9thclass WS-15 [Torque & Angular momentum]

Task

① Given
$\alpha = 5 \text{ rad/s}^2$
$I = 5 \text{ kg m}^2$
$\tau = I\alpha$
$\Rightarrow \tau = 5 \times 5$
$= 25 \text{ Nm}$

② Given $\tau = 100 \text{ Nm}$
$\alpha = 2 \text{ rad/s}^2$
$\tau = I\alpha$
$\Rightarrow 100 = I \times 2$
$\Rightarrow I = 50 \text{ kg m}^2$

③ Given $\alpha = 20 \text{ rad/s}^2$
$\omega_0 = 0$ $\omega = 100 \text{ rad/s}$
$\omega = \omega_0 + \alpha t$ $\tau = I\alpha$
$\Rightarrow 100 = 0 + 20t$ $\Rightarrow 40 = 2\alpha$
$\Rightarrow 100 = 20t$ $\Rightarrow \alpha = 20 \text{ rad/s}^2$
$\Rightarrow t = 5 \text{ sec}$

④ $\omega = 6000 \text{ rpm}$
$= 6000 \times \frac{2\pi}{60} \text{ rad/sec.}$
$\omega = 200\pi \text{ rad/sec.}$
$P = 2\pi k\omega = 2\pi \times 10^3 \omega$
$P = \tau\omega$
$\Rightarrow 2\pi \times 10^3 = \tau \times 200\pi$
$\Rightarrow \tau = 10 \text{ Nm}$

⑤ $\omega_0 = 0$; $t = 5 \text{ sec}$; $\omega = 60 \text{ rad/sec}$ $r = 10 \text{ cm} = 10^{-1} \text{ m}$
$\omega = \omega_0 + \alpha t$ $m = 2 \text{ kg}$ $I = \frac{1}{2} mr^2$
$\Rightarrow 60 = 0 + \alpha \times 5$ $\tau = I\alpha$
$\Rightarrow 60 = 5\alpha$ $= \frac{1}{2} MR^2 \alpha$
$\Rightarrow \alpha = 12 \text{ rad/s}^2$ $= \frac{1}{2} \times 2 \times (10^{-1})^2 \times 12$
$= 12 \times 10^{-2} = 0.12 \text{ Nm}$

⑥ Given $\frac{r_1}{r_2} = \frac{2}{1}$
$K.E = \frac{L^2}{2I}$
$\Rightarrow 2K.E\, I = L^2$
$\Rightarrow 2K.E \frac{1}{2} MR^2 = L^2$
K.E, M are constants
So $L^2 \propto R^2$
$\Rightarrow L \propto R$
$\Rightarrow \frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{2}{1}$

⑦ $D = 0.2 \text{ m}$; $m = 5 \text{ kg}$; $\omega = 2 \text{ rad/sec}$
$R = \frac{D}{2} = 0.1 = 10^{-1} \text{ m}$
$L = I\omega$
$\Rightarrow L = \frac{2}{5} MR^2 \omega$
$= \frac{2}{5} \times 10 \times (10^{-1})^2 \times 2$
$\Rightarrow L = \frac{4}{5} \times 10 \times 10^{-2}$
$= 0.8 \times 10^{-1}$
$= 0.08 \text{ kg m}^2/\text{sec.}$

⑧ Given $\omega = \text{constant}$
$r_1 = r$; $r_2 = \frac{r}{2}$
$L_1 = L \rightarrow L_2 = ?$
$L = I\omega = mr^2\omega$
$L \propto r^2$
$\Rightarrow \frac{L_1}{L_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{r}{\frac{r}{2}}\right)^2$
$\Rightarrow \frac{L}{L_2} = 4 \Rightarrow L_2 = \frac{L}{4}$

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(9) $m = 1\,kg$ ; $r = 1m$

$\omega = 4$ rev/sec.

$\omega = 4 \times 2\pi$ rad/sec.

$L = I\omega = mr^2\omega$

$= 1 \times 1^2 \times 4 \times 2\pi$

$L = 8\pi$ kgm$^2$/sec

(10) $I = 10\,kgm^2$

$\tau = I\alpha = 10^2\left[\frac{1}{5\pi}\right]$

$\tau = \frac{2}{\pi}$ N m.

Given $\omega = \frac{2}{\pi}$ rps $= \frac{2}{\pi} \times 2\pi = 4$ rad/s

$\omega^2 - \omega_0^2 = 2\alpha\theta$ $\quad \theta = 20$ rot

$\Rightarrow 0^2 - 4^2 = 2\alpha(40\pi)$ $\quad = 20 \times 2\pi$ rad

$= 40\pi$ rad

$\Rightarrow -16 = 80\pi\alpha$

$\Rightarrow \alpha = \frac{-16}{80\pi} = \frac{-1}{5\pi}$ rad/s$^2$.

(16) (i) Given

$\omega_0 = 300$ rpm $= \frac{300 \times 2\pi}{60}$

$\omega_0 = 10\pi$ rad/sec

$r = \frac{1}{2}$ m; $m = 50$ kg; $t = 5$ sec.

$\omega = 0$.

$I = \frac{1}{2}mr^2 = \frac{1}{2} \times 50 \times (\frac{1}{2})^2$

$\Rightarrow \frac{1}{8}50 = \frac{50}{8}$ kg m$^2$

(ii) $\omega = \omega_0 + \alpha t$

$0 = 10\pi + \alpha \times 5$

$\Rightarrow -10\pi = 5\alpha$

$\Rightarrow \alpha = -2\pi$ rad/sec

(iii) $\tau = I\alpha$

$= \frac{50}{8} \times (2\pi)$

$= \frac{25\pi}{2}$ Nm.

(17) Given, $r = 2m$; $m = 50$ kg

$t = 5$ sec $\quad \omega = 0$

$\tau = I\alpha$

$\Rightarrow \tau = \frac{1}{2}Mr^2\alpha$

$\Rightarrow \tau = \frac{1}{2} \times 50 \times 2^2 \times (2\pi)$

$= 200\pi$ Nm.

(18) Given $m = 1gm = 10^{-3}$ kg.

$r = 1m$, $v = 1$ m/sec

$\vec{L} = mvr$

$= 10^{-3} \times 1 \times 1$

$\vec{L} = 10^{-3}$ kgm$^2$/sec

$\omega_0 = 300$ rpm $= \frac{300 \times 2\pi}{60}$

$\omega_0 = 10\pi$ rad/sec

From $\omega = \omega_0 + \alpha t$

$\Rightarrow 0 = 10\pi + \alpha \times 5$

$\Rightarrow -10\pi = 5\alpha$

$\Rightarrow \alpha = -2\pi$ rad/s$^2$.

L-Task

(1) $I = \frac{2}{5}MR^2$ ; $d\omega = 2\omega - \omega = \omega$

$t = 3$ sec.

$\tau = I\alpha$.

$= I\frac{d\omega}{dt}$

$= \frac{I\omega}{3}$

(2) $d\omega = \omega - \omega_0$

$= 4\omega - 2\omega$

$= 2\omega$

$t = 3$ sec

$\tau = I\alpha$

$= \frac{ml^2}{3}\frac{d\omega}{dt} =$

$I = \frac{ml^2}{3}$ → M.O.I of rod about an axis Passing Through one end

$\tau = \frac{ml^2}{3}\frac{2\omega}{3}$

$\tau = \frac{2Ml^2\omega}{9}$

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(3) $L_1 = L$ ; $t = 2\text{sec}$

$L_2 = \frac{4L}{3}$

$\tau = \frac{dL}{dt} = \frac{L_2 - L_1}{t}$

$\tau = \frac{\frac{4L}{3} - L}{2} = \frac{\frac{L}{3}}{2}$

$\tau = \frac{L}{6}$

(4) $m = 16\text{ kg}$ ; $r = 25 \times 10^{-2}\text{ m}$

$\omega_0 = 0 \rightarrow \omega = 8\pi\text{ rad/sec}$

$t = 8\text{ sec}$

$I = \frac{mR^2}{2}$ ; $\alpha = \frac{\omega - \omega_0}{t}$

$\tau = I\alpha = \frac{MR^2}{2}\left(\frac{\omega - \omega_0}{t}\right)$

$\tau = \frac{16 \times (25 \times 10^{-2})^2}{2}\left(\frac{8\pi - 0}{8}\right)$

$= 8 \times 625 \times 10^{-4} \times \pi$

$\Rightarrow 0.5\pi = \frac{\pi}{2}\text{ Nm}$

(5) Given $m = 50\text{ kg}$

$\vec{v} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ ; $\vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$L = m(\vec{v} \times \vec{r})$

$\vec{v} \times \vec{r} = \begin{vmatrix} i & j & k \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix}$

$\Rightarrow i(4 \times 4 - 5 \times 3) - j(3 \times 4 - 5 \times 2) + k(3 \times 3 - 4 \times 2)$

$\Rightarrow \hat{i} - 2\hat{j} + \hat{k}$

$\therefore L = 50(\hat{i} - 2\hat{j} + \hat{k})$

$\Rightarrow 50\hat{i} - 100\hat{j} + 50\hat{k}$

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(6) Given

$L_S = L_H$

$L = I\omega$

$\Rightarrow \omega \propto \frac{1}{I}$

$\frac{\omega_S}{\omega_H} = \frac{I_H}{I_S} = \frac{\frac{2}{3}MR^2}{\frac{2}{5}MR^2}$

$= \frac{5}{3}$

(7) Given $K.E_S = K.E_H$

$T_S = T_H$

Both are brought to rest their final angular velocities are zero. $\omega_0^2 = 2\alpha\theta$.

$\Rightarrow \theta = \frac{\omega_0^2}{2\alpha}$

$\tau = I\alpha \Rightarrow I = \frac{\tau}{\alpha}$

$K.E = \frac{1}{2} I \omega_0^2$

$= \frac{1}{2}\frac{\tau}{\alpha} 2\alpha\theta$

$K.E = \tau\theta$

Since K.E and $\tau$ are same for both solid and hallow sphere.

$\therefore \frac{\theta_S}{\theta_H} = 1$

(8) $\tau = 100\text{ Nm}$ ; $I = 20\text{ kg m}^2$

$\omega = ?$ $t = 3\text{ sec}$

$\tau = I\alpha$. $\alpha = \frac{\omega}{t}$

$\Rightarrow 100 = 20\frac{\omega}{3}$ $= \frac{\omega}{3}$

$5 = \frac{\omega}{3} \Rightarrow \omega = 5 \times 3$

$= 15\text{ rad/sec}$.

(9) $\tau = 1000\text{ Nm}$

$P = 10\text{ k}\omega = 10 \times 10^3 \omega$

$P = \tau\omega$

$\Rightarrow 10^4 = 1000\omega$

$\Rightarrow \omega = 10\text{ rad/sec}$

(10) $K_1 = K$ $\omega_2 = 2\omega_1$.

$K_2 = \frac{K}{2}$.

$K.E = \frac{1}{2} L\omega$

$\Rightarrow \frac{K_1}{K_2} = \frac{L_1}{L_2}\frac{\omega_1}{\omega_2}$

$\Rightarrow \frac{K}{\frac{K}{2}} = \frac{L_1}{L_2}\frac{\omega_1}{2\omega_1}$

$\Rightarrow 2 = \frac{L_1}{L_2} \times \frac{1}{2}$

$\Rightarrow \frac{L_2}{L_1} = \frac{1}{4}$

(16) $R_S = R_H$ ; $L_S = L_H$ ; $\omega_S = \omega_H$

$L = I\omega$

$\frac{2}{5}M_S R_S^2 = \frac{2}{3}M_H R_H^2$

$\Rightarrow \frac{M_S}{5} = \frac{M_H}{3} \Rightarrow \frac{M_H}{M_S} = \frac{3}{5}$

(17) $R_1 = R$ ; $R_2 = \frac{R}{2}$

$I \propto \frac{1}{\omega}$ $\omega = \frac{2\pi}{T}$

$MR^2 \propto \frac{1}{\omega}$

$R^2 \propto T$ $\Rightarrow \left(\frac{R}{\frac{R}{2}}\right)^2 = \frac{T_1}{T_2}$ $\Rightarrow T_2 = \frac{T_1}{4}$

$\left(\frac{R_1}{R_2}\right)^2 = \frac{T_1}{T_2}$ $\Rightarrow 4 = \frac{T_1}{T_2}$ $= \frac{24}{4} = 6\text{ hrs}$

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(15) (i) Given

$\omega_0 = 0$; $\omega = 600$ rpm

$= \frac{600}{60} \times \frac{2\pi}{60} = 20\pi$ rad/sec

$t = 1\text{min} = 60$ sec.; $M = 10$ kg

$r = 0.4$ m.

$I = \frac{1}{2} M r^2 = \frac{1}{2} \times 10 \times (0.4)^2$

$= 5 \times 0.16 = 0.8$ kgm$^2$

(ii) $\omega = \omega_0 + \alpha t$

$\Rightarrow 20\pi = 0 + \alpha \times 60$

$\Rightarrow 20\pi = 60\alpha$

$\Rightarrow \alpha = \frac{\pi}{3}$ rad/s$^2$

(iii) $\tau = I\alpha$

$= 0.8 \times \frac{\pi}{3} = 0.84$ Nm



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