

FUNCTIONS - II ①

Class: IX : Mathematics
IIT. FOUNDATION

TEACHING TASK

01.

$$f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$$

$$f(-x) = \frac{-x}{e^{-x} - 1} - \frac{x}{2} + 1$$

$$= \frac{-xe^x}{1 - e^x} - \frac{x}{2} + 1$$

$$= \frac{-xe^x + x - x}{1 - e^x} - \frac{x}{2} + 1$$

$$= \frac{x(1 - e^x) - x}{1 - e^x} - \frac{x}{2} + 1$$

$$= x - \frac{x}{1 - e^x} - \frac{x}{2} + 1$$

$$= \frac{x}{e^x - 1} + \frac{x}{2} + 1 = f(x)$$

Hence, $f(x)$ is even function.

Ans: B

02.

$$y = [4 - (x-7)^3]^{1/5}$$

$$\Rightarrow y^5 = 4 - (x-7)^3$$

$$\Rightarrow (x-7)^3 = 4 - y^5$$

$$\Rightarrow x-7 = (4 - y^5)^{1/3}$$

$$x = 7 + (4 - y^5)^{1/3}$$

$$f^{-1}(x) = 7 + (4 - x^5)^{1/3}$$

Ans: A

03 let $y = \frac{e^x + e^{-x}}{2}$ (2)

let $e^x = a$

$\Rightarrow y = \frac{a + a^{-1}}{2}$

$\Rightarrow 2y = \frac{a^2 + 1}{a}$

$\Rightarrow 2ya = a^2 + 1$

$\Rightarrow a^2 - 2ya + 1 = 0$

$\Rightarrow a = \frac{2y + \sqrt{4y^2 - 4}}{2}$

$\Rightarrow a = y + \sqrt{y^2 - 1}$

$\Rightarrow e^x = y + \sqrt{y^2 - 1}$

$\Rightarrow x = \log(y + \sqrt{y^2 - 1})$

$\Rightarrow f^{-1}(x) = \log_e(x + \sqrt{x^2 - 1})$ Ans: C

04. $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

let $e^x = a$

$\Rightarrow y = \frac{a - a^{-1}}{a + a^{-1}}$

$\Rightarrow y = \frac{a^2 - 1}{a^2 + 1}$

$\Rightarrow a^2 y + y = a^2 - 1$

$\Rightarrow y + 1 = a^2 - a^2 y$

$\Rightarrow y + 1 = a^2(1 - y)$

$\Rightarrow a^2 = \frac{1 + y}{1 - y}$

$\Rightarrow a = \sqrt{\frac{1 + y}{1 - y}}$

04

$$y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\text{let } e^x = a$$

$$\therefore y = \frac{a - a^{-1}}{a + a^{-1}}$$

$$\Rightarrow y = \frac{a^2 - 1}{a^2 + 1}$$

$$\Rightarrow y a^2 + y = a^2 - 1$$

$$\Rightarrow y + 1 = a^2 - y a^2$$

$$\Rightarrow y + 1 = a^2(1 - y)$$

$$\Rightarrow a^2 = \frac{1+y}{1-y}$$

$$\Rightarrow a = \sqrt{\frac{1+y}{1-y}} \quad (3)$$

$$\Rightarrow e^x = \sqrt{\frac{1+y}{1-y}}$$

$$\Rightarrow x = \log\left(\sqrt{\frac{1+y}{1-y}}\right)$$

$$\Rightarrow f^{-1}(y) = \log_e\left(\sqrt{\frac{1+y}{1-y}}\right)$$

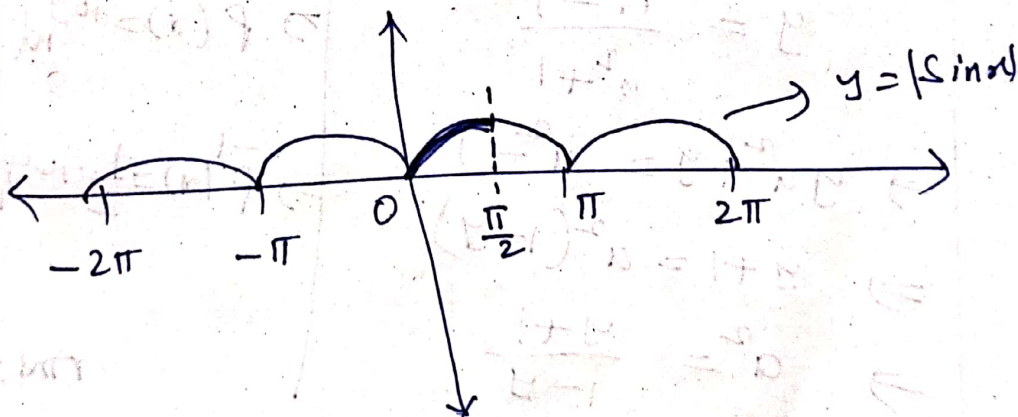
$$= \frac{1}{2} \log\left(\frac{1+y}{1-y}\right)$$

~~Ans. A~~

Ans. A

05

$$f(x) = |\sin x|$$



f is bijective in $[0, \frac{\pi}{2}]$

Ans. B

06

$$y = 2^{x(x-1)}$$

$$\Rightarrow \log y = x(x-1) \log 2$$

$$\Rightarrow \log_2 y = x^2 - x$$

$$\Rightarrow x^2 - x - \log_2 y = 0$$

$$x = \frac{1 + \sqrt{1 + 4 \log_2 y}}{2}$$

$$f^{-1}(y) = \frac{1 + \sqrt{1 + 4 \log_2 y}}{2}$$

Ans: B

07

$$f(x) = (a - x^n)^{1/n}$$

(4)

$$\begin{aligned} f \circ f(x) &= f(f(x)) \\ &= \left[a - \left[(a - x^n)^{1/n} \right]^n \right]^{1/n} \\ &= x \end{aligned}$$

Ans: A

08

$$\text{let } y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$$

$$\Rightarrow a = \sqrt{\frac{1+y}{1-y}}$$

$$\text{let } 10^x = a$$

$$\Rightarrow 10^x = \sqrt{\frac{1+y}{1-y}}$$

$$\Rightarrow x = \log_{10} a$$

$$\Rightarrow x = \log_{10} \left(\sqrt{\frac{1+y}{1-y}} \right)$$

$$\therefore y = \frac{a - a^{-1}}{a + a^{-1}}$$

$$y = \frac{a^2 - 1}{a^2 + 1}$$

$$\Rightarrow f^{-1}(x) = \log_{10} \left(\sqrt{\frac{1+x}{1-x}} \right)$$

$$\Rightarrow y a^2 + y = a^2 - 1$$

$$\Rightarrow y + 1 = a^2(1 - y)$$

$$\Rightarrow a^2 = \frac{y+1}{1-y}$$

$$\Rightarrow f^{-1}(x) = \frac{1}{2} \log_{10} \left(\frac{1+x}{1-x} \right)$$

Ans: B

09.

$$f(x) = [x], g(x) = x - [x]$$

$= \{x\}$ fractional part
lies b/w 0 to 1.

$$f \circ g(x) = f(g(x))$$

$$= f(\{x\})$$

$$= [\{x\}] = 0$$

Ans: D



10. $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cdot \cos\left(x + \frac{\pi}{3}\right)$

$$= \sin^2 x + \left[\sin x \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \cos x \right]^2 + \cos x \left[\cos x \cdot \frac{1}{2} - \sin x \cdot \frac{\sqrt{3}}{2} \right] \quad (5)$$

$$= \sin^2 x + \frac{1}{4} (\sin x + \sqrt{3} \cos x)^2 + \frac{\cos x}{2} (\cos x - \sqrt{3} \sin x)$$

$$= \frac{5}{4} (\sin^2 x + \cos^2 x) = \frac{5}{4}$$

Now $g(f(x)) = g\left(\frac{5}{4}\right)$

$$= 1$$

Ans: A

11. $f(x) = \sqrt{3} \sin x - \cos x + 2$

$$= 2 \left(\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x \right) + 2$$

$$= 2 \sin\left(x - \frac{\pi}{6}\right) + 2$$

$$\therefore 2 \sin\left(x - \frac{\pi}{6}\right) + 2 = y$$

$$\Rightarrow x = \sin^{-1}\left(\frac{y-2}{2}\right) + \frac{\pi}{6}$$

$$\therefore f^{-1}(x) = \sin^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$$

$$= \frac{\pi}{2} - \cos^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$$

$$= \frac{2\pi}{3} - \cos^{-1}\left(\frac{x-2}{2}\right)$$

Ans: B, C

12

$$f(x) = \frac{ax+2}{x-b}$$

(6)

$$\Rightarrow \frac{ax+2}{x-b} = y$$

$$\Rightarrow x = \frac{2+by}{y-a}$$

$$\Rightarrow f^{-1}(x) = \frac{2+bx}{x-a}$$

$$f^{-1}(1) = \frac{2+b}{1-a} = 0 \Rightarrow b = -2$$

f^{-1} is not defined at 3

$\therefore \frac{2+b \cdot 3}{3-a}$ ~~is~~ not defined.

$$\text{when } 3-a=0 \Rightarrow a=3$$

Ans: A, D

13

Statement I: e^x and $\log_e x$ both are inverse functions to each other. (True)

Statement II: Conceptual: True

A

14. Statement I:

$$y = (x+1)^2 - 1$$

①

$$\Rightarrow (x+1)^2 = 1+y$$

$$\Rightarrow x = \sqrt{1+y} - 1$$

$$\therefore f^{-1}(x) = \sqrt{1+x} - 1$$

Now $f(x) = f^{-1}(x)$

$$\Rightarrow (x+1)^2 - 1 = \sqrt{1+x} - 1$$

$$\Rightarrow (x+1)^2 = \sqrt{1+x}$$

$$\Rightarrow (x+1)^4 - (1+x) = 0$$

$$\Rightarrow (x+1) [(x+1)^3 - 1] = 0$$

$$\Rightarrow x+1=0 \quad \vee \quad (x+1)^3=1$$

$$\Rightarrow x=-1 \quad \vee \quad x=0$$

Given $\{0, 1\}$

hence statement I is false

Statement II: $y = (x+1)^2 - 1$

This graph is clearly a parabola.

Hence it is bijective

Ans: D.

15.

$$\text{Let } F = f_6$$

(2)

$$f_1 \circ F = f_1(F)$$

$$= f_1(f_6) = f_1\left(\frac{x}{x-1}\right)$$

$$= \frac{x}{x-1}$$

Ans: D

$$16. \text{ Let } G = f_5$$

$$G \circ f_3 = f_5 \circ f_3 = f_5(f_3)$$

$$= f_5(1-x)$$

$$= \frac{(1-x)-1}{1-x}$$

$$= \frac{x}{x-1} = f_6$$

Ans: A

$$17. \text{ Let } H = f_2$$

$$f_4 \circ H = f_4 \circ f_2$$

$$= f_4(f_2)$$

$$= f_4\left(\frac{1}{x}\right)$$

$$= \frac{1}{1-\frac{1}{x}}$$

$$= \frac{x}{x-1} = f_4$$

Ans: A

$$f(x) = \frac{x-1}{x-2}$$

(9)

$$\text{let } a=1, b=2$$

$$f(x) = \frac{x-1}{x-2}$$

$$\text{Now } f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1-1}{x_1-2} = \frac{x_2-1}{x_2-2}$$

$$\Rightarrow x_1 x_2 - 2x_1 - x_2 + 2 = x_1 x_2 - x_1 - 2x_2 + 2$$

$$\Rightarrow x_1 = x_2$$

hence f is one-one function.

$$\text{let } y = \frac{x-1}{x-2}$$

$$\Rightarrow x = \frac{2y-1}{y-1}$$

This is defined only when $y \neq 1$

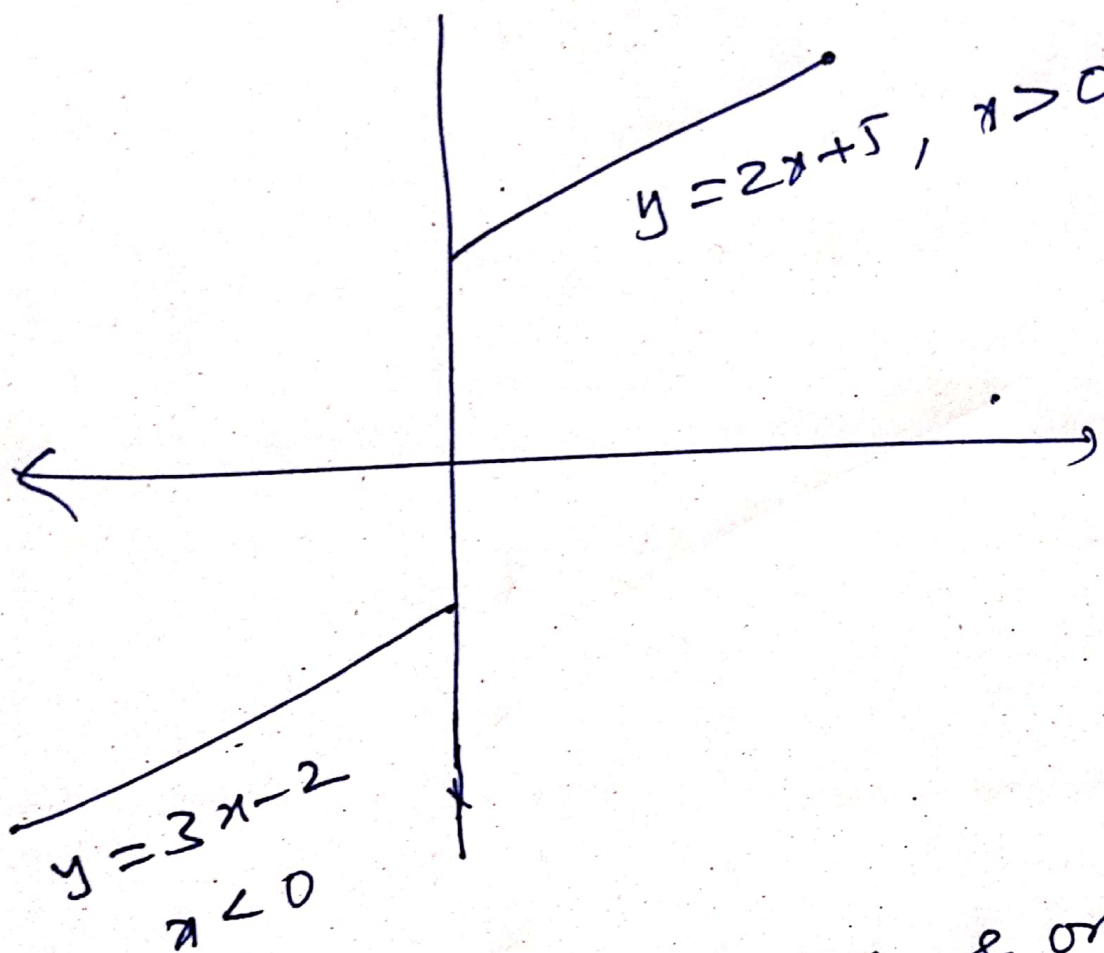
$$\text{If } y=1 \Rightarrow 1 = \frac{x-1}{x-2}$$

$$\Rightarrow x-2 = x-1 \quad (\text{Contradiction})$$

i.e. for $y=1$, there is no pre-image.
hence it is not on-to function.

Ans: A

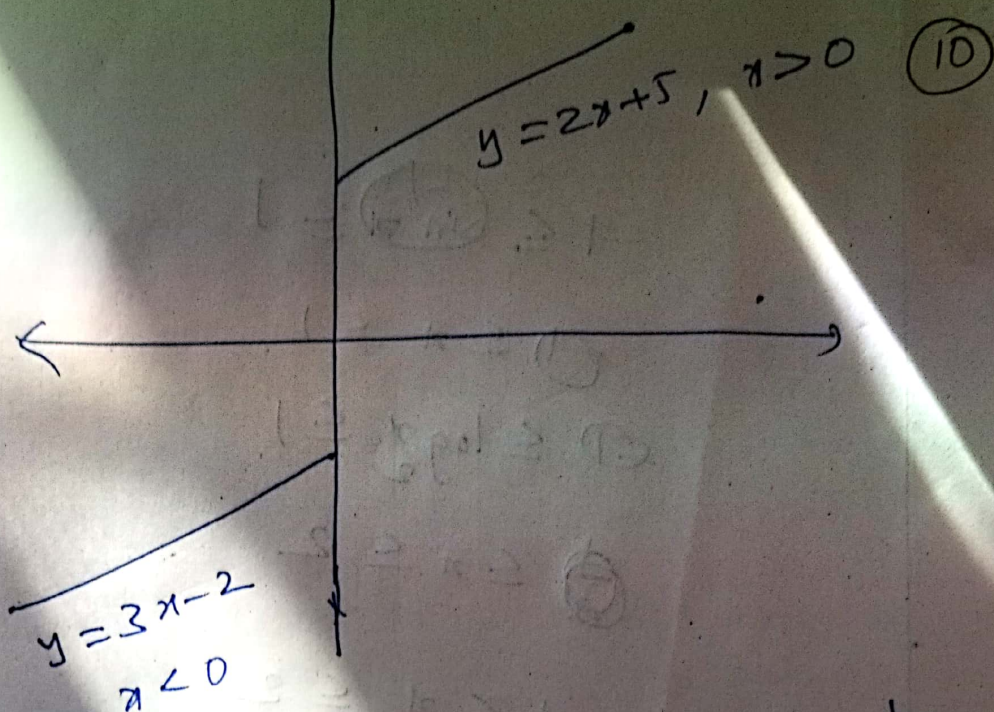
19.



(10)

clearly f is one-one & on-to
Ans: C

19.



clearly f is one-one & on-to
Ans: C

20. Let $f(x) = ax + b$, where $a \neq 0$

Given $f(f(x)) = x + f(x)$

$$f(ax + b) = x + ax + b$$

$$\Rightarrow a(ax + b) + b = x + ax + b$$

$$\Rightarrow a^2x + ab = x + ax$$

$$\text{Coeff. of } x^2 \Rightarrow a^2 = 1 + a$$

$$\Rightarrow a^2 - a - 1 = 0$$

$$\Rightarrow a = \frac{1 \pm \sqrt{5}}{2}$$

$$\left. \begin{array}{l} ab = 0 \\ a \neq 0, b = 0 \end{array} \right\}$$

Thus $f(x) = \left(\frac{1 \pm \sqrt{5}}{2}\right)x + 0$

Hence, there are two linear functions.

Ans: 2

21. $f \circ f(1) = f(f(1))$ (11)
 $= f(1)$ since $1 \in \mathcal{Q}$
 $= 1$ since $1 \in \mathcal{Q}$
 Ans: 1

22 a) $f(x) = \sqrt{\sin^{-1}(\log_2 x)}$

$\Rightarrow 0 \leq \log_2 x \leq 1$

$\Rightarrow 1 \leq x \leq 2$

Domain = $[1, 2]$

b) $g(x) = \sqrt{\cos(\sin x)}$

Domain = $\mathbb{R}$.

c) $h(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$

$\Rightarrow -1 \leq \frac{1+x^2}{2x} \leq 1$

$\Rightarrow \left| \frac{1+x^2}{2x} \right| \leq 1$

$\Rightarrow |1+x^2| \leq |2x|$

$\Rightarrow 1+x^2 \leq |2x|$ since $1+x^2 > 0$

$\Rightarrow x^2 - 2|x| + 1 \leq 0$

$\Rightarrow |x|^2 - 2|x| + 1 \leq 0$

$\Rightarrow (|x| - 1)^2 \leq 0$

But $(|x| - 1)^2$ is always either positive, or zero. $(|x| - 1)^2 = 0$ or $|x| = 1 \Rightarrow x = \pm 1$
 Domain = $\{-1, 1\}$

22 d) $f(x) = g(x) + h(x)$

(12)

$$= \sqrt{\cos(\sin x)} + \sin^{-1}\left(\frac{1+x^2}{2x}\right)$$

$\downarrow$ $\downarrow$
 $\mathbb{R}$ $\{-1, 1\}$

hence Domain = $\{-1, 1\}$ Ans: S, T, P, P

23 a) $f(x) = ax + b$

both injection and surjection.

b) $f(x) = [x]$

It is not injective but it is surjective.

because $[1.35] = 1$ & $[1.78] = 1$
 $\therefore$ for both 1.35, 1.78 it gives the same image

c) $f(x) = |x|$

it is not injective, but it is surjective

d) $f(x) = x^3$

Both injective and surjective.

Ans: S, T, P, S

LEARNERS TASK

(13)

1. CUQS
 $(\bar{f}')^{-1} = f$ Ans: A

2. $x-1=y$
 $\Rightarrow x=1+y$
 $\Rightarrow \bar{f}'(x)=1+x$ Ans: B

3. Range of $\sin^{-1}(x) = \left[-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right]$ Ans: A

4. $(g \circ f)^{-1} = \bar{f}' \circ \bar{g}'$ Ans: C

5. $(g \circ f)(x) = g(f(x))$ Ans: C

6. $\bar{f}'(b) = \{a, \frac{1}{2}\}$ Ans: A

7. $f(g(x)) = f(x+2)$
 $= (x+2)^2$
 $= x^2 + 4x + 4$ Ans: D

8. $f \circ g$ — one-one
 $g \rightarrow$ one-one Ans: B

9. $f \circ \bar{f}' = I_B$ Ans: B

10. $g \circ f: A \rightarrow C$
need not be bijection Ans: C

JEE MAINS LEVEL

14

01. $x^2 - 5x + 6 = 0$
 $\Rightarrow (x-2)(x-3) = 0$
 $\Rightarrow x = 2 \text{ or } 3$

Ans: B

02. $10x - 7 = y$: $g(x) = f^{-1}(x)$
 $\Rightarrow x = \frac{y+7}{10} = \frac{x+7}{10}$

$\Rightarrow f^{-1}(x) = \frac{x+7}{10}$

Ans: C

03. $f(g(a)) + f(g(b))$

$= f(a^3) + f(b^3)$

$= \log a^3 + \log b^3$

$= \log(ab)^3$

option: B $f(g(ab))$

$= f((ab)^3)$

$= \log(ab)^3$

Ans: B

04. $f(x) = \frac{1}{1-x}$

$\frac{1}{1-x} = y$

$\Rightarrow 1-x = \frac{1}{y}$

$\Rightarrow x = 1 - \frac{1}{y}$

$\Rightarrow x = \frac{y-1}{y}$

$\Rightarrow f^{-1}(x) = \frac{x-1}{x}$

Ans: D

06

$$f(f(x)) = x$$

(15)

$$\Rightarrow f\left(\frac{\alpha x}{x+1}\right) = x$$

$$\Rightarrow \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}\right) + 1} = x$$

$$\Rightarrow \frac{\alpha^2}{\alpha x + x + 1} = 1$$

$$\Rightarrow \alpha^2 = \alpha x + x + 1$$

$$\Rightarrow \alpha^2 = (\alpha + 1)x + 1$$

Comparing coeff of x on b/s

$$\alpha + 1 = 0 \Rightarrow \alpha = -1$$

Ans: D

07

$$(f \circ g)(\pi) + (g \circ f)(e)$$

$$= f(g(\pi)) + g(f(e))$$

 $\pi \rightarrow \text{Irrational}$ $e \rightarrow \text{rational}$

$$= f(0) + g(0)$$

$$= 0 + (-1)$$

Ans: A

$$= -1$$

08

opt: A $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$

$$g(f(x)) = g(\sin^2 x) = \sqrt{\sin^2 x} = |\sin x|$$

$$f(g(x)) = f(\sqrt{x}) = \sin^2 \sqrt{x} = (\sin \sqrt{x})^2$$

Ans: A



09. $n(A) = n, n(B) = 3$

No. of on to functions = $3^n - {}^3C_1(3-1)^n - {}^3C_2(3-2)^n$ (16)

$$= 3^n - 3 \cdot 2^n + 3$$

$$= 3^n - 3(2^n - 1)$$

Ans: D

10. $\therefore \text{Sof}\left(\frac{a+1}{4}\right) = \text{S}\left(f\left(\frac{a+1}{4}\right)\right)$

$$= \text{S}\left(4\left(\frac{a+1}{4}\right) - 1\right)$$

$$= \text{S}(a)$$

$$= a^3 + 2$$

Ans: C

11. $f \circ g(x) = f(g(x))$

$$= f(2)$$

$$= 1$$

Ans: D

12. A) $f(g(x)) = f\left(\frac{1}{x^2}\right)$

$$= x^2 \quad (\text{True})$$

B) $h(g(x)) = h\left(\frac{1}{x^2}\right) = \left(\frac{1}{x^2}\right)^2 = \frac{1}{x^4} = (g(x))^2$
(True)

C) $h\left(\frac{1}{x^2}\right) = \frac{1}{x^4} \quad (\text{True})$

D) $f(g(x)) = f\left(\frac{1}{x^2}\right) = x^2 = h(x) \quad (\text{True})$

Ans: A, B, C, D.

13.

Statement I
 $f(x) = \frac{1}{1-x}$

$$\Rightarrow \frac{1}{1-x} = y$$

$$\Rightarrow 1-x = \frac{1}{y}$$

$$\Rightarrow x = 1 - \frac{1}{y}$$

$$\Rightarrow x = \frac{y-1}{y}$$

$$f^{-1}(x) = \frac{x-1}{x}$$

(True)

(17)

Statement II:

$$\frac{x-1}{x+1} = y$$

$$\Rightarrow x-1 = xy+y$$

$$\Rightarrow x-xy = 1+y$$

$$\Rightarrow x(1-y) = 1+y$$

$$f(f^{-1}(x)) = f\left(\frac{1+x}{1-x}\right)$$

$$= \frac{\frac{1+x}{1-x} - 1}{\frac{1+x}{1-x} + 1} = \frac{\cancel{1}x - \cancel{1} + x}{1 + \cancel{x} + 1 - \cancel{x}} = \frac{2x}{2} = x$$

(True)

Ans: A

14

$$5x+3=y$$

$$x = \frac{y-3}{5}$$

$$f^{-1}(x) = \frac{x-3}{5} = S(x)$$

Statement I (True)

Statement II: Conceptual (True)

Ans: A

15. Given $f(-x) = -f(x)$, $f(x+1) = x+1$ (18)

and $f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2}$, $\forall x \neq 0$

The only function satisfies the above functions is $f(x) = x$

Ans: A

16. $f(x) = x \Rightarrow f^{-1}(x) = x$

$\therefore (f^{-1} \circ f^{-1} \circ \dots \circ f^{-1})(1) = 1$

Ans: D

17. $f^{-1}(2010) = 2010$

Ans: A

18. $f \circ g(x^2+1) = f(9(x^2+1))$

19.

$= f((x^2+1)^2+1)$

$= f(x^4 + 2x^2 + 2)$

$= 3(x^4 + 2x^2 + 2) - 2$

$= 3x^4 + 6x^2 + 4$

Ans: A

18. $f \circ g(-1) = f(9(-1))$

$= f(3(-1)-4)$

$= f(-7)$

$= (-7)^2 + 2(-7) - 3$

$= 49 - 14 - 3$

$= 32$

Ans: B

20

$$[g \circ (f \circ f)](2) = g(f(f(2)))$$

(19)

$$= g(f(2 \cdot 2 + 1))$$

$$= g(f(5))$$

$$= g(2 \cdot 5 + 1)$$

$$= g(11)$$

$$= (11)^2 + 1 = 122$$

Ans: 122

21.

$$f(g(x)) = 25$$

$$\Rightarrow f(x^2 + 7) = 25$$

$$\Rightarrow 2(x^2 + 7) + 3 = 25$$

$$\Rightarrow 2x^2 + 8 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

Ans: ± 2

22

$$f(f(f(0))) + 1$$

$$= f(f(3 \cdot 0 + 2)) + 1$$

$$= f(f(2)) + 1$$

$$= f(3 \cdot 2 + 2) + 1$$

$$= f(8) + 1$$

$$= 3 \cdot 8 + 2 + 1$$

$$= 27$$

Ans: 27

23

$$a) f(x) = 4x + 1$$

$$\Rightarrow f^{-1}(x) = \frac{x-1}{4}$$

$$f^{-1}(-3) = \frac{-3-1}{4} = -1$$

23 a) $f(x) = 4x + 1$

(20)

$$\Rightarrow f^{-1}(x) = \frac{x-1}{4}$$

$$\Rightarrow f^{-1}(-3) = \frac{-3-1}{4} = -1$$

b) $f(x) = x + 4$

$$\Rightarrow f^{-1}(x) = x - 4$$

$$\Rightarrow f^{-1}(5) = 5 - 4 = 1$$

c) $f(x) = 3x - 5$

$$\Rightarrow f^{-1}(x) = \frac{x+5}{3}$$

$$\Rightarrow f^{-1}(-23) = \frac{-23+5}{3} = -6$$

d) $f(x) = 5x + 2$

$$\Rightarrow f^{-1}(x) = \frac{x-2}{5}$$

$$\Rightarrow f^{-1}(-3) = \frac{-3-2}{5} = -1$$

Ans: s, t, p, s

~~24~~ a)

$$(1-n)(1) = (n)f$$

$$(1-n)(1) = n = (n)p = (n)e$$

$$(1) + (1-n)(1) =$$

$$(1-n)(1) =$$

24) a) $f(-5) + f(-4)$ (21)
 $= (-5+4) + 2(-4)+2$
 $= -11$

b) $f(|f(-6)|)$
 $= f(1-8+4)$
 $= f(1-4)$
 $= f(4)$
 $= 2 \cdot 4 + 2 = 14$

c) $f(f(-7)) + f(3)$
 $= f(-7+4) + f(3)$
 $= f(-3) + f(3)$
 $= 3(-3)+2 + 2(3)+2$
 $= 4$

d) $f(f(f(0))) + 2$
 $= f(f(3 \cdot 0 + 2)) + 2$
 $= f(f(2)) + 2$
 $= f(2 \cdot 2 + 2) + 2$
 $= f(6) + 2$
 $= 2 \cdot 6 + 2$
 $= 14$

Ans: 5, 9, 14, 6

ADDITIONAL PRACTICE QUESTIONS

01. $[h \circ (g \circ f)]\left(\sqrt{\frac{\pi}{4}}\right)$

(22)

$$= h\left[g\left(f\left(\sqrt{\frac{\pi}{4}}\right)\right)\right]$$

$$= h\left[g\left(\frac{\pi}{4}\right)\right]$$

$$= h[1]$$

$$= 0$$

Ans: C

02. $f(x) = \log(x + \sqrt{x^2 + 1})$

$$f(x) + f(-x) = \log(x + \sqrt{x^2 + 1}) + \log(\sqrt{x^2 + 1} - x)$$

$$= \log((x^2 + 1) - x^2)$$

$$= \log(1) = 0$$

$$\therefore f(-x) = -f(x)$$

f is odd function

03. $f(n) = (-1)^{n-1}(n-1)$, $g(n) = 1 - f(n)$

let $n=1$, $f(1) = 0$, $g(1) = 1$

$$\text{Now } g \circ g(1) = g(g(1))$$

$$= g(1)$$

$$= 1$$

Ans: 1

04 $f(x) = 5x - 3$

(23)

$$\Rightarrow f^{-1}(1) = \frac{1+3}{5} = 1$$

$$\Rightarrow f^{-1}(3) = \frac{3+3}{5} = \frac{6}{5}$$

$$(g \circ f^{-1})(3) = g(f^{-1}(3)) = g\left(\frac{6}{5}\right)$$

$$= \boxed{5\left(\frac{6}{5}\right) - 3 = 3}$$

$$= \left(\frac{6}{5}\right)^2 + 3 = \frac{111}{25}$$

Ans: B

05 $f(g(x)) = f\left(\frac{x}{\sqrt{1+x^2}}\right)$

$$= \frac{\left(\frac{x}{\sqrt{1+x^2}}\right)}{\sqrt{1+\left(\frac{x}{\sqrt{1+x^2}}\right)^2}}$$

$$\frac{x + x\sqrt{1+x^2}}{1 + x^2 + x^2} = \frac{x}{1+x^2}$$

$$\frac{(x+x\sqrt{1+x^2})}{1+x^2} = x$$

Ans: A

$\Rightarrow$ THE END $\Leftarrow$

$$(1+x^2)(1+x^2)$$

$$\boxed{5-9}$$

$$\boxed{5-9}$$