7TH CLASS WS-5 SOLUTIONS

SURDS

Teaching Task

1. Conceptual
2. Conceptual
3. Conceptual
4. $\sqrt[10]{3}, \sqrt[4]{2}, \sqrt[5]{5}$

   $\Rightarrow 3^{\frac{1}{10}}, 2^{\frac{1}{4}}, 5^{\frac{1}{5}}$

   Taking power 20 which is LCM of 10, 4, 5 = 20

   $\Rightarrow \left(3^{\frac{1}{10}}\right)^{20}, \left(2^{\frac{1}{4}}\right)^{20}, \left(5^{\frac{1}{5}}\right)^{20} \Rightarrow 3^2, 2^5, 5^4$

   We get $9 < 32 < 625$ $\therefore \sqrt[10]{3} < \sqrt[4]{2} < \sqrt[5]{5}$

5. Option A) $\left(\sqrt[5]{7}\right)^5 = \left((7)^{\frac{1}{5}}\right)^5 = 7$ true

   Option B) $\sqrt[5]{\frac{3}{2}} = \frac{\sqrt[5]{3}}{\sqrt[5]{2}}$ True

   Option C) $\sqrt[4]{\sqrt[3]{5}} = \sqrt[4]{5^{\frac{1}{3}}} = \left(5^{\frac{1}{3}}\right)^{\frac{1}{4}} = 5^{\frac{1}{12}} = \sqrt[12]{5}$ True

   Option D) $\sqrt[6]{729} = \left(3^6\right)^{\frac{1}{6}} = 3 \neq 1$ false

   Key – D

6. $i) p^6 = 2^x = 4^3$

   Let $2^x = 4^3 = \left(2^2\right)^3 = 2^6$

   $\therefore x = 6$

   And $p^6 = 2^x = 2^6$

   $\therefore p = 2$ (by comparing bases)

   $ii) q^{12} = 2^{y+1} = 8^4$

   Let $2^{y+1} = 8^4 = \left(2^3\right)^4 = 2^{12}$

   $\therefore y + 1 = 12 \Rightarrow y = 11$

   And $q^{12} = 2^{y+1} = 2^{12}$

   $\therefore q = 2$ (by comparing)

   iii) $r^{20} = 2^{z+2} = 16^5$

   Let $2^{z+2} = 16^5 = \left(2^4\right)^5 = 2^{20}$

   $\therefore z + 2 = 20 \Rightarrow z = 18$

   And $r^{20} = 2^{z+2} = 2^{20}$

   $\therefore r = 2$ by comparing

   $\therefore \sqrt[p]{x} = \sqrt[2]{6}$

   $\sqrt[q]{y} = \sqrt[2]{11}$

   $\sqrt[r]{z} = \sqrt[2]{18}$

   Descending order

   $\therefore \sqrt[r]{z}, \sqrt[q]{y}, \sqrt[p]{x}$

7. $\sqrt[4]{3}, \sqrt[6]{2}, \sqrt[3]{4} \Rightarrow 3^{\frac{1}{4}}, 2^{\frac{1}{6}}, 4^{\frac{1}{3}}$

Taking power which is LCM of 4, 6, 3 = 12

$\left(3^{\frac{1}{4}}\right)^{12}, \left(2^{\frac{1}{6}}\right)^{12}, \left(4^{\frac{1}{3}}\right)^{12} \Rightarrow 3^3, 2^2, 4^4$

We get 27, 4, 256

And $256 > 27 > 4$

$\therefore \sqrt[3]{4} > \sqrt[4]{3} > \sqrt[6]{2}$

8. $\sqrt[3]{a^2}\left\{\sqrt[3]{a}\left(\sqrt[4]{a}\right)^4\right\}^{\frac{1}{4}} = a^{\frac{2}{3}} \cdot \left(a^{\frac{1}{3}}\right)^{\frac{1}{4}}\left(\left(a^{\frac{1}{4}}\right)^4\right)^{\frac{1}{4}}$

$= a^{\frac{2}{3}} \cdot a^{\frac{1}{12}} \cdot a^{\frac{1}{4}} = a^{\frac{2}{3}+\frac{1}{12}+\frac{1}{4}} = a^1 = a$

9. $\sqrt[m]{a} = \sqrt[n]{b} = \sqrt[p]{c} = k, abc = 1$

$\sqrt[m]{a} = k \Rightarrow a = k^m$

Similarly, $b = k^n$ and c = k^p

Given that $abc = 1$

$\therefore k^m \cdot k^n \cdot k^p = 1$

$k^{m+n+p} = 1 = k^0$

$\therefore m + n + p = 0$

10. $4 \cdot \sqrt[3]{\frac{2010}{64}} = 4 \cdot \frac{\sqrt[3]{2010}}{\sqrt[3]{64}} = 4 \cdot \frac{\sqrt[3]{2010}}{4} = \sqrt[3]{2010} \rightarrow$ Pure surd

11. $\therefore \sqrt{\frac{2010 \times 402 \times 2}{125 \times 125}} = \sqrt{\frac{402 \times 402 \times 2 \times 5}{25 \times 25 \times 5 \times 5}}$

$= \frac{402}{25}\sqrt{\frac{2}{5}} \rightarrow$ mixed surd

12. $\sqrt[p]{\sqrt[q]{\left(x^3\right)^y}} = \sqrt[q]{x^3}$

$\sqrt[p]{\sqrt[q]{x^{3y}}} = x^{\frac{3}{q}}$

$\sqrt[p]{x^{\frac{3y}{q}}} = x^{\frac{3}{q}}$

$x^{\frac{3y}{pq}} = x^{\frac{3}{q}}$

$\therefore \frac{3y}{pq} = \frac{3}{q}$

$\therefore y = p$

13. $a - \sqrt[n]{b} = a \cdot \sqrt[n]{b} \Rightarrow a = a\sqrt[n]{b} + \sqrt[n]{b} = (a+1)\sqrt[n]{b}$

$\therefore \sqrt[n]{b} = \frac{a}{a+1}$

$\therefore b = \left(\frac{a}{a+1}\right)^n$

14. $\sqrt[4]{30} \div \sqrt[5]{6} = \frac{\sqrt[4]{30}}{\sqrt[5]{6}} = \frac{\sqrt[4]{5 \times 6}}{\sqrt[5]{6}}$

$= \sqrt[4]{5} \times 6^{\frac{1}{4}} \cdot 6^{-\frac{1}{5}} = \sqrt[4]{5} \times 6^{\frac{1}{4}-\frac{1}{5}}$

$= \sqrt[4]{5} \cdot 6^{\frac{5-4}{20}} = \sqrt[4]{5} \times 6^{\frac{1}{20}}$

$= \sqrt[4]{5} \times \sqrt[20]{6}$

15. Column II

$p)\sqrt[12]{256}=\left(2^{8}\right)^{\frac{1}{12}}=2^{\frac{8}{12}}=2^{\frac{2}{3}}=\sqrt[3]{2^{2}}=\sqrt[3]{4}$

$q)\sqrt[12]{8}=\left(2^{3}\right)^{\frac{1}{12}}=2^{\frac{1}{4}}=\sqrt[4]{2}$

$r)\sqrt[12]{64}=\left(2^{6}\right)^{\frac{1}{12}}=2^{\frac{1}{2}}=\sqrt{2}$

$s)\sqrt[12]{729}=\left(3^{6}\right)^{\frac{1}{12}}=3^{\frac{1}{2}}=\sqrt[2]{3}$

$t)\sqrt[12]{196}=\left(14^{2}\right)^{\frac{1}{12}}=\sqrt[6]{14}$

$a-s, b-r, c-q, d-p$

16. $a)\sqrt[3]{3}, \sqrt[4]{5} \Rightarrow 3^{\frac{1}{3}}, 5^{\frac{1}{4}}$

Taking power 12 which is LCM of 3 & 4

$\left(3^{\frac{1}{3}}\right)^{12}, \left(5^{\frac{1}{4}}\right)^{12} \Rightarrow 3^{4}, 5^{3} \Rightarrow 81, 125$

$125>81$

$\therefore \sqrt[4]{5}>\sqrt[3]{3}$ $\sqrt[4]{5}$ is greater

$a-s$

Similarly, we can do the remaining

We get $a-s, b-p, c-r, d-r$

LEARNER'S TASK

1. Conceptual
2. Conceptual
3. $\sqrt[2]{\sqrt[3]{6}}=\left(6^{\frac{1}{3}}\right)^{\frac{1}{2}}=6^{\frac{1}{6}}=\sqrt[6]{6}$

   Order = 6
4. Conceptual
5. $\sqrt[4]{\sqrt[3]{2^{6}}}=\sqrt[4]{\left(2^{6}\right)^{\frac{1}{3}}}=\sqrt[4]{2^{2}}=\left(2^{2}\right)^{\frac{1}{4}}=2^{\frac{1}{2}}=\sqrt{2}$
6. $\sqrt{\frac{5}{4}}=\frac{\sqrt{5}}{\sqrt{4}}=\frac{\sqrt{5}}{2}$
7. Order less, values greater

   For $\sqrt{2}$, order = 2

   For $\sqrt[5]{2}$, order = 5
8. Conceptual
9. $a^{n}=x^{a}=16$ $n>a$

   $\therefore 2^{4}=16$ by comparing

   a = 2 and n = 4

   $\therefore x^{2}=16=4^{2}$

   X = 4

   Substitute above values in options from option C)

   $2\sqrt{a^{n+x}}=2\sqrt{2^{4+4}}=2\sqrt{2^{8}}=2^{1}\cdot 2^{4}=32$

   It is greater or by comparing options directly, we get option c as answer.
10. $\sqrt[3]{343}=a=7; \sqrt[3]{216}=b=6$

    $\sqrt[3]{125}=c=5, \sqrt[3]{64}=d=4$

    $\therefore \sqrt[5]{a}, \sqrt[5]{b}, \sqrt[5]{c}, \sqrt[5]{d}$ are $\sqrt[5]{7}, \sqrt[5]{6}, \sqrt[5]{5}, \sqrt[5]{4}$

Descending order is also same by observing options directly we get the answer.

JEE MAINS LEVEL

1. Conceptual
2. $3\cdot\sqrt[4]{5}=\sqrt[4]{5\times3^4}=\sqrt[4]{405}$
3. $\sqrt[2]{\sqrt[3]{512}}=\left((512)^{\frac{1}{3}}\right)^{\frac{1}{2}}=\left(2^9\right)^{\frac{1}{6}}=(2)^{\frac{9}{6}}$

   $2^{\frac{3}{2}}=\left(2^3\right)^{\frac{1}{2}}=\sqrt{8}=2\sqrt{2}$
4. $\dfrac{\sqrt[3]{2009}}{\sqrt[3]{2010}}=\sqrt[3]{\dfrac{2009}{2010}}$
5. Conceptual
6. $\sqrt[5]{288}=\sqrt[5]{32\times9}=\sqrt[5]{32}\cdot\sqrt[5]{9}=2\cdot\sqrt[5]{9}$
7. $a^{\frac{m}{n}}\times b^{\frac{m}{n}}=(ab)^{\frac{1}{k}}$

   $\therefore(ab)^{\frac{m}{n}}=(ab)^{\frac{1}{k}}$

   $\therefore k=\dfrac{n}{m}$
8. Conceptual
9. $\dfrac{2}{3}\sqrt{32}=\sqrt{\dfrac{32\times4}{9}}=\sqrt{\dfrac{128}{9}}$
10. $\sqrt{1350}=\sqrt{225\times6}=15\sqrt{6}$

ADVANCED LEVEL QUESTIONS

11. Conceptual
12. $\sqrt[36]{11}=(11)^{\frac{1}{36}}=11^{\frac{1}{9\times4}}=\left(11^{\frac{1}{9}}\right)^{\frac{1}{4}}=\sqrt[4]{\sqrt[9]{11}}=\sqrt[9]{\sqrt[4]{11}}$
13. Conceptual
14. Conceptual
15. $\sqrt[5]{\sqrt[2]{\left(x^2\right)^y}}=\sqrt[2]{x^{\frac{5}{2}}}$

   $\left(\left(x^{2y}\right)^{\frac{1}{2}}\right)^{\frac{1}{5}}=\left(x^{\frac{5}{2}}\right)^{\frac{1}{2}}$

   $x^{\frac{2y}{10}}=x^{\frac{5}{4}}$

   $\therefore\dfrac{2y}{10}=\dfrac{5}{4}\Rightarrow y=\dfrac{25}{4}$

   Key – D
16. $\sqrt[5]{\sqrt[11]{a^{x+y}}}=\sqrt[11]{a^{x-y}}$

   $\left(\left(a^{x+y}\right)^{\frac{1}{11}}\right)^{\frac{1}{5}}=\left(a^{x-y}\right)^{\frac{1}{11}}$

   $a^{\frac{x+y}{55}}=a^{\frac{x-y}{11}}$

   $\therefore\dfrac{x+y}{55}=\dfrac{x-y}{11}$

   $11x+11y=55x-55y$

$55y+11y=55x-11x$
$66y=44x$
$\frac{x}{y}=\frac{66}{44}=\frac{3}{2}$
$\therefore 2x=3y$

17. Conceptual
18. Conceptual
19. $x=\sqrt[6]{64}=\left(2^6\right)^{\frac{1}{6}}=2$

$y=\sqrt[2]{64}=\left(8^2\right)^{\frac{1}{2}}=8$

$z=\sqrt[4]{256}=\left(4^4\right)^{\frac{1}{4}}=4$

$\frac{z}{xy}=\frac{4}{2\times 8}=\frac{1}{4}$

20. Column I

$a)\sqrt[5]{16}\times\sqrt[5]{2}=\sqrt[5]{16\times 2}=\sqrt[5]{2^5}=2$

$b)\frac{\sqrt[4]{243}}{\sqrt[4]{3}}=\sqrt[4]{\frac{243}{3}}=\sqrt[4]{81}=\sqrt[4]{3^4}=3$

$c)\sqrt[3]{4}\times\sqrt[3]{16}=\sqrt[3]{4\times 16}=\sqrt[3]{4^3}=4$

$d)\frac{\sqrt[4]{1250}}{\sqrt[4]{2}}=\sqrt[4]{\frac{1250}{2}}=\sqrt[4]{625}=\sqrt[4]{5^4}=5$

$\therefore a-r, b-q, c-s, d-p$

ADDITIONAL PRACTICE QUESTIONS

1. $\sqrt[n]{x^3y^2}=x^6y^4=\left(x^3y^2\right)^2$

$\left(x^3y^2\right)^{\frac{1}{n}}=\left(x^3y^2\right)^2$

$\therefore \frac{1}{n}=2$

$n=\frac{1}{2}$

2. $1838\frac{17}{64}=\frac{117649}{64}$ $\quad 1838\times 64=117632$

$\sqrt[6]{\frac{117649}{64}}=\frac{7}{2}$ $\quad 2^6=64$, $7^2=49$

By verification method $\quad 7^6=49\times 49\times 49=117649$

3. $2+\sqrt{1-\frac{2176}{2401}}=1+x$

$x=1+\sqrt{\frac{2401-2176}{2401}}=1+\sqrt{\frac{225}{2401}}=1+\frac{15}{49}=\frac{49+15}{49}=\frac{64}{49}$

4. $\frac{x}{\sqrt{0.0064}}=\sqrt[3]{0.008}$

$x=\sqrt[3]{0.008}\sqrt{0.0064}=(0.2)(0.08)=0.016$

5. Conceptual
6. Conceptual
7. Conceptual

8. Conceptual
9. Conceptual
10. Conceptual
11. Conceptual
12. Conceptual
13. $2008\cdot\sqrt[3]{2009}=\sqrt[3]{2009\times 2008^3}$
14. $\sqrt[4]{3^1\times 3^3\times 3^4}=\sqrt[4]{3^8}=\left(3^8\right)^{\frac{1}{4}}=3^2=9$
15. If $x\cdot\sqrt[n]{a}$ is a pure surd, then x = 1
16. Column -I

$a)\sqrt[2]{3}\times\sqrt[3]{4}=(3)^{\frac{1}{2}}\times(4)^{\frac{1}{3}}=\sqrt[6]{432}$

By taking power 6 we get $3^3\times 4^2=432$

$b)\sqrt[5]{30}\div\sqrt[5]{6}=\sqrt[5]{\frac{30}{6}}=\sqrt[5]{5}$

$c)\sqrt[3]{3}\times\sqrt[4]{5}=\left(3^{\frac{1}{3}}\right)\times\left(5^{\frac{1}{4}}\right)=\sqrt[12]{10125}$

By taking power 12, we get $3^4\times 5^3=10125$

$d)\sqrt[5]{\sqrt[7]{\left[(2010)^7\right]^5}}=2010$

$\therefore a-q, b-p, c-s, d-r$