

Real numbers exercise 1.4

Question 1:

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion: (i)

13/3125

(ii) $17/8$

(iii) $64/455$

(iv) $15/1600$

(v) $29/343$

(vi) $23/2^3 \times 5^2$

(vii) $129/2^2 \times 5^7 \times 7^5$

(viii) $6/15$

(ix) $35/50$

(x) $77/210$

Answer:

We know that for terminating decimal expansion of a rational number of form p/q , where q must be of the form $2^m \times 5^n$.

S.no	Rational Number	Denominator Factorization	Terminating/Non-terminating
i)	13/3125	Factorize the denominator we get $3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$	As denominator is in form of 5^m so it is terminating.
ii)	17/8	Factorize the denominator we get $8 = 2 \times 2 \times 2 = 2^3$	As denominator is in form of 2^m so it is terminating.
iii)	64/455	Factorize the denominator we get $455 = 5 \times 7 \times 13$	There are 7 and 13 also in denominator so denominator is not in form of $2^m \times 5^n$. so it is not terminating.
iv)	15/1600	Factorize the denominator we get $1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^8 \times 5^2$	As denominator is in form of $2^m \times 5^n$ Hence it is terminating
v)	29/343	Factorize the denominator we get $343 = 7 \times 7 \times 7 = 7^3$	There are 7 also in denominator so denominator is not in form of $2^m \times 5^n$ Hence it is non-terminating
vi)	23/ ($2^3 \times 5^2$)		Denominator is in form of $2^m \times 5^n$ Hence it is terminating.
vii)	129/ ($2^2 \times 5^7 \times 7^5$)		Denominator has 7 in denominator, so denominator is not in form of $2^m \times 5^n$ Hence it is none terminating.
viii)	6/15	divide nominator and denominator both by 3 we get 2/5	Denominator is in form of 5^m so it is terminating.
ix)	35/50	divide denominator and nominator both by 5 we get 7/10 Factorize the denominator we get $10 = 2 \times 5$	So, denominator is in form of $2^m \times 5^n$ so it is terminating.
x)	77/210	Simplify it by dividing nominator and denominator both by 7 we get 11/30 Factorize the denominator we get $30 = 2 \times 3 \times 5$	Denominator has 3 also in denominator so denominator is not in form of $2^m \times 5^n$ Hence it is none terminating.

Question 2:

Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer:

(i) $13/3125 = 13/5^5 = 13 \times 2^5 / 5^5 \times 2^5 = 416/10^5 = 0.00416$

- (ii) $17/8 = 17/2^3 = 17 \times 5^3 / 2^3 \times 5^3 = 17 \times 5^3 / 10^3 = 2125/10^3 = 2.125$
- (iv) $15/1600 = 15/2^4 \times 10^2 = 15 \times 5^4 / 2^4 \times 5^4 \times 10^2 = 9375/10^6 = 0.009375$
- (vi) $23/2^3 5^2 = 23 \times 5^3 \times 2^2 / 2^3 \times 5^2 \times 5^3 \times 2^2 = 11500/10^5 = 0.115$
- (viii) $6/15 = 2/5 = 2 \times 2 / 5 \times 2 = 4/10 = 0.4$
- (ix) $35/50 = 7/10 = 0.7.$

Question 3

The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form p/q you say about the prime factors of q ?

- (i) 43.123456789
- (ii) 0.120120012000120000...
- (iii) $43.\overline{123456789}$

Answer:

- (i) **43.123456789** : Since this number has a terminating decimal expansion, it is a rational number of the form p/q , and q is of the form $2^m \times 5^n$.
- (ii) **0.120120012000120000...**

The decimal expansion is neither terminating nor repeating. Therefore, the given number is an irrational number.

- (iii) $43.\overline{123456789}$: Since the decimal expansion is non-terminating but repeating, the given number is a rational number of the form p/q , and q is not of the form $2^m \times 5^n$.