9th class

MATHEMATICS

REAL NUMBERS

TEACHING TASK

JEE MAINS LEVEL QUESTIONS:

① Let $x = 1.142857$ —— ①

Multiply both side with 10,00,000

$1000000x = 1142857.142857$ —— ②

Substract eqn. ① from ②

$$\begin{array}{r} 1000000x = 1142857.142857 \\ x = 1.142857 \\ \hline 999999 = 1142856.00000 \end{array}$$

$$x = \frac{1142856}{999999} = 1.142$$

$\therefore \frac{8}{7}$ is 1.142 ANS: D

(2) $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \cdots + \frac{1}{\sqrt{99}+\sqrt{100}} =$

$\frac{1}{\sqrt{n}+\sqrt{n+1}} = (\sqrt{(n+1)}-\sqrt{n})/((\sqrt{n}+\sqrt{(n+1)})(\sqrt{(n+1)}-\sqrt{n}))$

Simplyfing, we get

$(\sqrt{(n+1)}-\sqrt{n})/(n+1-n) = \sqrt{(n+1)}-\sqrt{n}$

Now, add up the terms:

$\cancel{(\sqrt{2}-\sqrt{1})} + \cancel{(\sqrt{3}-\sqrt{2})} + \cdots + \cancel{\sqrt{100}-\sqrt{99}}$

leaving only the first and last terms

$\sqrt{2}-\sqrt{1}+\sqrt{100}-\sqrt{99} = \sqrt{100}-\sqrt{1} = 10-1=9.$

Ans: B

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(3) $\frac{1}{11} = 0.090909\cdots$

$\frac{1}{11} = 0.\overline{09}$ Ans : A

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(4) $x = 7+4\sqrt{3}, y = 7-4\sqrt{3}$

$\frac{1}{1+x} + \frac{1}{1+y}$

$= \frac{1}{1+7+4\sqrt{3}} + \frac{1}{1+7-4\sqrt{3}}$

$= \frac{1}{8+4\sqrt{3}} + \frac{1}{8-4\sqrt{3}}$

$= \frac{(8-4\sqrt{3})+8+4\sqrt{3}}{(8+4\sqrt{3})(8-4\sqrt{3})}$

$\Rightarrow \frac{8-\cancel{4\sqrt{3}}+8+\cancel{4\sqrt{3}}}{8^2-(4\sqrt{3})^2}$

$\Rightarrow \frac{8+8}{64-16\times 3} \Rightarrow \frac{16}{64-48}$

$\Rightarrow \frac{\cancel{16}^{1}}{\cancel{16}_{1}} = 1$

Ans: B

(5) $\sqrt{12+\sqrt{12+\sqrt{12+\sqrt{12 \ldots \infty}}}} =$

$\sqrt{n+\sqrt{n+\sqrt{n+\sqrt{n+\sqrt{n \ldots \infty}}}}} = \frac{(2n)^2}{4}$

$\Rightarrow \frac{2\times \cancel{12}^{2}}{\cancel{6}_{1}} = 2\times 2 = 4.$ Ans: D

(6) The product of two rational numbers can be rational or irration

Example: $\sqrt{2}\times\sqrt{2} = 2$ Ans: B

$\sqrt{2}\times\sqrt{3} = \sqrt{6}.$

(7) $2^x \times 5^x = 10^x$, and 10^x always ends in 0, never 5.

Therefore, there is no value of x that satisfies the condition. Ans: D

(8) $\sqrt{11+\sqrt{11+\sqrt{11+\sqrt{11 \ldots \infty}}}} = \frac{1+\sqrt{1+4n}}{2}$

$P \Rightarrow \frac{1+\sqrt{1+44}}{2} \Rightarrow \frac{1+\sqrt{45}}{2} = \frac{1+\sqrt{9\times 5}}{2} = \frac{1+3\sqrt{5}}{2}$

$Q = \sqrt{11-\sqrt{11-\sqrt{11-\sqrt{11}}}} \ldots = \frac{1\cancel{+}\sqrt{1+4n}}{2} = \frac{1-\sqrt{1+4\times 11}}{2} = \frac{1-\sqrt{45}}{2} = \frac{1-3\sqrt{5}}{2}$

$P-Q = \frac{1+3\sqrt{5}}{2} - \left(\frac{1-3\sqrt{5}}{2}\right) \Rightarrow \frac{1+3\sqrt{5}}{2} - \frac{1+3\sqrt{5}}{2} \Rightarrow \frac{\cancel{1}+3\sqrt{5}-\cancel{1}+3\sqrt{5}}{2}$

$\Rightarrow \frac{3\sqrt{5}+3\sqrt{5}}{2} = \left(\frac{\cancel{2}(3\sqrt{5})}{\cancel{2}}\right) = 3\sqrt{5}$

Ans: No correct option

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(9) $x = \sqrt{2 \times \sqrt[3]{4 \times \sqrt{2 \times \sqrt{4 \times \sqrt[3]{4 \cdots}}}}}$

$= \sqrt{2 \times \sqrt[3]{2^2 \times \sqrt{2 \times \sqrt{2^2 \times \sqrt[3]{2^2 \cdots}}}}}$

Consider the number which is in the base

$= 2 \times 2^2 \times 2 \times 2^2 \times 2^2$

$\therefore$ 2 is the answer.

Ans: A

(10) $\sqrt{a + \sqrt{a + \sqrt{a + \cdots \infty}}}$

Formula for the above problem

$\frac{1 + \sqrt{1+4a}}{2}$ Ans: B

JEE ADVANCED LEVEL QUESTIONS

Multicorrect answer type:

(11) $\sqrt[4]{216} = \sqrt[4]{2^3 \times 3^3}$

$\Rightarrow 2^3 \times 3^3 \times \sqrt[4]{2 \times 3}$

To rationalize this, we need to multiply by a factor that will eliminate the fourth root.

Rationalizing factor $= \sqrt[4]{2\times3} \times \dfrac{\sqrt[4]{6}}{\sqrt[4]{6}}$

$\Rightarrow \dfrac{\sqrt[4]{2\times3}}{\sqrt[4]{6}} = \dfrac{\sqrt[4]{\cancel{6}3}}{\sqrt[4]{6}} \Rightarrow 2 \times \dfrac{\sqrt[4]{6}}{\sqrt[4]{6}}$ (Since $\sqrt[4]{6} = 2$)

$\Rightarrow 2\sqrt[4]{6}.$

Rationalizing factor $= \sqrt[4]{(2\times3)} \times \dfrac{\sqrt[4]{6}}{\sqrt[4]{6}}$

$\Rightarrow \sqrt[4]{(2\times3)}/4\sqrt{6}$

$\Rightarrow \dfrac{\sqrt[4]{6}}{\sqrt[4]{6}} \Rightarrow 3 \times \dfrac{\sqrt[4]{6}}{\sqrt[4]{6}}$ (Since $\sqrt[4]{6} = 3$)

$\Rightarrow 3\sqrt[4]{6}.$ Ans: B and D

(12) $\frac{2\sqrt[3]{3}}{\sqrt[3]{25}} = \frac{2\sqrt[3]{3}}{\sqrt[3]{5^2}} \Rightarrow 2\sqrt[3]{3} \times (\sqrt[3]{5^2})^{-1}$

$\Rightarrow (2\sqrt[3]{3}) \times \frac{1}{\sqrt[3]{25}} \Rightarrow \left(\frac{2}{5}\right) \times \sqrt[3]{(3/5^2)}$

$\Rightarrow \left(\frac{2}{5}\right) \times \frac{(\sqrt[3]{3})}{(5^{2/3})} \Rightarrow \frac{(2\times\sqrt[3]{3})}{5\times 5^{2/3}} \Rightarrow \frac{(2\times\sqrt[3]{3})}{(5^{4/3})}$

$\Rightarrow \left(\frac{2}{5^{4/3}}\right) \times \sqrt[3]{3} \Rightarrow \left(\frac{2}{5} \times 5^{1/3}\right) \times \sqrt[3]{3} \Rightarrow \left(\frac{2}{5}\right) \times (\sqrt[3]{5^3 \times 3})$

$\Rightarrow \left(\frac{2}{5}\right) \times (\sqrt[3]{125\times 3}) = \frac{2}{5} \times \sqrt[3]{375} \Rightarrow \frac{2}{5} \times 5 \times \sqrt[3]{3}$

$\Rightarrow 2\times\sqrt[3]{3} \Rightarrow 2\times\sqrt[3]{3\times 5} \Rightarrow 2\times\sqrt[3]{15}$.

and option D: $\sqrt[3]{15}$, $\frac{2}{5}$ can be simplified to

$\frac{2\times\sqrt[3]{15}}{5} \times \sqrt[3]{15} = \sqrt[3]{15}$ // Ans: B and D

STATEMENT TYPE:

(13) $\frac{68}{2^2\times 5^2\times 7^2} = \frac{68}{(4\times 25\times 49)} = \frac{68}{4900}$, this rational number has a terminating decimal expansion because the denominator 4900 is of the form $2^m \times 5^m$.

Statement-II: If P/q is a rational number, where q is of the form $2^m \times 5^m$, then P/q has a terminating decimal expansion. This is because the prime factors of the denominator are only 2 and 5, which are the prime factors of 10.

Statement-I is false, Statement-II is true.

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(14) Statement-1: To rationalize the denominator of $\frac{1}{\sqrt{5}+\sqrt{3}-\sqrt{8}}$, we need to multiply the numerator and denominator by the conjugate of the denominator, which is $\sqrt{5}+\sqrt{3}+\sqrt{8}$.

After multiplying and simplifying we get

$\frac{5\sqrt{3}+3\sqrt{5}+2\sqrt{30}}{30}$, so, this statement is true.

Statement-II: The rationalizing factor of $a+\sqrt{6}$ is indeed $a-\sqrt{6}$, because multiplying these two expressions eliminates the radical.

$$(a+\sqrt{6})(a-\sqrt{6}) = a^2 - \cancel{a\sqrt{6}} + \cancel{a\sqrt{6}} - (\sqrt{6})^2 = a^2 - 6$$

$\therefore$ This is a rational number, so, the rationalizing factor is correct.

Both statements are true.

COMPREHENSION TYPE:

(15) The given expression is $\sqrt[3]{2}+\sqrt[3]{3}$.

Using the given comprehension, we can see that:

$(\sqrt[3]{2}+\sqrt[3]{3}) \times (2^{2/3} - 2^{1/3} \times 3^{1/3} + 3^{2/3})$

$\Rightarrow (2^{1/3}+3^{1/3}) * (2^{2/3} - 2^{1/3} \times 3^{1/3} + 3^{2/3})$

$\Rightarrow 2+3 = 5$ (is a rational number)

Ans: C

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(16) $4^{2/3} + 4^{1/3} . 5^{1/3} + 5^{2/3}$

Using the given comprehension, we can see that:

$(4^{2/3} + 4^{1/3} \times 5^{1/3} + 5^{2/3}) \times (4^{1/3} + 5^{1/3})$

$\Rightarrow (4^{1/3} + 5^{1/3}) \times (4^{2/3} + 4^{1/3} \times 5^{1/3} + 5^{2/3})$

$\Rightarrow 5+4 = 9$ is a rational number

Ans: C

COMPREHENSION-II:

(17) $\frac{23}{175} = 0.1\overline{31}$

It is a non-terminating decimal.

Ans: B

(18) $\frac{73}{40} = 1.825$

It is a terminating decimal.

Ans: A

INTEGER ANSWER TYPE:

(19) $\frac{2-\sqrt{3+x}}{x-1}$ => Rationalize the numerator is

$2+\sqrt{3+x}$.

$\Rightarrow (2-\sqrt{3+x})(2+\sqrt{3+x}) = 4 + \cancel{2\sqrt{3+x}} - \cancel{2\sqrt{3+x}} - (\sqrt{3+x})^2$

$= 4-3+x = 1+x$

(20) $\frac{3+\sqrt{5}}{3-\sqrt{5}} = a+b\sqrt{5}$

Multiply the numerator and denominator by the conjugate of the denominator.

$\frac{(3+\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{(9+6\sqrt{5}+5)}{(9-5)} = \frac{14+6\sqrt{5}}{4}$

$a+b\sqrt{5} = \frac{14+6\sqrt{5}}{4}$

comparing coefficients, we get:

$a = \frac{14}{4} = 7/2$; $b = \frac{6}{4} = 3/2$

$a+b = \frac{7}{2} + \frac{3}{2} = \frac{10}{2} = 5$

MATRIX MATCHING TYPE:

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(i) $\frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}} \Rightarrow \frac{(2+\sqrt{5})^2}{(2-\sqrt{5})(2+\sqrt{5})} = \frac{2^2+2\times2\times\sqrt{5}+(\sqrt{5})^2}{2^2-(\sqrt{5})^2}$

$\Rightarrow \frac{4+4\sqrt{5}+5}{4-5} = \frac{9+4\sqrt{5}}{-1} \Rightarrow -9-4\sqrt{5}$

Ans: t

(ii) $\frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{(3+\sqrt{5})^2}{(3-\sqrt{5})(3+\sqrt{5})} \Rightarrow \frac{3^2+2\times3\times\sqrt{5}+(\sqrt{5})^2}{3^2-(\sqrt{5})^2}$

$\Rightarrow \frac{9+6\sqrt{5}+5}{9-5} = \frac{14+6\sqrt{5}}{4} = \frac{\cancel{14}^{7}}{\cancel{4}_{2}} + \frac{\cancel{6}^{3}\sqrt{5}}{\cancel{4}_{2}} = \frac{7}{2}+\frac{3}{2}\sqrt{5}$

Ans: P

(iii) $\frac{3+2\sqrt{3}}{5-2\sqrt{3}} \times \frac{5+2\sqrt{3}}{5+2\sqrt{3}} = \frac{(3+2\sqrt{3})(5+2\sqrt{3})}{(5-2\sqrt{3})(5+2\sqrt{3})} = \frac{3\times5+3\times2\sqrt{3}+5\times2\sqrt{3}+2\sqrt{3}\times2\sqrt{3}}{(5)^2-(2\sqrt{3})^2}$

$= \frac{15+6\sqrt{3}+10\sqrt{3}+4\times3}{25-4\times3} = \frac{27+16\sqrt{3}}{13}$ Ans: q

(iv) $\frac{1}{3\sqrt{2}-3\sqrt{3}} \times \frac{3\sqrt{2}+3\sqrt{3}}{3\sqrt{2}+3\sqrt{3}} = \frac{3\sqrt{2}+3\sqrt{3}}{(3\sqrt{2}-3\sqrt{3})(3\sqrt{2}+3\sqrt{3})}$

$= \frac{3\sqrt{2}+3\sqrt{3}}{9\times2+\cancel{(3\sqrt{2})(3\sqrt{3})}-\cancel{(3\sqrt{3})(3\sqrt{2})}-(3\sqrt{3})^2}$

$= \frac{3\sqrt{2}+3\sqrt{3}}{18-9\times3} = \frac{3\sqrt{2}+3\sqrt{3}}{18-27} = \frac{3\sqrt{2}+3\sqrt{3}}{-9} = \frac{\sqrt{2}+\sqrt{3}}{-3}$

Ans: No correct option.

(22) $x = 2^{1/3} - 2$

(i) $x = \frac{8}{3} - 2 \Rightarrow \frac{8-6}{3} = \frac{2}{3}$

$x^3 + 6x^2 + 12x \Rightarrow \left(\frac{2}{3}\right)^3 + 6\left(\frac{2}{3}\right)^2 + 12\left(\frac{2}{3}\right)$

$\Rightarrow \left(\frac{8}{27}\right) + 6\left(\frac{4}{9}\right) + \frac{24}{3} \Rightarrow \frac{8}{27} + \frac{24}{9} + 8$

$\Rightarrow \frac{8}{27} + \frac{8}{3} + 8 \Rightarrow \frac{8+72+216}{27} = \frac{296}{27}$

Ans: No correct option

(ii) $y = 3^{1/3} + 3$

$y = \frac{10}{3} + 3 \Rightarrow \frac{10+9}{3} = \frac{19}{3}$

$y^3 - 9y^2 + 27y \Rightarrow \left(\frac{19}{3}\right)^3 - 9\left(\frac{19}{3}\right)^2 + \cancel{27}^{9}\left(\frac{19}{\cancel{3}_{1}}\right)$

$\Rightarrow \frac{6859}{27} - 9\left(\frac{361}{27}\right) + 171$

$\Rightarrow \frac{6859}{27} - \frac{3249}{27} + 171 = \frac{6859 - 3249 + 171 \times 27}{27}$

$\Rightarrow \frac{6859 - 3249 + 4617}{27} = \frac{11476 - 3249}{27} = \frac{8227}{27}$

Ans: No correct option.

(iv) $x = \frac{1}{2-\sqrt{3}}$ Rationalize the denominator by multiplying the numerator and denominator.

$$\Rightarrow \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \Rightarrow \frac{2+\sqrt{3}}{2^2-(\sqrt{3})^2} = \frac{2+\sqrt{3}}{4-3} = \frac{2+\sqrt{3}}{1} = 2+\sqrt{3}$$

Substitute x value in the following expression.

$$x^3-2x^2-7x+10 = (2+\sqrt{3})^3-2(2+\sqrt{3})^2-7(2+\sqrt{3})+10$$

$$\Rightarrow (2+\sqrt{3})^3 = 8+12\sqrt{3}+18+3\sqrt{3} \Rightarrow 26+15\sqrt{3}$$

$$(2+\sqrt{3})^2 = 4+4\sqrt{3}+3 \Rightarrow 7+4\sqrt{3}$$

$$\Rightarrow (26+15\sqrt{3})-2(7+4\sqrt{3})-7(2+\sqrt{3})+10$$

$$= 26+\cancel{15\sqrt{3}}-14-\cancel{8\sqrt{3}}-14-\cancel{7\sqrt{3}}+10$$

$$= 36-28 = 8.$$

Ans: 8

(iii) $x = 1+5^{1/3}+5^{2/3}$

$$x = 1+\frac{16}{3}+\frac{17}{3} \Rightarrow 1+\frac{33}{3} = 1+ \;\; = 12$$

Substitute x value in the following expression

$$\Rightarrow x^3-3x^2-12x+6 = (12)^3-3(12)^2-12(12)+6$$

$$\Rightarrow 1728-3(144)-144+6$$

$$\Rightarrow 1728-432-144+6 \Rightarrow 1734-576$$

$$= 1158$$

Ans: No correct option.

STUDENT'S TASK

Conceptual understanding questions:

① Through options

A) $\sqrt{2}$ is an irrational number, as it cannot be expressed as a finite decimal or fraction.

B) 3.1413721 is a rational approximation of the irrational number π. While this specific value is rational, pi itself is an irrational number.

C) $3\sqrt[3]{5}$ is an irrational, as the square root of 5 is an irrational number

Ans: D

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② $\frac{a}{c} = \frac{b}{d} \Rightarrow ad = bc$

Now, we can add ac and bd to both sides

$ad + ac = bc + bd$

factoring out a and b:

$a(d+c) = b(c+d)$

dividing both sides by $(a-c)(c-d)$

$$\frac{a+b}{a-c} = \frac{c+d}{c-d}$$

Ans: A

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③ Through the options

A) $-\frac{3}{2}$ is a rational number, as it is a finite decimal or fraction.

B) 1.12411242.. is a repeating decimal, which means it can be expressed as a fraction, making it a rational number.

C) −7 is an integer, and all integers are rational number. Ans: D

④ Through options

A): Closure: The Sum, difference, product and quotient of two rational numbers are always rational numbers.

B: Commutative: The order of rational number does not change the result of addition and multiplication. $a+b=b+a$, $a\times b=b\times a$.

C: Distributive: The distributive property holds for rational numbers.

$$a\times(b+c)=a\times b+a\times c$$

Ans: D

(5) Through options:

A) π (pi) is an irrational number, as it is a non-repeating and non-terminating decimal.

B) Euler's number (e) is also an irrational number, approximately equal to 2.71828, but it goes on indefinitely without repeating.

C) The Golden Ratio (φ) is an irrational number, approximately equal to 1.61803, but it is a non-repeating and non-terminating decimal.

Ans: D

(6) A Surd is a type of irrational number that cannot be expressed as a finite decimal or fraction, and π (pi) is indeed a surd.

In mathematics, a surd is defined as a root of a number that cannot be expressed as a finite decimal or fraction, such as $\sqrt{3}$, $\sqrt{2}$ or π.

Ans: B

⑦ If $\frac{a}{b}$ is a rational number, then multiplying both the numerator and denominator by a non-zero integer m will result in an equivalent fraction: $\frac{ma}{mb} = \frac{a}{b}$.

B) $\frac{m+a}{m+b} \neq \frac{a}{b}$

Ans: A

C) $\frac{m-a}{m-b} \neq \frac{a}{b}$

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⑧ $\sqrt{a} \times \sqrt{a} \times \sqrt{a} \times \sqrt{a} \ldots \text{n times} = \sqrt{a^n}$

Using the property of exponents that states

$\sqrt{a^n} = (a^{1/2})^n = a^{n/2} = a^{1/2n}$

So, the result of multiplying $\sqrt{a}$ by itself n times is $a^{1/2n}$.

Ans: D

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⑨ $\sqrt[3]{5}$ is an irrational number because it cannot be expressed as a finite decimal or fraction.

→ It is also a surd because it is an expression containing a root that cannot be simplified to a rational number.

Ans: D

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(18) $\sqrt{125}+\sqrt{125}-\sqrt{845}$

$\Rightarrow \sqrt{25\times5}+\sqrt{25\times5}-\sqrt{169\times5}$

$\Rightarrow 5\sqrt{5}+5\sqrt{5}-13\sqrt{5} \Rightarrow 10\sqrt{5}-13\sqrt{5}$

$= -3\sqrt{5}$ Ans: D

JEE MAINS LEVEL QUESTIONS

(1) $\sqrt{3+\sqrt{5}} = 2.88$

Throughoptions

A) $\sqrt{2}+1 = 2.41$

B) $\sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}} = \frac{\sqrt{10}+\sqrt{2}}{2} = 2.28$

C) $\sqrt{\frac{7}{2}}-\sqrt{\frac{1}{2}} = \frac{\sqrt{14}-\sqrt{2}}{2} = 1.16$ Ans: A

D) $\sqrt{\frac{9}{2}}-\sqrt{\frac{3}{2}} = \frac{-\sqrt{6}+3\sqrt{2}}{2} = 0.89$

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(20) $\sqrt{20-\sqrt{20-\sqrt{20-\sqrt{20\cdots\infty}}}}$

here we need to take the number i.e. 20

we need to find out the roots for 20, and difference between roots should be 1.

$\therefore$ 4 and 5 are the roots of 20, 20 are in negative

so take small number i.e. 4

Ans: C

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(3) here, A^3 and A are same for the following

* $1 \Rightarrow 1^3 = 1$
* $5 \Rightarrow 5^3 = 125$ Ans: D
* $6 \Rightarrow 6^3 = 216$.

(4) Since n is an odd natural number, we can write $n = 2k+1$, where k is an integer.

Now, let's examine the expression.

$3^{2n} + 2^{2n}$

$\Rightarrow 3^{(4k+2)} + 2^{(4k+2)}$

$\Rightarrow (3^2)^{(2k+1)} + (2^2)^{(2k+1)}$

$\Rightarrow 9^{(2k+1)} + 4^{(2k+1)}$.

Let's subtract one term from the other.

$9^{(2k+1)} - 4^{(2k+1)}$

$= (9-4) = 5$

Let add the two terms again:

$9^{(2k+1)} + 4^{(2k+1)} = 9+4 = 13$

$\therefore 3^{2n} + 2^{2n}$ is always divisible by 13.

Ans: A

(5) $x = 7-4\sqrt{3}$

$x^2 = (7-4\sqrt{3})^2 = 49-56\sqrt{3}+48$

$= 97-56\sqrt{3}$

Now, let's find $\frac{1}{x}$:

$\frac{1}{x} = \frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{7+4\sqrt{3}}{7^2-(4\sqrt{3})^2} = \frac{7+4\sqrt{3}}{49-48}$

$\Rightarrow \frac{7+4\sqrt{3}}{1} = 7+4\sqrt{3}$

$\frac{x^2+1}{x} \Rightarrow \frac{97-56\sqrt{3}}{7+4\sqrt{3}} = 2.$ Ans: A

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(6)

Given $A^{1/A} = B^{1/B} = C^{1/C} = K$ (where K is a constant)

$A = KA, B = KB, C = KC$

$A^{BC} + B^{AC} + C^{AB} = 729$

substituting $A = KA, B = KB, C = KC$

$(KA)^{BC} + (KB)^{AC} + (KC)^{AB} = 729$

$K^{(BC+AC+AB)}(A^{BC}+B^{AC}+C^{AB}) = 729$

Since $A^{BC}+B^{AC}+C^{AB} = 729$, we get

$K^{A(BC+AC+AB)} = 1$

$BC+AC+AB = 0$ (since $K^0 = 1$)

Now, let's find $A^{1/A}$

$A^{1/A} = K = ABC\sqrt{BC+AC+AB+1}$

$= ABC(\sqrt{1})$

$= ABC\sqrt{9}.$ Ans: D

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(4)

$\frac{3}{7} = 0.4286$

Through options

A) $\frac{4}{9} = 0.44$, $\frac{5}{9} = 0.55$.

B) $\frac{43}{99} = 0.43$, $\frac{9}{99} = 0.09$.

C) $\frac{42}{99} = 0.42$, $\frac{4}{9} = 0.44$

D) $\frac{41}{99} = 0.41$, $\frac{42}{99} = 0.42$

Ans: C.

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(8) $x = \frac{2}{\sqrt{3}-\sqrt{5}}$ $y = \frac{2}{\sqrt{3}+\sqrt{5}}$,

$x+y = \frac{2}{\sqrt{3}-\sqrt{5}} + \frac{2}{\sqrt{3}+\sqrt{5}} \Rightarrow \frac{2(\sqrt{3}+\sqrt{5})+2(\sqrt{3}-\sqrt{5})}{(\sqrt{3}-\sqrt{5})(\sqrt{3}+\sqrt{5})}$

$\Rightarrow \frac{2\sqrt{3}+\cancel{2\sqrt{5}}+2\sqrt{3}-\cancel{2\sqrt{5}}}{(\sqrt{3})^2-(\sqrt{5})^2} = \frac{4\sqrt{3}}{3-5} = \frac{\cancel{4}^{2}\sqrt{3}}{\cancel{-2}_{1}} = -2\sqrt{3}$

Ans: C

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(9) $x = \frac{1}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}} = \frac{5-2\sqrt{6}}{5^2-(2\sqrt{6})^2} = \frac{5-2\sqrt{6}}{25-24} = 5-2\sqrt{6}$

$x^2-10x+1 = (5-2\sqrt{6})^2 - 10(5-2\sqrt{6})+1$

$\Rightarrow 25-20\sqrt{6}-24-50+20\sqrt{6}+1$

$\Rightarrow -48+20\sqrt{6} \Rightarrow 20\sqrt{6}-48 \approx 1$. Ans: A

(10) $x = \frac{1}{\sqrt{3}+2}$

$\frac{1}{x} = \frac{1}{\frac{1}{(\sqrt{3}+2)}} = \frac{\sqrt{3}}{1+2\sqrt{3}}$

$x + \frac{1}{x} = \frac{1}{(\sqrt{3}+2)} + \frac{\sqrt{3}}{(1+2\sqrt{3})} \Rightarrow \frac{(\sqrt{3}+2+2\sqrt{3}+3)}{1(1+2\sqrt{3})}$

$= \frac{(5+3\sqrt{3})}{1+2\sqrt{3}}$

$\left(x+\frac{1}{x}\right)^2 = \left(\frac{5+3\sqrt{3}}{1+2\sqrt{3}}\right)^2 = \frac{25+30\sqrt{3}+27}{1+4\sqrt{3}+12} = \frac{50+30\sqrt{3}}{13+4\sqrt{3}} = 16$

Ans: A

JEE ADVANCED LEVEL QUESTIONS:

Multicorrect answer type:

(11) $\sqrt[x]{3} \times \sqrt[y]{5} \Rightarrow (3)^{1/x} \times (5)^{1/y} = 10125$

$3^{1/x} \times 5^{1/y} = 81 \times 125 \Rightarrow 3^{1/x} \times 5^{1/y} = 3^4 \times 5^3$

comparing both sides $\frac{1}{x} = 4, \frac{1}{y} = 3$

$4x = 1, 3y = 1$

$\Rightarrow 12xy = 4x \times 3y$

$= 1 \times 1 = 1$

Ans: A

(12) Given $a = \sqrt{17} - \sqrt{16} \Rightarrow \sqrt{17} - 4$

$b = \sqrt{16} - \sqrt{15} \Rightarrow 4 - \sqrt{15}$

Now compare a and b:

$a - b = (\sqrt{17} - 4) - (4 - \sqrt{15})$

$= \sqrt{17} - \cancel{4} + \cancel{4} + \sqrt{15} = \sqrt{17} + \sqrt{15}$

$\sqrt{17} > 4 \quad \sqrt{15} > 3,$

$\therefore \sqrt{17} + \sqrt{15} > 4 + 3 \Rightarrow 7$

$a - b > 0.$

Ans: B

Statement Type:

(13) Statement 1: $\frac{3+2\sqrt{3}}{5-2\sqrt{3}} = a+b\sqrt{3}$

To rationalize the denominator, multiply by the conjugate:

$$\frac{(3+2\sqrt{3})}{5-2\sqrt{3}} = \frac{3+2\sqrt{3}}{5-2\sqrt{3}} \times \frac{5+2\sqrt{3}}{5+2\sqrt{3}} = \frac{15+6\sqrt{3}+10\sqrt{3}+12}{25-12}$$

$$\Rightarrow \frac{27+16\sqrt{3}}{13} \Rightarrow \frac{27}{13} + \frac{16\sqrt{3}}{13}$$

$a = \frac{27}{13}$ and $b = \frac{161}{13}$ are correct

Statement-I is true.

Statement-II: If $a+b\sqrt{n} = x+y\sqrt{n}$ then $a=x$ and $b=y$, this statement also true, as the equality of two expressions with the same radical term ($\sqrt{n}$) implies that the corresponding coefficients a and x, b and y are equal.

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(14) Statement 1: $\frac{\sqrt{2}-1}{\sqrt{3}-5} \Rightarrow \frac{1.414-1}{1.732-5} = \frac{0.414}{-3.268} \approx -0.127$

-0.822, so statement-I is false

Statement-II: The conjugate surd of $\sqrt{6}-a$ is indeed $\sqrt{6}+a$. For example, the conjugate of $\sqrt{2}-1$ is $\sqrt{2}+1$, This statement is true.

Comprehension type questions :- 1:

(15) Given $\frac{\sqrt[5]{4}}{\sqrt[5]{50}} \Rightarrow \frac{5\times2=10}{\sqrt{25\times2}=5\sqrt{2}} = \frac{10}{5\sqrt{2}}$.

Now, rationalize the denominator by multiplying the numerator and denominator by $\sqrt{2}$.

$$\frac{10\times\sqrt{2}}{(15\sqrt{2}\times\sqrt{2})} = \frac{10\sqrt{2}}{30} = \frac{\sqrt{2}}{3}$$

$$\frac{\sqrt{2}}{3} = \frac{\sqrt{2}\times\sqrt{2}}{3\times\sqrt{2}} \Rightarrow \frac{2}{3\sqrt{2}} \Rightarrow \frac{2}{\sqrt{18}} \Rightarrow \frac{2}{3\sqrt{2}}.$$

Now, multiply the numerator and denominator by $\sqrt{2}$ again.

$$(2\times\sqrt{2})/(3\sqrt{2}\times\sqrt{2}) = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}.$$

Now multiply the numerator and denominator by 5. $\frac{5\times\sqrt{2}}{5\times3} = \frac{5\sqrt{2}}{15}$

Now, Multiply the numerator and denominator by 16.

$$= \frac{5\sqrt{2}\times16}{15\times16} \Rightarrow \frac{80\sqrt{2}}{240} = \frac{\sqrt[5]{8000}}{80}.$$

ANS: B

(16) The rational form (RF) of $\sqrt[n]{\frac{a}{b}}$ is obtained by multiplying and dividing by $b^{(n+1)}$, which eliminates the radical in the denominator.

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a} \times b^{n-1}}{b \times b^{n-1}} \Rightarrow \sqrt[n]{\frac{a^{(n-1)} \times b^{(n-1)}}{b^{n}}} = \sqrt[n]{\frac{a^{(n-1)}}{b^{n-1}}}.$$

Ans: B

Comprehension-II:

(17) Given : $\sqrt{3}+\sqrt{10}+\sqrt{5}$

Its conjugate is : $\sqrt{3}+\sqrt{10}-\sqrt{5}$

$\Rightarrow (\sqrt{3}) + \sqrt{10} + \sqrt{5}) * (\sqrt{3} + \sqrt{10} - \sqrt{5})$

$= (\sqrt{3})^2 + (\sqrt{10})^2 + (\sqrt{5})^2 + 2(\sqrt{3} \times \sqrt{10}) - 2(\sqrt{3} \times \sqrt{5}) + 2(\sqrt{10} + \sqrt{5})$

$\Rightarrow 3 + 10 + 5 + 2\sqrt{30} + 2\sqrt{15} + 2\sqrt{50}$

$= 18 + 2\sqrt{30} \cancel{+} 2\sqrt{15} + 2\sqrt{50}$ Ans: A

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(18) $\sqrt{3}+\sqrt{10}+\sqrt{5}$, conjugate : $\sqrt{3}+\sqrt{10}-\sqrt{5}$

$\Rightarrow (\sqrt{3}+\sqrt{10}+\sqrt{5})^2 - (\sqrt{5})^2 + 2(\sqrt{3} \times \sqrt{10}) - 2(\sqrt{3} \times \sqrt{5}) + 2(\sqrt{10} + \sqrt{5})$

$= 3 + 10 + 5 + 2\sqrt{30} - 2\sqrt{15} + 2\sqrt{50}$

Ans: B

Integer Answer type questions:

(19) The rationalizing factor is given as $\sqrt[5]{a^2b^3c}$

$\Rightarrow \sqrt[5]{a^2b^3c} = \sqrt[5]{a^mb^nc^p}$

$\Rightarrow$ equalizing both sides

$\cancel{\sqrt[5]{}}a^2b^3c = \cancel{\sqrt[5]{}}a^mb^nc^p \Rightarrow a^2b^3c^1 = a^mb^nc^p$

Both side bases are equal so, power also equal

$\therefore m = 2, n = 3, p = 1$

$\therefore m+n+p = 2+3+1 = 6.$

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(20) $\dfrac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}+\sqrt{6}} = \sqrt{a}+\sqrt{b}-\sqrt{c}$

multiply the numerator and denominator by the conjugate of the denominator. $(\sqrt{2}-\sqrt{3}-\sqrt{6})$

$\dfrac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}+\sqrt{6}} \times \dfrac{\sqrt{2}-\sqrt{3}-\sqrt{6}}{\sqrt{2}-\sqrt{3}-\sqrt{6}} = \dfrac{2\sqrt{12}-2\sqrt{18}-2\sqrt{36}}{2-3-6+2\sqrt{12}-\sqrt{12}+2\sqrt{18}+\sqrt{18}-2\sqrt{36}}$

$\Rightarrow \dfrac{2\sqrt{3}-6-12}{-7+\sqrt{6}} \Rightarrow \dfrac{-16+2\sqrt{3}}{-7+\sqrt{6}} \times \dfrac{-7-\sqrt{6}}{-7-\sqrt{6}}$

$\Rightarrow \dfrac{(112+16\sqrt{6}-14\sqrt{3}-6\sqrt{18})}{49-6} = \dfrac{(112+16\sqrt{6}-14\sqrt{3}-12\sqrt{2})}{43}$

$\Rightarrow \dfrac{112-12\sqrt{2}+16\sqrt{6}-14\sqrt{3}}{43} = \sqrt{a}+\sqrt{b}-\sqrt{c}$

By comparing both side $a=2, b=6, c=3$

$a+b-c = 2+6-3 = 8-3 = 5.$

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Matrix matching type:

(21)

(i) $\frac{2^{5x}}{2^x} = \sqrt[5]{2^{10}}$

Now Simplyfy LHS Side $\frac{2^{5x}}{2^x} = 2^{5x} \cdot 2^{-x} \Rightarrow 2^{5x-x} = 2^{4x}$

Now, equate this to the right-hand side.

$2^{4x} = \sqrt[5]{2^{10}} \Rightarrow 5 \times 2^5 \Rightarrow 5 \times 32 = 160$

Now, equate the exponents:

$4x = 5+10 \Rightarrow 4x = 15$

$x = \frac{15}{4} \Rightarrow \frac{5}{2} \times \frac{3}{3} \Rightarrow \frac{5}{6}$.

(ii) $\frac{\sqrt{50}+\sqrt{75}}{\sqrt{72}+\sqrt{108}} = \frac{\sqrt{25\times2}+\sqrt{3\times25}}{\sqrt{9\times8}+\sqrt{12\times9}} = \frac{5\sqrt{2}+5\sqrt{3}}{3\sqrt{8}+3\sqrt{12}} = \frac{5\sqrt{2}+5\sqrt{3}}{3\sqrt{4\times2}+3\sqrt{4\times3}}$

$\Rightarrow \frac{5\sqrt{2}+5\sqrt{3}}{6\sqrt{2}+6\sqrt{3}} = \frac{5\cancel{(\sqrt{2}+\sqrt{3})}}{6\cancel{(\sqrt{2}+\sqrt{3})}} = \frac{5}{6}$

Ans: P

(iii) $\frac{3-2\sqrt{5}}{6-\sqrt{5}} = a+b\sqrt{c}$

$\frac{3-2\sqrt{5}}{6-\sqrt{5}} \times \frac{6+\sqrt{5}}{6+\sqrt{5}} \Rightarrow \frac{(3-2\sqrt{5})(6+\sqrt{5})}{(6-\sqrt{5})(6+\sqrt{5})} = \frac{18+3\sqrt{5}-12\sqrt{5}-2\times5}{6^2-(\sqrt{5})^2}$

$\Rightarrow \frac{18-9\sqrt{5}-10}{36-5} \Rightarrow \frac{8-9\sqrt{5}}{31} \Rightarrow$ Now equalize terms

$a+b\sqrt{c} = \frac{8-9\sqrt{5}}{31}$; $a+b\sqrt{c} = \frac{8}{31}, \frac{-9\sqrt{5}}{31}$; $a = \frac{8}{31}, b = \frac{-9}{31}$

$a+b = \frac{8}{31} + \left(\frac{-9}{31}\right) = \frac{8}{31} - \frac{9}{31} = \frac{8-9}{31} = \frac{-1}{31}$

Ans: Q

(vi) $\sqrt{13-x\sqrt{10}} = \sqrt{8}+\sqrt{5}$

Squareing on both sides

$13-x\sqrt{10} = (\sqrt{8}+\sqrt{5})^2$

$13-x\sqrt{10} = 8+5+2\sqrt{40}$

$13-x\sqrt{10} = 13+2\sqrt{40}$

$= 13+2\sqrt{10\times4}$

$13-x\sqrt{10} = 13+4\sqrt{10}$

Comparing both side $x = -4$

(22) $P = 7-4\sqrt{3}$; $\frac{P^2+1}{7P}$

(i) $\frac{P^2+1}{7P} = \frac{1}{7}\left(\frac{P^2+1}{P}\right) \Rightarrow \frac{1}{7}\left(P+\frac{1}{P}\right)$

$\Rightarrow \frac{1}{P} = \frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{7+4\sqrt{3}}{49-48} = 7+4\sqrt{3}$

$\Rightarrow \frac{1}{7}\left(P+\frac{1}{P}\right) = \frac{1}{7}\left(7-\cancel{4\sqrt{3}}+7+\cancel{4\sqrt{3}}\right) = \frac{1}{\cancel{7}}\left(\cancel{14}^{2}\right)$

$\therefore \frac{P^2+1}{7P} = 2.$ Ans: S

(ii) $x = 2^{1/3} - 2 \Rightarrow x+2 = 2^{1/3}$

on taking cubes both sides

$(x+2)^3 = (2^{1/3})^3 \Rightarrow x^3+8+6x(x+2) = 2$

$\Rightarrow x^3+6x^2+12x = 2-8 = -6.$ Ans: P

(iii) $\frac{1}{\sqrt{8+2\sqrt{15}}} = \frac{1}{\sqrt{5+3+2\sqrt{15}}}$

$= \frac{1}{\sqrt{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}\cdot\sqrt{3}}} = \frac{1}{\sqrt{(\sqrt{5}+\sqrt{3})^{\cancel{2}}}}$

$\Rightarrow \frac{1}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}-\sqrt{3}}{5-3} = \frac{1}{2}(\sqrt{5}-\sqrt{3})$

Ans: Q

(iv) $\sqrt{7+2\sqrt{6}} + \sqrt{7-2\sqrt{6}}$

$\sqrt{6+1+2\sqrt{6}} + \sqrt{6+1-2\sqrt{6}}$

$\Rightarrow \sqrt{(\sqrt{6}+1)^2} + \sqrt{(\sqrt{6}-1)^2}$

$\Rightarrow \sqrt{6}\cancel{+1} + \sqrt{6}\cancel{-1} = 2\sqrt{6}$

Ans: 8