Rectilinear Motion

MECHANIC

The branch of physics in which motion and the forces causing motion are studies is called mechanics. As a first step is studying mechanics, we describe the motion of particles and bodies in terms of space and time, without studying the cause of motion. This part of mechanics is displacement, velocity and acceleration, then using the objects moving under different conditions. The force causing motion will be discussed later in Dynamics. Mechanics is classified under two stream namely statics and dynamics. Dynamics is further divided into kinematics and kinetics.

MECHANICS

- **Statics** (Study of force and their effect on object at rest)
- **Dynamics** (Study of force and their effect on object in motion)
  - **Kinematics** (The word kinematics means 'Science of motion'. Branch of mechanics which deal with study of motion without going into the cause of motion, i.e. force, torque etc.)
  - **Kinetics** (It is branch of mechanics which is concerned about the cause (i.e. the force, torque) that cause motion of bodies.)

REST & MOTION

Motion is a combined property of the object and observer. There is no meaning of rest or motion without the observer. Nothing is in absolute rest or in absolute motion.

An object is said to be in motion with respect to a observer. It may happen by both ways either observer moves or object moves.

Motion is broadly classified into three categories

1. Rectilinear and translatory motion
2. Circular and rotatory motion
3. Oscillatory and vibratory motion

Type of Translation motion

A body in translation motion can move on either a straight -line path or curvilinear path.

1. Rectilinear motion

Translation motion on straight-line path is known as rectilinear translation. It is also known as one-dimensional motion. A car running on a straight road, train running on a straight track and a ball throw vertically upward or dropped from a height etc. are very common examples of rectilinear translation.

2. Curvilinear motion

Translation motion of a body on curvilinear path is known as curvilinear translation. If the trajectory is in a plane, the motion is known as two-dimensional motion. A ball thrown at some angle with the horizontal describes a curvilinear trajectory in a vertical plane; a stone tied to a string when whirled describes a circular path and an insect crawling on the floor or on a wall are examples of two-dimensional motion.

Translation motion
- Rectilinear motion or one dimensional motion
- Curvilinear motion
  - Motion in a plane or Two dimensional motion
  - Motion in space or Three dimensional motion

Reference Frame

Motion of a body can only be observed if it changes its position with respect to some other body. Therefore, for a motion to be observed there must be a body, which is changing its position with respect to other body and a person who is observing motion. The person observing motion is known as observer. The observer for the purpose of investigation must have its own clock to measure time and a point in the space attached with the other body as origin and a set of coordinate axes. These two things the time measured by the clock and the coordinate system are collectively known as reference frame.

In this way, motion of the moving body is expressed in terms of its position coordinates changing with time.

Rectilinear Motion 1- D motion

When a particle is moving along a straight line, then its motion is a rectilinear motion

parameter of rectilinear motion or translatory motion or plane motion :-

(A) Time

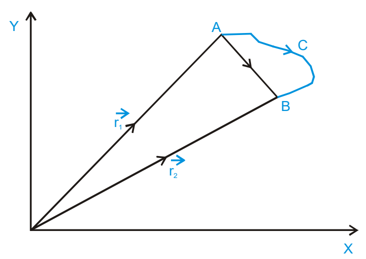
(a) It is a scalar quantity and S.I units second (s)

(b) At a particular instant of time, a physical object can be present at one location only

(c) Time can never decrease.

(B) Position or location

It is defined with respect to some reference point (origin) of given frame of reference. Consider a particle which moves from location $\vec{r}_1$ (at time t_1) to location $\vec{r}_2$ (at time t_2) following path ACB



It is concerned with the question- "where is the particle at a particular moment of time ?"

(C) Displacement :

The change in position vector of the particle for a given time interval is known as its displacement.

$$\overrightarrow{AB} = \vec{r} = \vec{r}_2 - \vec{r}_1$$

(a) Displacement is a vector quantity and its SI unit is meter.

(b) It can decrease can be negative positive or zero.

(D) Distance :

The length of the actual path travelled by a particle during a given time interval is called as distance.

Distance = length of path ACB

(a) Its SI unit is meter and it is a scalar quantity

(b) It can never decrease with time.

KEY POINTS

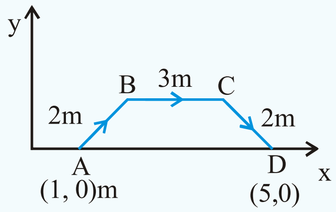
For a moving particle in a given interval of time :-

(i) Displacement can be +ve, –ve or o, but distance would be always +ve

(ii) Distance $\geq$ Magnitude of displacement

(iii) Distance is always equal to displacement only and only if particle is moving along a straight line without any change in direction.

Eg. Suppose a particle moves from position A to B as shown after travelling from A to B to C to D.



Here Displacement $\vec{S} = \overrightarrow{AD} = 5\hat{i} - \hat{i} = 4\hat{i}m$

$\therefore$ |displacement| = 4 m

Also distance covered,

$$l = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right| + \left|\overrightarrow{CD}\right| = 2 + 3 + 2 = 7m$$

(iv) Here |displacement| < Distance

(v) Magnitude of displacement would be equal to distance travelled if there is no change in direction during the whole motion.

In general, |Displacement| $\leq$ Distance

(E) Average Velocity (in an interval) :

The average velocity of a moving particle over a certain time interval is defined as the displacement by the lapsed time.

$$\text{Average Velocity} = \frac{\text{displacement}}{\text{time interval}}$$

for straight line motion, along x-axis, we have

$$v_{av} = \bar{v} = < v > = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The dimension of velocity is $\left[LT^{-1}\right]$ and its SI unit is m/s.

The average velocity is a vector can be taken care of by +ve and –ve sign of the quantity.

(F) Instantaneous Velocity (at an instant) :

The velocity at a particular instant of time is known as instantaneous velocity. The term "velocity" usually means instantaneous velocity.

$$V_{inst.} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

In other words, the intantaneous velocity at a given moment (say, t) is the limiting value of the average velocity as we let Δt approach zero. The limit as $\Delta t \to 0$ is written in calculus notation as dx/dt and is called the derivative of x with respect to t.

(G) Average Speed (in an interval) :-

Average speed is defined as the total path length traveled divided by the total time interval during which the motion has taken place. It helps in describing the motion along the actual path.

$$\text{Average Speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

The dimension of velocity is $\left[LT^{-1}\right]$ and its SI unit is m/s.

For moving particle in a given interval of time

(a) Average speed can be a many valued function but average velocity would be always a single valued function.

(b) Average velocity can be positive, negative or o but average speed would be always positive.

(H) Average Acceleration (in an interval) :

The average acceleration for a finite time interval is defined as :

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

Average acceleration is a vector quantity whose direction is same as that of the change in velocity.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

(I) Instantaneous Acceleration (at an instant) :

The instantaneous acceleration of a particle is its acceleration at a particular instant of time. It is defined as the derivation (rate of change) of velocity with respect to time. We usually mean instantaneous acceleration when we say "acceleration". For straight motion we defined instantaneous acceleration as :

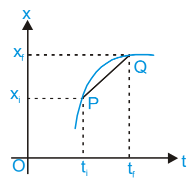
$$a = \frac{dv}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right) \quad \text{and in general} \quad \vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{v}}{\Delta t}\right)$$

The dimension of acceleration is $\left[LT^{-2}\right]$ and its SI unit is m/s^2.

Graphical Interpretation of Some Quantities :-

Average velocity :

If a particle passes a point P $\left(x_i\right)$ at time t = t_i and reaches Q $\left(x_i\right)$ at a later time instant t = t_f, its average velocity in the interval PQ is $V_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$



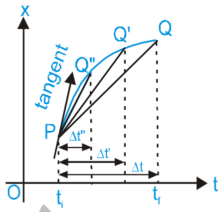
This expression suggests that the average velocity is equal to the slope of the line (chord) joining the points corresponding to P and Q on the x-t graph.

Instantaneous Velocity :

Consider the motion of the particle between the two points P and Q on the x - t graph shown. As the point Q is brought closer and closer to the point P, the time interval between PQ $(\Delta t, \Delta t', \Delta t''......)$ get progressively smaller.

The average velocity for each time interval is the slope of the appropriate dotted line $(PQ, PQ', PQ''......)$.

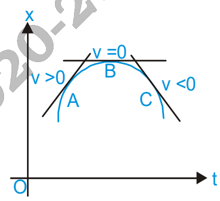
As the point Q approaches P, the time interval approaches zero, but at the same time the slope of the dotted line approaches that of the tangent to the curve at the point P.



As $\Delta t \to 0, V_{av}\left(=\Delta x/\Delta t\right) \to V_{inst}$

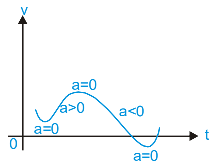
Geometrically, as $\Delta t \to 0$, chord PQ $\to$ tangent at P.

Hence the intantaneous velocity at P is the slope of the tangent at P in the x -t graph. When the slope of the x - t graph is positive, v is positive (as at the point A in figure). At C, v is negative because the tangent has negative slope. The instantaneous velocity at point B (turning point) is zero as the slope is zero.



Instantaneous Acceleration :

The derivation of velocity with respect to time is the slope of the tangent in velocity time (v - t) graph.



Ex. Position of a particle as a function of time is given as $x = 5t^2 + 4t + 3$. Find the velocity and acceleration of the particle at t = 2 s?

Sol. : Velocity; $v = \dfrac{dx}{dt} = 10t + 4$

At $t = 2$ s

$v = 10(2) + 4$

$v = 24$ m/s

Acceleration; $a = \dfrac{d^2x}{dt^2} = 10$

Acceleration is constant, so at t = 2 s $a = 10\ m/s^2$

We may divided Rectilinear motion in the following Situations :-

(1) Motion with constant velocity
(2) Motion with variable velocity
(3) Motion with variable acceleration

(1) Motion with constant velocity :

consider a particle moving along x -axis with uniform velocity u starting from the point x = x; at t = 0

$$V = \frac{dx}{dt} \Rightarrow \int_{x_i}^{x} dx = \int_0^t u dt$$

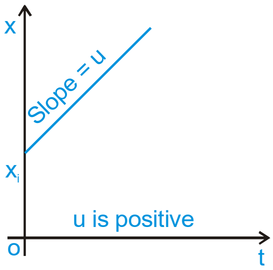
Since velocity is constant, it comes out of the integration

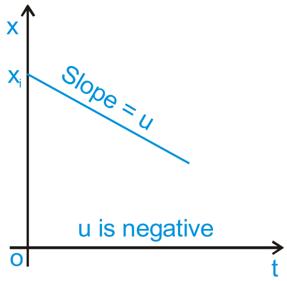
$$\int_{x_i}^{x} dx = \mu \int_0^t dt \qquad \Rightarrow \qquad [x]_{x_i}^{x} = v[t]_0^t$$

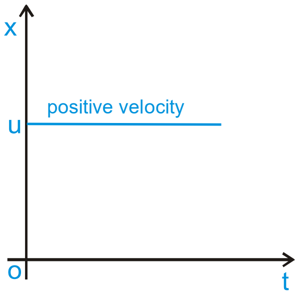
$x - x_i = ut$, displacement $\Delta x = vt$

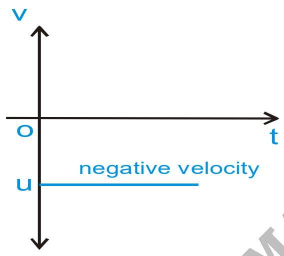
$x = x_i + ut$

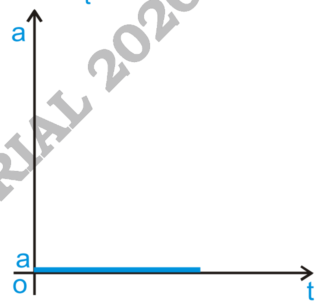
- x -t graph is a straight line slope u through x_i
- as velocity is constant, v - t graph is a horizontal line
- a -t graph coincides with time axis because a = 0 at all time instants











(2) Motion with variable velocity but constant acceleration :

(i) $a = \frac{dv}{dt}$

(ii) $a = \frac{dv}{dx}.\frac{dx}{dt} = \frac{vdv}{dx}$ (By chain rule)

from formula (i)

$a = \frac{dv}{dt} \Rightarrow dv = adt; \int_u^v dv = \int_0^t adt$

Since acceleration is constant so it comes out of the integration

$[v]_v^u = a\int dt$

v - u = at $\Rightarrow$ $\therefore$ v = u + at . ...(i)

$\frac{dx}{dt} = u + at \Rightarrow dx = udt + atdt$ on further integration

$\int_{x_0}^{x} dx = u\int_0^t dt + a\int_0^t tdt \Rightarrow [x]_{x_0}^{x} = ut + \frac{at^2}{2} \Rightarrow x - x_0 = ut + \frac{1}{2}at^2$

$\Delta x = ut + \frac{1}{2}at^2$ ......(ii)

from formula (ii)

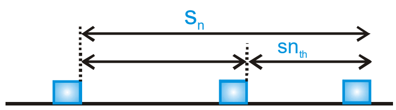
$a = v\frac{dv}{dx} \Rightarrow \int_u^v vdv = a\int_{x_0}^{x} dx \Rightarrow \frac{v^2}{2} - \frac{u^2}{2} = a(x - x_0)$

$$v^2 = u^2 + 2a(\Delta x) \quad ......(iii)$$

Taking $a = v - u$ from equation (i) and putting it in equation (ii), we get

$$\Delta x = ut + \frac{1}{2}\left(\frac{v-u}{t}\right)t^2 \quad \Rightarrow \quad \Delta x = \left(\frac{v+u}{2}\right)t \quad ......(iv)$$

Displacement in n^{th} seconds :



Displacement in n^{th} second = Displacement in n sec - Displacement in (n-1) sec.

$$S_{n^{th}} = S_n - S_{n-1^{th}}$$

$$= \left[u(n) + \frac{1}{2}an^2\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$S_{n^{th}} = u + \frac{a}{2}(2n-1)$$

Motion with variable acceleration :

(i) $\frac{dv}{dt} = a \Rightarrow \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a dt$ **(ii)** $\frac{dx}{dt} = v \Rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v dt$

(iii) $a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$ (By chain rule)

$\therefore \quad a = v\frac{dv}{dx}$ $\therefore \quad \int_{v_1}^{v_2} v dv = \int_{x_1}^{x_2} a dx$

Ex. : The acceleration of a particle is given by $a = 2t^2\ m/s^2$. If it is at rest at the origin at time t = 0, find its position, velocity, and acceleration at time t = 1s.

Sol : $a = 2t^2$

$\therefore \quad a = 2 \times 1^2 = 2\ m/s^2$ (at t = 1 sec)

Formula for v,

$$\frac{dv}{dt} = 2t^2 \quad \text{or} \quad \int_0^v dv = \int_0^t 2t^2 dt \quad \text{or} \quad v = \frac{2t^3}{3}$$

At t = 1 sec, t = 1, v $= \frac{2 \times 1^3}{3} = \frac{2}{3}$ m/s

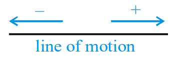
Formula for x, $\frac{dx}{dt} = \frac{2}{3}t^3$

or $\int_0^x dx = \int_0^t \frac{2}{3}t^3 dt$ or, $x = \frac{t^4}{6}$ At t = 1 sec, $x = \frac{1}{6}m$

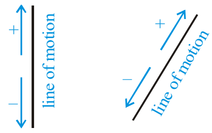
Directions of vectors in straight line motion :-

In straight line motion, all the vectors (position, displacement, velocity & acceleration) will have only one component (along the line of motion) and there will be only two possible directions for each vector.

(i) For example, if a particle is moving in a horizontal line (x-axis), the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number.



(ii) For vertical or inclined motion, upward direction can be taken +ve and down ward as -ve



(iii) For objects moving vertically near the surface of the earth, the only force acting on the particle is its weight (mg) i.e. the gravitational pull of the earth. Hence acceleration for this type of motion will always be a = -g i.e. a = -9.8 m/s^2 (-ve, sign, because the force and acceleration are directed downward, if we select upward direction as positive).

Note :

1. If acceleration is in same direction as velocity, then speed of the particle increases.
2. If acceleration is in opposite direction to the velocity then speed decreases i.e. the particle slow down. This situation is known as retardation.

Ex. A particle moving rectilinearly with constant acceleration is having initial velocity of 10 m/s. After some time, its velocity becomes 30 m/s. Find out velocity of the particle at the mid point of its path?

Sol. : Let the total distance be 2x.

$\therefore$ distance upto midpoint = x

Let the velocity at the mid point be v

and acceleration be a.

From equations of motion

$v^2 = 10^2 + 2ax$ .... (i)

$30^2 = v^2 + 2ax$ .... (ii)

(ii) - (i) gives

$v^2 - 30^2 = 10^2 - v^2$

$\Rightarrow$ $v^2 = 500$ $\Rightarrow$ $v = 10\sqrt{5}$ m/s

Ex. Mr. Sharma brakes his car with constant acceleration from a velocity of 25 m/s to 15 m/s over a distance of 200 m.

(a) How much time elapses during this interval?

(b) What is the acceleration?

(c) If he has to continue braking with the same constant acceleration, how much longer would it take for him to stop and how much additional distance would he cover?

Sol. : **(a)** We select positive direction for our coordinate system to be the direction of the velocity and choose the origin so that $x_i = 0$ when the braking begins. Then the initial velocity is $u_x = +25$ m/s at t = 0, and the $v_x = +15$ m/s and x = 200 m at time t.

Since the acceleration is constant, the average velocity in the interval can be found from the average of the initial and final velocities.

$$\therefore \quad v_{av,x} = \frac{1}{2}(u_x + v_x) = \frac{1}{2}(15 + 25) = 20 \text{ m/s.}$$

The average velocity can also be expressed as

$$v_{av,x} = \frac{\Delta x}{\Delta t}. \text{ With } \Delta x = 200 \text{ m}$$

and $\Delta t = t - 0$, we can solve for t : $t = \frac{\Delta x}{v_{av,x}} = \frac{200}{20} = 10$ s.

(b) We can now find the acceleration using $v_x = u_x + a_x t$

$$a_x = \frac{v_x - u_x}{t} = \frac{15-25}{10} = -1 \text{ m/s}^2.$$

The acceleration is negative, which means that the positive velocity is becoming smaller as brakes are applied (as expected).

(c) Now with known acceleration, we can find the total time for the car to go from velocity $u_x = 25$ m/s to $v_x = 0$. Solving for t, we find

$$t = \frac{v_x - u_x}{a_x} = \frac{0-25}{-1} = 25 \text{ s.}$$

The total distance covered is

$$x = x_i + u_x t + \frac{1}{2}a_x t^2$$

$$= 0 + (25)(25) + \frac{1}{2}(-1)(25)^2 \quad = 625 - 312.5 \quad = 312.5 \text{ m.}$$

Additional distance covered

$$= 312.5 - 200 = 112.5 \text{ m.}$$

Ex. A police inspector in a jeep is chasing a pickpocket an a straight road. The jeep is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance d away, and the motorcycle starts with a constant acceleration a. Show that the pick pocket will be caught if v ³ $\sqrt{2ad}$ ·

Sol : Suppose the pickpocket is caught at a time t after motorcycle starts. The distance travelled by the motorcycle during this interval is

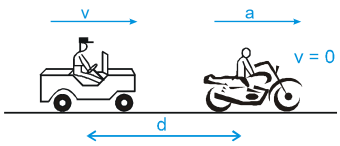
$$s = \frac{1}{2}at^2 \qquad \text{....... (1)}$$

During this interval the jeep travels a distance

$$s + d = vt \qquad \text{....... (2)}$$

By (1) and (2),

$$\frac{1}{2}at^2 + d = vt$$



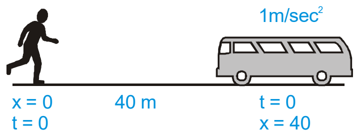
or,

$$t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$$

The pickpocket will be caught if t is real and positive.
This will be possible if

$$v^2 \geq 2ad \qquad \text{or,} \qquad v \geq \sqrt{2ad}$$

Ex. A man is standing 40 m behind the bus. Bus starts with 1 m/sec^2 constant acceleration and also at the same instant the man starts moving with constant speed 9 m/s. Find the time taken by man to catch the bus.



Sol. : Let after time 't' man will catch the bus

For bus

$x = x_0 + ut + \frac{1}{2}at^2$, $\quad x = 40 + 0(t) + \frac{1}{2}(1)t^2$

$x = 40 + \frac{t^2}{2}$ .... (i)

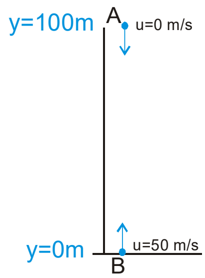
For man, $x = 9t$ .... (ii)

From (i) & (ii)

$40 + \frac{t^2}{2} = 9t$ or $t = 8$ s or $t = 10$s.

Ex. A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 m/s from the ground along the same line. Find out the position where two particle will meet? (take g = 10 m/s^2)

Sol : Let the upward direction is positive.



Let the particles meet at a distance y from the ground.

For particle A,

$y_0 = +100$ m

$u = 0$ m/s

$a = -10$ m/s^2

$y = 100 + 0(t) - \frac{1}{2}\times 10 \times t^2$ $\quad [y = y_0 + ut + \frac{1}{2}at^2]$

$= 100 - 5t^2$ .... (1)

For particle B,

$y_0 = 0$ m $\Rightarrow$ $u = +50$ m/s $\Rightarrow$ $a = -10$ m/s^2

$y = 50(t) - \frac{1}{2}\times 10 \times t^2 = 50t - 5t^2$ .... (2)

According to the problem;

$50t - 5t^2 = 100 - 5t^2$

$t = 2$ sec

Putting t = 2 sec in eqn. (1),

$y = 100 - 20 = 80$ m

Hence, the particles will meet at a height 80 m above the ground.

Ex. A particle is dropped from a tower. It is found that it travels 45 m in the last second of its journey. Find out the height of the tower? (take g = 10 m/s^2)

Sol. :

Let the total time of journey be n seconds.

Using; $s_n = u + \frac{a}{2}(2n-1)$ $\Rightarrow$ $45 = 0 + \frac{10}{2}(2n-1)$ $\Rightarrow$ $n = 5$ sec

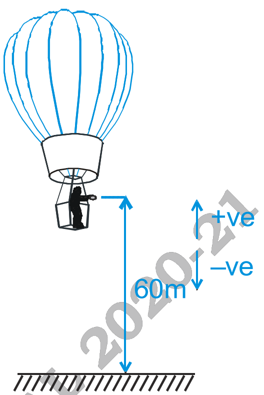
Height of tower; $h = \frac{1}{2}gt^2 = \frac{1}{2}\times 10 \times 5^2 = 125$ m

Reaction time :-

When a situation demands our immediate action. It takes some time before we really respond Reaction time is the time a person takes to observe, think and act.

Ex. A stone is dropped from a balloon going up with a uniform velocity of 5 m/s. If the balloon was 60 m high when the stone was dropped, find its height when the stone hits the ground. Take $g = 10\ m/s^2$.

Sol. $S = ut + \frac{1}{2}at^2$



$-60 = 5(t) + \frac{1}{2}(-10)t^2$

$-60 = 5t - 5t^2$

$5t^2 - 5t - 60 = 0$

$t^2 - t - 12 = 0$

$t^2 - 4t + 3t - 12 = 0$

$(t-4)(t+3) = 0$

$\therefore \quad t = 4$

Height of balloon from ground at this instant

$= 60 + 4 \times 5 = 80$ m

Ex.. A balloon is rising with constant acceleration $2\ m/sec^2$. Two stones are released from the balloon at the interval of 2 sec. Find out the distance between the two stones 1 sec. after the release of second stone.

Sol. Acceleration of balloon = $2\ m/sec^2$

Let at t = 0, y = 0 when the first stone is released.

By the question, $y_1 = 0\,t_1 + \frac{1}{2}gt_1^2$

(taking vertical upward as – ve and downward as + ve)

$\therefore \quad$ Position of I^{st} stone = $\frac{9}{2}g$

(1 second after release of second stone will be the 3^{rd} second for the 1^{st} stone)

For second stone $y_2 = ut_2 + \frac{1}{2}gt_2^2$

$u = 0 + at = -2 \times 2 = -4$m/s (taking vertical upward as – ve and downward as + ve)

$\therefore \quad y_2 = -4 \times 1 + \frac{1}{2}g \times (1)^2 \ (t_2 = 1 \text{ second})$

The 2^{nd} stone is released after 2 second

$\therefore \quad y = -\frac{1}{2}at^2 = -\frac{1}{2} \times 2 \times 4 = -4$

So, Position of second stone from the origin $= -4 + \frac{1}{2}g - 4$

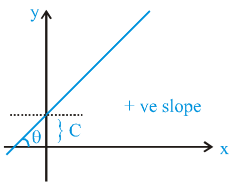
Distance between two stones $= \frac{1}{2}g \times 9 - \frac{1}{2}g \times 1 + 8 = 48$ m.

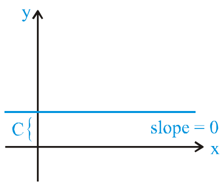
Straight Line-Equation, Graph Slope (+ve, -ve zero slope).

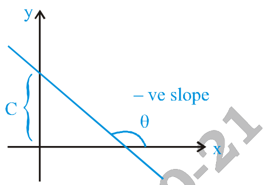
If θ is the angle at which a straight line is inclined to the positive direction of x-axis, & $0^o \le \theta < 180^o, \theta \ne 90^o$, then the slope of the line, denoted by m, is defined by m = tan θ. If θ is 90^o, m does not exist, but the line is parallel to the y-axis. If θ = 0, then m = 0 & the line is parallel to the x-axis.

Slope - Intercept form : y = mx + c is the equation of a straight line whose slope is m & which makes an intercept c on the y-axis.

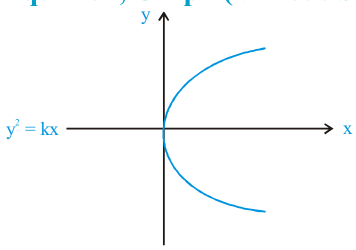
$$m = \text{slope} = \tan\theta = \frac{dy}{dx}$$

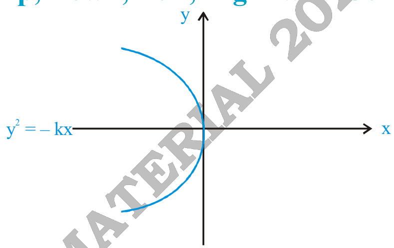


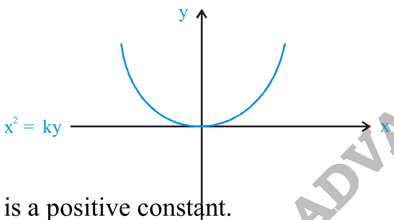


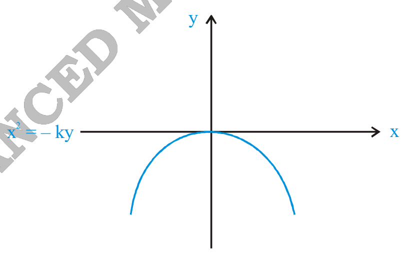


Parabolic Curve-Equation, Graph (Various Situation Ip, Down, Left, Right with Conditions) :









Where k is a positive constant.

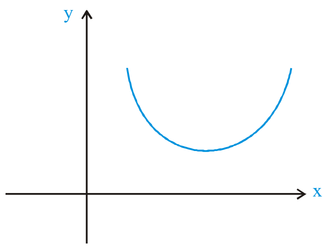
Equation of parabola :

Case (i) : $y = ax^2 + bx + c$

For a > 0

The nature of the parabola will be like that of the of nature x^2 = ky

Minimum value of y exists at the vertex of the parabola.

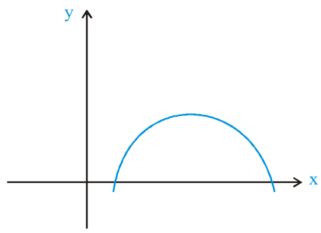


$$y_{\min} = \frac{-D}{4a} \text{ where } D = b^2 - 4ac$$

Coordinates of vertex = $\left(\frac{-b}{2a}, \frac{D}{4a}\right)$

Case (ii) : a < 0

The nature of the parabola will be like that of the nature of $x^2 = -ky$

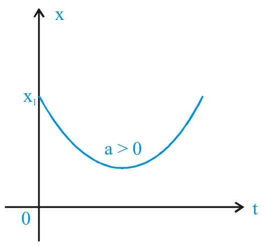


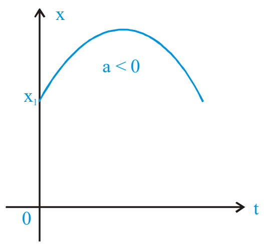
Maximum value of y exists at the vertex of the parabola

$$y_{\max} = \frac{D}{4a} \text{ where } D = b^2 - 4ac$$

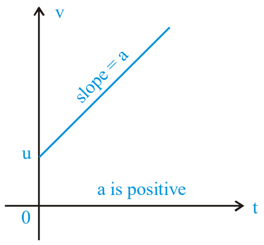
Graph in Uniformly Accelerated Motion $(a \neq 0)$

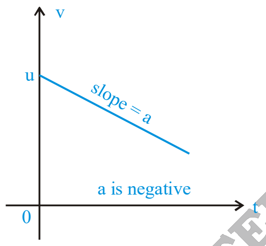
(a) x is a quadratic polynomial in terms of t. Hence x-t graph is a parabola.



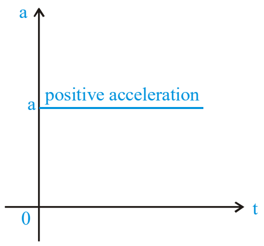


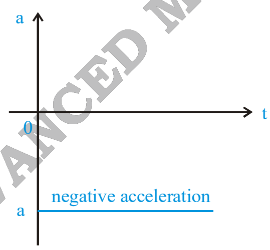
(b) v is a linear polynomial in term of t. Hence v -t graph is a straight line of slope a.





(c) a -t graph is a horizontal line because a is constant





INTERPRETATION OF SOME MORE GRAPHS

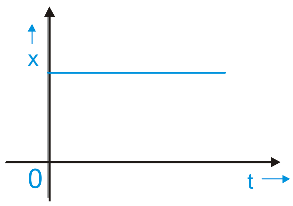
Position vs Time graph

(i) Zero Velocity

As position of particle is fixed at all the time, so the body is at rest.

Slope; $\frac{dx}{dt} = \tan\theta = \tan 0^\circ = 0$

Velocity of particle is zero

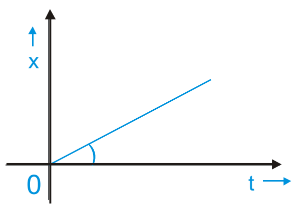


(ii) Uniform Velocity

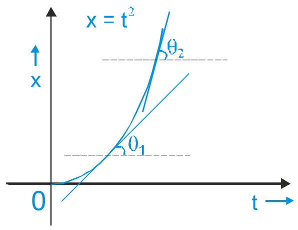
Here $\tan\theta$ is constant $\quad \tan\theta = \frac{dx}{dt}$

$\therefore \quad \frac{dx}{dt}$ is constant.

$\therefore$ velocity of particle is constant.



(iii) Non uniform velocity (increasing with time)



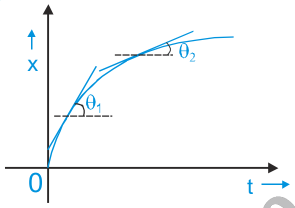
In this case;

As time is increasing, θ is also increasing.

$\therefore \quad \frac{dx}{dt} = \tan\theta$ is also increasing

Hence, velocity of particle is increasing.

(iv) Non uniform velocity (decreasing with time)



In this case;

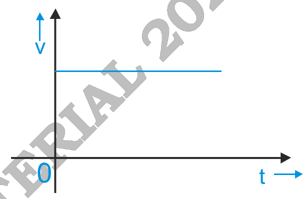
As time increases, θ decreases.

$\therefore \quad \frac{dx}{dt} = \tan\theta$ also decreases.

Hence, velocity of particle is decreasing.

VELOCITY VS TIME GRAPH

(i) Zero acceleration



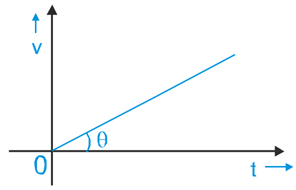
Velocity is constant.

$\tan\theta = 0$

$\therefore \quad \frac{dv}{dt} = 0$

Hence, acceleration is zero.

(ii) Uniform acceleration

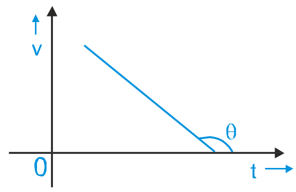


$\tan\theta$ is constant.

$\therefore \quad \frac{dv}{dt} = \text{constant}$

Hence, it shows constant acceleration.

(iii) Uniform retardation



Since $\theta > 90^\circ$

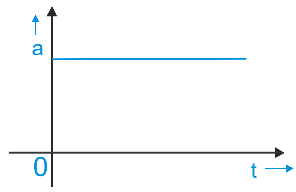
$\therefore$ $\tan\theta$ is constant and negative.

$\therefore \quad \frac{dv}{dt} = \text{negative constant}$

Hence, it shows constant retardation.

ACCELERATION VS TIME GRAPH

(i) Constant acceleration

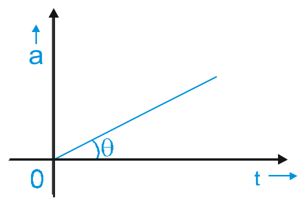


$\tan\theta = 0$

$\therefore \quad \frac{da}{dt} = 0$

Hence, acceleration is constant.

(ii) Uniformly increasing acceleration



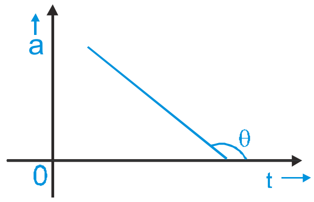
θ is constant.

$0^\circ < \theta < 90^\circ \Rightarrow \tan\theta > 0$

$\Rightarrow \frac{da}{dt} = \tan\theta = \text{constant} > 0$

Hence, acceleration is uniformly increasing with time.

(iii) Uniformly decreasing acceleration

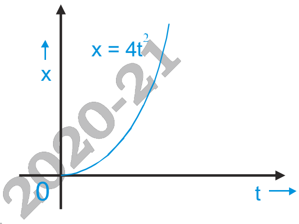


Since $\theta > 90°$

$\therefore$ $\tan\theta$ is constant and negative.

$\therefore \frac{da}{dt} = \text{negative constant}$

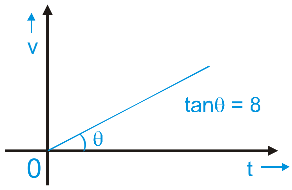
Hence, acceleration is uniformly decreasing with time



Ex. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.

Sol. : $x = 4t^2$

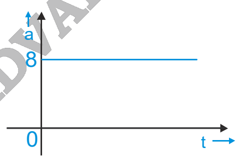
$v = \frac{dx}{dt} = 8t$



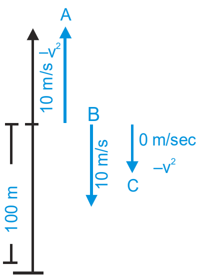
Hence, velocity-time graph is a straight line having slope i.e. $\tan\theta = 8$.

$a = \frac{dv}{dt} = 8$

Hence, acceleration is constant throughout and is equal to 8.



Ex. At the height of 100 m, a particle A is thrown up with V = 10 m/s, B particle is thrown down with V =10m/s and C particle released with V = 0 m/s. Draw graphs of each particle.



(i) Displacement–time

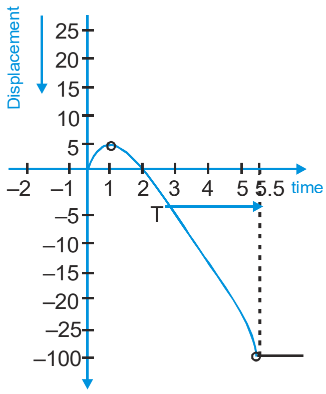
(ii) Speed–time

(iii) Velocity–time

(iv) Acceleration–time

Sol. : For particle A :

(i) Displacement vs time graph is



$y = ut + \frac{1}{2}at^2$

$u = +10 \text{ m/sec}^2$

$y = 10t - \frac{1}{2} \times 10t^2$

$= 10t - 5t^2$

$v = \frac{dy}{dt} = 10 - 10\,t = 0$

$t = 1$; hence, velocity is zero at $t = 1$

$10\,t - 5\,t^2 = -100$

$t^2 - 2t - 20 = 0$

$t = 5.5$ sec.

i.e. particle travels up till 5.5 seconds.

(ii) Speed vs time graph :

Particle has constant acceleration = g ↓ throughout the motion, so v-t curve will be straight line.

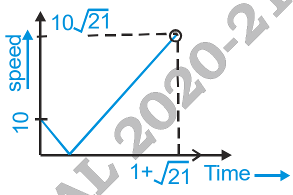
when moving up, $v = u + at$

$0 = 10 - 10\,t$ or $t = 1$ is the time at which speed is zero.

there after speed increases at constant rate of 10 m/s^2 .

Resulting Graph is : (speed is always positive).
This shows that particle travels till a time of

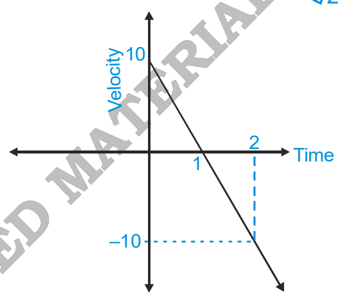
$1 + \sqrt{21}$ seconds



(iii) Velocity vs time graph :

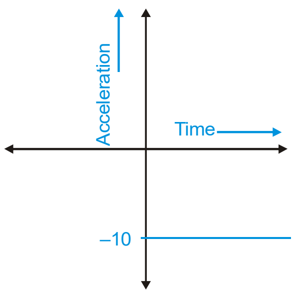
$V = u + at$

$V = 10 - 10t$; this shows that velocity becomes zero at $t = 1$ sec
and thereafter the velocity is negative with slope g.



(iv) Acceleration vs time graph :

throughout the motion, particle has constant acceleration = –10 m/s^2.



For particle B :

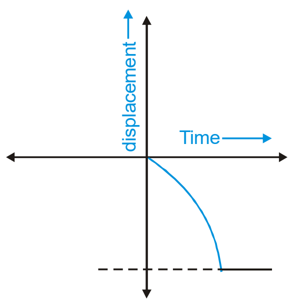
$u = -10$ m/s. $y = -10t - \frac{1}{2}(10)\,t^2 = -10t - 5t^2$

(i) Displacement time graph :

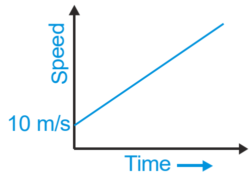
$y = 10t - 5t^2$

$\frac{dy}{dt} = -10t - 5t^2 = -10 - 10t$

this shows that slope becomes more negative with time.



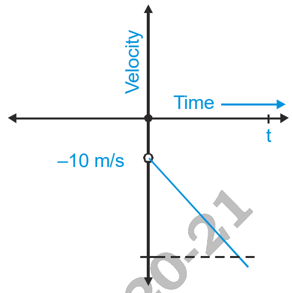
(ii) **Speed time graph :**



$$\frac{dy}{dt} = -10t - 5t^2 = -10 - 10t$$

hence, speed is directly proportional to time with slope of 10. initial speed = 10 m/s

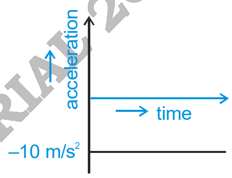
(iii) **Velocity time graph :**



$$\frac{dy}{dt} = -10t - 5t^2 = -10 - 10t$$

hence, velocity is directly proportional to time with slope of –10. Initial velocity = –10 m/s

(iv) **Acceleration vs time graph :**

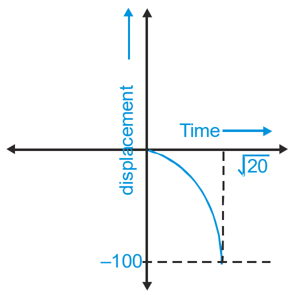


throughout the motion,
particle has constant acceleration = –10 m/s^2.

$$a = \frac{dv}{dt} = -10$$

For Particle C :

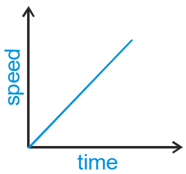
(i) **Displacement time graph :**



$$u = 0 \quad , y = -\frac{1}{2} \times 10t^2 = -5t^2$$

this shows that slope becomes more negative with time.

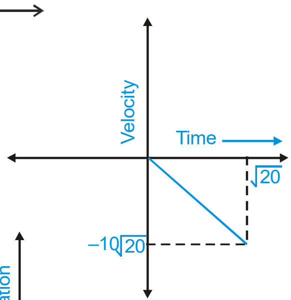
(ii) **Speed vs time graph :**



$$v = \frac{dy}{dt} = -10\,t$$

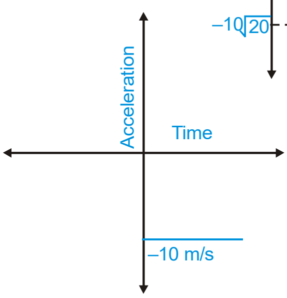
hence, speed is directly proportional to time with slope of 10.

(iii) **Velocity time graph :**



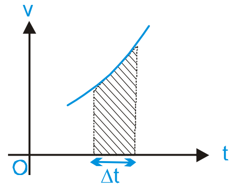
V = u + at
V = – 10t ;
hence, velocity is directly proportional to time with slope of –10.

(iv) **Acceleration vs time graph :**



throughout the motion,
particle has constant acceleration = –10 m/s^2.

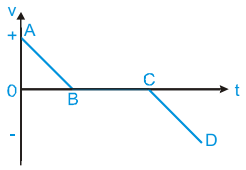
Displacement from v - t graph & change in velocity from a -t graph



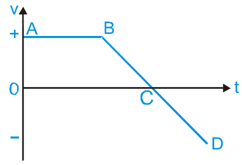
Displacement = Δx = area under v–t graph.
Since a negative velocity causes a negative displacement, areas below the time axis are taken negative. In similar way, can see that $\Delta v = a \Delta t$ leads to the conclusion that **area under a – t graph gives the change in velocity Δv during that interval.**

Ex. Describe the motion shown by the following velocity-time graphs.

(a)



(b)



Sol : (a) **During interval AB:** velocity is +ve so the particle is moving in +ve direction, but it is slowing down as acceleration (slope of v-t curve) is negative. **During interval BC:** particle remains at rest as velocity is zero. Acceleration is also zero. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

(b) **During interval AB:** particle is moving in +ve direction with constant velocity and acceleration is zero. **During interval BC:** particle is moving in +ve direction as velocity is +ve, but it slows down until it comes to rest as acceleration is negative. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

KEY POINTS

(i) For uniformly accelerated motion ($a \neq 0$), x–t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.

(ii) For uniformly accelerated motion ($a \neq 0$), v–t graph is a straight line whose slope gives the acceleration of the particle.

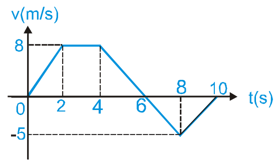
(iii) In general, the slope of tangent in x–t graph is velocity and the slope of tangent in v–t graph is the acceleration.

(iv) The area under a–t graph gives the change in velocity.

(v) The area between the v–t graph gives the distance travelled by the particle, if we take all areas as positive.

(vi) Area under v–t graph gives displacement, if areas below the t–axis are taken negative.

Ex. For a particle moving along x-axis, velocity-time graph is as shown infigure. Find the distance travelled and displacement of the particle?



Sol. : Distance travelled = Area under v-t graph (taking all areas as +ve.)
Distance travelled = Area of trapezium + Area of triangle

$$= \frac{1}{2}(2+6)\times 8 + \frac{1}{2}\times 4\times 5$$

= 32 + 10 = 42 m

Displacement = Area under v-t graph (taking areas below time axis as –ve.)

Displacement = Area of trapezium – Area of triangle

$= \frac{1}{2}(2+6)\times 8 - \frac{1}{2}\times 4\times 5$

= 32 – 10

= 22 m

Hence, distance travelled = 42 m and displacement = 22 m.

Vertical motion under gravity (free fall) :

Motion that occurs safely under the influences of gravity is called free fall, thus a body projected upward or downward or released from rest are all under free fall.

In the absence of air resistance all falling bodies have the same acceleration due to gravity, regardless of their sizes or shapes.

The value of the acceleration due to gravity depends on both latitude and altitude. It is approximately 9.8 m/s^2 near the surface of the earth. For simplicity a value of 10 m/s^2 is used.

Sign Convention :

To do calculation regarding motion under gravity, we follow a proper sign convention. We are taking upward direction as positive and downward as negative, thus acceleration is taken $a = -g = 10\, m/s^2$ no matter whether body is moving upward or downwards, since always acts downward.

Thus the equation of kinematics may be modified as

$v = u - gt$ ....(i)

$\Delta y = y - y_0 = ut - \frac{1}{2}gt^2$ ....(ii)

$v^2 = u^2 - 2g(y - y_0)$ ....(iii)

These y_0 = position of particle at time t = 0

y = position of particle at time t

u = velocity of particle at time t = 0

v = velocity of particle at time t

Some results

1. Maximum Height : - $H = \frac{u^2}{2g}$

Derivation : At maximum height v = 0

$\therefore$ from equation (iii), $v^2 - u^2 - 2gH = 0 \Rightarrow H = \frac{u^2}{2g}$

2. Time to reach maximum height : - $t = \frac{u}{g}$

Derivation : At maximum height v = 0 = u – gt [equation (i)]

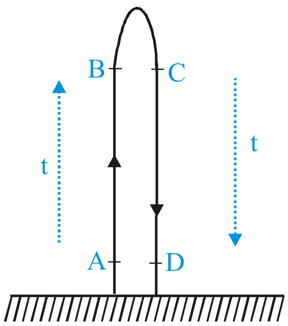
$\therefore$ $t = \frac{u}{g}$

3. Total time of flight = time to go up + time to move down (to reach the same horizontal level again)

T = 2t

$T = \frac{2u}{g}$

4. Time of ascent = Time of descent for motion between two points at same horizontal level for example between A & B and between C & D shown in the figure.



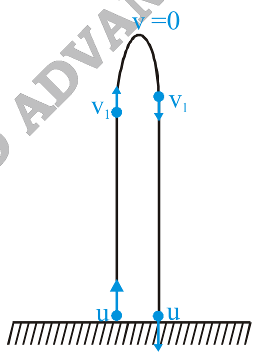
5. If an object is dropped (means initial velocity is zero) from Height h. Its speed on reaching ground is $v = \sqrt{2gh}$ and time taken to reach ground is $t = \sqrt{\frac{2h}{g}}$.

Derivation : From equation (iii) $0 - 2g(-h) = v^2$ [$\therefore \Delta y = -h$]

Also from equation (ii) $\Delta y = -h = 0 - \frac{1}{2}gt^2$

$\therefore$ $t = \sqrt{\frac{2h}{g}}$

6. A particle has the same special at a point on the path. While going vertically up and down.



Ex. A man is standing on the top of a building, throws a ball with speed 5 m/s from 30 height above the ground kevel. How much time it takes to reach the ground.

Sol. u = 5m/s

When it reaches the ground, $\Delta y = -30m$

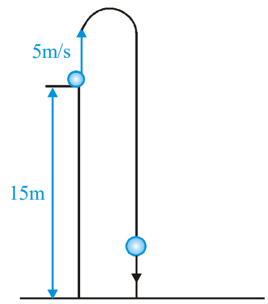
$\therefore$ from equation (ii)

$$-30 = 5t - \frac{1}{2}(10)t^2$$

$\Rightarrow$ $t^2 - t - 6 = 0$

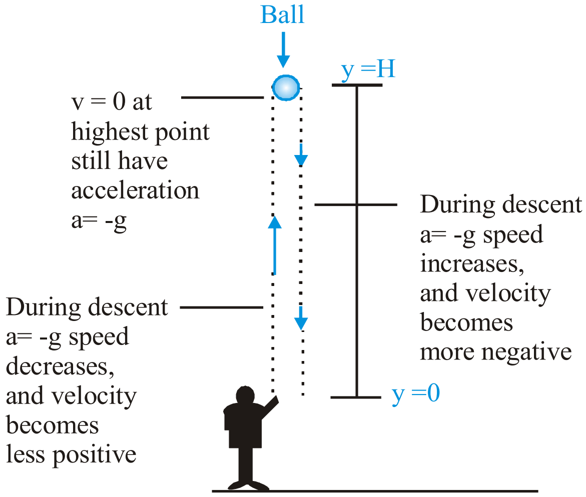
On solving, we get t = 3 & –2

Rejecting t = –2 sec, we get t = 3 sec



Ex. A kid throws a ball up, with some initial speed. Comment on magnitudes and signs of acceleration and velocity of the ball.

Sol.



Here : (i) During ascent, a = –g, velocity becomes less positive i.e., speed decreases
(ii) During descend, a = –g, but now it is in the direction of velocity so it is not retardation. It makes velocity becomes more negative i.e. increases v in negative direction.

Ex. A ball is released from the top of a building. It travels 25 m in last second of its motion before striking the ground. Find height of the building. take $g = 10\ m/s^2$.

Sol. Let it takes 't' time to strike the ground.

$|\Delta y$ in t sec$| - |\Delta y$ in (t–1) sec$| = 25$

$$\frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2 = 25$$

on solving, t = 3 sec

$\therefore$ height of the building, $h = \frac{1}{2}g(3)^2$

h = 45 m

Ex. A balloon is moving up with an acceleration $a_0 = 4\ m/s^2$ starting from rest. A coin is dropped from the balloon 5 sec after the start balloon. Find :

(a) The initial velocity of the dropped coin.

(b) The height attained by the lift till the time of drop.

(c) The time after the drop when the coin reaches ground.

Sol. Till t = 5 sec, the coin shares the same motion as that of the balloon and for 1 > 5 sec (after release) the coin has motion under gravity only.

(a) Velocity of the coin just after it is dripped

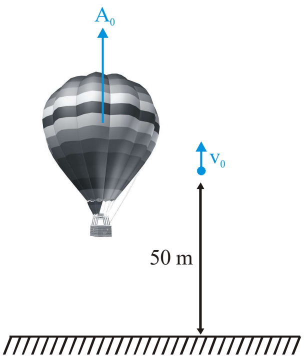
V_0 = velocity of the lift at 5 sec

$= 0 + a_0(5) = 20$ m/s

(b) The height attained by the lift till 5 sec.

$$h = \frac{1}{2}a_0(5)^2 = 50\ m$$

(c) Let it takes t_0 time to reach the ground after the drop i.e. for the time t, its displacement is 50 m in downward direction.



$$\therefore \quad \Delta y = -50 = 20t_0 - \frac{1}{2}gt_0^2$$

on solving, $t_0 = 5.74$ sec.

MOTION IN TWO DIMENSIONS :

Whatever we have studies in kinematics of one dimensional motion, we apply the same for motion in two and three dimensional motion, for x, y, and Z components separately.

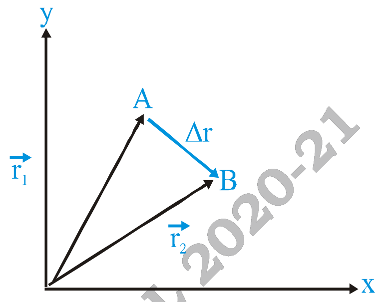
Suppose a particle has position coordinates (x, y) at any instant, then its position vector is given by,

$$\vec{r} = x\hat{i} + y\hat{j}$$

If particle moves from point A to B, through any path, then its displacement is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$= \Delta x\hat{i} + \Delta y\hat{j}$$



Now at any instant, its velocity is given by

$$\vec{V} = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j}$$

i.e., $V_x = \frac{dx}{dt}$ i.e., x -component of velocity

and $V_y = \frac{dy}{dt}$ i.e., y-component of velocity

Similarly $\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j}$

where $a_x = \frac{dv_x}{dt}$ & $a_y = \frac{dv_y}{dt}$