WS-18 For 6th class

Tbasic

(1) At $t_1 = 0^\circ C \Rightarrow d_1 = 10\ gm/cm^3$

$t_2 = 100^\circ C$; $d_2 = 9.4\ gm/cm^3$

$d_1 = d_2[1 + \gamma(t_2 - t_1)]$

$\Rightarrow 90 = 9.4[1 + \gamma(100 - 0)]$

$\Rightarrow 90 - 9.4 = 9.4 \gamma 100$

$\Rightarrow 0.6 = 940\gamma$

$\Rightarrow \gamma = \frac{1}{940}$

$\Rightarrow 3\alpha = \frac{1}{940}$

$\Rightarrow \alpha = \frac{1}{3 \times 940}$

$= 2.12 \times 10^{-4}/^\circ C$

(2) at $t_1 = 0^\circ C$; $d_1 = 9\ gm/cc$

$t_2 = 20^\circ C$. $d_2 = ?$

$\alpha = 17 \times 10^{-6}/^\circ C$

$\because \gamma = \frac{d_1 - d_2}{d_2(t_2 - t_1)}$

$\Rightarrow 3\alpha = \frac{9 - d_2}{d_2(20 - 0)}$

$\Rightarrow 3 \times 17 \times 10^{-6} = \frac{9 - d_2}{20 d_2}$

$\Rightarrow 20 d_2 \times 51 \times 10^{-6} = 9 - d_2$

$\Rightarrow 1.01 d_2 = 9$

$d_2 = \frac{9}{1.01} = 8.99\ gm/cc$

(3) at $t_1 = 0^\circ C \rightarrow d_1 = ?$

$t_2 = 800^\circ C \rightarrow d_2 = 18.06\ gm/cc$

$\alpha = 14 \times 10^{-6}/^\circ C$; $\gamma = 3\alpha$

$\gamma = \frac{d_1 - d_2}{d_2(t_2 - t_1)}/^\circ C$

$\Rightarrow d_1 = d_2[1 + \gamma(t_2 - t_1)]$

$= 18.06[1 + 42(800 - 0) \times 10^{-6}$

$\Rightarrow d_1 = 18.06[1 + 42 \times 800 \times 10^{-6}]$

$\Rightarrow d_1 = 18.6 \times 1.0336$

$= 19.39\ gm/cc$

(4) at $t_1 = 0^\circ C$; $a_1 = 100 cm^2$

$t_2 = 100^\circ C$; $\gamma = 6 \times 10^{-5} /^\circ C$

$\gamma = \frac{3}{2}\beta$; $\Delta a = ?$

$\beta = \frac{2}{3}\gamma$

$\Rightarrow \frac{\Delta a}{a \Delta t} /^\circ C = \frac{2}{3}\gamma$

$\Rightarrow \Delta a = \frac{2}{3}\gamma a \Delta t$

$= \frac{2}{3} \times 6 \times 10^{-5} \times 100 (100 - 0)$

$= 4 \times 10^{-5} \times 100 \times 100$

$= 0.4 cm^2$

(5) Given $\frac{\Delta a}{a} = 0.08\%$

$= 8 \times 10^{-4}$;

$\Delta t = 80^\circ C$

$\beta = \frac{\Delta a}{a \Delta t}$

$\beta = \frac{8 \times 10^{-4}}{80} = 10^{-5} /^\circ C$

$\gamma = \frac{3}{2}\beta$

$= \frac{3}{2} \times 10^{-5}$

$= 1.5 \times 10^{-5} /^\circ C$

$= 15 \times 10^{-6} /^\circ C$

(6) $\alpha = \frac{\Delta l}{l \Delta t} = \frac{\Delta D}{D \times \Delta t}$

Given $\alpha_{Al} = 25 \times 10^{-6} /^\circ C$

$\alpha_s = 10 \times 10^{-6} /^\circ C$

$(\alpha_{Al} - \alpha_s) = \frac{\Delta D}{D \times \Delta t}$

$\Rightarrow 15 \times 10^{-6} = \frac{10^{-3}}{1 \cdot \Delta t}$

$\Rightarrow \Delta t = \frac{1000}{15} = 66.6^\circ C$

(7) at $t_1 = 20^\circ C$; $l = 1m$

$t_2 = 50^\circ C$; $\Delta l = 10^{-3} m$

If $\Delta l = 10^{-3} m$; $t' = ?$

From $\alpha = \frac{\Delta l}{l(t_2 - t_1)}$

$\Rightarrow \alpha = \frac{10^{-3}}{1(50-20)} = \frac{10^{-3}}{30}$ $\rightarrow$ (1)

Also $\alpha = \frac{\Delta l}{l(t' - t)}$ $\rightarrow$ (2)

$\Rightarrow$ From (1) & (2)

$\frac{10^{-3}}{30} = \frac{\Delta l}{l(t' - t)}$

$\Rightarrow \frac{10^{-3}}{30} = \frac{-10^{-3}}{t' - 20}$

$\Rightarrow t' - 20 = -30$

$\Rightarrow t' = -10^\circ C$

(8) Given $l = 10m$; $\Delta l = ?$

$\Delta t = 15^\circ C$; $\alpha = 12 \times 10^{-6} /^\circ C$

From $\alpha = \frac{\Delta l}{l \cdot \Delta t}$

$\Rightarrow \Delta l = \alpha l \Delta t$

$= 12 \times 10^{-6} \times 10 \times 15$

$\Rightarrow 180 \times 10 \times 10^{-6}$

$= 0.0018 m$

(9) Given $t_1 = 20^\circ C$; $l_1 = 10m$

$t_2 = 40^\circ C$ $\Delta l = 0.22 cm$

$= 22 \times 10^{-4} m$

$\alpha = \frac{\Delta l}{l \Delta t} = \frac{22 \times 10^{-4}}{10(40-20)}$

$\alpha = \frac{22 \times 10^{-4}}{200} = 11 \times 10^{-6} /^\circ C$

(10) $t_1 = 20^\circ C$; $D_1 = 2cm$

$t_2 = 120^\circ C$; $D_2 = ?$

We know that

$\Rightarrow D_2 = D_1[1 + \alpha \Delta t]$

$\Rightarrow = 2[1 + 23 \times 10^{-6}(120-20)]$

$= 2[1 + 23 \times 10^{-4}]$

$= 2[1 + 0.0023]$

$= 2(1.0023)$

(11) Given $l = 200m$; $t_1 = 10^\circ C$

$t_2 = 40^\circ C$, $\alpha = 11 \times 10^{-6} /^\circ C$

$\alpha = \frac{\Delta l}{l \Delta t} /^\circ C$

$\Delta l = \alpha l \Delta t$

$= 11 \times 10^{-6} \times 200 \times (40-10)$

$= 11 \times 10^{-3} \times 2 \times 3$

$= 66 \times 10^{-3} m$

$\Delta l = 0.066 m$

(17) Given $t_1 = 0^\circ C \rightarrow d_1 = 10\ gm/cc$

$t_2 = 100^\circ C \rightarrow d_2 = 9.7\ gm/cc$

$$\gamma = \frac{d_1 - d_2}{d_2(t_2 - t_1)}$$

$$\Rightarrow \gamma = \frac{10 - 9.7}{9.7(100 - 0)}$$

$$\Rightarrow 3\alpha = \frac{0.3}{9.7 \times 100}$$

$$\Rightarrow \alpha = 3 \times 10^{-4}/^\circ C$$

(18) Given $\alpha = 2 \times 10^{-6}/^\circ C$, $l = 10\ m$, $t_1 = 20^\circ C$

(i) $\Delta l = -1\ mm = -10^{-3}\ m$, $t_2 = ?$

$$\Rightarrow \alpha = \frac{\Delta l}{l \Delta t}$$

$$\Rightarrow \Delta t = \frac{\Delta l}{l \alpha}$$

$$\Rightarrow \Delta t = \frac{-10^{-3}}{10 \times 2 \times 10^{-6}}$$

$$\Rightarrow \Delta t = -\frac{1}{20} \times 10^3$$

$$\Rightarrow \Delta t = -50^\circ C$$

$$\Rightarrow t_2 - t_1 = -50^\circ C$$

$$t_2 - 20 = -50^\circ C$$

$$t_2 = -30^\circ C$$

(ii) $\gamma = 3\alpha$

$= 3 \times 2 \times 10^{-6}$

$= 6 \times 10^{-6}/^\circ C$

Tasks

(1) Given $t_1 = 20^\circ C \rightarrow 70^\circ C$

$\Delta l = 10^{-3}\ m$; $l_1 = l$

$$\alpha = \frac{\Delta l}{l_1 \Delta t}$$

$$\Rightarrow \alpha l = \frac{\Delta l}{\Delta t} = \frac{10^{-3}}{50}$$

$$\Rightarrow \alpha l = \frac{1}{5} \times 10^{-4}$$

Also Given $l_2 = 4l$.

From $\alpha = \frac{\Delta l'}{l_2(t_2' - t_1')}$

$$\Rightarrow \alpha = \frac{\Delta l'}{4l(95 - 20)}$$

$$\Rightarrow \alpha l = \frac{\Delta l'}{4 \times 75}$$

$$\Rightarrow \frac{1}{5} \times 10^{-4} = \frac{\Delta l'}{4 \times 75}$$

$$\Delta l' = 60 \times 10^{-4} = 6 \times 10^{-3}\ m$$

(2) Given $\frac{\Delta V}{V} = 0.30\%$

$= 3 \times 10^{-3}$

$$\gamma = \frac{3}{2}\beta$$

$$\frac{\Delta V}{V \Delta t} = \frac{3}{2}\beta$$

$$\Rightarrow \frac{3 \times 10^{-3}}{100} = \frac{3}{2}\beta$$

$$\Rightarrow \beta = 2 \times 10^{-5}/^\circ C$$

(3) Given $\alpha_x = 10 \times 10^{-7}/^\circ C$

$\alpha_z = \alpha_y = 220 \times 10^{-7}/^\circ C$.

∴ The coefficient of cubical expansion :-

$= 10 \times 10^{-7} + 220 \times 10^{-7} + 220 \times 10^{-7}$

$= 450 \times 10^{-7}/^\circ C$

$= 45 \times 10^{-6}/^\circ C$

(4) Given $t_1 = 27^\circ C \rightarrow l_1 = 10\ m$

$\Delta l = -1.1\ mm = -1.1 \times 10^{-3}\ m$

$t_2 = ?$

$$\alpha = \frac{\Delta l}{l \Delta t}$$

$$\Rightarrow 11 \times 10^{-6} = \frac{-1.1 \times 10^{-3}}{10 \times \Delta t}$$

$$\Rightarrow \Delta t = \frac{-11 \times 10^{-4}}{11 \times 10 \times 10^{-6}}$$

$$\Rightarrow \Delta t = -10^\circ C.$$

$t_2 - t_1 = -10^\circ C$

$\Rightarrow t_2 - 27 = -10^\circ C$

$\Rightarrow t_2 = -10 + 27$

$= 17^\circ C$.

(5) $d_2 = 20\ gm/cc$; $t_2 = 100°C$

$t_1 = 0°C$ $\quad d_1 = ?$

Given $\alpha = 20 \times 10^{-6}/°C$

$\gamma = 3\alpha = 3 \times 20 \times 10^{-6}$

$= 60 \times 10^{-6}$

From

$d_1 = d_2[1 + \gamma \Delta t]$

$\Rightarrow d_1 = 20 + 20\,(100-0)\,60 \times 10^{-6}$

$\Rightarrow d_1 = 20[1 + 100 \times 60 \times 10^{-6}]$

$= 20[1 + 0.006]$

$= 20[1.006]$

$= 20.12\ gm/cc$

(6) Given $\alpha_s = 11 \times 10^{-6}/°C$

$t_2 = 250 + 20 = 270°C$

$D = 1.02\ m$; $t_1 = 20°C$

$\Delta D_2 = 3.3\ mm_2$

$= 3.3 \times 10^{-3}\ m$

$= 33 \times 10^{-4}\ m$

From $\alpha = \dfrac{\Delta D}{D \times \Delta t}$

$\Rightarrow 11 \times 10^{-6} = \dfrac{33 \times 10^{-4}}{1.02 \times (t_2 - 20)}$

$\Rightarrow 1 \cdot 10^{-2} = \dfrac{3}{12 \times 10^{-1} \times (t_2 - 20)}$

$\Rightarrow t_2 - 20 = \dfrac{1000}{4} = 250$

$\Rightarrow t_2 = 270°C$

(7) Given $\alpha = 10 \times 10^{-6}/°C$

$t_1 = 20°C$; $d_1 = 1\ gm/c$

$t_2 = 40°C$; $d_2 = ?$

$\gamma = \dfrac{d_1 - d_2}{d_2 \Delta t}$

$\Rightarrow d_1 = d_2(1 + \gamma \Delta t)$

$\Rightarrow 1 = d_2(1 + 30 \times 10^{-6} \times (40-20))$

$\Rightarrow 1 = d_2[1 + 600 \times 10^{-6}]$

$\Rightarrow d_2 = \dfrac{1}{1.006}$

$\Rightarrow d_2 = 0.998\ gm/c$

(8) Given $\dfrac{l_1}{l_2} = \dfrac{2}{3}$; $\dfrac{\alpha_1}{\alpha_2} = \dfrac{3}{2}$; $\dfrac{\Delta l_1}{\Delta l_2} = ?$

From $\alpha = \dfrac{\Delta l}{l \Delta t}$

$\Rightarrow \Delta l = \alpha l \Delta t$

$\Rightarrow \dfrac{\alpha_1 l_1}{\alpha_2 l_2} = \dfrac{\Delta l_1}{\Delta l_2}$

$\Rightarrow \dfrac{2}{3} \times \dfrac{3}{2} = \dfrac{\Delta l_1}{\Delta l_2}$

$\Rightarrow \dfrac{\Delta l_1}{\Delta l_2} = \dfrac{1}{1}$

(9) Given $\dfrac{\Delta V}{V} = 0.4\% = 4 \times 10^{-3}$

$\Delta t = 80°C$.

From $\gamma = \dfrac{\Delta V}{V \Delta t}$

$\beta = \dfrac{2}{3}\gamma$

$\Rightarrow \beta = \dfrac{2}{3} \dfrac{\Delta V}{V \Delta t}$

$\Rightarrow \beta = \dfrac{2}{3} \times \dfrac{4 \times 10^{-3}}{80}$

$\Rightarrow \beta = \dfrac{1}{3} \times 10^{-4}$

$\Rightarrow \beta = 0.33 \times 10^{-4}$

$\Rightarrow \beta = 3.3 \times 10^{-5}/°C$.

(10) at $t_1 = 27°C \rightarrow l_1 = 2m$

$t_2 = 77°C \rightarrow l_2 = 2.0024m$

$\alpha = \dfrac{\Delta l}{l \Delta t} = \dfrac{l_2 - l_1}{l(t_2 - t_1)}$

$\alpha = \dfrac{2.0024 - 2}{2(77-27)}$

$\alpha = \dfrac{0.0024}{2 \times 50}$

$\alpha = \dfrac{24 \times 10^{-4}}{2 \times 50}$

$\alpha = 24 \times 10^{-6}/°C$

(17) Given $d_1 = 10\ gm/cc \rightarrow 6°C$

$d_2 = 9.7\ gm/cc \rightarrow t_2 = 100°C$

$\gamma = \dfrac{d_1 - d_2}{d_2 (t_2 - t_1)} = \dfrac{10 - 9.7}{9.7(100-0)}$

$3\alpha = \dfrac{0.3}{9.7 \times 100}$

$\Rightarrow \alpha = 10^{-4}/°C$

(18) We know that $\alpha = \dfrac{1}{3}\beta$

Given $\dfrac{\Delta V}{V} = 0.12\% = 12 \times 10^{-4}$

$\Delta t = 30°$

$\therefore \alpha = \dfrac{1}{3} \dfrac{\Delta V}{V \Delta t}$

$= \dfrac{1}{3} \dfrac{12 \times 10^{-4}}{30}$

$= \dfrac{4}{3} \times 10^{-5}/°C$

(ii) $\beta = 2\alpha$

$\beta = 2(0.33 \times 10^{-5})$

$= 2.66 \times 10^{-5}/°C$