SOLVED EXAMPLES

Ex. 1 The sum of first four terms of an A.P. is 56 and the sum of it's last four terms is 112. If its first term is 11 then find the number of terms in the A.P.

Sol. $a + a + d + a + 2d + a + 3d = 56$

$4a + 6d = 56$

$44 + 6d = 56$ (as $a = 11$)

$6d = 12$ hence $d = 2$

Now sum of last four terms.

$a + (n - 1)d + a + (n - 2)d + a + (n - 3)d + a + (n - 4)d = 112$

$\Rightarrow$ $4a + (4n - 10)d = 112$ $\Rightarrow$ $44 + (4n - 10)2 = 112$

$\Rightarrow$ $4n - 10 = 34$

$\Rightarrow$ $n = 11$

Ex. 2 Find the sum of all the three digit natural numbers which on division by 7 leaves remainder 3.

Sol. All these numbers are 101, 108, 115, ..., 997

$997 = 101 + (n - 1)\,7$

$\Rightarrow$ $n = 129$

So $S = \frac{129}{2}[101 + 997] = 70821.$

Ex. 3 If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots, a_n$ are in A.P. where $a_i > 0$ for all i, show that :

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{(n-1)}{\sqrt{a_1}+\sqrt{a_n}}$$

Sol. L.H.S. $= \frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$

$$=\frac{1}{\sqrt{a_2}+\sqrt{a_1}}+\frac{1}{\sqrt{a_3}+\sqrt{a_2}}+\ldots\ldots+\frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$$

$$=\frac{\sqrt{a_2}-\sqrt{a_1}}{(a_2-a_1)}+\frac{\sqrt{a_3}-\sqrt{a_2}}{(a_3-a_2)}+\ldots\ldots+\frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n-a_{n-1}}$$

Let 'd' is the common difference of this A.P.

then $a_2 - a_1 = a_3 - a_2 = \ldots\ldots. = a_n - a_{n-1} = d$

Now L.H.S.

$$=\frac{1}{d}\left\{\sqrt{a_2}-\sqrt{a_1}+\sqrt{a_3}-\sqrt{a_2}+\ldots\ldots+\sqrt{a_{n-1}}-\sqrt{a_{n-2}}+\sqrt{a_n}-\sqrt{a_{n-1}}\right\}=\frac{1}{d}\left\{\sqrt{a_n}-\sqrt{a_1}\right\}$$

$$=\frac{a_n-a_1}{d\left(\sqrt{a_n}+\sqrt{a_1}\right)}=\frac{a_1+(n-1)d-a_1}{d\left(\sqrt{a_n}+\sqrt{a_1}\right)}=\frac{1}{d}\,\frac{(n-1)d}{(\sqrt{a_n}+\sqrt{a_1})}=\frac{n-1}{\sqrt{a_n}+\sqrt{a_1}}=\text{R.H.S.}$$

Ex. 4 If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P., then prove that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P.

Sol. Given $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P.

Add 2 to each term

$\Rightarrow$ $\frac{b+c+a}{a}, \frac{c+a+b}{b}, \frac{a+b+c}{c}$ are in A.P.

divide each by a + b + c $\Rightarrow$ $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Ex. 5 A number consists of three digits which are in G.P. the sum of the right hand and left hand digits exceeds twice the middle digit by 1 and the sum of the left hand and middle digits is two third of the sum of the middle and right hand digits. Find the numbers.

Sol. Let the three digits be a, ar and ar^2 then number is

$$100a + 10ar + ar^2 \quad \text{....(i)}$$

Given, $a + ar^2 = 2ar + 1$

or $a(r^2 - 2r + 1) = 1$

or $a(r-1)^2 = 1$ ....(ii)

Also given $a + ar = \frac{2}{3}(ar + ar^2)$

$\Rightarrow$ $3 + 3r = 2r + 2r^2$

$\Rightarrow$ $2r^2 - r - 3 = 0$

$\Rightarrow$ $(r+1)(2r-3) = 0$

$\therefore$ $r = -1, 3/2$

for $r = -1$, $a = \frac{1}{(r-1)^2} = \frac{1}{4} \notin I$ $\quad \therefore r \neq -1$

for $r = 3/2$, $a = \frac{1}{\left(\frac{3}{2}-1\right)^2} = 4$ {from (ii)}

From (i), number is $400 + 10.4.\frac{3}{2} + 4.\frac{9}{4} = 469$

Ex. 6 Evaluate 7 + 77 + 777 + ... upto n terms.

Sol. Let S = 7 + 77 + 777 + ...upto n terms.

$= \frac{7}{9}[9 + 99 + 999 + ...]$

$= \frac{7}{9}[(10-1) + (10^2 - 1) + (10^3 - 1) + \text{........} + \text{upto n terms}]$

$= \frac{7}{9}[10 + 10^2 + 10^3 + \text{..........} + 10^n - n]$

$= \frac{7}{9}\left(\frac{10(10^n - 1)}{9} - n\right)$

$= \frac{7}{81}[10^{n+1} - 9n - 10]$

Ex. 7 If $n > 0$, prove that $2^n > 1 + n\sqrt{2^{n-1}}$

Sol. Using the relation A.M. ≥ G.M. on the numbers $1, 2, 2^2, 2^3, ..., 2^{n-1}$, we have

$$\frac{1+2+2^2+.......+2^{n-1}}{n} > (1.2.\,2^2.\,2^3.\,.........\,.2^{n-1})^{1/n}$$

Equality does not hold as all the numbers are not equal.

$$\Rightarrow \quad \frac{2^n-1}{2-1} > n\left(2^{\frac{(n-1)\,n}{2}}\right)^{\frac{1}{n}}$$

$$\Rightarrow \quad 2^n - 1 > n.\,2^{\frac{(n-1)}{2}}$$

$$\Rightarrow \quad 2^n > 1 + n.\,2^{\frac{(n-1)}{2}}$$

Ex. 8 If $a_i > 0$ " i Î N such that $\prod_{i=1}^{n} a_i = 1$, then prove that $(1+a_1)(1+a_2)(1+a_3).....(1+a_n)$ ³ 2^n

Sol. Using A.M. ≥ G.M.

$1 + a_1$ ³ $2\sqrt{a_1}$

$1 + a_2$ ³ $2\sqrt{a_2}$

⋮

$1 + a_n$ ³ $2\sqrt{a_n}$ $\Rightarrow$ $(1+a_1)(1+a_2)........(1+a_n)$ ³ $2^n(a_1a_2a_3.....a_n)^{1/2}$

As $a_1a_2a_3.....a_n = 1$

Hence $(1+a_1)(1+a_2)..........(1+a_n)$ ³ 2^n.

Ex. 9 Sum to n terms of the series $\frac{1}{(1+x)(1+2x)} + \frac{1}{(1+2x)(1+3x)} + \frac{1}{(1+3x)(1+4x)} + ...$

Sol. Let T_r be the general term of the series

$$T_r = \frac{1}{(1+rx)(1+(r+1)x)}$$

So $$T_r = \frac{1}{x}\left[\frac{(1+(r+1)x)-(1+rx)}{(1+rx)(1+(r+1)x)}\right] = \frac{1}{x}\left[\frac{1}{1+rx} - \frac{1}{1+(r+1)x}\right]$$

$T_r = f(r) - f(r+1)$

$\therefore$ $S = \sum T_r = T_1 + T_2 + T_3 + ... + T_n$

$$= \frac{1}{x}\left[\frac{1}{1+x} - \frac{1}{1+(n+1)x}\right] = \frac{n}{(1+x)[1+(n+1)x]}$$

Ex. 10 If a, b, x, y are positive natural numbers such that $\frac{1}{x}+\frac{1}{y}=1$ then prove that $\frac{a^x}{x}+\frac{b^y}{y}\geq ab$.

Sol. Consider the positive numbers a^x, a^x,.......y times and b^y, b^y,......x times
For all these numbers,

$$AM = \frac{\{a^x + a^x + ...y\ time\} + \{b^y + b^y + ...x\ times\}}{x+y} = \frac{ya^x + xa^y}{(x+y)}$$

$$GM = \left\{\left(a^x.a^x......y\ times\right)\left(b^y.b^y......x\ times\right)\right\}^{\frac{1}{(x+y)}} = \left[\left(a^{xy}\right).\left(b^{xy}\right)\right]^{\frac{1}{(x+y)}} = (ab)^{\frac{xy}{(x+y)}}$$

As $\frac{1}{x}+\frac{1}{y}=1$, $\frac{x+y}{xy}=1$, i.e, $x + y = xy$

So using AM ≥ GM $\frac{ya^x + xa^y}{x+y} \geq (ab)^{\frac{xy}{x+y}}$

$\therefore$ $\frac{ya^x + xa^y}{xy} \geq ab$ or $\frac{a^x}{x}+\frac{a^y}{y} \geq ab.$

Ex. 11 If p^{th}, q^{th}, r^{th} terms of an H.P. be a, b, c respectively, prove that
$$(q - r)bc + (r - p)\ ac + (p - q)ab = 0$$

Sol. Let 'x' be the first term and 'd' be the common difference of the corresponding A.P..

So $\frac{1}{a} = x + (p - 1)d$ .....**(i)**

$\frac{1}{b} = x + (q - 1)\ d$ .....**(ii)**

$\frac{1}{c} = x + (r - 1)\ d$ .....**(iii)**

(i) - (ii) $\Rightarrow$ $ab(p - q)d = b - a$ .....**(iv)**
(ii) - (iii) $\Rightarrow$ $bc\ (q - r)d = c - b$ .....**(v)**
(iii) - (i) $\Rightarrow$ $ac\ (r - p)\ d = a - c$ .....**(vi)**
(iv) + **(v)** + **(vi)** gives
$bc(q - r) + ac(r - p) + ab\ (p - q) = 0.$

Ex. 12 Find the sum of series upto n terms $\left(\frac{2n+1}{2n-1}\right)+3\left(\frac{2n+1}{2n-1}\right)^2+5\left(\frac{2n+1}{2n-1}\right)^3+.........$

Sol. For $x \neq 1$, let

$S = x + 3x^2 + 5x^3 + ... + (2n - 3)x^{n-1} + (2n - 1)x^n$ .....**(i)**

$\Rightarrow$ $xS = x^2 + 3x^3 + ... + (2n - 5)x^{n-1} + (2n - 3)x^n + (2n - 1)x^{n+1}$ .....**(ii)**

Subtracting **(ii)** from **(i)**, we get

$$(1 - x)S = x + 2x^2 + 2x^3 + ... + 2x^{n-1} + 2x^n - (2n - 1)x^{n+1} = x + \frac{2x^2(1-x^{n-1})}{1-x} - (2n-1)x^{n+1}$$

$$= \frac{x}{1-x}\left[1 - x + 2x - 2x^n - (2n - 1)x^n + (2n - 1)x^{n+1}\right]$$

$\Rightarrow$ $$S = \frac{x}{(1-x)^2}\left[(2n-1)x^{n+1} - (2n + 1)x^n + 1 + x\right]$$

Thus $$\left(\frac{2n+1}{2n-1}\right)+3\left(\frac{2n+1}{2n-1}\right)^2+........+(2n-1)\left(\frac{2n+1}{2n-1}\right)^n$$

$$= \left(\frac{2n+1}{2n-1}\right)\left(\frac{2n-1}{2}\right)^2\left[(2n-1)\left(\frac{2n+1}{2n-1}\right)^{n+1} - (2n+1)\left(\frac{2n+1}{2n-1}\right)^n + 1 + \frac{2n+1}{2n-1}\right] = \frac{4n^2-1}{4}.\frac{4n}{2n-1} = n(2n + 1)$$

Ex. 13 Sum to n terms of the series $\frac{4}{1.2.3}+\frac{5}{2.3.4}+\frac{6}{3.4.5}+.........$

Sol. Let $T_r = \frac{r+3}{r(r+1)(r+2)} = \frac{1}{(r+1)(r+2)} + \frac{3}{r(r+1)(r+2)}$

$$= \left[\frac{1}{r+1}-\frac{1}{r+2}\right]+\frac{3}{2}\left[\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}\right]$$

$$\therefore \quad S = \left[\frac{1}{2}-\frac{1}{n+2}\right]+\frac{3}{2}\left[\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right]$$

$$= \frac{5}{4}-\frac{1}{n+2}\left[1+\frac{3}{2(n+1)}\right] = \frac{5}{4}-\frac{1}{2(n+1)(n+2)}[2n+5]$$

Ex. 14 The series of natural numbers is divided into groups (1), (2, 3, 4), (5, 6, 7, 8, 9) .... and so on. Show that the sum of the numbers in n^{th} group is $n^3 + (n-1)^3$

Sol. The groups are (1), (2, 3, 4), (5, 6, 7, 8, 9) ....

The number of terms in the groups are 1, 3, 5......

$\therefore$ The number of terms in the n^{th} group = $(2n-1)$ the last term of the n^{th} group is n^2

If we count from last term common difference should be -1

So the sum of numbers in the n^{th} group $=\left(\frac{2n-1}{2}\right)\left\{2n^2+(2n-2)(-1)\right\}$

$$= (2n-1)(n^2-n+1) = 2n^3-3n^2+3n-1 = n^3+(n-1)^3$$

Ex. 15 Find the natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n-1)$, where the function f satisfied $f(x+y) = f(x).\ f(y)$ for all natural number x,y and further $f(1) = 2$.

Sol. It is given that

$f(x+y) = f(x)\, f(y)$ and $f(1) = 2$

$f(1+1) = f(1)\, f(1)$

$\Rightarrow \quad f(2) = 2^2, f(1+2) = f(1)\, f(2) \qquad \Rightarrow \quad f(3) = 2^3, \quad f(2+2) = f(2)\, f(2)$

$\Rightarrow \quad f(4) = 2^4$

Similarly $f(k) = 2^k$ and $f(a) = 2^a$

Hence, $\sum_{k=1}^{n} f(a+k) = \sum_{k=1}^{n} f(a)f(k) = f(a)\sum_{k=1}^{n} f(k) = 2^a\sum_{k=1}^{n} 2^k = 2^a\{2^1+2^2+.........+2^n\}$

$$= 2^a\left\{\frac{2(2^n-1)}{2-1}\right\} = 2^{a+1}(2^n-1)$$

But $\sum_{k=1}^{n} f(a+k) = 16(2^n-1)$

$2^{a+1}(2^n-1) = 16\,(2^n-1)$

$\therefore \quad 2^{a+1} = 2^4$

$\therefore \quad a+1 = 4 \quad \therefore \quad a = 3$

Exercise # 1 [Single Correct Choice Type Questions]

1. If $\ln(a+c)$, $\ln(c-a)$, $\ln(a-2b+c)$ are in A.P., then :
(A) a, b, c are in A.P. **(B)** a^2, b^2, c^2 are in A.P
(C) a, b, c are in G.P. **(D)** a, b, c are in H.P.

2. The quadratic equation whose roots are the A.M. and H.M. between the roots of the equation, $2x^2-3x+5=0$ is -
(A) $4x^2-25x+10=0$ **(B)** $12x^2-49x+30=0$
(C) $14x^2-12x+35=0$ **(D)** $2x^2+3x+5=0$

3. If a, b and c are three consecutive positive terms of a G.P. then the graph of $y = ax^2 + bx + c$ is
(A) a curve that intersects the x-axis at two distinct points.
(B) entirely below the x-axis.
(C) entirely above the x-axis.
(D) tangent to the x-axis.

4. If $x \in R$, the numbers $5^{1+x} + 5^{1-x}$, a/2, $25^x + 25^{-x}$ form an A.P. then 'a' must lie in the interval:
(A) [1, 5] **(B)** [2, 5] **(C)** [5, 12] **(D)** $[12, \infty)$

5. If a, b, c are distinct positive real in H.P., then the value of the expression, $\frac{b+a}{b-a} + \frac{b+c}{b-c}$ is equal to
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

6. The maximum value of the sum of the A.P. 50, 48, 46, 44, ... is -
(A) 325 **(B)** 648 **(C)** 650 **(D)** 652

7. Let s_1, s_2, s_3 .... and t_1, t_2, t_3 .... are two arithmetic sequences such that $s_1 = t_1 \neq 0$; $s_2 = 2t_2$ and $\sum_{i=1}^{10} s_i = \sum_{i=1}^{15} t_i$.

Then the value of $\frac{s_2 - s_1}{t_2 - t_1}$ is
(A) 8/3 **(B)** 3/2 **(C)** 19/8 **(D)** 2

8. For a sequence $\{a_n\}$, $a_1 = 2$ and $\frac{a_{n+1}}{a_n} = \frac{1}{3}$. Then $\sum_{r=1}^{20} a_r$ is
(A) $\frac{20}{2}[4 + 19 \times 3]$ **(B)** $3\left(1 - \frac{1}{3^{20}}\right)$ **(C)** $2(1 - 3^{20})$ **(D)** none of these

9. The interior angles of a convex polygon are in AP . The smallest angle is 120° & the common difference is 5°. Find the number of sides of the polygon -
(A) 9 **(B)** 16 **(C)** 12 **(D)** none of these

10. The sum $\sum_{k=1}^{100} \frac{k}{k^4 + k^2 + 1}$ is equal to
(A) $\frac{4950}{10101}$ **(B)** $\frac{5050}{10101}$ **(C)** $\frac{5151}{10101}$ **(D)** none

11. Consider an A.P. with first term 'a' and the common difference 'd'. Let S_k denote the sum of its first K terms.
If $\frac{S_{kx}}{S_x}$ is independent of x, then
(A) a = d/2 **(B)** a = d **(C)** a = 2d **(D)** none of these

12. If $a_1, a_2, a_3, ..., a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + a_3 + ... + a_{n-1} + 2a_n$ is

(A) $n(2c)^{1/n}$ **(B)** $(n + 1)\, c^{1/n}$ **(C)** $2nc^{1/n}$ **(D)** $(n + 1)(2c)^{1/n}$

13. The first term of an infinitely decreasing G.P. is unity and its sum is S. The sum of the squares of the terms of the progression is -

(A) $\frac{S}{2S-1}$ **(B)** $\frac{S^2}{2S-1}$ **(C)** $\frac{S}{2-S}$ **(D)** S^2

14. The sum of the first n-terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n(n+1)^2}{2}$, when n is even.

When n is odd, the sum is

(A) $\frac{n(n+1)^2}{4}$ **(B)** $\frac{n^2(n+2)}{4}$ **(C)** $\frac{n^2(n+1)}{2}$ **(D)** $\frac{n(n+2)^2}{4}$

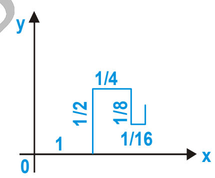
15. If p, q, r in harmonic progression and p & r be different having same sign then the roots of the equation $px^2 + qx + r = 0$ are -

(A) real and equal **(B)** real and distinct **(C)** irrational **(D)** imaginary

16. The arithmetic mean of the nine numbers in the given set {9, 99, 999, ... 999999999} is a 9 digit number N, all whose digits are distinct. The number N does not contain the digit

(A) 0 **(B)** 2 **(C)** 5 **(D)** 9

17. A particle begins at the origin and moves successively in the following manner as shown, 1 unit to the right, 1/2 unit up, 1/4 unit to the right, 1/8 unit down, 1/16 unit to the right etc. The length of each move is half the length of the previous move and movement continues in the 'zigzag' manner indefinitely. The co-ordinates of the point to which the 'zigzag' converges is -



(A) (4/3, 2/3) **(B)** (4/3, 2/5) **(C)** (3/2, 2/3) **(D)** (2, 2/5)

18. For which positive integers *n* is the ratio, $\frac{\sum_{k=1}^{n} k^2}{\sum_{k=1}^{n} k}$ an integer ?

(A) odd *n* only **(B)** even *n* only

(C) $n = 1 + 6k$ only, where $k \geq 0$ and $k \in I$ **(D)** $n = 1 + 3k$, integer $k \geq 0$

19. If A, G & H are respectively the A.M., G.M. & H.M. of three positive numbers a, b, & c, then the equation whose roots are a, b, & c is given by:

(A) $x^3 - 3Ax^2 + 3G^3x - G^3 = 0$ **(B)** $x^3 - 3Ax^2 + 3(G^3/H)x - G^3 = 0$

(C) $x^3 + 3Ax^2 + 3(G^3/H)x - G^3 = 0$ **(D)** $x^3 - 3Ax^2 - 3(G^3/H)x + G^3 = 0$

20. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+........+\text{to } \infty=\frac{\pi^4}{90}$, then $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+......+\text{to } \infty$ is equals to -

(A) $\frac{\pi^4}{96}$ (B) $\frac{\pi^4}{45}$ (C) $\frac{89\pi^4}{90}$ (D) none of these

21. a, b be the roots of the equation $x^2 - 3x + a = 0$ and g, d the roots of $x^2 - 12x + b = 0$ and numbers a, b, g, d (in this order) form an increasing G.P., then

(A) a = 3, b = 12 (B) a = 12, b = 3 (C) a = 2, b = 32 (D) a = 4, b = 16

22. If a, b, c are positive numbers in G.P. and $\log\left(\frac{5c}{a}\right)$, $\log\left(\frac{3b}{5c}\right)$ and $\log\left(\frac{a}{3b}\right)$ are in A.P., then a, b, c forms the sides of a triangle which is -

(A) equilateral (B) right angled (C) isosceles (D) none of these

23. Suppose a, b, c are in A.P. & $|a|, |b|, |c| < 1$. If $x = 1 + a + a^2 + ...$ to ∞; $y = 1 + b + b^2 + ...$ to ∞ & $z = 1 + c + c^2 + ...$ to ∞, then x, y, z are in:

(A) A.P. (B) G.P. (C) H.P. (D) none

24. $\frac{1}{2.4}+\frac{1.3}{2.4.6}+\frac{1.3.5}{2.4.6.8}+\frac{1.3.5.7}{2.4.6.8.10}+...............\infty$ is equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1

25. If a, b, c, d are positive real numbers such that a + b + c + d = 2, then M = (a + b) (c + d) satisfies the relation:

(A) $0 \le M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$

26. The sum to n terms of the series $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+.........$ is -

(A) $\frac{3n}{n+1}$ (B) $\frac{6n}{n+1}$ (C) $\frac{9n}{n+1}$ (D) $\frac{12n}{n+1}$

27. Let a_n, $n \in N$ is an A.P. with common difference 'd' and all whose terms are non-zero. If n approaches infinity, then the sum $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+......+\frac{1}{a_na_{n+1}}$ will approach

(A) $\frac{1}{a_1d}$ (B) $\frac{2}{a_1d}$ (C) $\frac{1}{2a_1d}$ (D) a_1d

28. If $3+\frac{1}{4}(3+d)+\frac{1}{4^2}(3+2d)+......+\text{upto } \infty = 8$, then the value of d is:

(A) 9 (B) 5 (C) 1 (D) none of these

29. If the $(m+1)^{th}$, $(n+1)^{th}$ & $(r+1)^{th}$ terms of an AP are in GP & m, n, r are in HP, then the ratio of the common difference to the first term of the AP is -

(A) $\frac{1}{n}$ (B) $\frac{2}{n}$ (C) $-\frac{2}{n}$ (D) none of these

30. If $\frac{1+3+5+....\text{upto n terms}}{4+7+10+....\text{upto n terms}}=\frac{20}{7\log_{10}x}$ and $n = \log_{10}x+\log_{10}x^{\frac{1}{2}}+\log_{10}x^{\frac{1}{4}}+\log_{10}x^{\frac{1}{8}}+.......+\infty$, then x is equal to

(A) 10^3 (B) 10^5 (C) 10^6 (D) 10^7

Exercise # 2

Part # I [Multiple Correct Choice Type Questions]

1. Let a, b, g be the roots of the equation $x^3 + 3ax^2 + 3bx + c = 0$. If a, b, g are in H.P. then b is equal to -

(A) $-c/b$ **(B)** c/b **(C)** $-a$ **(D)** a

2. x_1, x_2 are the roots of the equation $x^2 - 3x + A = 0$; x_3, x_4 are roots of the equation $x^2 - 12x + B = 0$, such that x_1, x_2, x_3, x_4 form an increasing G.P., then

(A) $A = 2$ **(B)** $B = 32$ **(C)** $x_1 + x_3 = 5$ **(D)** $x_2 + x_4 = 10$

3. If $a_1, a_2, \ldots\ldots a_n \in R^+$ and $a_1.a_2 \ldots\ldots a_n = 1$ then the least value of $(1 + a_1 + a_1^2)(1 + a_2 + a_2^2)\ldots..(1 + a_n + a_n^2)$ is -

(A) 3^n **(B)** $n3^n$ **(C)** 3^{3n} **(D)** data inadequate

4. If sum of the infinite G.P., $p, 1, \frac{1}{p}, \frac{1}{p^2}, \frac{1}{p^3}, \ldots\ldots$ is $\frac{9}{2}$, then value of p is

(A) 3 **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{1}{3}$

5. If a, $a_1, a_2, \ldots\ldots a_{10}$, b are in A.P. and a, $g_1, g_2, \ldots\ldots g_{10}$, b are in G.P. and h is the H.M. between a and b, then $\frac{a_1 + a_2 + \ldots.. + a_{10}}{g_1 g_{10}} + \frac{a_2 + a_3 + \ldots.. + a_9}{g_2 g_9} + \ldots\ldots + \frac{a_5 + a_6}{g_5 g_6}$ is -

(A) $\frac{10}{h}$ **(B)** $\frac{10}{h}$ **(C)** $\frac{30}{h}$ **(D)** $\frac{5}{h}$

6. Let $a_1, a_2, a_3 \ldots\ldots$ and $b_1, b_2, b_3 \ldots\ldots$ be arithmetic progressions such that $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$. Then

(A) the difference between successive terms in progression 'a' is opposite of the difference in progression 'b'.

(B) $a_n + b_n = 100$ for any n.

(C) $(a_1 + b_1), (a_2 + b_2), (a_3 + b_3), \ldots\ldots$ are in A.P.

(D) $\sum_{r=1}^{100} (a_r + b_r) = 10000$

7. For the A.P. given by $a_1, a_2, \ldots\ldots\ldots\ldots, a_n, \ldots\ldots$, with non-zero common difference, the equations satisfied are-

(A) $a_1 + 2a_2 + a_3 = 0$ **(B)** $a_1 - 2a_2 + a_3 = 0$

(C) $a_1 + 3a_2 - 3a_3 - a_4 = 0$ **(D)** $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

8. If $(1 + 3 + 5 + \ldots + a) + (1 + 3 + 5 + \ldots + b) = (1 + 3 + 5 + \ldots. + c)$, where each set of parentheses containsthe sum of consecutive odd integers as shown such that - **(i)** $a + b + c = 21$, **(ii)** $a > 6$

If G = Max{a, b, c} and L = Min{a, b, c}, then -

(A) $G - L = 4$ **(B)** $b - a = 2$ **(C)** $G - L = 7$ **(D)** $a - b = 2$

9. The p^{th} term T_p of H.P. is $q(q + p)$ and q^{th} term T_q is $p(p + q)$ when $p > 1, q > 1$, then -

(A) $T_{p+q} = pq$ **(B)** $T_{pq} = p + q$ **(C)** $T_{p+q} > T_{pq}$ **(D)** $T_{pq} > T_{p+q}$

10. If a, b and c are distinct positive real numbers and $a^2 + b^2 + c^2 = 1$, then ab + bc + ca is -

(A) equal to 1 **(B)** less than 1 **(C)** greater than 1 **(D)** any real number

11. If $\sum_{r=1}^{n} r(r+1)\ (2r+3) = an^4 + bn^3 + cn^2 + dn + e$, then

(A) $a + c = b + d$
(B) $e = 0$
(C) $a, b - 2/3, c - 1$ are in A.P.
(D) c/a is an integer

12. Let $a_1, a_2, \ldots\ldots., a_{10}$ be in A.P. & $h_1, h_2, \ldots\ldots.h_{10}$ be in H.P. . If $a_1 = h_1 = 2$ & $a_{10} = h_{10} = 3$ then a_4h_7 is -

(A) 2 (B) 3 (C) 5 (D) 6

13. If first and $(2n - 1)^{th}$ terms of an A.P., G.P. and H.P. are equal and their n^{th} terms are a, b, c respectively, then -

(A) $a + c = 2b$ (B) $a \geq b \geq c$ (C) $a + c = b$ (D) $b^2 = ac$

14. If the roots of the equation, $x^3 + px^2 + qx - 1 = 0$ form an increasing G.P. where p and q are real, then

(A) $p + q = 0$
(B) $p \in (-3, \infty)$
(C) one of the roots is unity
(D) one root is smaller than 1 and one root is greater than 1

15. If $x, |x + 1|, |x - 1|$ are three terms of an A.P., then its sum upto 20 terms is –

(A) 180 (B) 350 (C) 90 (D) 720

16. Let a, x, b be in A.P. ; a, y, b be in G.P. and a, z, b be in H.P. If $x = y + 2$ and $a = 5z$ then -

(A) $y^2 = xz$ (B) $x > y > z$ (C) $a = 9, b = 1$ (D) $a = \frac{9}{4}, b = \frac{1}{4}$

17. If the arithmetic mean of two positive numbers a & b $(a > b)$ is twice their geometric mean, then a: b is:

(A) $2+\sqrt{3} : 2-\sqrt{3}$ (B) $7+4\sqrt{3} : 1$ (C) $1 : 7-4\sqrt{3}$ (D) $2 : \sqrt{3}$

18. If $\sin(x - y)$, $\sin x$ and $\sin(x + y)$ are in H.P., then $\sin x . \sec\frac{y}{2} =$

(A) 2 (B) $\sqrt{2}$ (C) $-\sqrt{2}$ (D) -2

19. The sum of the first 100 terms common to the series 17, 21, 25, ... and 16, 21, 26, ...is -

(A) 101100 (B) 111000 (C) 110010 (D) 100101

20. a, b, c are three distinct real numbers, which are in G.P. and $a + b + c = xb$, then -

(A) $x < -1$ (B) $-1 < x < 2$ (C) $2 < x < 3$ (D) $x > 3$

21. If $a_1, a_2, \ldots\ldots., a_n$ are distinct terms of an A.P., then

(A) $a_1 + 2a_2 + a_3 = 0$
(B) $a_1 - 2a_2 + a_3 = 0$
(C) $a_1 + 3a_2 - 3a_3 - a_4 = 0$
(D) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

22. Let $p, q, r \in R^+$ and $27\,pqr \geq (p + q + r)^3$ and $3p + 4q + 5r = 12$ then $p^3 + q^4 + r^5$ is equal to -

(A) 2 (B) 6 (C) 3 (D) none of these

Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** The sum of the first 30 terms of the sequence 1, 2, 4, 7, 11, 16, 22,...... is 4520.
Statement-II : If the successive differences of the terms of a sequence form an A.P., then general term of sequence is of the form $an^2 + bn + c$.

2. **Statement-I :** n^{th} term (T_n) of the sequence (1, 6, 18, 40, 75, 126,....) is $an^3 + bn^2 + cn + d$, and $6a + 2b - d$ is $= 4$.
Statement-II : If the second successive differences (Differences of the differences) of a series are in A.P., then T_n is a cubic polynomial in n.

3. **Statement-I :** 1, 2, 4, 8, ... is a G.P., 4, 8, 16, 32 is a G.P. and $1+4, 2+8, 4+16, 8+32$, ... is also a G.P.
Statement-II : Let general term of a G.P. (with positive terms) with common ratio r be T_{k+1} and general term of another G.P. (with positive terms) with common ratio r be T'_{k+1}, then the series whose general term $T''_{k+1} = T_{k+1} + T'_{k+1}$ is also a G.P. with common ratio r.

4. **Statement-I :** For $n \in N$, $2^n > 1 + n\left(\sqrt{(2^{n-1})}\right)$
Statement-II : G.M. > H.M. and (AM) (HM) = $(GM)^2$

5. **Statement-I :** Circumradius and inradius of a triangle can not be 12 and 8 respectively.
Statement-II : Circumradius $\geq$ 2 (inradius)

6. **Statement-I :** Minimum value of $\dfrac{\sin^3 x + \cos^3 x + 3\sin^2 x + 3\sin x + 2}{(\sin x + 1)\cos x}$ for $x \in \left[0, \dfrac{\pi}{2}\right)$ is 3
Statement-II : The least value of $a \sin q + b \cos q$ is $-\sqrt{a^2 + b^2}$

7. **Statement-I :** The format of n^{th} term (T_n) of the sequence (ln2, ln4, ln32, ln1024.......) is $an^2 + bn + c$.
Statement-II : If the second successive differences between the consecutive terms of the given sequence are in G.P., then $T_n = a + bn + cr^{n-1}$, where a, b, c are constants and r is common ratio of G.P.

8. **Statement-I :** If $27\,abc \geq (a+b+c)^3$ and $3a + 4b + 5c = 12$ then $\dfrac{1}{a^2} + \dfrac{1}{b^3} + \dfrac{1}{c^5} = 10$; where a, b, c are positive real numbers.
Statement-II : For positive real numbers A.M. $\geq$ G.M.

9. **Statement-I :** The series for which sum to n terms, S_n, is given by $S_n = 5n^2 + 6n$ is an A.P.
Statement-II : The sum to n terms of an A.P. having non–zero common difference is a quadratic in n, i.e., $an^2 + bn$.

10. **Statement-I :** In any ΔABC, maximum value of $r_1 + r_2 + r_3 = \dfrac{9R}{2}$.
Statement-II : In any ΔABC, $R \geq 2r$.

11. **Statement-I :** If a, b, c are three distinct positive number in H.P., then $\left(\frac{a+b}{2a-b}\right)+\left(\frac{c+b}{2c-b}\right)>4$

Statement-II : Sum of any number and it's reciprocal is always greater than or equal to 2.

12. **Statement-I :** 3, 6, 12 are in G.P., then 9, 12, 18 are in H.P.

Statement-II : If three consecutive terms of a G.P. are positive and if middle term is added in these terms, then resultant will be in H.P.

13. **Statement-I :** If $x^2y^3 = 6(x, y > 0)$, then the least value of $3x + 4y$ is 10

Statement-II : If $m_1, m_2 \in N$, $a_1, a_2 > 0$ then $\frac{m_1a_1+m_2a_2}{m_1+m_2} \geq (a_1^{m_1}a_2^{m_2})^{\frac{1}{m_1+m_2}}$ and equality holds when $a_1 = a_2$.

14. **Statement-I :** The difference between the sum of the first 100 even natural numbers and the sum of the first 100 odd natural numbers is 100.

Statement-II : The difference between the sum of the first *n* even natural numbers and sum of the first *n* odd natural numbers is *n*.

15. **Statement-I :** If a, b, c are three positive numbers in G.P., then $\left(\frac{a+b+c}{3}\right).\left(\frac{3abc}{ab+bc+ca}\right) = \left(\sqrt[3]{abc}\right)^2$

Statement-II : (A.M.) (H.M.) = $(G.M.)^2$ is true for any set of positive numbers.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following questions contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one** statement in **Column-II**.

1.

	Column-I		Column-II
(A)	If a_i's are in A.P. and $a_1 + a_3 + a_4 + a_5 + a_7 = 20$, a_4 is equal to	(p)	21
(B)	Sum of an infinite G.P. is 6 and it's first term is 3. then harmonic mean of first and third terms of G.P. is	(q)	4
(C)	If roots of the equation $x^3 - ax^2 + bx + 27 = 0$, are in G.P. with common ratio 2, then a + b is equal to	(r)	24
(D)	If the roots of $x^4 - 8x^3 + ax^2 + bx + 16 = 0$ are positive real numbers then a is	(s)	6/5

2.

	Column – I		Column – II
(A)	If $\log_x y$, $\log_z x$, $\log_y z$ are in G.P., $xyz = 64$ and x^3, y^3, z^3 are in A.P., then $\frac{3x}{y}$ is equal to	(p)	2
(B)	The value of $2^{\frac{1}{4}} \cdot 4^{\frac{1}{8}} \cdot 8^{\frac{1}{16}}$ - - - ∞ is equal to	(q)	1
(C)	If x, y, z are in A.P., then $(x+2y-z)(2y+z-x)(z+x-y) = kxyz$, where $k \in N$, then k is equal to	(r)	3
(D)	There are m A.M. between 1 and 31. If the ratio of the 7^{th} and $(m-1)^{th}$ means is 5 : 9, then $\frac{m}{7}$ is equal to	(s)	4

3.

	Column – I		Column – II
(A)	If $\log_5 2$, $\log_5(2^x - 5)$ and $\log_5(2^x - 7/2)$ are in A.P., then value of 2x is equal to	(p)	6
(B)	Let S_n denote sum of first n terms of an A.P. If $S_{2n} = 3S_n$, then $\frac{S_{3n}}{S_n}$ is	(q)	9
(C)	Sum of infinite series $4 + \frac{8}{3} + \frac{12}{3^2} + \frac{16}{3^3} + - - -$ is	(r)	3
(D)	The length, breadth, height of a rectangular box are in G.P. The volume is 27, the total surface area is 78. Then the length is	(s)	1

4. **Column-I** | **Column-II**

	Column-I		Column-II
(A)	n^{th} term of the series 4, 11, 22, 37, 56, 79,.......	(p)	$2n^2 + n$
(B)	$\|1^2 - 2^2 + 3^2 - 4^2 ....$ 2n terms$\|$ is equal to	(q)	$2n^2 + n + 1$
(C)	sum to n terms of the series 3, 7, 11, 15,....... is	(r)	$-(n^2 + n)$
(D)	coefficient of x^n in $2x(x-1)(x-2)$ .... $(x-n)$ is	(s)	$\frac{1}{2}(n^2+n)$

Part # II [Comprehension Type Questions]

Comprehension # 1

There are 4n + 1 terms in a sequence of which first 2n + 1 are in Arithmetic Progression and last 2n + 1 are in Geometric Progression the common difference of Arithmetic Progression is 2 and common ratio of Geometric Progression is 1/2. The middle term of the Arithmetic Progression is equal to middle term of Geometric Progression. Let middle term of the sequence is T_m and T_m is the sum of infinite Geometric Progression whose sum of first two terms is $\left(\frac{5}{4}\right)^2$ n and ratio of these terms is $\frac{9}{16}$.

1. Number of terms in the given sequence is equal to -

(A) 9 (B) 17 (C) 13 (D) none

2. Middle term of the given sequence, i.e. T_m is equal to -

(A) 16/7 (B) 32/7 (C) 48/7 (D) 16/9

3. First term of given sequence is equal to -

(A) –8/7, –20/7 (B) –36/7 (C) 36/7 (D) 48/7

4. Middle term of given A. P. is equal to -

(A) 6/7 (B) 10/7 (C) 78/7 (D) 11

5. Sum of the terms of given A. P. is equal to -

(A) 6/7 (B) 7 (C) 3 (D) 6

Comprehension # 2

In a sequence of (4n + 1) terms the first (2n + 1) terms are in AP whose common difference is 2, and the last (2n + 1) terms are in GP whose common ratio 0.5. If the middle terms of the AP and GP are equal, then

1. Middle term of the sequence is

(A) $\frac{n \cdot 2^{n+1}}{2^n - 1}$ (B) $\frac{n \cdot 2^{n+1}}{2^{2n} - 1}$ (C) $n \cdot 2^n$ (D) None of these

2. First term of the sequence is

(A) $\frac{4n + 2n \cdot 2^n}{2^n - 1}$ (B) $\frac{4n - 2n \cdot 2^n}{2^n - 1}$ (C) $\frac{2n - n \cdot 2^n}{2^n - 1}$ (D) $\frac{2n + n \cdot 2^n}{2^n - 1}$

3. Middle term of the GP is

(A) $\frac{2^n}{2^n - 1}$ (B) $\frac{n \cdot 2^n}{2^n - 1}$ (C) $\frac{n}{2^n - 1}$ (D) $\frac{2n}{2^n - 1}$

Comprehension # 3

Let a_m (m = 1, 2, ...,p) be the possible integral values of a for which the graphs of $f(x) = ax^2 + 2bx + b$ and $g(x) = 5x^2 - 3bx - a$ meets at some point for all real values of b.

Let $t_r = \prod_{m=1}^{p}(r - a_m)$ and $S_n = \sum_{r=1}^{n} t_r$, $n \in N$.

1. The minimum possible value of a is

(A) $\frac{1}{5}$ **(B)** $\frac{5}{26}$ **(C)** $\frac{3}{38}$ **(D)** $\frac{2}{43}$

2. The sum of values of n for which S_n vanishes is

(A) 8 **(B)** 9 **(C)** 10 **(D)** 15

3. The value of $\sum_{r=5}^{\infty} \frac{1}{t_r}$ is equal to

(A) $\frac{1}{3}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{15}$ **(D)** $\frac{1}{18}$

Comprehension # 4

We know that $1 + 2 + 3 + ... = \frac{n(n+1)}{2} = f(n)$,

$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6} = g(n)$,

$1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2 = h(n)$

1. $g(n) - g(n - 1)$ must be equal to

(A) n^2 **(B)** $(n - 1)^2$ **(C)** $n - 1$ **(D)** n^3

2. Greatest even natural number which divides $g(n) - f(n)$, for every $n \geq 2$, is

(A) 2 **(B)** 4 **(C)** 6 **(D)** none of these

3. $f(n) + 3\,g(n) + h(n)$ is divisible by $1 + 2 + 3 + ... + n$

(A) only if n = 1 **(B)** only if n is odd **(C)** only if n is even **(D)** for all $n \in N$

Comprehension # 5

If $a_i > 0$, i = 1, 2, 3, ... n and $m_1, m_2, m_3, \ldots, m_n$ be positive rational numbers, then

$$\left(\frac{m_1a_1+m_2a_2+\ldots+m_na_n}{m_1+m_2+\ldots+m_n}\right) \geq \left(a_1^{m_1}a_2^{m_2}\ldots a_n^{m_n}\right)^{1/(m_1+m_2+\ldots m_n)} \geq \frac{(m_1+m_2+\ldots+m_n)}{\frac{m_1}{a_1}+\frac{m_2}{a_2}+\ldots+\frac{m_n}{a_n}}$$

is called weighted mean theorem

where $A^* = \dfrac{m_1a_1+m_2a_2+\ldots+m_na_n}{m_1+m_2+\ldots+m_n}$ = Weighted arithmetic mean

$G^* = \left(a_1^{m_1}a_2^{m_2}\ldots a_n^{m_n}\right)^{1/(m_1+m_2+\ldots+m_n)}$ = Weighted geometric mean

and $H^* = \dfrac{m_1+m_2+\ldots+m_n}{\frac{m_1}{a_1}+\frac{m_2}{a_2}+\ldots\frac{m_n}{a_n}}$ = Weighted harmonic mean

i.e., $A^* \geq G^* \geq H^*$

Now, let a + b + c = 5(a, b, c > 0) and $x^2y^3 = 243(x > 0, y > 0)$

1. The greatest value of ab^3c is -

(A) 3 (B) 9 (C) 27 (D) 81

2. Which statement is correct -

(A) $\frac{1}{5} \geq \frac{1}{\frac{1}{a}+\frac{3}{b}+\frac{1}{c}}$ (B) $\frac{1}{25} \geq \frac{1}{\frac{1}{a}+\frac{9}{b}+\frac{1}{c}}$ (C) $\frac{1}{5} \geq \frac{1}{\frac{1}{a}+\frac{9}{b}+\frac{1}{c}}$ (D) $\frac{1}{25} \geq \frac{1}{\frac{1}{a}+\frac{6}{b}+\frac{1}{c}}$

3. The least value of $x^2 + 3y + 1$ is -

(A) 15 (B) greater than 15 (C) 3 (D) less than 15

4. Which statement is correct -

(A) $\frac{2x+3y}{5} \geq 3 \geq \frac{5}{\frac{3}{x}+\frac{2}{y}}$ (B) $\frac{2x+3y}{5} \geq 3 \geq \frac{5xy}{3x+2y}$

(C) $\frac{2x+3y}{5} \geq 3 \geq \frac{5xy}{3x+4y}$ (D) $\frac{2x+3y}{5} \geq 3 \geq \frac{5xy}{2x+3y}$

Exercise # 4 [Subjective Type Questions]

1. Let $a_1, a_2, a_3 \ldots\ldots a_n$ be an AP . Prove that :

$$\frac{1}{a_1 a_n}+\frac{1}{a_2 a_{n-1}}+\frac{1}{a_3 a_{n-2}}+\ldots\ldots\ldots+\frac{1}{a_n a_1}=\frac{2}{a_1+a_n}\left[\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\ldots\ldots\ldots+\frac{1}{a_n}\right]$$

2. If the sum to first n terms of a series, the r^{th} term of which is given by $(2r + 1)2^r$ can be expressed as $R(n \cdot 2^n) + S \cdot 2^n + T$, then find the value of $(R + S + T)$.

3. Find the sum of 35 terms of the series whose p^{th} term is $\frac{p}{7}+2$.

4. If one AM 'a' & two GM's p & q be inserted between any two given numbers then show that $p^3+ q^3 = 2\,apq$.

5. If the p^{th} , q^{th} , r^{th} terms of a G.P. be a, b, c respectively, prove that $a^{q-r}\, b^{r-p}\, c^{p-q} = 1$.

6. Find the sum of n terms of the series the r^{th} term of which is $(2r + 1)2^r$.

7. Prove that the number $\underbrace{444\ldots\ldots\ldots4}_{n\text{ digits}}\ \underbrace{888\ldots\ldots\ldots8}_{(n-1)\text{ digits}}9$ is a perfect square of the number $\underbrace{666\ldots\ldots\ldots6}_{(n-1)\text{ digits}}7$.

8. If $x > 0$, then find greatest value of the expression $\frac{x^{100}}{1+x+x^2+x^3+\ldots..+x^{200}}$.

9. Find the sum of the first n terms of the series : $1+2\left(1+\frac{1}{n}\right)+3\left(1+\frac{1}{n}\right)^2+4\left(1+\frac{1}{n}\right)^3+\ldots\ldots\ldots$

10. The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer.

11. The value of $x + y + z$ is 15, if a, x, y, z, b are in A.P. while the value of ; $(1/x)+(1/y)+(1/z)$ is 5/3 if a, x, y, z, b are in H.P. Find a & b.

12. If a, b, c, d are in G.P., prove that :

(i) $(a^2 - b^2), (b^2 - c^2), (c^2 - d^2)$ are in G.P. (ii) $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}$ are in G.P.

13. The harmonic mean of two numbers is 4 . The arithmetic mean A & the geometric mean G satisfy the relation $2A + G^2 = 27$. Find the two numbers.

14. The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1 & the third is increased by 1, we obtain three consecutive terms of a G.P., find the numbers.

15. If the p^{th}, q^{th} & r^{th} terms of an AP are in GP. Find the common ratio of the GP.

16. Find the sum $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1$.

Exercise # 5

Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If 1, $\log_3\sqrt{3^{1-x}+2}$, $\log_3(4.3^x - 1)$ are in A.P. then x equals. **[AIEEE 2002]**

(A) $\log_3 4$ **(B)** $1 - \log_3 4$ **(C)** $1 - \log_4 3$ **(D)** $\log_4 3$

2. Sum of infinite number of terms in G.P. is 20 and sum of their square is 100. The common ratio of G.P. is- **[AIEEE 2002]**

(A) 5 **(B)** 3/5 **(C)** 8/5 **(D)** 1/5

3. Fifth term of a G.P. is 2, then the product of its 9 terms is- **[AIEEE 2002]**

(A) 256 **(B)** 512 **(C)** 1024 **(D)** None of these

4. The sum of the series $1^3 - 2^3 + 3^3 - ..... + 9^3 =$ **[AIEEE 2002]**

(A) 300 **(B)** 125 **(C)** 425 **(D)** 0

5. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers m,n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals **[AIEEE 2004]**

(A) 0 **(B)** 1 **(C)** $\frac{1}{mn}$ **(D)** $\frac{1}{m}+\frac{1}{n}$

6. If AM and GM of two roots of a quadratic equation are 9 and 4 respectively, then this quadratic equation is- **[AIEEE 2004]**

(A) $x^2 - 18x + 16 = 0$ **(B)** $x^2 + 18x - 16 = 0$ **(C)** $x^2 + 18x + 16 = 0$ **(D)** $x^2 - 18x - 16 = 0$

7. If a_1 , a_2, a_3 , ... a_n , ... are in G.P. then the value of the $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ determinant, is- **[AIEEE 2004]**

(A) 0 **(B)** 1 **(C)** 2 **(D)** –2

8. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} C^n$ where a, b, c are in A.P. and |a| < 1, |b| < 1, |c| < 1 then x, y, z are in- **[AIEEE 2005]**

(A) HP **(B)** Arithmetic - Geometric Progression
(C) AP **(D)** GP

9. Let a_1, a_2, a_3....... be terms of an A.P. If $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}$, $p \neq q$ then $\frac{a_6}{a_{21}}$ equals- **[AIEEE-2006]**

(A) $\frac{2}{7}$ **(B)** $\frac{11}{41}$ **(C)** $\frac{41}{11}$ **(D)** $\frac{7}{2}$

10. If a_1, a_2 ,......, a_n are in H.P., then the expression $a_1a_2 + a_2a_3 + ...+ a_{n-1}a_n$ is equal to- **[AIEEE-2006]**

(A) na_1a_n **(B)** $(n - 1)a_1a_n$ **(C)** $n(a_1 - a_n)$ **(D)** $(n - 1)(a_1 - a_n)$

11. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals- [AIEEE-2007]

(A) $\frac{1}{2}\sqrt{5}$ (B) $\sqrt{5}$ (C) $\frac{1}{2}(\sqrt{5}-1)$ (D) $\frac{1}{2}(1-\sqrt{5})$

12. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is [AIEEE 2008]

(A) –4 (B) –12 (C) 12 (D) 4

13. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots$ is :- [AIEEE-2009]

(A) 4 (B) 6 (C) 2 (D) 3

14. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in an AP with common difference –2, then the time taken by him to count all notes is :- [AIEEE-2010]

(A) 24 minutes (B) 34 minutes (C) 125 minutes (D) 135 minutes

15. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after :- [AIEEE-2011]

(A) 20 months (B) 21 months (C) 18 months (D) 19 months

16. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is:

(A) $\frac{\alpha-\beta}{200}$ (B) a – b (C) $\frac{\alpha-\beta}{100}$ (D) b – a [AIEEE-2011]

17. **Statement–1 :** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement–2 : $\sum_{k=1}^{n}\left(k^3-(k-1)^3\right)=n^3$, for any natural number n. [AIEEE-2012]

(A) Statement–1 is true, Statement–2 is false.
(B) Statement–1 is false, Statement–2 is true.
(C) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(D) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

18. If 100 times the 100^{th} term of an A.P. with non-zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this A.P. is : [AIEEE-2012]

(A) zero (B) –150 (C) 150 times its 50^{th} term (D) 150

19. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is : **[JEE-MAIN 2013]**

(A) $\frac{7}{81}(179-10^{-20})$ **(B)** $\frac{7}{9}(99-10^{-20})$ **(C)** $\frac{7}{81}(179+10^{-20})$ **(D)** $\frac{7}{9}(99-10^{-20})$

20. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is **[JEE Main 2014]**

(A) $\frac{\sqrt{61}}{9}$ **(B)** $\frac{2\sqrt{17}}{9}$ **(C)** $\frac{\sqrt{34}}{9}$ **(D)** $\frac{2\sqrt{13}}{9}$

21. Three positive numbers from an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is : **[JEE Main 2014]**

(A) $\sqrt{2}+\sqrt{3}$ **(B)** $3+\sqrt{2}$ **(C)** $2-\sqrt{3}$ **(D)** $2+\sqrt{3}$

22. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9 = k(10)^9$, then k is equal to **[JEE Main 2014]**

(A) $\frac{121}{10}$ **(B)** $\frac{441}{100}$ **(C)** 100 **(D)** 110

23. The sum of first 9 terms of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + ...$ is : **[JEE Main 2015]**

(A) 142 **(B)** 192 **(C)** 71 **(D)** 96

24. If m is the A.M. of two distinct real numbers l and $n (l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals. **[JEE Main 2015]**

(A) $4\,lmn^2$ **(B)** $4\,l^2m^2n^2$ **(C)** $4\,l^2mn$ **(D)** $4\,lm^2n$

25. The mean of the data set comprising of 16 observations is 16. If one of the three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is : **[JEE Main 2015]**

(A) 15.8 **(B)** 14.0 **(C)** 16.8 **(D)** 16.0

26. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is :

(A) 4/3 **(B)** 1 **(C)** 7/4 **(D)** 8/5 **[JEE Main 2016]**

27. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + ...$, is $\frac{16}{5}$ m, then m is equal to :

(A) 101 **(B)** 100 **(C)** 99 **(D)** 102 **[JEE Main 2016]**

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. (A) Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is 3/4, then - **[JEE 2000]**

(A) $a = \frac{7}{4}$, $r = \frac{3}{7}$ **(B)** $a = 2,\ r = \frac{3}{8}$ **(C)** $a = \frac{3}{2}$, $r = \frac{1}{2}$ **(D)** $a = 3,\ r = \frac{1}{4}$

(B) If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation -

(A) $0 \le M \le 1$ **(B)** $1 \le M \le 2$ **(C)** $2 \le M \le 3$ **(D)** $3 \le M \le 4$

(C) The fourth power of the common difference of an arithmetic progression with integer entries is added to the Product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. **[JEE 2000]**

2.(A) Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integer values of p and q respectively, are - **[JEE 2001]**

(A) –2, –32 **(B)** –2, 3 **(C)** –6, 3 **(D)** –6, –32

(B) If the sum of the first 2n terms of the A.P. 2, 5, 8 .... is equal to the sum of the first n terms of the A.P. 57, 59, 61, ... then n equals -

(A) 10 **(B)** 12 **(C)** 11 **(D)** 13

(C) Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are

(A) not in A.P./G.P./H.P. **(B)** in A.P. **(C)** in G.P. **(D)** in H.P.

(D) Let a_1, a_2........... be positive real numbers in G.P. For each n, let A_n, G_n, H_n, be respectively, the arithmetic mean, geometric mean and harmonic mean of a_1, a_2, a_3,.......... a_n. Find an expression for the G.M. of G_1, G_2,.............G_n in terms of A_1, A_2, ...A_n, H_1, H_2,..........H_n **[JEE 2001]**

3.(A) Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is -

(A) $\frac{1}{2\sqrt{2}}$ **(B)** $\frac{1}{2\sqrt{3}}$ **(C)** $\frac{1}{2} - \frac{1}{\sqrt{3}}$ **(D)** $\frac{1}{2} - \frac{1}{\sqrt{2}}$ **[JEE 2002]**

(B) Let a, b be positive real numbers. If a, A_1, A_2, b are in A.P. ; a, G_1, G_2, b are in G.P. and a, H_1, H_2, b are in H.P., show that $\frac{G_1G_2}{H_1H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a + b)(a + 2b)}{9ab}$. **[JEE 2002]**

4. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P., then prove that either $a = b = c$ or a, b, $-\frac{c}{2}$ form a G.P. **[JEE 2003]**

5. If a, b, c are positive real numbers, then prove that $[(1 + a)(1 + b)(1 + c)]^7 > 7^7 a^4 b^4 c^4$. **[JEE 2004]**

6. The first term of an infinite geometric progression is x and its sum is 5. Then - **[JEE 2004]**

(A) $0 \le x \le 10$ **(B)** $0 < x < 10$ **(C)** $-10 < x < 0$ **(D)** $x > 10$

7. If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k^{th} match are given by k. 2^{n+1-k}, where $1 \le k \le n$. Find n. **[JEE-2005]**

8. In quadratic equation $ax^2 + bx + c = 0$, if a, b are roots of equation, $\Delta = b^2 - 4ac$ and $a + b$, $a^2 + b^2$, $a^3 + b^3$ are in G.P. then **[JEE 2005]**

(A) $\Delta \neq 0$ **(B)** $\beta\Delta = 0$ **(C)** $\chi\Delta = 0$ **(D)** $\Delta = 0$

9. If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + ...(-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$ then find the minimum natural number n_0 such that $b_n > a_n \ \forall\ n\ ^3\ n_0$ **[JEE 2006]**

Comprehension Based Question

Comprehension # 1

Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r - 1)$.

Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for r = 1,2,........

10. The sum $V_1 + V_2 + ... + V_n$ is : **[JEE 2007]**

(A) $\frac{1}{12}n(n + 1)(3n^2 - n + 1)$ **(B)** $\frac{1}{12}n(n + 1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ **(D)** $\frac{1}{3}(2n^3 - 2n + 3)$

11. T_r is always : **[JEE 2007]**

(A) an odd number **(B)** an even number

(C) a prime number **(D)** a composite number

12. Which one of the following is a correct statement ? **[JEE 2007]**

(A) Q_1, Q_2, Q_3,...are in A.P. with common difference 5

(B) Q_1, Q_2, Q_3,...are in A.P. with common difference 6

(C) Q_1, Q_2, Q_3,...are in A.P. with common difference 11

(D) $Q_1 = Q_2 = Q_3 = ...$

Comprehension # 2

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively :

13. Which one of the following statements is correct ? **[JEE 2007]**

(A) $G_1 > G_2 > G_3 > ...$ **(B)** $G_1 < G_2 < G_3 < ...$

(C) $G_1 = G_2 = G_3 = ...$ **(D)** $G_1 < G_2 < G_3 < ...$ and $G_4 > G_5 > G_6 > ...$

14. Which one of the following statements is correct ? **[JEE 2007]**

(A) $A_1 > A_2 > A_3 > ...$ **(B)** $A_1 < A_2 < A_3 < ...$

(C) $A_1 > A_3 > A_5 > ...$ and $A_2 < A_4 < A_6 < ...$ **(D)** $A_1 < A_3 < A_5 < ...$ and $A_2 > A_4 > A_6 > ...$

15. Which one of the following statements is correct ? **[JEE 2007]**

(A) $H_1 > H_2 > H_3 > ...$ **(B)** $H_1 < H_2 < H_3 < ...$

(C) $H_1 > H_3 > H_5 > ...$ and $H_2 < H_4 < H_6 > ...$ **(D)** $H_1 < H_3 < H_5 < ...$ and $H_2 > H_4 > H_6 > ...$

16. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

Statement -I : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

Statement -II : The numbers b_1, b_2, b_3, b_4 are in H.P. **[JEE 2008]**

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

17. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is [JEE 2009]

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$ (C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

18. Let S_k, k = 1,2,.......,100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right|$ is [JEE 2010]

19. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying
$a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3,4,........,11.
If $\frac{a_1^2+a_2^2+....+a_{11}^2}{11}=90$, then the value of $\frac{a_1+a_2+...+a_{11}}{11}$ is equal to [JEE 2010]

20. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with a > 0 is [JEE 2014]

21. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p=\sum_{i=1}^{p}a_i, 1\le p\le 100$. For any integer n with $1 \le n \le 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is [JEE 2011]

22. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is [JEE 2012]

(A) 22 (B) 23 (C) 24 (D) 25

23. Let $S_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$. Then S_n can take value(s) [JEE-Ad. 2013]

(A) 1056 (B) 1088 (C) 1120 (D) 1332

24. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller to the numbers on the removed cards is k, then k – 20 = [JEE-Ad. 2013]

25. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2+a-14}{a+1}$ is [JEE Ad. 2014]

26. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is. [JEE Ad. 2015]

27. Let $b_i > 1$ for i = 1, 2, ..., 101. Suppose $\log_e b_1, \log_e b_2, \ldots, \log_e b_{101}$ are in Arithmetic Progression (A.P) with the common difference $\log_e 2$. Suppose $a_1, a_2, \ldots, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$.
If $t = b_1 + b_2 + \ldots + b_{51}$ and $s = a_1 + a_2 + \ldots, a_{51}$, then [JEE Ad. 2016]

(A) s > t and $a_{101} > b_{101}$ (B) s > t and $a_{101} < b_{101}$
(C) s < t and $a_{101} > b_{101}$ (D) s < t and $a_{101} < b_{101}$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If $1^2 + 2^2 + 3^2 + ... + 2003^2 = (2003)(4007)(334)$ and
$(1)(2003) + (2)(2002) + (3)(2001) + ... + (2003)(1) = (2003)(334)(x).$, then x equals

(A) 2005 **(B)** 2004 **(C)** 2003 **(D)** 2001

2. If S, P and R are the sum, product and sum of the reciprocals of n terms of an increasing G.P. and $S^n = R^n . P^k$, then k is equal to

(A) 1 **(B)** 2 **(C)** 3 **(D)** none of these

3. The common difference 'd' of the A.P. in which $T_7 = 9$ and $T_1T_2T_7$ is least, is

(A) $\frac{33}{2}$ **(B)** $\frac{5}{4}$ **(C)** $\frac{33}{20}$ **(D)** none of these

4. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is

(A) $\alpha - \beta$ **(B)** $\beta - \alpha$ **(C)** $\frac{\alpha - \beta}{2}$ **(D)** none of these

5. If 1, 2, 3 ... are first terms; 1, 3, 5 .... are common differences and S_1, S_2, S_3 .... are sums of n terms of given p AP's; then $S_1 + S_2 + S_3 + ... + S_p$ is equal to

(A) $\frac{np(np+1)}{2}$ **(B)** $\frac{n(np+1)}{2}$ **(C)** $\frac{np(p+1)}{2}$ **(D)** $\frac{np(np-1)}{2}$

6. If the sum to infinity of the series, $1 + 4x + 7x^2 + 10x^3 + ...$, is $\frac{35}{16}$, where $|x| < 1$, then 'x' equals to :

(A) 19/7 **(B)** 1/5 **(C)** 1/4 **(D)** none of these

7. If a and b are p^{th} and q^{th} terms of an AP, then the sum of its (p + q) terms is

(A) $\frac{p+q}{2}\left[a-b+\frac{a+b}{p-q}\right]$ **(B)** $\frac{p+q}{2}\left[a+b+\frac{a-b}{p-q}\right]$

(C) $\frac{p-q}{2}\left[a+b+\frac{a+b}{p+q}\right]$ **(D)** none of these

8. If the length of sides of a right triangle are in A.P., then the sines of the acute angles are

(A) $\frac{3}{5}, \frac{4}{5}$ **(B)** $\sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}$ **(C)** $\sqrt{\frac{\sqrt{5}-1}{2}}, \sqrt{\frac{\sqrt{5}+1}{2}}$ **(D)** $\sqrt{\frac{\sqrt{3}-1}{2}}, \sqrt{\frac{\sqrt{3}+1}{2}}$

9. If $S_n = \sum_{r=1}^{n} t_r = \frac{1}{6}n(2n^2+9n+13)$, then $\sum_{r=1}^{\infty} \frac{1}{r.\sqrt{t_r}}$ equals

(A) 1 **(B)** 2 **(C)** $\frac{3}{2}$ **(D)** $\frac{1}{2}$

10. **S_1 :** Let $a_1, a_2, a_3, \ldots\ldots\ldots a_8$ be 8 non-negative real numbers such that $a_1 + a_2 + \ldots\ldots + a_8 = 16$ and $P = a_1a_2 + a_2a_3 + a_3a_4 + \ldots\ldots + a_7a_8$, then the maximum value of P is 64.

S_2 : If x, y, r and s are positive real numbers such that $x^2 + y^2 = r^2 + s^2 = 1$, then the maximum value of $(xr + ys)$ is 2.

S_3 : If A.M. and G.M. between two positive numbers are respectively A and G, then the numbers are

$A + \sqrt{A^2 - G^2}\ , A - \sqrt{A^2 - G^2}$

S_4 : If p, q, r be three distinct real numbers in A.P. then $p^3 + r^3$ equals – 6 pqr

(A) TTFF **(B)** FTFT **(C)** TFTF **(D)** FFTT

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. The value of $\sum_{r=1}^{n} \frac{1}{\sqrt{a + rx} + \sqrt{a + (r-1)x}}$ is

(A) $\frac{n}{\sqrt{a} + \sqrt{a + nx}}$ **(B)** $\frac{n}{\sqrt{a} - \sqrt{a + nx}}$ **(C)** $\frac{\sqrt{a + nx} - \sqrt{a}}{x}$ **(D)** $\frac{\sqrt{a} + \sqrt{a + nx}}{x}$

12. For the series $S = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2 + \frac{1}{(1+3+5+7)}(1+2+3+4)^2 + \ldots\ldots$

(A) 7^{th} term is 16 **(B)** 7^{th} term is 18

(C) sum of first ten terms is $\frac{505}{4}$ **(D)** sum of first ten term is $\frac{405}{4}$

13. If 1, $\log_y x$, $\log_z y$, $-15 \log_x z$ are in A.P., then

(A) $z^3 = x$ **(B)** $x = y^{-1}$ **(C)** $z^{-3} = y$ **(D)** $x = y^{-1} = z^3$

14. If $\sum_{r=1}^{n} r(r+1) = \frac{(n+a)(n+b)(n+c)}{3}$, where a < b < c, then

(A) 2b = c **(B)** $a^3 - 8b^3 + c^3 = 8abc$ **(C)** c is prime number **(D)** $(a + b)^2 = 0$

15. Let $a_n = \frac{(111\ldots\ldots1)}{n \text{ times}}$, then

(A) a_{912} is not prime **(B)** a_{951} is not prime **(C)** a_{480} is not prime **(D)** a_{91} is not prime

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** If a, b, c are non zero real numbers such that $3(a^2 + b^2 + c^2 + 1) = 2(a + b + c + ab + bc + ca)$, then a, b, c are in A.P. as well as in G.P.

Statement-II : A series is in A.P. as well as in G.P. if all the terms in the series are equal and non zero.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. **Statement-I :** Equations $x^2 - 4x + 1 = 0$ and $x^2 - ax + b = 0$, where a, b are rational numbers, have atleast one common root, then $a = 4$ and $b = 1$

Statement-II : If two equations $ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$, where a, b, c, a_1, b_1, c_1 are non-zero rational numbers, have common irrational root, then $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement-I :** The sum of the first 30 terms of the sequence 1, 2, 4, 7, 11, 16, 22,...... is 4520.

Statement-II : If the successive differences of the terms of a sequence form an A.P., then general term of sequence is of the form $an^2 + bn + c$.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** Let a, b, c be positive integers, then $a^{\frac{a}{a+b+c}} . b^{\frac{b}{a+b+c}} . c^{\frac{c}{a+b+c}} \geq \frac{1}{3}(a+b+c)$

Statement-II : Let $a_1, a_2, ... a_n$ be positive numbers in A.P. If A & G are the arithmetic and the geometric means of a_1 and a_n respectively then, $G^n < a_1.a_2.....a_n < A^n$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. **Statement-I :** If one A.M. 'A' and two G.M.'s p and q be inserted between any two numbers, then $p^3 + q^3 = 2Apq$

Statement-II : If x, y, z are in G.P., then $y^2 = xz$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21. Match the column

Column – I		Column – II	
(A)	Suppose that $F(n+1) = \frac{2F(n)+1}{2}$ for $n = 1, 2, 3, \ldots$ and $F(1) = 2$. Then $F(101)$ equals	**(p)**	42
(B)	If $a_1, a_2, a_3, \ldots\ldots\ldots a_{21}$ are in A.P. and $a_3 + a_5 + a_{11} + a_{17} + a_{19} = 10$ then the value of $\sum_{i=1}^{21} a_i$ is	**(q)**	1620
(C)	10^{th} term of the sequence $S = 1 + 5 + 13 + 29 + \ldots\ldots$, is	**(r)**	52
(D)	The sum of all two digit numbers which are not divisible by 2 or 3 is	**(s)**	2045
		(t)	$2 + 4 + 6 + \ldots + 12$

22. Match the column

Column – I		Column – II	
(A)	The arithmetic mean of two positive numbers is 6 and their geometric mean G and harmonic mean H satisfy the relation $G^2 + 3H = 48$, then product of the two number is.	**(p)**	$\frac{2}{7}$
(B)	The sum of the series $\frac{5}{1^2.4^2} + \frac{11}{4^2.7^2} + \frac{17}{7^2.10^2} + \ldots$ is.	**(q)**	32
(C)	If the first two terms of a Harmonic Progression be $\frac{1}{2}$ and $\frac{1}{3}$, then the Harmonic Mean of the first four terms is	**(r)**	$\frac{1}{3}$
(D)	Geometric mean of – 4 and – 9	**(s)**	6
		(t)	– 6

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

Let $A_1, A_2, A_3, \ldots\ldots, A_m$ be arithmetic means between –2 and 1027 and $G_1, G_2, G_3, \ldots\ldots, G_n$ be geometric means between 1 and 1024. Product of geometric means is 2^{45} and sum of arithmetic means is 1025×171.

1 The value of n is

(A) 7 **(B)** 9 **(C)** 11 **(D)** none of these

2 The value of m is

(A) 340 **(B)** 342 **(C)** 344 **(D)** 346

3 The value of $G_1 + G_2 + G_3 + \ldots\ldots + G_n$ is

(A) 1022 **(B)** 2044 **(C)** 512 **(D)** none of these

24. **Read the following comprehension carefully and answer the questions.**

There are two sets A and B each of which consists of three numbers in A.P. whose sum is 15 and where D and d are the common differences such that $D - d = 1$. If $\frac{p}{q} = \frac{7}{8}$ where p and q are the product of the numbers respectively and $d > 0$, in the two sets

1. Value of p is

(A) 100 **(B)** 120 **(C)** 105 **(D)** 110

2. Value of q is

(A) 100 **(B)** 120 **(C)** 105 **(D)** 110

3. Value of $D + d$ is

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

25. **Read the following comprehension carefully and answer the questions.**

Four different integers form an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Then

1. The smallest number is :

(A) -2 **(B)** 0 **(C)** -1 **(D)** 2

2. The common difference of the four numbers is

(A) 2 **(B)** 1 **(C)** 3 **(D)** 4

3. The sum of all the four numbers is

(A) 10 **(B)** 8 **(C)** 2 **(D)** 6

SECTION - VI : INTEGER TYPE

26. Find the sum to infinity of a decreasing G.P. with the common ratio x such that $|x| < 1$; $x \neq 0$. The ratio of the fourth term to the second term is $\frac{1}{16}$ and the ratio of third term to the square of the second term is $\frac{1}{9}$

27. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid he dies leaving a third of the debt unpaid. Find the value of the first installment.

28. If $(1^2 - a_1) + (2^2 - a_2) + (3^2 - a_3) + ... + (n^2 - a_n) = \frac{1}{3} n (n^2 - 1)$, then find the value of a_7 .

29. The sum of first p-terms of an A.P. is q and the sum of first q terms is p, find the sum of first (p + q) terms.

30. Circles are inscribed in the acute angle α so that every neighbouring circles touch each other. If the radius of the first circle is R, then find the sum of the radii of the first n circles in terms of R and α.

ANSWER KEY

EXERCISE - 1

1. D **2.** B **3.** C **4.** D **5.** B **6.** D **7.** C **8.** B **9.** A **10.** B **11.** A **12.** A **13.** B
14. C **15.** D **16.** A **17.** B **18.** D **19.** B **20.** A **21.** C **22.** D **23.** C **24.** C **25.** A **26.** B
27. A **28.** A **29.** C **30.** B

EXERCISE - 2 : PART # I

1. A **2.** ABCD **3.** A **4.** AC **5.** C **6.** ABCD **7.** BD **8.** AD **9.** ABC
10. B **11.** ABCD **12.** D **13.** BD **14.** ACD **15.** AB **16.** ABC **17.** ABC **18.** BC
19. A **20.** AD **21.** BD **22.** C

PART - II

1. D **2.** A **3.** A **4.** C **5.** A **6.** B **7.** B **8.** D **9.** A **10.** D **11.** C **12.** A **13.** A
14. A **15.** C

EXERCISE - 3 : PART # I

1. $A\to q, B\to s, C\to p, D\to r$ **2.** $A\to r, B\to p, C\to s, D\to p$ **3.** $A\to p, B\to p, C\to q, D\to q$
4. $A\to q, B\to p, C\to p, D\to r$

PART - II

Comprehension # 1 : **1.** C **2.** C **3.** B **4.** A **5.** D **Comprehension # 2 :** **1.** A **2.** B **3.** D
Comprehension # 3 : **1.** B **2.** C **3.** D **Comprehension # 4 :** **1.** A **2.** A **3.** D
Comprehension # 5 : **1.** C **2.** C **3.** B **4.** B

EXERCISE - 5 : PART # I

1. B **2.** B **3.** B **4.** C **5.** A **6.** B **7.** A **8.** A **9.** B **10.** B **11.** C **12.** B **13.** D
14. B **15.** B **16.** C **17.** C **18.** A **19.** C **20.** D **21.** D **22.** C **23.** D **24.** D **25.** B **26.** A
27. A

PART - II

1. **A** D, **B** A **2.** **A** A, **B** C, **C** D, **D** $\left[(A_1, A_2 \ldots\ldots\ldots A_n)\ (H_1, H_2\ldots\ldots\ldots H_n)\right]^{\frac{1}{2n}}$

3. $a\to D$ **6.** B **7.** $(n=7)$ **8.** C **9.** 6 **10.** B **11.** D **12.** B **13.** C **14.** A **15.** B **16.** C
17. C **18.** 3 **19.** 0 **20.** 8 **21.** 9 or 3 **22.** D **23.** A, D **24.** 5 **25.** 4 **26.** 9
27. B

MOCK TEST

1. A	**2.** B	**3.** C	**4.** D	**5.** A	**6.** B	**7.** B	**8.** A	**9.** A
10. ?	**11.** AC	**12.** AC	**13.** ABCD	**14.** ABC	**15.** ABCD	**16.** A	**17.** A	**18.** D

19. A **20.** B **21.** $A \to r, B \to pt, C \to sq, D \to q$ **22.** $A \to q, B \to r, C \to p, D \to t$

23. **1.** B **2.** B **3.** A **4.** A **5.** A **24.** **1.** C **2.** B **3.** C **25.** **1.** C **2.** B **3.** C

26. 12 **27.** Rs. 51 **28.** 7 **29.** $-(p+q)$ **30.** $\dfrac{R\left(1-\sin\frac{\alpha}{2}\right)}{2\sin\frac{\alpha}{2}}\left[\left(\dfrac{1+\sin\frac{\alpha}{2}}{1-\sin\frac{\alpha}{2}}\right)^n - 1\right]$