$

$\Rightarrow \tan 20^0 + \tan 40^0 = \sqrt{3} - \sqrt{3}\tan 20^0 \cdot \tan 40^0$

$\Rightarrow \tan 20^0 + \tan 40^0 + \sqrt{3}\tan 20^0 \cdot \tan 40^0 = \sqrt{3}$

$b) \tan 75^0 - \tan 30^0 - \tan 75^0 \cdot \tan 30^0$

$= \left(2 + \sqrt{3}\right) - \frac{1}{\sqrt{3}} - \left(2 + \sqrt{3}\right) \cdot \frac{1}{\sqrt{3}}$

$$= 2 + \sqrt{3} - \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} - 1$$

$$= \frac{2\sqrt{3} + 3 - 1 - 2 - \sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} = 1$$

$$c)\ \frac{\cos 13^0 - \sin 13^0}{\cos 13^0 + \sin 13^0} + \frac{1}{\cot 148^0}$$

$$= \frac{1 - \tan 13^0}{1 + \tan 13^0} + \tan 148^0$$

$$= \frac{\tan 45^0 - \tan 13^0}{1 + \tan 45^0 . \tan 13^0} + \tan\left(180^0 - 32^0\right)$$

$$= \tan\left(45^0 - 13^0\right) - \tan 32^0$$

$$= \tan 32^0 - \tan 32^0$$

$$= 0$$

$$d)\ \frac{\tan\alpha + \tan\beta}{\tan(\alpha + \beta)} + \frac{\tan\alpha - \tan\beta}{\tan(\alpha - \beta)}$$

$$= \frac{\tan\alpha + \tan\beta}{\left(\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}\right)} + \frac{\tan\alpha - \tan\beta}{\left(\dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta}\right)}$$

$$= 1 - \tan\alpha \cdot \tan\beta + 1 + \tan\alpha \cdot \tan\beta$$

$$= 2$$

ANS: a – s, b – p, c – r, d – t

ADDITIONAL PRACTICE QUESTIONS FOR STUDENTS

1. $\operatorname{cosec} 15^0 - \sec 15^0$

$$= \sqrt{2}\left(\sqrt{3} + 1\right) - \sqrt{2}\left(\sqrt{3} - 1\right)$$

$$= \sqrt{6} + \sqrt{2} - \sqrt{6} + \sqrt{2} = 2\sqrt{2}$$

ANS: A

2. Given $4x + 5x = 9x$

$$\Rightarrow \tan(4x + 5x) = \tan 9x$$

$$\Rightarrow \frac{\tan 4x + \tan 5x}{1 - \tan 4x \cdot \tan 5x} = \tan 9x$$

$$\Rightarrow \tan 4x + \tan 5x = \tan 9x - \tan 4x \cdot \tan 5x \cdot \tan 9x$$

$$\Rightarrow \tan 4x + \tan 5x - \tan 9x = -\tan 4x \cdot \tan 5x \cdot \tan 9x$$

$$\therefore K = -1$$

ANS: B

3. $\tan\left(\frac{\pi}{4} + \theta\right) \cdot \tan\left(\frac{3\pi}{4} + \theta\right)$

$$=\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\cdot\tan\theta}\right)\left(\frac{\tan\frac{3\pi}{4}+\tan\theta}{1-\tan\frac{3\pi}{4}\cdot\tan\theta}\right)$$

$$=\left(\frac{1+\tan\theta}{1-\tan\theta}\right)\left(\frac{-1+\tan\theta}{1+\tan\theta}\right)$$

$$=\frac{\tan^2\theta-1}{1-\tan^2\theta}=-1$$

ANS: B

4. Given $\cos\alpha+\cos\beta=0=\sin\alpha+\sin\beta$

$$\Rightarrow\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta=0\rightarrow (1)$$

$$\sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta=0\rightarrow (2)$$

Adding both the equations, we get

$$\left(\cos^2\alpha+\sin^2\alpha\right)+\left(\cos^2\beta+\sin^2\beta\right)+2\left(\cos\alpha\cos\beta+\sin\alpha\sin\beta\right)=0$$

$$\Rightarrow 1+1+2\cos\left(\alpha-\beta\right)=0$$

$$\Rightarrow\cos\left(\alpha-\beta\right)=-1$$

ANS: C

5. Given $\frac{\cos\left(A-B\right)}{\cos\left(A+B\right)}+\frac{\cos\left(C+D\right)}{\cos\left(C-D\right)}=0$

$$\Rightarrow\frac{\cos A\cdot\cos B+\sin A\cdot\sin B}{\cos A\cdot\cos B-\sin A\cdot\sin B}+\frac{\cos C\cdot\cos D-\sin C\cdot\sin D}{\cos C\cdot\cos D+\sin C\cdot\sin D}=0$$

$$\Rightarrow\frac{1+\tan A\cdot\tan B}{1-\tan A\cdot\tan B}+\frac{1-\tan C\cdot\tan D}{1+\tan C\cdot\tan D}=0$$

$$\Rightarrow(1+\tan A\cdot\tan B)(1+\tan C\cdot\tan D)+(1-\tan C\cdot\tan D)(1-\tan A\cdot\tan B)=0$$

$$\Rightarrow 1+\tan C\cdot\tan D+\tan A\cdot\tan B+\tan A\cdot\tan B\cdot\tan C\cdot\tan D+1-\tan A\cdot\tan B-\tan C\cdot\tan D+\tan A\cdot\tan B\cdot\tan C\cdot\tan D=0$$

$$\Rightarrow 2+2\cdot\tan A\cdot\tan B\cdot\tan C\cdot\tan D=0$$

$$\Rightarrow\tan A\cdot\tan B\cdot\tan C\cdot\tan D=-1$$

ANS: C