

Dot ProductTask

(1)

Given line equation $y = x + 5$ compare with $y = mx + 5$ $r = 5$ Given $v = 4\sqrt{2} \text{ m/s}$; $m = 8 \text{ kg}$; where slope $m = 1$

$\tan \theta = 1$

$\Rightarrow \theta = 45^\circ$

Angular momentum

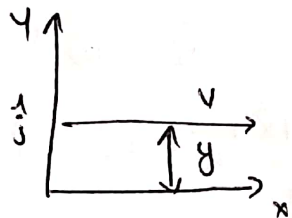
$$\vec{L} = mvr \sin \theta$$

$$= 8 \times 4\sqrt{2} \times 5 \times \sin 45^\circ$$

$$= 32\sqrt{2} \times 5 \times \frac{1}{\sqrt{2}}$$

$$= 160 \text{ kg m}^2/\text{s}$$

(2)



$$\vec{p} = mv \hat{i}$$

Angular momentum

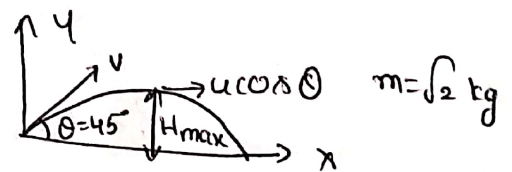
$$\vec{L} = \vec{r} \times \vec{p}$$

$$= y \hat{j} \times mv \hat{i}$$

$$= mvy (\hat{j} \times \hat{i})$$

$$= -mvy \hat{k}$$

(3)



$$\vec{r} = H_{\max} \hat{j}$$

$$= \frac{v^2 \sin^2 \theta}{2g} \hat{j}$$

$$= \frac{v^2 \sin^2 45^\circ}{2g} \hat{j} \Rightarrow \frac{v^2 (\frac{1}{2})^2}{2g} \hat{j}$$

$$= \frac{v^2}{4g} \hat{j}$$

$$\text{momentum } \vec{p} = mv \cos \theta \hat{i}$$

$$= \sqrt{2} v \cos 45^\circ \hat{i}$$

$$= v \hat{i}$$

Angular momentum $\vec{L} = \vec{r} \times \vec{p}$

$$\vec{L} = \frac{v^2}{4g} \hat{j} \times v \hat{i} = \frac{v^3}{4g} (\hat{j} \times \hat{i})$$

$$\vec{L} = \frac{v^3}{4g} \hat{k}$$



(4)

$$\vec{v} = (3\hat{i} + 2\hat{j}) \times 10^5 \text{ m/s}$$

$$\vec{B} = 2\hat{i} + 3\hat{k} \text{ wb/m}^2$$

$$\text{Force} = q (\vec{v} \times \vec{B})$$

$$\Rightarrow ma = 1.6 \times 10^{-19} (6\hat{i} - 9\hat{j} - 4\hat{k}) \times 10^5$$

$$\Rightarrow a = \frac{1.6}{m} \times 10^{-14} (6\hat{i} - 9\hat{j} - 4\hat{k})$$

$$\Rightarrow a = \frac{1.6 \times 10^{-14}}{1.6 \times 10^{-19}} [6\hat{i} - 9\hat{j} - 4\hat{k}]$$

$$\Rightarrow a = 9.6 \times 10^{12} (6\hat{i} - 9\hat{j} - 4\hat{k})$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$

$$= (\hat{i}(6-0) - \hat{j}(9-0) + \hat{k}(0-4)) \times 10^5$$

$$= (6\hat{i} - 9\hat{j} - 4\hat{k}) \times 10^5$$

$$\frac{q}{m} = 9.6 \times 10^7 \text{ C/kg}$$

$$\Rightarrow m = \frac{1.6 \times 10^{-19}}{9.6 \times 10^7} \text{ kg}$$

(5)

Given $m = 5 \text{ kg}$: $\vec{r}_1 = 6\hat{i} + 5\hat{j} - 3\hat{k} \text{ m}$

$$\vec{r}_2 = 10\hat{i} - 2\hat{j} + 7\hat{k} \text{ m.}$$

$$\vec{s} = \vec{r}_2 - \vec{r}_1 = (10\hat{i} - 2\hat{j} + 7\hat{k}) - (6\hat{i} + 5\hat{j} - 3\hat{k})$$

$$\vec{s} = 4\hat{i} - 7\hat{j} + 10\hat{k}$$

and given $\vec{F} = 10\hat{i} - 3\hat{j} + 6\hat{k} \text{ N}$

$$W = \vec{F} \cdot \vec{s} = (10\hat{i} - 3\hat{j} + 6\hat{k}) \cdot (4\hat{i} - 7\hat{j} + 10\hat{k})$$

$$= 10 \times 4 + (-3)(-7) + 6 \times 10$$

$$= 40 + 21 + 60$$

$$= 121 \text{ J}$$



6

Given two vectors are

$$\vec{A} = \hat{i} + 2\hat{j} + n\hat{k} \quad ; \quad \vec{B} = 4\hat{i} + 2\hat{j} + 2\hat{k}$$

Given $\vec{A}$ and $\vec{B}$ are perpendicular

$$\therefore \vec{A} \cdot \vec{B} = 0$$

$$(\hat{i} + 2\hat{j} + n\hat{k}) \cdot (4\hat{i} + 2\hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow 4 + 4 + 2n = 0$$

$$\Rightarrow 2n = -8 \Rightarrow n = -4.$$

7

Given

$$\vec{F} = 8\hat{i} + 4\hat{j} \text{ N.}$$

$$\vec{s} = 3\hat{i} + 3\hat{j} \text{ m}$$

$$P = \frac{W}{t} = \frac{\vec{F} \cdot \vec{s}}{t} \quad \text{Given Power} = 0.6 \text{ W}$$

$$\Rightarrow 0.6 = \frac{(8\hat{i} + 4\hat{j}) \cdot (3\hat{i} + 3\hat{j})}{t}$$

$$\Rightarrow 0.6 = \frac{24 + 12}{t}$$

$$\Rightarrow t = \frac{36}{0.6} = 60 \text{ s}$$

8

$$\text{Let } \vec{A} = -4\hat{i} + 2\hat{j} \quad ; \quad \vec{B} = 9\hat{k}.$$

$$\vec{A} \cdot \vec{B} = (-4\hat{i} + 2\hat{j}) \cdot 9\hat{k} = 0.$$

$$\text{Angle between two vectors } \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\cos \theta = \frac{0}{|\vec{A}| |\vec{B}|} = 0$$

$$\cos \theta = 0 \Rightarrow \theta = 90^\circ$$

9)

component of $\vec{B}$ along $\vec{A}$ is

$$A \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}, \quad B \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|}$$

$$\text{Given } \vec{A} \cos \theta = \vec{B} \cos \theta$$

$$\Rightarrow \vec{A} = \vec{B}$$

The angle between $\vec{A}$ and $\vec{B}$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{\vec{A} \cdot \vec{A}}{|\vec{A}| |\vec{A}|}$$

$$\cos \theta = \frac{A}{A^2} = 0 \Rightarrow \theta = 90^\circ$$

10)

$$\text{Given } \vec{F}_1 = 2\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\vec{F}_2 = 3\hat{i} - 5\hat{j} + 4\hat{k}$$

$$\begin{aligned} \vec{F}_{\text{net}} &= \vec{F}_1 + \vec{F}_2 = 2\hat{i} + 2\hat{j} + 5\hat{k} + 3\hat{i} - 5\hat{j} + 4\hat{k} \\ &= 5\hat{i} - 3\hat{j} + 9\hat{k} \end{aligned}$$

$$\text{Given } \vec{S} = 18\hat{i} + 17\hat{j} - 7\hat{k}$$

$$\therefore W = \vec{F} \cdot \vec{S}$$

$$= (5\hat{i} - 3\hat{j} + 9\hat{k}) \cdot (18\hat{i} + 17\hat{j} - 7\hat{k})$$

$$= 18 \times 5 + (-3) \times 17 + 9 \times (-7)$$

$$= 90 - 51 - 63$$

$$= -24$$

(3)

(17) (i), (ii), (iii)

Given $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{c} = 3\hat{i} + \hat{j}$

$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$$\vec{r} = \vec{a} + \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 1 \end{vmatrix}$$

$$\vec{r} = \hat{i}(2-6) - \hat{j}(1+3) + \hat{k}(2-(-2))$$

$$\Rightarrow \vec{r} = -4\hat{i} - 4\hat{j} + 4\hat{k} = -4(\hat{i} + \hat{j} - \hat{k})$$

(ii)

$$|\vec{r}| = 4\sqrt{1^2 + 1^2 + 1^2}$$

$$= 4\sqrt{3}$$

Angle between $\vec{r}$ and $\vec{c}$ $\cos \theta = \frac{\vec{r} \cdot \vec{c}}{|\vec{r}| |\vec{c}|}$

$$|\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$\cos \theta = \frac{-4(\hat{i} + \hat{j} - \hat{k}) \cdot (3\hat{i} + \hat{j})}{4\sqrt{3} \sqrt{10}}$$

$$= - \frac{[1 \times 3 + 1 \times 1]}{\sqrt{30}} = -\frac{4}{\sqrt{30}}$$

$$\theta = \cos^{-1} \left[-\frac{4}{\sqrt{30}} \right]$$

(19)

Given Force $\vec{F} = 3\hat{i} + 4\hat{k}$ N.

Displacement $\vec{s} = 2\hat{i} + 4\hat{j}$ m ; $t = 3$ sec

$$\text{Power} = \frac{\vec{F} \cdot \vec{s}}{t} = \frac{(3\hat{i} + 4\hat{k}) \cdot (2\hat{i} + 4\hat{j})}{3} = \frac{3 \times 2 + 4 \times 4 (\hat{k} \cdot \hat{j})}{3}$$

$$= \frac{6 + 16(0)}{3} = \frac{6}{3} = 2 \text{ W}$$



(18)

Given Angular velocity $\vec{\omega} = 4\hat{i} + \hat{j} - 2\hat{k}$ Linear velocity $\vec{v} = 2\hat{i} - 3\hat{j} + \hat{k}$

$$\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -2 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= \hat{i}(1 - 6) - \hat{j}(4 + 4) + \hat{k}(-12 - 2)$$

$$= -5\hat{i} - 8\hat{j} - 14\hat{k}$$

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(1)

Let the two vectors are.

$$\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{B} = \hat{i} + 4\hat{j} - 5\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 1 & 4 & -5 \end{vmatrix}$$

$$= \hat{i} [(-3)(-5) - 4 \times 4] - \hat{j} [2(-5) - 1 \times 4] + \hat{k} [2 \times 8 + 1 \times (-3)]$$

$$= \hat{i} [15 - 16] - \hat{j} [-10 - 4] + \hat{k} (16 - 3)$$

$$= -\hat{i} + 14\hat{j} + 11\hat{k}$$

(4)

(2)

Given $m = 0.2 \text{ kg}$:

$$\text{let } \vec{v} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{r} = 4\hat{i} + \hat{j} - \hat{k}$$

$$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$\Rightarrow \hat{i}[(1 \times 2 - (-1)(-1))] - \hat{j}[4 \times 2 - 1(-1)] + \hat{k}[1 \times (-1) - 4(-1)]$$

$$\Rightarrow \hat{i}(2-1) - \hat{j}(8+1) + \hat{k}(-1+4)$$

$$\Rightarrow +\hat{i} - 9\hat{j} + 3\hat{k} = \sqrt{1^2 + (-9)^2 + (3)^2}$$

$$= \sqrt{107}$$

$$\text{Angular momentum } \vec{L} = m(\vec{r} \times \vec{v})$$

$$= 0.2 \times \sqrt{107} = \sqrt{428} \text{ units.}$$

(3)

let two vectors are $\vec{A}$ and $\vec{B}$

$$\text{Given } \vec{A} \cdot \vec{B} = AB \cos \theta = 48\sqrt{3}$$

$$\text{and } \vec{A} \times \vec{B} = AB \sin \theta = 144$$

$$\therefore \frac{\vec{A} \cdot \vec{B}}{\vec{A} \times \vec{B}} = \frac{AB \cos \theta}{AB \sin \theta} = \frac{48\sqrt{3}}{144} = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \cot \theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 60^\circ$$

(4)

Given adjacent sides

$$\vec{P} = 2\hat{i} - 3\hat{j} + \hat{k} ; \quad \vec{Q} = -2\hat{i} + 4\hat{j} - \hat{k}$$

$$\text{Area} = \vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ -2 & 4 & -1 \end{vmatrix}$$

$$= \hat{i} [(-3)(-1) - (1 \times 4)] - \hat{j} [2(-1) - (-2)(1)] + \hat{k} [2 \times 4 - (-2)(-3)]$$

$$= \hat{i} [3 - 4] - \hat{j} [-2 + 2] + \hat{k} [8 - 6]$$

$$= -\hat{i} + 2\hat{k}$$

$$|\vec{P} \times \vec{Q}| = \sqrt{(-1)^2 + 2^2} = \sqrt{5} \text{ m}^2$$

(5)

Let the vectors are $\vec{A} = \underset{A_x}{2}\hat{i} + \underset{A_y}{3}\hat{j} - \underset{A_z}{4}\hat{k}$

$$\vec{B} = \underset{B_x}{3}\hat{i} - \underset{B_y}{a}\hat{j} + \underset{B_z}{b}\hat{k}$$

If two vectors are Parallel

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

$$\Rightarrow \frac{2}{3} = \frac{3}{-a} = \frac{-4}{b}$$

$$\Rightarrow \frac{2}{3} = \frac{3}{-a}$$

$$\frac{2}{3} = \frac{-4}{b}$$

$$\Rightarrow a = -\frac{9}{2}$$

$$\Rightarrow b = -6$$

$$= -4.5$$



(7)

Given $\vec{A} = -4 \text{ N}$; $\vec{B} = 3 \text{ N}$.

$$\begin{aligned}
 |\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 &= (AB \sin \theta)^2 + (AB \cos \theta)^2 \\
 &= A^2 B^2 \sin^2 \theta + A^2 B^2 \cos^2 \theta \\
 &= A^2 B^2 [\sin^2 \theta + \cos^2 \theta] \\
 &= (-4)^2 (3)^2 (1) \\
 &= 16 \times 9 \times 1 = 144 \text{ N}
 \end{aligned}$$

(8)

Given $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ & $\vec{C} = [6\hat{i} + 2\hat{j} - 3\hat{k}]k$
 $\vec{B} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

Given $\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$

$$= \hat{i} [3 \times 2 - (6 \times -6)] - \hat{j} (2 \times 2 - 6 \times 3) + \hat{k} [2 \times (-6) - 3 \times 3]$$

$$= \hat{i} [6 + 36] - \hat{j} [4 - 18] + \hat{k} [-12 - 9]$$

$$= 42\hat{i} + 14\hat{j} - 21\hat{k}$$

$$\vec{C} = 7 [6\hat{i} + 2\hat{j} - 3\hat{k}]$$

Given $\vec{C} = k [6\hat{i} + 2\hat{j} - 3\hat{k}]$

$$k = 7$$

(9)

$$\text{Let } \vec{a} = \hat{i} + 3\hat{j} + 2\hat{k} \quad ; \quad \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \\ \vec{c} = -\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{AB} = \vec{B} - \vec{A} = (2\hat{i} - \hat{j} + \hat{k}) - (\hat{i} + 3\hat{j} + 2\hat{k}) \\ = 2\hat{i} - \hat{j} + \hat{k} - \hat{i} - 3\hat{j} - 2\hat{k} \\ = \hat{i} - 4\hat{j} - \hat{k}$$

$$\vec{BC} = \vec{C} - \vec{B} = (-\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) \\ = -\hat{i} + 2\hat{j} + 3\hat{k} - 2\hat{i} + \hat{j} - \hat{k} \\ = -3\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\text{Area} = \frac{1}{2} | \vec{AB} \times \vec{BC} |$$

$$= \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & -1 \\ -3 & 3 & 2 \end{vmatrix}$$

$$= \frac{1}{2} \left[\hat{i}(-4 \times 2 - 3 \times (-1)) - \hat{j}(1 \times 2 - (-3)(-1)) + \hat{k}(1 \times 3 - (-3)(-4)) \right]$$

$$= \frac{1}{2} [\hat{i}(-8+3) - \hat{j}(2-3) + \hat{k}(3-12)]$$

$$= \frac{1}{2} [-5\hat{i} + \hat{j} - 9\hat{k}]$$

$$= \frac{1}{2} \sqrt{5^2 + 1^2 + 9^2}$$

$$= \frac{1}{2} \sqrt{107}$$

(6)

$$\vec{a} \times \vec{b} = 0$$

$$ab \sin \theta = 0$$

$$\Rightarrow \theta = 0$$

$$\vec{a} \cdot \vec{b} = ab \cos \theta$$

$$= ab \cos 0$$

$$= ab$$



⑥

(16) (i) (ii)

$$\text{let } \vec{A} = 2\hat{i} - 3\hat{j} + 6\hat{k}; \quad \vec{B} = 6\hat{i} + 3\hat{j} - 2\hat{k}$$

angle between $\vec{A}$ and $\vec{B}$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{(2\hat{i} - 3\hat{j} + 6\hat{k}) \cdot (6\hat{i} + 3\hat{j} - 2\hat{k})}{\sqrt{2^2 + (-3)^2 + 6^2} \sqrt{6^2 + 3^2 + (-2)^2}}$$

$$\cos \theta = \frac{2 \cdot 6 + (-3)(3) + 6(-2)}{\sqrt{49} \sqrt{49}}$$

$$\cos \theta = \frac{12 - 9 - 12}{49} = -\frac{9}{49}$$

(ii)

unit vector $\perp$ to $\vec{A}$ and $\vec{B}$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}; \quad \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 6 \\ 6 & 3 & -2 \end{vmatrix}$$

$$= \hat{i} [-3 \times -2 - 6 \times 3] - \hat{j} [2 \times -2 - 6 \times 6] + \hat{k} [2 \times 3 - 6 \times (-3)]$$

$$= \hat{i} [6 - 18] - \hat{j} [-4 - 36] + \hat{k} [6 + 18]$$

$$= -12\hat{i} + 40\hat{j} + 24\hat{k} = 4(-3\hat{i} + 10\hat{j} + 6\hat{k})$$

$$|\vec{A} \times \vec{B}| = 4\sqrt{(-3)^2 + 10^2 + 6^2} = 4\sqrt{9 + 100 + 36} = 4\sqrt{145}$$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{4(-3\hat{i} + 10\hat{j} + 6\hat{k})}{4\sqrt{145}}$$

$$\hat{n} = \frac{-3\hat{i} + 10\hat{j} + 6\hat{k}}{\sqrt{145}}$$

(17)

$$\text{let } \vec{A} = \hat{i} + 4\hat{j} \quad ; \quad \vec{B} = 2\hat{i} + 3\hat{j}$$

$$\text{Area } \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 0 \\ 2 & 3 & 0 \end{vmatrix}$$

$$= \hat{i}[4 \times 0 - 3 \times 0] - \hat{j}(1 \times 0 - 2 \times 0) + \hat{k}[3 - 8]$$

$$= \hat{i}(0) - \hat{j}(0) - 5\hat{k} = -5\hat{k}$$

$$|\vec{A} \times \vec{B}| = 5 \text{ units}$$

(18)

$$\text{let } \vec{A} = 2\hat{i} - 2\hat{j} + c\hat{k} \quad ; \quad \vec{B} = 3\hat{i} + 6\hat{j} + 4\hat{k}$$

Given $\vec{A}$ and $\vec{B}$ are perpendicular to each

$$\text{other } \vec{A} \cdot \vec{B} = 0$$

$$(2\hat{i} - 2\hat{j} + c\hat{k}) \cdot (3\hat{i} + 6\hat{j} + 4\hat{k}) = 0$$

$$= 2 \times 3 + (-2) \times 6 + c \times 4 = 0$$

$$= 6 - 12 + c = 0$$

$$\Rightarrow c = 6$$