WS-17 Vertical Circular Motion

Thask

1

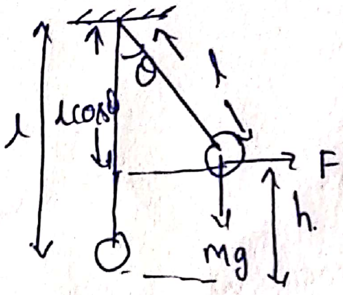
The forces acting on the bob are; mg, F and T

$W_T = 0$ [T is always perpendicular to $\vec{s}$]

$W_g = -mgh = -mgl(1-\cos\theta)$

Since the bob is moved slowly.

According to WE theorem $W_{Total} = \Delta k.E$



$\Rightarrow W_T + W_g + W_F = KE_F - k.E_I$

$\Rightarrow 0 - mgl(1-\cos\theta) + W_F = 0$

$W_F = mgl(1-\cos\theta)$

2

The Pendulum can oscillate without string becoming slack when the bob describes a semi circle.

To enable the bob rise through height L.

k.E at bottom should be

$\Rightarrow \frac{1}{2}mv^2 = mgl$

$\Rightarrow v = \sqrt{2gl}$.

3

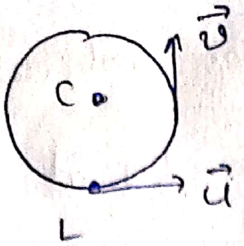
In case of Pendulum Tension in the string and displacement $\vec{s}$ are Perpendicular to each other

So $W_T = Fs\cos\theta$

$= Ts\cos 90°$

$= 0$

4



According to W-E theorem

$W = \Delta K.E$

$\Rightarrow MgL = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$\Rightarrow mgL = \frac{1}{2}m(u^2 - v^2)$

$\Rightarrow u^2 - v^2 = 2gL \Rightarrow v^2 = u^2 - 2gL \rightarrow (1)$

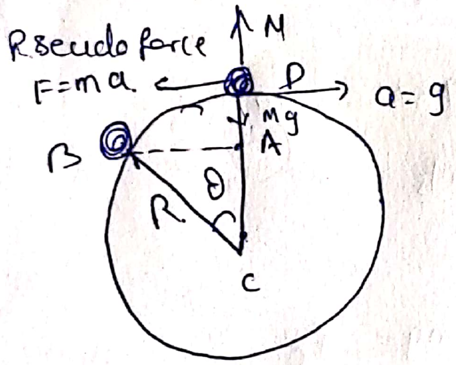
Magnitude of change in velocity $= |\vec{v} - \vec{u}| = \sqrt{v^2 + u^2 - 2vu\cos\theta}$

From fig clearly angle between $\vec{u}$ & $\vec{v}$ is 90°

$\therefore |\vec{v} - \vec{u}| = \sqrt{v^2 + u^2}$

From (1) $|\vec{v} - \vec{u}| = \sqrt{u^2 - 2gL + u^2} = \sqrt{2u^2 - 2gL} = \sqrt{2(u^2 - gL)}$

5



Three forces are acting on the particle with respect to the sphere

(i) Pseudo force = $ma = mg$ [$a = g$]

(ii) weight = mg

(iii) Normal reaction = N

of these first two forces are constant and will do work. the third force is not constant and it does not perform any work.

From W-E theorem $W = \Delta K.E \Rightarrow W_{pseudo} + W_g = \frac{1}{2}mV_r^2$

Here V_r = speed of particle relative to sphere

$\frac{1}{2}mV_r^2 = ma(AB) + mg(AD)$

$\Rightarrow \frac{1}{2}V_r^2 = g[R\sin\theta + R(1-\cos\theta)]$

$\Rightarrow V_r = \sqrt{2gR[1 + \sin\theta - \cos\theta]}$

From fig $\sin\theta = \frac{AB}{BC}$

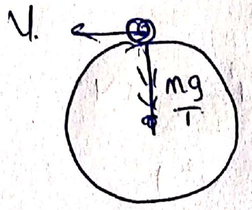
$AB = BC\sin\theta = R\sin\theta$

$AC = BC\cos\theta = R\cos\theta$

$AD = CD - AC = R - R\cos\theta = R(1-\cos\theta)$

(6)

when the bob ~~reaches~~ reaches vertical position



$T + Mg = \frac{mv^2}{r}$ — v at highest point $= \sqrt{3gl}$.

$T = \frac{mv^2}{r} - mg$ — here $l = r$

$T = \frac{m(\sqrt{3gr})^2}{r} - mg$ — $v = \sqrt{3gr}$

$\Rightarrow T = 2mg = 2 \times 2 \times 9.8 = 39.2\ N$

(7)

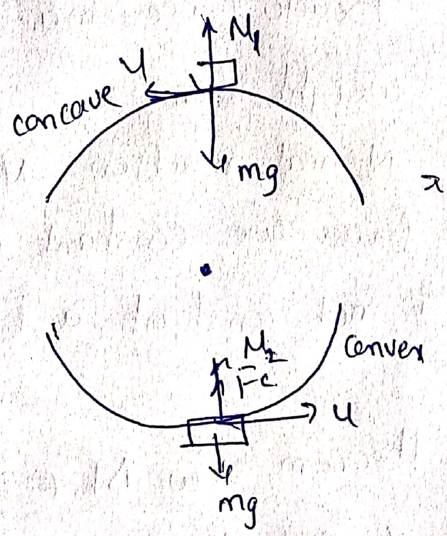
The minimum horizontal speed given to the bob To make one rotation is

$V_L = \sqrt{5gL}$ — here given $l = 100\ cm = 1m$

$\Rightarrow V_L = \sqrt{5 \times 9.8 \times 1} = \sqrt{49}$

$= 7\ m/s$

(8)



$N - mg = F_c$

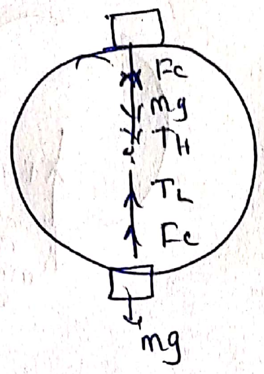
$\Rightarrow N_1 = Mg + F_c = Mg + \frac{mv^2}{r}$

$N + Mg = F_c$

$N_2 = F_c - mg = \frac{mu^2}{r} - mg$

clearly on concave surface normal reaction is Maximum

9



T_H = Tension at heighest point

$$T + mg = F_c \Rightarrow T = F_c - mg$$

Tension at lowest point

$$T_L = F_c + mg$$

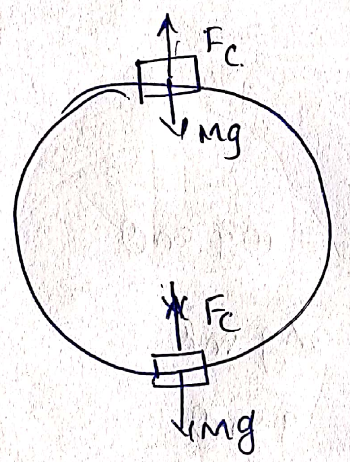
$$\Rightarrow T_L = \frac{mv^2}{r} + mg$$

$$\Rightarrow v^2 = \frac{(T_L - mg)\,r}{m} = \frac{(140.6 - 2\times 9.8)\times \cancel{2}}{\cancel{2}}$$

$$\Rightarrow v^2 = 140.6 - 19.6 = 121$$

$$\Rightarrow v = \sqrt{121} = 11\ m/s$$

10



$$T_{max} = \frac{mv^2}{r} + mg$$

$= 7mg$ Given $T_{max} = 7mg$

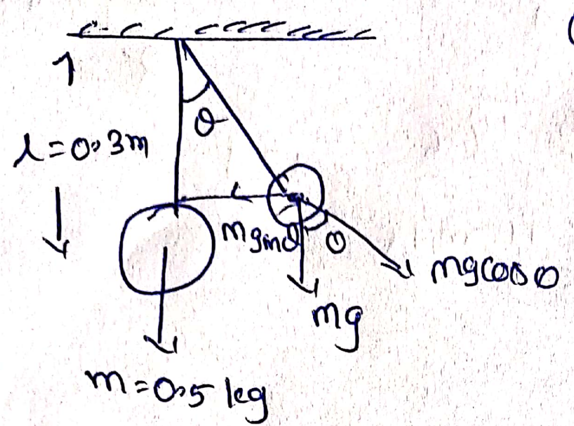
$$\therefore 7mg = \frac{mv^2}{r} + mg$$

$$\Rightarrow \frac{mv^2}{r} = 6mg$$

$$\Rightarrow r_{max} = \frac{v^2}{6g} = \frac{210\times 210}{6\times 9.8}$$

$$\Rightarrow r_{max} = 750m$$

(14), (15), (16)



$\theta = 60^\circ$

$T = mg\cos\theta$

$= 0.5 \times 10 \times \cos 60$

$\Rightarrow 2.5N$

(15)

Net force acting $= mg\sin\theta$

$\therefore ma = mg\sin\theta$

$\Rightarrow a = g\sin\theta$

$\Rightarrow a = 10\sin 60 = 5\sqrt{3}\ m/s^2$

$F = ma$

$= 0.5 \times 5\sqrt{3}$

$= 2.5\sqrt{3}\ N.$

(17)

The minimum velocity at bottom $= \sqrt{5 g l}$

Given $l = r = 5m$

$m = 2\ kg$

$V_{min} = \sqrt{5 \times 9.8 \times 5} = \sqrt{245}$

$\simeq 15\ m/s$

(18)

Given $r = 5m = l$

The velocity at highest point $= \sqrt{g l}$

$= \sqrt{9.8 \times 5}$

$= \sqrt{49}$

$= 7\ m/s$

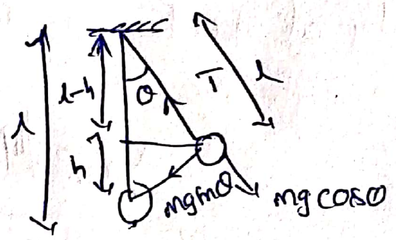
(19)

Given ~~$u = 5\ m/s$~~ $u = 5\ m/s$

when normal reaction is zero at highest point then only the particle comples one cycle of rotation.

LTask

(1)



$V_L = \sqrt{147}$ m/s

$$T - mg\cos\theta = \frac{mv^2}{l}$$

Given $T = mg$

$$\cos\theta = \frac{l-h}{l}$$

$$h = l(1-\cos\theta) \qquad \Rightarrow mg - mg\cos\theta = \frac{mv^2}{l}$$

$$\Rightarrow \cancel{m}g[1-\cos\theta] = \frac{\cancel{m}v^2}{l}$$

$$\Rightarrow v^2 = gl(1-\cos\theta)$$

According to law of conservation of energy

loss of K.E = Gain in P.E

$$\Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mgh$$

$$\Rightarrow \frac{1}{2}\cancel{m}((\sqrt{147})^2 - u^2) = \cancel{m}gh$$

$$\Rightarrow 147 - u^2 = 2gl(1-\cos\theta)$$

$$\Rightarrow 147 - u^2 = 2u^2$$

$$\Rightarrow 147 = 3u^2$$

$$\Rightarrow u = \sqrt{\frac{147}{3}} = 7 \text{ m/s}$$

(2)

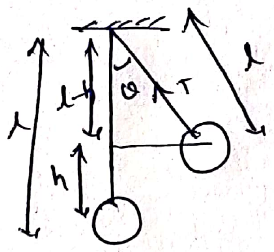
Tension at mean position $T = mg + \frac{mv^2}{r} \rightarrow (1)$

According to Law of conservation of Energy

K.E = P.E

$$\Rightarrow \frac{1}{2}mu^2 = mgh \rightarrow (2)$$

Given $\Rightarrow m = 50$ gm
$= 0.05$ kg



$$\cos\theta = \frac{l-h}{l} \Rightarrow h = l(1-\cos\theta)$$
$$= l(1-\cos 60)$$
$$= 0.5L$$

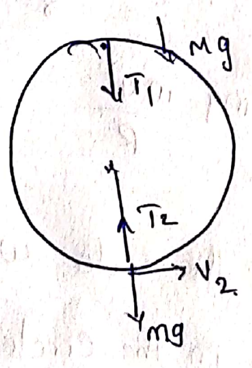
∴ From (2)

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$$
$$\Rightarrow \frac{v^2}{2} = 10 \times 0.5L$$
$$\Rightarrow v^2 = 10L$$

From (1)

$$T = mg + \frac{mv^2}{l} \Rightarrow mg + m\frac{(10\cancel{L})}{\cancel{L}}$$
$$\Rightarrow 0.05\times10 + 0.05\times10$$
$$\Rightarrow 0.5 + 0.5$$
$$= 1\,N.$$

(3)



Net centripetal force $= \frac{mv^2}{R}$

$$T_1 + mg = \frac{mv_1^2}{R} \rightarrow (1)$$
$$T_2 - mg = \frac{mv_2^2}{R} \rightarrow (2)$$

Tension is always acting ~~to~~ radially inwards.
Displacement is always Tangential $\Rightarrow T \perp^{r} S$

$$\Rightarrow W = TS\cos\theta,\ \theta = 90^\circ \Rightarrow W_T = 0$$

Total ~~P~~ M.E = 0

$$\frac{1}{2}mv_2^2 + mgh_2 = \frac{1}{2}mv_1^2 + mgh_1$$
$$\Rightarrow \frac{1}{2}mv_2^2 + 0 = \frac{1}{2}mv_1^2 + 2mgR$$

$$m v_2^2 - m v_1^2 = 4mgR$$

$$\Rightarrow \frac{m v_2^2}{R} - \frac{m v_1^2}{R} = 4mg$$

$$\Rightarrow (T_2 - mg) - (T_1 + mg) = 4mg$$

$$\Rightarrow T_2 - T_1 = 6mg$$

(4)

Given mass of the car $= M$

velocity of the car $= V$

The radius of the convex bridge $= r$

The gravitational acceleration be g.

let F be the exerted force.

Force of gravity $-$ Exerted force $=$ centripetal force

$$\Rightarrow Mg - F = \frac{mv^2}{r}$$

$$\Rightarrow F = mg - \frac{mv^2}{R}.$$

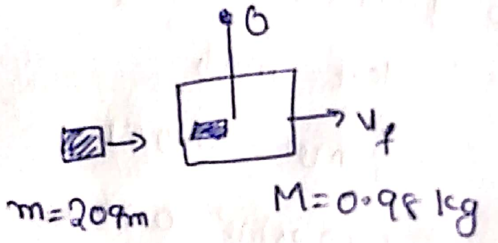
(6)

As the bottle rotates, the soda will experience different forces at different points in the circle. At the top of the circle, the soda is pushed outward due to inertia, while at bottom of the circle, it is pushed upward due to the centripetal force.

The air bubbles in the soda are less dense than the liquid, therefore, they will tend to rise to the top of the liquid. when the bottle is rotated, the soda moves in a circular path, and the bubbles will move in the opposite direction to the soda.

Since the bubbles will rise to the top of the liquid, and considering the rotation, the bubbles will collect at the neck of the bottle. this is because when the soda is at the top of the circle. the bubbles will be pushed downwards and, when the soda is at the bottom, the bubbles will rise to the neck.

7



As the block - bullet system just completes the vertical circle it means $v_f = \sqrt{5gl}$

$$\Rightarrow v_f = \sqrt{5 \times 10 \times 1} = 5\sqrt{2}\ m/s$$

By law of conservation of linear momentum

$$mu + M(0) = (m+M)v_f$$

$$\Rightarrow 20 \times 10^{-3} v + 0 = (20 \times 10^{-3} + 0.98)\, 5\sqrt{2}$$

$$\Rightarrow 0.02 v = 5\sqrt{2}$$

$$v = \frac{5\sqrt{2}}{0.02} = 353.55\ m/s$$

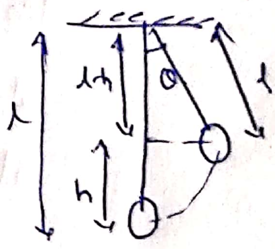
After collision Both bullet + block move with common velocity $v_1 = v_2 = v_f$.

(8)

Given $\theta = 60^\circ$; $m = 50\,gm = 5\times10^{-2}\,kg$.

The height h of the bob above the lowest point can be calculated using $h = L - L\cos\theta$

$h = L(1-\cos\theta)$



$\Rightarrow h = L[1-\cos60^\circ]$

$\Rightarrow h = \frac{L}{2} = 0.5\,L$

$\cos\theta = \frac{L-h}{L}$

$P.E = mgh = mg(0.5\,L) = = 0.25\,L$.

Acc to Law of conservation of Energy

$K.E = mgh$

$\Rightarrow \frac{1}{2}mv^2 = 0.25\,L$

$\Rightarrow \frac{1}{2}(0.05)v^2 = 0.25\,L$

$\Rightarrow v^2 = \frac{0.5\,L}{0.05} = 10L$

Tension $T = mg + \frac{mv^2}{L} \Rightarrow mg + m\frac{10L}{L}$

$T = 0.05\times10 + 10\times0.05 = \underline{1\,N}$.

(9)

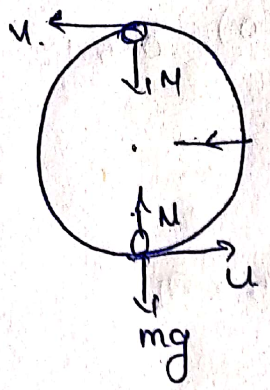
we know that velocity at any point in a vertical circular motion is $v = \sqrt{gR(3+2\cos\theta)}$; Given $\theta_1 = 60^\circ$

$\theta_2 = 120$

$$\frac{v_1}{v_2} = \sqrt{\frac{(3+2\cos\theta_1}{3+2\cos\theta_2}} = \sqrt{\frac{3+2\cos60}{3+2\cos120^\circ}}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{3+2(\frac{1}{2})}{3+2(-\frac{1}{2})}} = \sqrt{\frac{3+1}{3-1}} = \frac{\sqrt{2}}{1}$$

(14), (15), (16)



Normal reaction at the highest point $N=0$
then only the particle completes one rotation

At lowest point

$$N - mg = \frac{mu^2}{r}.$$

At lowest point $u = \sqrt{5gr}$

$$\Rightarrow N - mg = \frac{m}{r}(\sqrt{5gr})^2$$

$$\Rightarrow N - mg = 5mg$$

$$\Rightarrow \boxed{N = 6mg}$$

we know At highest point $N=0$

$$\Rightarrow mg = \frac{mv^2}{r}$$

$$\Rightarrow \boxed{v = \sqrt{gr}}$$

According to law of Conservation of Energy

$$T.E_L = T.E_H$$

$$\Rightarrow K.E_L = K.E_H + P.E_H$$

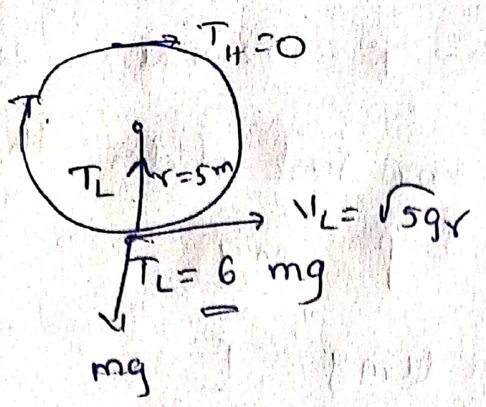
$$\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgH$$

$$\Rightarrow u^2 = v^2 + 2gH$$

$$\Rightarrow u^2 = gr + 2g(2r) = 5gr$$

$$\Rightarrow \boxed{u = \sqrt{5gr}}$$

(17)



At lowest point

$$T_L - mg = \frac{mv^2}{r}$$

$$\Rightarrow T_L - mg = \frac{m}{r}(\sqrt{5gr})^2$$

$$\Rightarrow T_L - mg = \frac{m}{r} \times 5gr$$

$$\Rightarrow T_L - mg = 5mg$$

$$T_L = 6mg$$

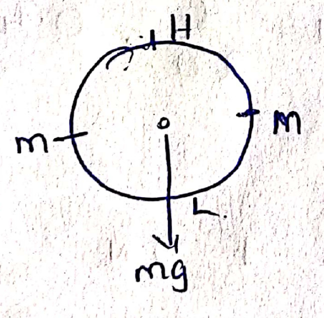
(18)

Given $v = 2\,m/s$; mass $= 1$ kg ; $r = 5m$.

The min value of normal force $= 0$ because the Particle completes one rotation

(19)

At L:



$$\Rightarrow T_L - mg = \frac{mv^2}{R}$$

Given $v = 2\ m/s$

$m = 5$ kg ; $l = 1m$

$$\Rightarrow T_L - 50 = \frac{5 \times 4}{1}$$

$$\Rightarrow T_L - 50 = 20 \Rightarrow T_L = 70\ N \uparrow$$

At H:-

$$T + mg = \frac{mv^2}{R}$$

$$\Rightarrow T + 50 = \frac{5(2)^2}{1}$$

$$\Rightarrow T + 50 = 20$$; $w = mg$ $= 50 \downarrow$

$$\Rightarrow T = 30\,N \downarrow$$