SOLVED EXAMPLES

Ex. 1 Prove that

(i) $\sin(45^\circ + A)\cos(45^\circ - B) + \cos(45^\circ + A)\sin(45^\circ - B) = \cos(A - B)$

(ii) $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{3\pi}{4}+\theta\right) = -1$

Sol. **(i)** Clearly $\sin(45^\circ + A)\cos(45^\circ - B) + \cos(45^\circ + A)\sin(45^\circ - B)$

$= \sin(45^\circ + A + 45^\circ - B) = \sin(90^\circ + A - B) = \cos(A - B)$

(ii) $\tan\left(\frac{\pi}{4}+\theta\right) \times \tan\left(\frac{3\pi}{4}+\theta\right) = \frac{1+\tan\theta}{1-\tan\theta} \times \frac{-1+\tan\theta}{1+\tan\theta} = -1$

Ex. 2 Prove that $\sin 5A + \sin 3A = 2\sin 4A \cos A$

Sol. L.H.S. $\sin 5A + \sin 3A = 2\sin 4A\cos A$ = R.H.S.

[$\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

Ex. 3 Find the value of $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$.

Sol. $2[(\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)] - 3[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta] + 1$

$= 2[1 - 3\sin^2\theta\cos^2\theta] - 3[1 - 2\sin^2\theta\cos^2\theta] + 1$

$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1 = 0$

Ex. 4 If the arcs of same length in two circles subtend angles of 60° and 75° at their centres. Find the ratio of their radii.

Sol. Let r_1 and r_2 be the radii of the given circles and let their arcs of same length s subtend angles of 60° and 75° at their centres.

Now, $60^\circ = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$ and $75^\circ = \left(75 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{12}\right)^c$

$\therefore \quad \frac{\pi}{3} = \frac{s}{r_1}$ and $\frac{5\pi}{12} = \frac{s}{r_2}$

$\Rightarrow \quad \frac{\pi}{3}r_1 = s$ and $\frac{5\pi}{12}r_2 = s \Rightarrow \frac{\pi}{3}r_1 = \frac{5\pi}{12}r_2 \Rightarrow 4r_1 = 5r_2 \Rightarrow r_1 : r_2 = 5 : 4$

Ex. 5 Find the value of θ for $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$.

Sol. Let us first find out θ lying between 0 and 360°.

Since $\sin\theta = -\frac{1}{2} \Rightarrow \theta = 210^\circ$ or 330° and $\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$ or 210°

Hence, $\theta = 210^\circ$ or $\frac{7\pi}{6}$ is the value satisfying both.

Ex. 6 Prove that

(i) $\dfrac{\sin 8\theta\cos\theta - \sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta - \sin 3\theta\sin 4\theta} = \tan 2\theta$ **(ii)** $\dfrac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta\cos 4\theta$

Sol. **(i)** $\dfrac{2\sin 8\theta\cos\theta - 2\sin 6\theta\cos 3\theta}{2\cos 2\theta\cos\theta - 2\sin 3\theta\sin 4\theta}$

$$= \frac{\sin 9\theta + \sin 7\theta - \sin 9\theta - \sin 3\theta}{\cos 3\theta + \cos\theta - \cos\theta + \cos 7\theta} = \frac{2\sin 2\theta\cos 5\theta}{2\cos 5\theta\cos 2\theta} = \tan 2\theta$$

(ii) $\dfrac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = \dfrac{\sin 5\theta\cos 3\theta + \sin 3\theta\cos 5\theta}{\sin 5\theta\cos 3\theta - \sin 3\theta\cos 5\theta} = \dfrac{\sin 8\theta}{\sin 2\theta} = 4\cos 2\theta\cos 4\theta$

Ex. 7 Prove that $\sqrt{3}$ cosec20° – sec20° = 4.

Sol. L.H.S. $= \dfrac{\sqrt{3}}{\sin 20^\circ} - \dfrac{1}{\cos 20^\circ} = \dfrac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ.\cos 20^\circ}$

$$= \frac{4\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{2\sin 20^\circ\cos 20^\circ} = \frac{4(\sin 60.\cos 20^\circ - \cos 60^\circ.\sin 20^\circ)}{\sin 40^\circ}$$

$$= 4.\frac{\sin(60^\circ - 20^\circ)}{\sin 40^\circ} = 4.\frac{\sin 40^\circ}{\sin 40^\circ} = 4 = \text{R.H.S.}$$

Ex. 8 Which of the following is greatest ?

(A) tan 1 **(B)** tan 4 **(C)** tan 7 **(D)** tan 10

Sol. (A) $\tan 4 = \tan(\pi + (4 - \pi)) = \tan(4 - \pi) = \tan(0.86)$

$\tan 7 = \tan(2\pi + (7 - 2\pi)) = \tan(7 - 2\pi) = \tan(0.72)$

$\tan 10 = \tan(3\pi + (10 - 3\pi)) = \tan(10 - 3\pi) = \tan(0.58)$

Now, $1 > 0.86 > 0.72 > 0.58$

$\Rightarrow$ $\tan 1 > \tan(0.86) > \tan(0.72) > \tan(0.58)$ [as 1, 0.86, 0.72, 0.58 lie in the first quadrant and tangent functions increase in all the quadrant]

Hence, tan 1 is greatest

Ex. 9 Find the value of $\cos^2\dfrac{\pi}{16} + \cos^2\dfrac{3\pi}{16} + \cos^2\dfrac{5\pi}{16} + \cos^2\dfrac{7\pi}{16}$.

Sol. LHS $= \cos^2\dfrac{\pi}{16} + \cos^2\dfrac{3\pi}{16} + \cos^2\left(\dfrac{\pi}{2} - \dfrac{3\pi}{16}\right) + \cos^2\left(\dfrac{\pi}{2} - \dfrac{\pi}{16}\right)$

$$= \cos^2\frac{\pi}{16} + \cos^2\frac{3\pi}{16} + \sin^2\frac{3\pi}{16} + \sin^2\frac{\pi}{16}$$

$$= \left(\cos^2\frac{\pi}{16} + \sin^2\frac{\pi}{16}\right) + \left(\cos^2\frac{3\pi}{16} + \sin^2\frac{3\pi}{16}\right)$$

$$= 1 + 1 = 2$$

Ex. 10 Simplify $\dfrac{\sin 5\theta + \sin 2\theta - \sin\theta}{\cos 5\theta + 2\cos 3\theta + 2\cos^2\theta + \cos\theta}$.

Sol. L.H.S.$= \dfrac{2\sin 2\theta\cos 3\theta + \sin 2\theta}{2\cos 3\theta.\cos 2\theta + 2\cos 3\theta + 2\cos^2\theta} = \dfrac{\sin 2\theta[2\cos 3\theta + 1]}{2\left[\cos 3\theta(\cos 2\theta + 1) + (\cos^2\theta)\right]}$

$$= \frac{\sin 2\theta[2\cos 3\theta + 1]}{2\left[\cos 3\theta(2\cos^2\theta) + \cos^2\theta\right]} = \frac{\sin 2\theta(2\cos 3\theta + 1)}{2\cos^2\theta(2\cos 3\theta + 1)} = \tan\theta$$

Ex. 11 If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, prove that $\tan\dfrac{\alpha-\beta}{2} = \pm\sqrt{\dfrac{4-a^2-b^2}{a^2+b^2}}$

Sol. Given, $\sin\alpha + \sin\beta = a$ ....**(i)**

and $\cos\alpha + \cos\beta = b$ ....**(ii)**

Now, $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = b^2 + a^2$

or $\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta = b^2 + a^2$

or $(\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = a^2 + b^2$

or $2 + 2\cos(\alpha-\beta) = a^2 + b^2$

or $\cos(\alpha-\beta) = \dfrac{a^2+b^2-2}{2}$

use $\tan\theta = \pm\sqrt{\dfrac{1-\cos 2\theta}{1+\cos 2\theta}}$

here $2\theta = \alpha - \beta \;\Rightarrow\; \theta = (\alpha-\beta)/2$

$$\tan\frac{\alpha-\beta}{2} = \pm\sqrt{\frac{1-(a^2+b^2-2)/2}{1+(a^2+b^2-2)/2}} \Rightarrow \tan\frac{\alpha-\beta}{2} = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

Ex. 12 Prove that : $\tan A + \tan(60° + A) + \tan(120° + A) = 3\tan 3A$

Sol. L.H.S. $= \tan A + \tan(60° + A) + \tan(120° + A)$

$= \tan A + \tan(60° + A) + \tan\{180° - (60° - A)\}$

$= \tan A + \tan(60° + A) - \tan(60° - A)$ [$\because$ $\tan(180° - \theta) = -\tan\theta$]

$$= \tan A + \frac{\tan 60° + \tan A}{1 - \tan 60°\tan A} - \frac{\tan 60° - \tan A}{1 + \tan 60°\tan A} = \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A} - \frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}$$

$$= \tan A + \frac{\sqrt{3} + \tan A + 3\tan A + \sqrt{3}\tan^2 A - \sqrt{3} + \tan A + 3\tan A - \sqrt{3}\tan^2 A}{(1 - \sqrt{3}\tan A)(1 + \sqrt{3}\tan A)}$$

$$= \tan A + \frac{8\tan A}{1 - 3\tan^2 A} = \frac{\tan A - 3\tan^3 A + 8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{9\tan A - 3\tan^3 A}{1 - 3\tan^2 A} = 3\left(\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}\right) = 3\tan 3A = \text{R.H.S.}$$

Ex. 13 Find maximum and minimum values of following :

(i) $3\sin x + 4\cos x$ **(ii)** $1 + 2\sin x + 3\cos^2 x$

Sol. **(i)** We know

$$-\sqrt{3^2+4^2} \le 3\sin x + 4\cos x \le \sqrt{3^2+4^2}$$

$$-5 \le 3\sin x + 4\cos x \le 5$$

(ii) $1 + 2\sin x + 3\cos^2 x = -3\sin^2 x + 2\sin x + 4$

$$= -3\left(\sin^2 x - \frac{2\sin x}{3}\right) + 4 = -3\left(\sin x - \frac{1}{3}\right)^2 + \frac{13}{3}$$

Now $0 \le \left(\sin x - \frac{1}{3}\right)^2 \le \frac{16}{9}$

$\Rightarrow$ $-\frac{16}{3} \le -3\left(\sin x - \frac{1}{3}\right)^2 \le 0$

$$-1 \le -3\left(\sin x - \frac{1}{3}\right)^2 + \frac{13}{3} \le \frac{13}{3}$$

Ex. 14 In any triangle ABC, sin A – cos B = cos C, then angle B is

Sol. We have , sin A – cos B = cos C

sin A = cos B + cos C

$\Rightarrow$ $2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$

$\Rightarrow$ $2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{\pi-A}{2}\right)\cos\left(\frac{B-C}{2}\right)$ $\because A+B+C=\pi$

$\Rightarrow$ $2\sin\frac{A}{2}\cos\frac{A}{2} = 2\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$

$\Rightarrow$ $\cos\frac{A}{2} = \cos\frac{B-C}{2}$ or $A = B - C$; But $A + B + C = \pi$

Therefore $2B = \pi$ $\Rightarrow$ $B = \pi/2$

Ex. 15 Find the summation of the following series

(i) $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

(ii) $\cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7}$

(iii) $\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$

Sol. **(i)** $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=\dfrac{\cos\dfrac{\left(\frac{2\pi}{7}+\frac{6\pi}{7}\right)}{2}\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}}$

$$=\frac{\cos\frac{4\pi}{7}\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}}=\frac{-\cos\frac{3\pi}{7}\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}}=-\frac{\sin\frac{6\pi}{7}}{2\sin\frac{\pi}{7}}=-\frac{1}{2}$$

(ii) $\cos\frac{\pi}{7}+\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{5\pi}{7}+\cos\frac{6\pi}{7}$

$$=\frac{\cos\left(\dfrac{\frac{\pi}{7}+\frac{6\pi}{7}}{2}\right)\sin\frac{6\pi}{14}}{\sin\frac{\pi}{14}}=\frac{\cos\frac{\pi}{2}\sin\frac{6\pi}{14}}{\sin\frac{\pi}{14}}=0$$

(iii) $\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$

$$=\frac{\cos\frac{10\pi}{22}\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}}=\frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}=\frac{1}{2}$$

Ex. 16 If $\tan\beta=\dfrac{\tan\alpha+\tan\gamma}{1+\tan\alpha\tan\gamma}$, prove that $\sin 2\beta=\dfrac{\sin 2\alpha+\sin 2\gamma}{1+\sin 2\alpha\sin 2\gamma}$

Sol. $\tan\beta=\dfrac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\gamma}{\cos\gamma}}{1+\frac{\sin\alpha}{\cos\alpha}.\frac{\sin\gamma}{\cos\gamma}}=\dfrac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}$

$$\sin 2\beta=\frac{2\tan\beta}{1+\tan^2\beta}=\frac{2\frac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}}{1+\frac{\sin^2(\alpha+\gamma)}{\cos^2(\alpha-\gamma)}}$$

$$=\frac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{\cos^2(\alpha-\gamma)+\sin^2(\alpha+\gamma)}$$

$$=\frac{\sin 2\alpha+\sin 2\gamma}{\frac{1+\cos 2(\alpha-\gamma)}{2}+\frac{1-\cos 2(\alpha+\gamma)}{2}}$$

$$=\frac{\sin 2\alpha+\sin 2\gamma}{1+\frac{1}{2}\times 2\sin 2\alpha\sin\gamma}=\frac{\sin 2\alpha+\sin 2\gamma}{1+\sin 2\alpha\sin 2\gamma}.$$

Ex. 17 Prove that

(i) $\dfrac{\sin 2A}{1+\cos 2A} = \tan A$ **(ii)** $\tan A + \cot A = 2 \operatorname{cosec} 2A$

(iii) $\dfrac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)} = \tan\dfrac{A}{2}\cot\dfrac{B}{2}$

Sol. **(i)** L.H.S. $\dfrac{\sin 2A}{1+\cos 2A} = \dfrac{2\sin A\cos A}{2\cos^2 A} = \tan A$

(ii) L.H.S. $\tan A + \cot A = \dfrac{1+\tan^2 A}{\tan A} = 2\left(\dfrac{1+\tan^2 A}{2\tan A}\right) = \dfrac{2}{\sin 2A} = 2\operatorname{cosec} 2A$

(iii) L.H.S. $\dfrac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)} = \dfrac{2\sin^2\frac{A}{2}+2\sin\frac{A}{2}\sin\left(\frac{A}{2}+B\right)}{2\cos^2\frac{A}{2}-2\cos\frac{A}{2}\cos\left(\frac{A}{2}+B\right)}$

$$= \tan\frac{A}{2}\left[\frac{\sin\frac{A}{2}+\sin\left(\frac{A}{2}+B\right)}{\cos\frac{A}{2}-\cos\left(\frac{A}{2}+B\right)}\right] = \tan\frac{A}{2}\left[\frac{2\sin\frac{A+B}{2}\cos\left(\frac{B}{2}\right)}{2\sin\frac{A+B}{2}\sin\left(\frac{B}{2}\right)}\right] = \tan\frac{A}{2}\cot\frac{B}{2}$$

Ex. 18 Evaluate cos a cos 2a cos 3a .... cos 999a, where $a = \dfrac{2\pi}{1999}$.

Sol. Let $P = \cos a \cos 2a \cos 3a \ldots \cos 999a$

$Q = \sin a \sin 2a \sin 3a \ldots \sin 999a.$

Then, $2^{999} PQ = (2\sin a\cos a)(2\sin 2a\cos 2a)\ldots(2\sin 999a\cos 999a)$

$= \sin 2a \sin 4a \ldots \sin 1998a$

$= (\sin 2a \sin 4a \ldots \sin 998a)\,[-\sin(2\pi - 1000a)]\cdot[-\sin(2\pi - 1002a)]\ldots[-\sin(2\pi - 1998a)]$

$= \sin 2a \sin 4a \ldots \sin 998a \sin 999a \sin 997a \ldots \sin a = Q.$

It is easy to see that $Q \neq 0$. Hence, the desired product is $P = \dfrac{1}{2^{999}}$.

Ex. 19 If $x + y + z = xyz$, Prove that $\dfrac{2x}{1-x^2}+\dfrac{2y}{1-y^2}+\dfrac{2z}{1-z^2} = \dfrac{2x}{1-x^2}\cdot\dfrac{2y}{1-y^2}\cdot\dfrac{2z}{1-z^2}$.

Sol. Put $x = \tan A$, $y = \tan B$ and $z = \tan C$,

So that we have

$\tan A + \tan B + \tan C = \tan A \tan B \tan C \quad \Rightarrow \quad A + B + C = n\pi$, where $n \in I$

Hence L.H.S.

$\therefore \quad \dfrac{2x}{1-x^2}+\dfrac{2y}{1-y^2}+\dfrac{2z}{1-z^2} = \dfrac{2\tan A}{1-\tan^2 A}+\dfrac{2\tan B}{1-\tan^2 B}+\dfrac{2\tan C}{1-\tan^2 C}$.

$= \tan 2A + \tan 2B + \tan 2C \quad [\because A + B + C = n\pi]$

$= \tan 2A \tan 2B \tan 2C = \dfrac{2x}{1-x^2}\cdot\dfrac{2y}{1-y^2}\cdot\dfrac{2z}{1-z^2}$

Ex. 20 Find the maximum value of $1 + \sin\left(\frac{\pi}{4}+\theta\right) + 2\cos\left(\frac{\pi}{4}-\theta\right)$ -

Sol. We have $1 + \sin\left(\frac{\pi}{4}+\theta\right) + 2\cos\left(\frac{\pi}{4}-\theta\right)$

$$= 1 + \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta) + \sqrt{2}\,(\cos\theta + \sin\theta) = 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)(\cos\theta + \sin\theta)$$

$$= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right).\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right)$$

$\therefore$ maximum value $= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right).\sqrt{2} = 4$

Ex. 21 Evaluate $\sum_{r=1}^{n-1}\cos^2\left(\frac{r\pi}{n}\right)$; $n \geq 2$

Sol. Sum $= \frac{1}{2}\sum_{r=1}^{n-1}\left(1 + \cos\frac{2r\pi}{n}\right) = \frac{1}{2}(n-1) + \frac{1}{2}\left\{\cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + \ldots\ldots + \cos\frac{(2n-2)\pi}{n}\right\}$

$$= \frac{1}{2}(n-1) + \frac{1}{2}\left\{\frac{\sin(n-1)\frac{2\pi}{2n}}{\sin\frac{2\pi}{n.2}}.\cos\left\{\frac{2\left(\frac{2\pi}{n}\right) + (n-2)\frac{2\pi}{n}}{2}\right\}\right\}$$

$$\left\{\text{Using, } \cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots\ldots + \cos(\alpha+(n-1)\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}.\cos\left\{\frac{2\alpha+(n-1)\beta}{2}\right\}\right\}$$

$$= \frac{1}{2}(n-1) + \frac{1}{2}\left\{\frac{\sin\frac{(n-1)\pi}{n}.\cos\pi}{\sin\left(\frac{\pi}{n}\right)}\right\} = \frac{1}{2}(n-1) - \frac{1}{2} = \frac{n}{2} - 1$$

$\therefore$ $\sum_{r=1}^{n-1}\cos^2\left(\frac{r\pi}{n}\right) = \frac{n-2}{2}$

Ex. 22 Prove that

$\tan\alpha + 2\tan 2\alpha + 2^2\tan^2\alpha + \ldots\ldots + 2^{n-1}\tan 2^{n-1}\alpha + 2^n\cot 2^n\alpha = \cot\alpha$

Sol. We know $\tan\theta = \cot\theta - 2\cot 2\theta$ .....**(i)**

Putting $\theta = \alpha, 2\alpha, 2^2\alpha$, ...in **(i)**, we get

$\tan\alpha = (\cot\alpha - 2\cot 2\alpha)$

$2(\tan 2\alpha) = 2(\cot 2\alpha - 2\cot 2^2\alpha)$

$2^2(\tan 2^2\alpha) = 2^2(\cot 2^2\alpha - 2\cot 2^3\alpha)$

....

$2^{n-1}(\tan 2^{n-1}\alpha) = 2^{n-1}(\cot 2^{n-1}\alpha - 2\cot 2^n\alpha)$

Adding,

$\tan\alpha + 2\tan 2\alpha + 2^2\tan^2\alpha + \ldots\ldots + 2^{n-1}\tan 2^{n-1}\alpha = \cot\alpha - 2^n\cot 2^n\alpha$

$\therefore$ $\tan\alpha + 2\tan 2\alpha + 2^2\tan^2\alpha + \ldots\ldots + 2^{n-1}\tan 2^{n-1}\alpha + 2^n\cot 2^n\alpha = \cot\alpha$

Exercise # 1

[Single Correct Choice Type Questions]

1. The value of the expression

$$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right) \text{ is}$$

(A) $\frac{1}{8}$ (B) $\frac{1}{16}$ (C) $\frac{1}{4}$ (D) 0

2. Which of the following is correct ?

(A) $\sin 1° > \sin 1$ (B) $\sin 1° < \sin 1$ (C) $\sin 1° = \sin 1$ (D) $\sin 1° = \frac{\pi}{180}\sin 1$

3. If $x + y = 3 - \cos 4\theta$ and $x - y = 4\sin 2\theta$ then

(A) $x^4 + y^4 = 9$ (B) $\sqrt{x} + \sqrt{y} = 16$

(C) $x^3 + y^3 = 2(x^2 + y^2)$ (D) $\sqrt{x} + \sqrt{y} = 2$

4. If $\tan B = \frac{n\sin A\cos A}{1 - n\cos^2 A}$ then $\tan(A + B)$ equals

(A) $\frac{\sin A}{(1-n)\cos A}$ (B) $\frac{(n-1)\cos A}{\sin A}$ (C) $\frac{\sin A}{(n-1)\cos A}$ (D) $\frac{\sin A}{(n+1)\cos A}$

5. If $A = \sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$ and $B = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7}$ then $\sqrt{A^2 + B^2}$ is equal to

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{3}$

6. The expression $\frac{\sin 22°\cos 8° + \cos 158°\cos 98°}{\sin 23°\cos 7° + \cos 157°\cos 97°}$ when simplified reduces to -

(A) 1 (B) -1 (C) 2 (D) none

7. If $\tan\theta = \sqrt{\frac{a}{b}}$ where a, b are positive reals then the value of $\sin\theta\sec^7\theta + \cos\theta\,\mathrm{cosec}^7\theta$ is -

(A) $\frac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}$ (B) $\frac{(a+b)^3(a^4-b^4)}{(ab)^{7/2}}$ (C) $\frac{(a+b)^3(b^4-a^4)}{(ab)^{7/2}}$ (D) $-\frac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}$

8. If $\frac{\sin 2\alpha - \sin 3\alpha + \sin 4\alpha}{\cos 2\alpha - \cos 3\alpha + \cos 4\alpha} = \tan k\alpha$ is an identity then the value of k is equal to -

(A) 2 (B) 3 (C) 4 (D) 6

9. Exact value of $\cos 20° + 2\sin^2 55° - \sqrt{2}\sin 65°$ is -

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) zero

10. If $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$ then $\cos 3\theta$ in terms of 'a' =

(A) $\frac{1}{4}\left(a^3 + \frac{1}{a^3}\right)$ (B) $4\left(a^3 + \frac{1}{a^3}\right)$ (C) $\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$ (D) none

11. The product cot 123°. cot 133° . cot 137° . cot 147°, when simplified is equal to -
(A) – 1 (B) tan 37° (C) cot 33° (D) 1

12. If 5 tan θ = 4, then $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ is equal to
(A) 0 (B) 1 (C) 1/6 (D) 6

13. The roots of the equation $4x^2 - 2\sqrt{5}\,x + 1 = 0$ are
(A) sin 36°, sin 18° (B) sin 18°, cos 36° (C) sin 36°, cos 18° (D) cos 18°, cos 36°

14. If cos(A – B) = 3/5 and tanA tanB = 2, then
(A) cosA cosB = 1/5 (B) sinA sinB = –2/5 (C) cosA cosB = –1/5 (D) sinA sinB = –1/5

15. If $\cos\alpha = \frac{2\cos\beta - 1}{2 - \cos\beta}$ then $\tan\frac{\alpha}{2}\cdot\cot\frac{\beta}{2}$ has the value equal to {where α, β ∈ (0, π)}
(A) 2 (B) $\sqrt{2}$ (C) 3 (D) $\sqrt{3}$

16. The graphs of y = sinx, y = cosx, y = tanx and y = cosecx are drawn on the same axes from 0 to π/2. A vertical line is drawn through the point where the graphs of y = cosx and y = tanx cross, intersecting the other two graphs at points A and B. The length of the line segment AB is:
(A) 1 (B) $\frac{\sqrt{5}-1}{2}$ (C) $\sqrt{2}$ (D) $\frac{\sqrt{5}+1}{2}$

17. If $\frac{5\pi}{2} < x < 3\pi$, then the value of the expression $\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}$ is
(A) $-\cot\frac{x}{2}$ (B) $\cot\frac{x}{2}$ (C) $\tan\frac{x}{2}$ (D) $-\tan\frac{x}{2}$

18. The value of $\cot x + \cot(60^\circ + x) + \cot(120^\circ + x)$ is equal to :
(A) cot 3x (B) tan 3x (C) 3 tan 3x (D) $\frac{3 - 9\tan^2 x}{3\tan x - \tan^3 x}$

19. The value of $\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8}$ is
(A) 1 (B) 2 (C) $1\frac{1}{8}$ (D) $2\frac{1}{8}$

20. The value of tan9° – tan27° – tan63° + tan81° is
(A) 2 (B) 3 (C) 4 (D) None of these

Exercise # 2

Part # I [Multiple Correct Choice Type Questions]

1. The value of $\cos\frac{\pi}{10}\cos\frac{2\pi}{10}\cos\frac{4\pi}{10}\cos\frac{8\pi}{10}\cos\frac{16\pi}{10}$ is :

(A) $\frac{\sqrt{10+2\sqrt{5}}}{64}$ **(B)** $-\frac{\cos(\pi/10)}{16}$ **(C)** $\frac{\cos(\pi/10)}{16}$ **(D)** $-\frac{\sqrt{10+2\sqrt{5}}}{64}$

2. If x + y = z, then $\cos^2 x + \cos^2 y + \cos^2 z - 2\cos x \cos y \cos z$ is equal to

(A) $\cos^2 z$ **(B)** $\sin^2 z$ **(C)** $\cos(x + y - z)$ **(D)** 1

3. Let m = tan3 & n = sec6, then which of following statement(s) does/do not hold good ?

(A) m & n both are positive **(B)** m & n both are negative

(C) m is positive & n is negative **(D)** m is negative & n is positive

4. In a triangle tan A + tan B + tan C = 6 and tan A tan B = 2, then the values of tan A, tan B and tan C are

(A) 1, 2, 3 **(B)** 2, 1, 3 **(C)** 1, 2, 0 **(D)** none

5. If $\sin\theta + \sqrt{\sin\theta + \sqrt{\sin\theta + \sqrt{\sin\theta + ...\infty}}} = \sec^4\alpha$, then $\sin\theta$ is equal to -

(A) $\sec^2\alpha\tan^2\alpha$ **(B)** $2\frac{(1-\cos 2\alpha)}{(1+\cos 2\alpha)^2}$ **(C)** $2\frac{(1+\cos 2\alpha)}{(1-\cos 2\alpha)^2}$ **(D)** $\cot^2\alpha\,\mathrm{cosec}^2\alpha$

6. Factors of $\cos 4\theta - \cos 4\phi$ are -

(A) $(\cos\theta + \cos\phi)$ **(B)** $(\cos\theta - \cos\phi)$ **(C)** $(\cos\theta + \sin\phi)$ **(D)** $(\cos\theta - \sin\phi)$

7. If $\cos(A-B) = \frac{3}{5}$ & tanAtanB = 2, then -

(A) $\cos A\cos B = \frac{1}{5}$ **(B)** $\sin A\sin B = -\frac{2}{5}$ **(C)** $\cos(A+B) = -\frac{1}{5}$ **(D)** $\sin A\sin B = \frac{2}{5}$

8. If $\sqrt{\frac{1-\sin A}{1+\sin A}} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$, for all permissible values of A, then A belongs to -

(A) first quadrant **(B)** second quadrant **(C)** third quadrant **(D)** fourth quadrant

9. Which of the following when simplified reduces to unity ?

(A) $\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$ **(B)** $\frac{\sin(\pi-\alpha)}{\sin\alpha - \cos\alpha\tan\frac{\alpha}{2}} + \cos(\pi-\alpha)$

(C) $\frac{1}{4\sin^2\alpha\cos^2\alpha} + \frac{(1-\tan^2\alpha)^2}{4\tan^2\alpha}$ **(D)** $\frac{1+\sin 2\alpha}{(\sin\alpha+\cos\alpha)^2}$

10. If $A + B = \frac{\pi}{3}$ and cosA + cosB = 1, then -

(A) $\cos(A-B) = 1/3$ **(B)** $|\cos A - \cos B| = \sqrt{\frac{2}{3}}$ **(C)** $\cos(A-B) = -\frac{1}{3}$ **(D)** $|\cos A - \cos B| = \frac{1}{2\sqrt{3}}$

11. $f(x)=\frac{\sin x}{\sqrt{1+\tan^2 x}}+\frac{\cos x}{\sqrt{1+\cot^2 x}}$ is constant in which of following interval -

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{2}, \pi\right)$ (C) $\left(\pi, \frac{3\pi}{2}\right)$ (D) $\left(\frac{3\pi}{2}, 2\pi\right)$

12. For a positive integer n, let $f_n(\theta)=\left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)....(1+\sec 2^n\theta)$. Then

(A) $f_2\left(\frac{\pi}{16}\right)=1$ (B) $f_3\left(\frac{\pi}{32}\right)=1$ (C) $f_4\left(\frac{\pi}{64}\right)=1$ (D) $f_5\left(\frac{\pi}{128}\right)=1$

13. $(a + 2)\sin\alpha + (2a - 1)\cos\alpha = (2a + 1)$ if $\tan\alpha =$

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{2a}{a^2+1}$ (D) $\frac{2a}{a^2-1}$

14. If $\tan x = \frac{2b}{a-c}$, $(a \neq c)$

$y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$

$z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$, then

(A) $y = z$ (B) $y + z = a + c$ (C) $y - z = a - c$ (D) $y - z = (a - c)^2 + 4b^2$

15. The equation $\sin^6 x + \cos^6 x = a^2$ has real solution if

(A) $a \in (-1, 1)$ (B) $a \in \left(-1, -\frac{1}{2}\right)$ (C) $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ (D) $a \in \left(\frac{1}{2}, 1\right)$

Part # II [Assertion & Reason Type Questions]

Each question has four choices (A), (B), (C) and (D) out of which only one is correct. These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** $\cos^3\alpha + \cos^3\left(\alpha+\frac{2\pi}{3}\right) + \cos^3\left(\alpha+\frac{4\pi}{3}\right) = 3\cos\alpha\cos\left(\alpha+\frac{2\pi}{3}\right)\cos\left(\alpha+\frac{4\pi}{3}\right)$

Statement-II : If $a + b + c = 0 \Leftrightarrow a^3 + b^3 + c^3 = 3abc$

2. **Statement-I :** If A is obtuse angle in ΔABC, then $\tan B\tan C < 1$

Statement-II : In ΔABC, $\tan A = \frac{\tan B+\tan C}{\tan B\tan C-1}$

3. **Statement-I :** $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is positive for all real values of x and y only when x = y

Statement-II : $t^2 \geq 0\ \forall\ t \in R$

4. **Statement-I :** If $\sin\theta + \text{cosec}\theta = 2$, then $\sin^n\theta + \text{cosec}^n\theta = 2^n$.
Statement-II : If $a + b = 2$, $ab = 1$, then $a = b = 1$

5. **Statement-I :** $\tan 5\theta - \tan 3\theta - \tan 2\theta = \tan 5\theta \tan 3\theta \tan 2\theta$
Statement-II : $x = y + z \Rightarrow \tan x - \tan y - \tan z = \tan x \tan y \tan z$.

6. **Statement-I :** If $x + y + z = xyz$, then at most one of the numbers can be negative,
Statement-II : In a triangle ABC, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ and there can be at most one abtuse angle in a triangle.

7. **Statement-I :** $\cos 1 < \sin 1$.
Statement-II : In the first quadrant, cosine decreases but sine increases.

8. Let f be any one of the six trigonometric functions. Let $A, B \in R$ satisfying $f(2A) = f(2B)$.
Statement-I : $A = n\pi + B$, for some $n \in Z$.
Statement-II : 2π is one of the period of f.

Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one** statement in **Column-II**.

1.

column-I		column-II	
(A)	$\text{cosec}10° - \sqrt{3}\sec 10° =$	(p)	$-\frac{1}{2}$
(B)	$4\cos 20° - \sqrt{3}\cot 20° =$	(q)	-1
(C)	$\frac{2\cos 40° - \cos 20°}{\sin 20°} =$	(r)	$\sqrt{3}$
(D)	$2\sqrt{2}\sin 10°\left[\frac{\sec 5°}{2} + \frac{\cos 40°}{\sin 5°} - 2\sin 35°\right] =$	(s)	4

2.

Column – I		Column – II	
(A)	If for some real x, the equation $x + \frac{1}{x} = 2\cos\theta$ holds, then $\cos\theta$ is equal to	(p)	2
(B)	If $\sin\theta + \text{cosec}\,\theta = 2$, then $\sin^{2008}\theta + \text{cosec}^{2008}\theta$ is equal to	(q)	1
(C)	Maximum value of $\sin^4\theta + \cos^4\theta$ is	(r)	0
(D)	Least value of $2\sin^2\theta + 3\cos^2\theta$ is	(s)	– 1

3.

Column - I		Column - II	
(A)	$\sin 420° \cos 390° + \cos(-660°)\sin(-330°)$	(p)	0
(B)	$\tan 315° \cot(-405°) + \cot 495° \tan(-585°)$	(q)	1
(C)	The value of $\frac{(1+\tan 8°)(1+\tan 37°)}{(1+\tan 22°)(1+\tan 23°)} =$	(r)	2
(D)	Value of $\left[\frac{\pi}{4}\right] + \left[\frac{1}{3}\sin^2 x\right]$ is (where [.] represents greatest integer function)	(s)	5

Part # II [Comprehension Type Questions]

Comprehension # 1

If $\sin\alpha = A\sin(\alpha+\beta), A \neq 0$, then

1. The value of $\tan\alpha$ is

(A) $\frac{A\sin\beta}{1-A\cos\beta}$ (B) $\frac{A\sin\beta}{1+A\cos\beta}$ (C) $\frac{A\cos\beta}{1-A\sin\beta}$ (D) $\frac{A\sin\beta}{1+A\cos\beta}$

2. The value of $\tan\beta$ is

(A) $\frac{\sin\alpha(1+A\cos\beta)}{A\cos\alpha\cos\beta}$ (B) $\frac{\sin\alpha\cos\alpha}{A\cos\beta-\sin^2\alpha}$ (C) $\frac{\cos\alpha(1-A\cos\beta)}{A\cos\alpha\cos\beta}$ (D) $\frac{\cos\alpha(1+A\sin\beta)}{A\cos\alpha\cos\beta}$

3. Which of the following is not the value of $\tan(\alpha+\beta)$?

(A) $\frac{\sin\beta}{\cos\beta-A}$ (B) $\frac{\sin\alpha\cos\alpha}{A\cos\beta-\sin^2\alpha}$ (C) $\frac{\sin\alpha\cos\alpha}{A\cos\beta+\sin^2\alpha}$ (D) none of these

Comprehension # 2

The measure of an angle in degrees, grades and radians be D, G and C respectively, then the relation between them

$$\frac{D}{90} = \frac{G}{100} = \frac{2C}{\pi} \text{ but } 1^c = \left(\frac{180}{\pi}\right)^\circ$$

$\simeq$ **57°**,17', 44.8"

and sum of interior angles of a n-sided regular polygon is $(2n - 4)\pi/2$

On the basis of above information, answer the following questions :

1. Which of the following are correct -

(A) $\sin 1^\circ < \sin 1$ (B) $\cos 1^\circ > \cos 1$ (C) $\cos 1^\circ < \cos 1$ (D) $\sin 1^\circ < \frac{\pi}{180}\sin 1$

2. The angles between the hour hand and minute hand of a clock at half past three is -

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

3. The number of sides of two regular polygon are as 5 : 4 and the difference between their angles is $\frac{\pi}{20}$, then the number of sides in the polygons respectively are-

(A) 25, 20 (B) 20, 16 (C) 15, 12 (D) 10, 8

4. One angle of a triangle is $\frac{4x}{3}$ grades and another is 3x degrees, while the third is $\frac{2\pi x}{75}$ radians. Then the angles in degrees are-

(A) 20°, 60°, 100° (B) 24°, 60°, 96° (C) 36°, 60°, 84° (D) 20°, 40°, 120°

Comprehension # 3

Continued product $\cos\alpha\cos2\alpha\cos2^2\alpha$ .... $\cos2^{n-1}\alpha$

$$= \begin{cases} \dfrac{\sin 2^n\alpha}{2^n \sin\alpha}, & \text{if } \alpha \neq n\pi \\ \dfrac{1}{2^n}, & \text{if } \alpha = \dfrac{\pi}{2^n+1} \quad \text{i.e. } 2^n\alpha = \pi - \alpha \\ -\dfrac{1}{2^n}, & \text{if } \alpha = \dfrac{\pi}{2^n-1} \quad \text{i.e. } 2^n\alpha = \pi + \alpha \end{cases}$$

Where, $n \in I$ (Integer)

On the basis of above information, answer the following questions :

1. The value of $\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}\cos\dfrac{6\pi}{7}$ is -

(A) –1/2 **(B)** 1/2 **(C)** 1/4 **(D)** 1/8

2. If $\alpha = \dfrac{\pi}{15}$, then the value of $\prod_{r=1}^{7}\cos r\alpha$ is -

(A) $\dfrac{1}{128}$ **(B)** $-\dfrac{1}{128}$ **(C)** $\dfrac{1}{64}$ **(D)** $\dfrac{1}{32}$

3. The value of $\sin\left(\dfrac{\pi}{14}\right)\sin\left(\dfrac{3\pi}{14}\right)\sin\left(\dfrac{5\pi}{14}\right)\sin\left(\dfrac{7\pi}{14}\right)\sin\left(\dfrac{9\pi}{14}\right)\sin\left(\dfrac{11\pi}{14}\right)\sin\left(\dfrac{13\pi}{14}\right)$ is -

(A) 1 **(B)** $\dfrac{1}{8}$ **(C)** $\dfrac{1}{32}$ **(D)** $\dfrac{1}{64}$

Exercise # 4 [Subjective Type Questions]

1. If $m\tan(\theta - 30°) = n\tan(\theta + 120°)$, show that $\cos 2\theta = \dfrac{m+n}{2(m-n)}$.

2. If $\tan\beta = \dfrac{\tan\alpha + \tan\gamma}{1 + \tan\alpha \cdot \tan\gamma}$, prove that $\sin 2\beta = \dfrac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \cdot \sin 2\gamma}$.

3. If $\sin x + \sin y = a$ & $\cos x + \cos y = b$, show that,

$\sin(x + y) = \dfrac{2ab}{a^2 + b^2}$ and $\tan\dfrac{x-y}{2} = \pm\sqrt{\dfrac{4 - a^2 - b^2}{a^2 + b^2}}$.

4. If $\sin(\theta + \alpha) = a$ & $\sin(\theta + \beta) = b$ $(0 < \alpha, \beta, \theta < \pi/2)$ then find the value of $\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta)$

5. Prove that $\sin 3x . \sin^3 x + \cos 3x . \cos^3 x = \cos^3 2x$.

6. If $\tan\alpha = \dfrac{p}{q}$ where $\alpha = 6\beta$, α being an acute angle, prove that; $\dfrac{1}{2}(p\,\mathrm{cosec}\,2\beta - q\sec 2\beta) = \sqrt{p^2 + q^2}$.

7. Show that:

(i) $\cot 7\dfrac{1°}{2}$ or $\tan 82\dfrac{1°}{2} = \left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{2}+1\right)$ or $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

(ii) $\tan 142\dfrac{1°}{2} = 2 + \sqrt{2} - \sqrt{3} - \sqrt{6}$.

8. Prove that, $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$.

9. Calculate the following without using trigonometric tables:

(i) $\tan 9° - \tan 27° - \tan 63° + \tan 81°$

(ii) $\mathrm{cosec}\,10° - \sqrt{3}\sec 10°$

(iii) $2\sqrt{2}\,\sin 10°\left[\dfrac{\sec 5°}{2} + \dfrac{\cos 40°}{\sin 5°} - 2\sin 35°\right]$

(iv) $\cot 70° + 4\cos 70°$

(v) $\tan 10° - \tan 50° + \tan 70°$

10. If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = \dfrac{-3}{2}$, prove that

$\cos\alpha + \cos\beta + \cos\gamma = 0$, $\sin\alpha + \sin\beta + \sin\gamma = 0$.

11. Let $A_1, A_2, \ldots\ldots, A_n$ be the vertices of an n–sided regular polygon such that; $\dfrac{1}{A_1A_2} = \dfrac{1}{A_1A_3} + \dfrac{1}{A_1A_4}$.

Find the value of n.

12. If $\dfrac{ax}{\cos\theta} + \dfrac{by}{\sin\theta} = a^2 - b^2$, $\dfrac{ax\sin\theta}{\cos^2\theta} - \dfrac{by\cos\theta}{\sin^2\theta} = 0$.

Show that $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$

13. If $P_n = \cos^n\theta + \sin^n\theta$ and $Q_n = \cos^n\theta - \sin^n\theta$, then show that

$P_n - P_{n-2} = -\sin^2\theta\cos^2\theta\, P_{n-4}$ $\quad$ $Q_n - Q_{n-2} = -\sin^2\theta\cos^2\theta\, Q_{n-4}$

and hence show that

$P_4 = 1 - 2\sin^2\theta\cos^2\theta$ $\quad$ $Q_4 = \cos^2\theta - \sin^2\theta$

14. If $A + B + C = \pi$, prove that $\tan B \tan C + \tan C \tan A + \tan A \tan B = 1 + \sec A.\sec B.\sec C$.

15. If $\tan^2\alpha + 2\tan\alpha.\tan 2\beta = \tan^2\beta + 2\tan\beta.\tan 2\alpha$, then prove that each side is equal to 1 or $\tan\alpha = \pm\tan\beta$.

16. Find the general solution of $\sec 4\theta - \sec 2\theta = 2$.

17. Solve the equation $\cot x - 2\sin 2x = 1$.

18. Solve the equation $\sin 5x = 16\sin^5 x$.

19. Solve for x, the equation $\sqrt{13 - 18\tan x} = 6\tan x - 3$, where $-2\pi < x < 2\pi$.

20. If α & β are two distinct roots of the equation $a\tan\theta + b\sec\theta = c$, then prove that $\tan(\alpha + \beta) = \dfrac{2ac}{a^2 - c^2}$.

21. Solve the equation for $0 \le \theta \le 2\pi$; $\left(\sin 2\theta + \sqrt{3}\cos 2\theta\right)^2 - 5 = \cos\left(\dfrac{\pi}{6} - 2\theta\right)$.

22. If $\tan\theta + \sin\phi = \dfrac{3}{2}$ & $\tan^2\theta + \cos^2\phi = \dfrac{7}{4}$, then find the general value of θ & ϕ.

23. If α & β satisfy the equation $a\cos 2\theta + b\sin 2\theta = c$ then prove that: $\cos^2\alpha + \cos^2\beta = \dfrac{a^2 + ac + b^2}{a^2 + b^2}$.

$\cos^2\alpha + \cos^2\beta = \dfrac{a^2 + ac + b^2}{a^2 + b^2}$.

24. Solve the equation $3 - 2\cos\theta - 4\sin\theta - \cos 2\theta + \sin 2\theta = 0$

25. Solve the equation $1 + 2\,\text{cosec}\, x = -\dfrac{\sec^2 \frac{x}{2}}{2}$.

26. Solve the equation $\sin^2 4x + \cos^2 x = 2\sin 4x.\cos^4 x$

27. Solve the equation $2\sin x = 3x^2 + 2x + 3$.

Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

1. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to

(1) $\frac{24}{25}$ (2) $-\frac{24}{25}$ (3) $\frac{13}{18}$ (4) $-\frac{13}{18}$ **[AIEEE 2002]**

2. The upper $\left(\frac{3}{4}\right)$th portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is- **[AIEEE 2003]**

(1) 20 m (2) 40 m (3) 60 m (4) 80 m

3. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60º and when he retires 40 m away from the tree, the angle of elevation becomes 30º. The breadth of the river is- **[AIEEE 2004]**

(1) 20 m (2) 30 m (3) 40 m (4) 60 m

4. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by **[AIEEE 2004]**

(1) $2(a^2 + b^2)$ (2) $2\sqrt{a^2 + b^2}$ (3) $(a + b)^2$ (4) $(a - b)^2$

5. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\left(\frac{\alpha - \beta}{2}\right)$ is **[AIEEE 2004]**

(1) $\frac{-3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$ (3) $\frac{6}{65}$ (4) $\frac{-6}{65}$

6. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$; $a \neq 0$ then **[AIEEE 2005]**

(1) $b = a + c$ (2) $b = c$ (3) $c = a + b$ (4) $a = b + c$

7. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is **[AIEEE 2006]**

(1) $\frac{4-\sqrt{7}}{3}$ (2) $-\left(\frac{4+\sqrt{7}}{3}\right)$ (3) $\frac{1+\sqrt{7}}{4}$ (4) $\frac{1-\sqrt{7}}{4}$

8. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is **[AIEEE 2006]**

(1) 6 (2) 1 (3) 2 (4) 4

9. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is **[AIEEE 2006]**

(1) $\frac{(4-\sqrt{7})}{3}$ (2) $-\frac{(4+\sqrt{7})}{3}$ (3) $\frac{(1+\sqrt{7})}{4}$ (4) $\frac{(1-\sqrt{7})}{4}$

10. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60º at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30º. The height of the tower is- **[AIEEE 2007]**

(1) $\frac{2a}{\sqrt{3}}$ (2) $2a\sqrt{3}$ (3) $\frac{a}{\sqrt{3}}$ (4) $\sqrt{3}$

11. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60º. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45º. Then the height of the pole is- **[AIEEE 2008]**

(1) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}+1}\right)$ m (2) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}-1}\right)$ m (3) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m (4) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m

12. Let A and B denote the statements **[AIEEE 2009]**

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then :

(1) A is false and B is true (2) both A and B are true

(3) both A and B are false (4) A is true and B is false

13. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$ **[AIEEE 2010]**

(1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

14. If $A = \sin^2 x + \cos^4 x$, then for all real x : **[AIEEE 2011]**

(1) $\frac{3}{4} \le A \le 1$ (2) $\frac{13}{16} \le A \le 1$ (3) $1 \le A \le 2$ (4) $\frac{3}{4} \le A \le \frac{13}{16}$

15. In a ΔPQR, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to :

(1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$ **[AIEEE 2012]**

16. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, BC = p and CD = q, then AB is equal to : **[AIEEE 2013]**

(1) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (2) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$ (3) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (4) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

17. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as : **[JEE MAIN 2013]**

(1) sinA cosA + 1 **(2)** secA cosecA + 1 **(3)** tanA + cotA **(4)** secA + cosecA

18. If $f_k(x)=\frac{1}{k}(\sin^k+\cos^k x)$, where $x \in R, k \geq 1$, then $f_4(x)-f_6(x)$ is equal to **[JEE MAIN 2014]**

(1) $\frac{1}{6}$ **(2)** $\frac{1}{3}$ **(3)** $\frac{1}{4}$ **(4)** $\frac{1}{12}$

19. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. It flies off horizontally straigth away from the point O. After 1s, the elevation of the bird from O is reduced to 30 °. Then, the speed (in m/s) of the bird is **[JEE MAIN 2014]**

(1) $40(\sqrt{2}-1)$ **(2)** $40(\sqrt{3}-\sqrt{2})$ **(3)** $20\sqrt{2}$ **(4)** $20(\sqrt{3}-1)$

20. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is : **[JEE MAIN 2015]**

(1) $1:\sqrt{3}$ **(2)** 2 : 3 **(3)** $\sqrt{3}:1$ **(4)** $\sqrt{3}:\sqrt{2}$

21. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is : **[JEE MAIN 2016]**

(1) 10 **(2)** 20 **(3)** 5 **(4)** 6

Part # II [Previous Year Questions][IIT-JEE ADVANCED]

1. The maximum value of $(\cos\alpha_1)(\cos\alpha_2)$ .... $(\cos\alpha_n)$ under the restrictions $0 \leq \alpha_1, \alpha_2, ..., \alpha_n \leq \pi/2$ and $(\cot\alpha_1)(\cot\alpha_2)$ .... $(\cot\alpha_n) = 1$ is

(A) $1/2^{n/2}$ **(B)** $1/2^n$ **(C)** 1/2n **(D)** 1 **[IIT JEE 2001]**

2. If $\sin\alpha = 1/2$ and $\cos\theta = 1/3$, then the values of $\alpha + \theta$ (if θ, α are both acute) will lie in the interval **[IIT JEE 2004]**

(A) $\left[\frac{\pi}{3},\frac{\pi}{2}\right]$ **(B)** $\left[\frac{\pi}{2},\frac{2\pi}{3}\right]$ **(C)** $\left[\frac{2\pi}{3},\frac{5\pi}{6}\right]$ **(D)** $\left[\frac{5\pi}{6},\pi\right]$

3. Find the range of values of 't' for which $2\sin t=\frac{1-2x+5x^2}{3x^2-2x-1}$, $t \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. **[IIT JEE 2005]**

4. Let $\theta \in \left(0,\frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then **[IIT JEE 2006]**

(A) $t_1 > t_2 > t_3 > t_4$ **(B)** $t_2 < t_1 < t_3 < t_4$ **(C)** $t_3 > t_1 > t_2 > t_4$ **(D)** $t_2 > t_3 > t_1 > t_4$

5. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then **[IIT JEE 2009]**

(A) $\tan^2 x = \frac{2}{3}$ **(B)** $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(C) $\tan^2 x = \frac{1}{3}$ **(D)** $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

6. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is **[IIT JEE 2010]**

7. The positive integer value of n > 3 satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$$ is **[IIT JEE 2011]**

8. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \varnothing$ **(B)** $Q \not\subset P$

(C) $P \not\subset Q$ **(D)** $P = Q$ **[IIT JEE 2011]**

9. Let $\theta, \phi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\phi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\phi - 1$, $\tan(2\pi - \theta) > 0$

and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then ϕ cannot satisfy **[IIT JEE 2012]**

(A) $0 < \phi < \frac{\pi}{2}$ **(B)** $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$ **(C)** $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ **(D)** $\frac{3\pi}{2} < \phi < 2\pi$

10. If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$, then possible value of $\cos\frac{x-y}{2}$ is **[JEE Ad. 2013]**

11. If $\cos\left(\frac{\pi}{4} - x\right)\cos 2x + \sin x \sin 2x \sec x = \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$, then possible value of sec x is **[JEE Ad. 2013]**

12. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals **[JEE Ad. 2016]**

(A) $2(\sec\theta - \tan\theta)$ **(B)** $2\sec\theta$ **(C)** $-2\tan\theta$ **(D)** 0

13. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to **[JEE Ad. 2016]**

(A) $3 - \sqrt{3}$ **(B)** $2(3 - \sqrt{3})$ **(C)** $2(\sqrt{3} - 1)$ **(D)** $2(2 + \sqrt{3})$

MOCK TEST

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If 2 cos x + sin x = 1, then value of 7 cos x + 6 sin x is equal to
(A) 2 or 6 (B) 1 or 3 (C) 2 or 3 (D) None of these

2. The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is equal to
(A) $\frac{4}{3}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) 3

3. Value of $\frac{3+\cot 80^\circ \cot 20^\circ}{\cot 80^\circ + \cot 20^\circ}$ is equal to
(A) cot 20° (B) tan 50° (C) cot 50° (D) $\cot\sqrt{20^\circ}$

4. The value of $\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + \ldots\ldots + \cos\frac{17\pi}{19}$ is equal to:
(A) 1/2 (B) 0 (C) 1 (D) None

5. If $\sin x + \cos x = \sqrt{2}\cos x$, then cosx – sinx is equal to
(A) $\sqrt{2}\cos x$ (B) $-\sqrt{2}\cos x$ (C) $\sqrt{2}\sin x$ (D) $-\sqrt{2}\sin x$

6. If $f(\theta) = \sin^2\theta + \sin^2\left(\theta+\frac{2\pi}{3}\right) + \sin^2\left(\theta+\frac{4\pi}{3}\right)$, then $f\left(\frac{\pi}{15}\right)$ is equal to
(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

7. If sin2β is the geometric mean between sinα and cosα, then cos4β is equal to
(A) $2\sin^2\left(\frac{\pi}{4}-\alpha\right)$ (B) $2\cos^2\left(\frac{\pi}{4}-\alpha\right)$ (C) $2\cos^2\left(\frac{\pi}{2}+\alpha\right)$ (D) $2\sin^2\left(\frac{\pi}{4}+\alpha\right)$

8. If $0^\circ < x < 90^\circ$ & $\cos x = \frac{3}{\sqrt{10}}$, then the value of $\log_{10}\sin x + \log_{10}\cos x + \log_{10}\tan x$ is
(A) 0 (B) 1 (C) – 1 (D) None of these

9. If sin α sin β – cos α cos β + 1 = 0, then the value of 1 + cot α tan β is
(A) 1 (B) – 1 (C) 2 (D) none of these

10. If $\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2}$ and $\frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2}$, $0 < A, B < \pi/2$, then tan A + tan B is equal to
(A) $\sqrt{3}/\sqrt{5}$ (B) $\sqrt{5}/\sqrt{3}$ (C) 1 (D) $(\sqrt{5}+\sqrt{3})/\sqrt{5}$

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If $2\sec^2\alpha - \sec^4\alpha - 2\operatorname{cosec}^2\alpha + \operatorname{cosec}^4\alpha = 15/4$, then $\tan\alpha$ is equal to

(A) $1/\sqrt{2}$ **(B)** $1/2$ **(C)** $1/2\sqrt{2}$ **(D)** $-1/\sqrt{2}$

12. If $3\sin\beta = \sin(2\alpha+\beta)$, then $\tan(\alpha+\beta) - 2\tan\alpha$ is

(A) independent of α **(B)** independent of β

(C) dependent of both α and β **(D)** independent of α but dependent of β

13. Which of following functions have the maximum value unity ?

(A) $\sin^2x - \cos^2x$ **(B)** $\sqrt{\frac{6}{5}}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{3}}\cos x\right)$

(C) $\cos^6 + \sin^6x$ **(D)** $\cos^2x + \sin^4x$

14. If $\tan A + \tan B + \tan C = \tan A.\ \tan B.\ \tan C$, then

(A) A, B, C may be angles of a triangle **(B)** $A + B + C$ is an integral multiple of π

(C) sum of any two of A, B, C is equal to third **(D)** none of these

15. $\cos4x\cos8x - \cos5x\cos9x = 0$ if

(A) $\cos12x = \cos14x$ **(B)** $\sin13x = 0$ **(C)** $\sin x = 0$ **(D)** $\cos x = 0$

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** $\sin 2 > \sin 3$

Statement-II : If $x, y \in \left(\frac{\pi}{2}, \pi\right)$, $x < y$, then $\sin x > \sin y$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. **Statement-I :** The number of integral values of λ, for which the equation $7\cos x + 5\sin x = 2\lambda + 1$ has a solution, is 8

Statement-II : $a\cos\theta + b\sin\theta = c$ has atleast one solution if $|c| > \sqrt{a^2+b^2}$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement-I :** The maximum value of $\sin\theta + \cos\theta$ is 2

Statement-II : The maximum value of $\sin\theta$ is 1 and that of $\cos\theta$ is also 1.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** If $\sin\theta + \text{cosec}\theta = 2$, then $\sin^n\theta + \text{cosec}^n\theta = 2^n$.

Statement-II : If $a + b = 2$, $ab = 1$, then $a = b = 1$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. Let $\alpha, \beta, \gamma > 0$ and $\alpha + \beta + \gamma = \frac{\pi}{2}$.

Statement-I : $\left|\tan\alpha\tan\beta - \frac{a!}{6}\right| + \left|\tan\beta\tan\gamma - \frac{b!}{2}\right| + \left|\tan\gamma\tan\alpha - \frac{c!}{3}\right| \le 0$, where $n! = 1.2 \ldots\ldots\ldots\ldots n$, then $\tan\alpha\tan\beta$, $\tan\beta\tan\gamma$, $\tan\gamma\tan\alpha$ are in A.P.

Statement-II : $\tan\alpha\tan\beta + \tan\beta\tan\gamma + \tan\gamma\tan\alpha = 1$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21.

	Column I		Column II
(A)	The number of solutions of the equation $\lvert\cot x\rvert = \cot x + \frac{1}{\sin x}$ $(0 < x < \pi)$ is	**(p)**	No solution
(B)	If $\sin\theta + \sin\phi = \frac{1}{2}$ and $\cos\theta + \cos\phi = 2$, then value of $\cot\left(\frac{\theta+\varphi}{2}\right)$ is	**(q)**	$\frac{1}{3}$
(C)	The value of $\sin^2\alpha + \sin\left(\frac{\pi}{3}-\alpha\right)\sin\left(\frac{\pi}{3}+\alpha\right)$ is	**(r)**	1
(D)	If $\tan\theta = 3\tan\phi$, then maximum value of $\tan^2(\theta - \phi)$ is	**(s)**	2
		(t)	4

22.

Column I		Column II	
(A)	If maximum and minimum values of $\dfrac{7+6\tan\theta-\tan^2\theta}{(1+\tan^2\theta)}$ for all real values of $\theta \sim \dfrac{\pi}{2}$ are λ and μ respectively, then	**(p)**	$\lambda+\mu=2$
(B)	If maximum and minimum values of $5\cos\theta+3\cos\left(\theta+\dfrac{\pi}{3}\right)+3$ for all real values of θ are λ and μ respectively, then	**(q)**	$\lambda-\mu=6$
		(r)	$\lambda+\mu=6$
(C)	If maximum and minimum values of $1+\sin\left(\dfrac{\pi}{4}+\theta\right)+2\cos\left(\dfrac{\pi}{4}-\theta\right)$ for all real values of θ are λ and μ respectively, then	**(s)**	$\lambda-\mu=10$
		(t)	$\lambda-\mu=14$

SECTION - V : COMPREHENSION TYPE

23. **Read the following comprehension carefully and answer the questions.**

If $P_n = \sin^n\theta + \cos^n\theta$ where $n \in W$ (whole number) and $\theta \in R$ (real number)

1. If $P_1 = m$, then the value of $4(1 - P_6)$ is

(A) $3(m-1)^2$ **(B)** $3(m^2-1)^2$ **(C)** $3(m+1)^2$ **(D)** $3(m^2+1)^2$

2. The value of $2P_6 - 3P_4 + 10$ is

(A) 0 **(B)** 6 **(C)** 9 **(D)** 15

3. The value of $6P_{10} - 15P_8 + 10P_6 + 7$ is

(A) 8 **(B)** 6 **(C)** 4 **(D)** 2

24. **Read the following comprehension carefully and answer the questions.**

If θ is an angle which measured in radian and $\theta \in [0, 2\pi]$, then $r\theta$ is length of arc AB, of circle of radius r, subtending angle θ at the centre O, of the circle. Area of sector OAB is $\dfrac{1}{2}r^2\theta$.

1. The angle between minute hand and hour hand of a clock at "half past 4" equals

(A) 42° **(B)** 43° **(C)** 44° **(D)** none of these

2. The wheel of a train is 1 meter in diameter and it makes 5 revolutions per second. Then the speed of the train is approximately equal to

(A) 57 km/hr **(B)** 66 km/hr **(C)** 68 km/hr **(D)** 42.6 km/hr.

3. Two lines drawn through a point on the circumference of a circle divide the circle into three regions of equal area. Then the angle θ between the lines is given by

(A) $3\theta + 3\sin\theta = \pi$ **(B)** $6\theta + 3\sin\theta = \pi$ **(C)** $2\theta + \sin\theta = \pi$ **(D)** $\theta + \sin\theta = \pi/2$

25. Read the following comprehension carefully and answer the questions.

Given $\cos 2^m\theta \cos 2^{m+1}\theta \ldots\ldots \cos 2^n\theta = \dfrac{\sin 2^{n+1}\theta}{2^{n-m+1}\sin 2^m\theta}$, where $2^m\theta \neq k\pi$, n, m, k $\in$ I

Solve the following :

1. $\sin\dfrac{9\pi}{14}\cdot\sin\dfrac{11\pi}{14}\sin\dfrac{13\pi}{14}$ is equal to

(A) $\dfrac{1}{64}$ **(B)** $-\dfrac{1}{64}$ **(C)** $\dfrac{1}{8}$ **(D)** $-\dfrac{1}{8}$

2. $\cos 2^3\dfrac{\pi}{10}\cos 2^4\dfrac{\pi}{10}\cos 2^5\dfrac{\pi}{10} \ldots\ldots\ldots \cos 2^{10}\dfrac{\pi}{10}$ is equal to

(A) $\dfrac{1}{128}$ **(B)** $\dfrac{1}{256}$ **(C)** $\dfrac{1}{512}\sin\dfrac{\pi}{10}$ **(D)** $\dfrac{\sqrt{5}-1}{512}\sin\dfrac{3\pi}{10}$

3. $\cos\dfrac{\pi}{11}\cos\dfrac{2\pi}{11}\cos\dfrac{3\pi}{11} \ldots\ldots \cos\dfrac{11\pi}{11}$ is equal to

(A) $-\dfrac{1}{32}$ **(B)** $\dfrac{1}{512}$ **(C)** $\dfrac{1}{1024}$ **(D)** $-\dfrac{1}{2048}$

SECTION - VI : INTEGER TYPE

26. In a triangle ABC, if $\sin 10A + \sin 10B + \sin 10C = \lambda \sin 5A \sin 5B \sin 5C$, then find the value of λ.

27. Find the absolute value of the expression $\tan\dfrac{\pi}{16} + \tan\dfrac{5\pi}{16} + \tan\dfrac{9\pi}{16} + \tan\dfrac{13\pi}{16}$.

28. If $\tan\beta = \dfrac{\tan\alpha + \tan\gamma}{1+\tan\alpha\tan\gamma}$ and $\sin 2\beta = \dfrac{\sin 2\alpha + \sin 2\gamma}{\lambda + \sin 2\alpha \sin 2\gamma}$, then find the value of λ.

29. The value of $64\sqrt{3}\sin\dfrac{\pi}{48}\cos\dfrac{\pi}{48}\cos\dfrac{\pi}{24}\cos\dfrac{\pi}{12}\cos\dfrac{\pi}{6}$ is

30. Let $f(\theta) = \dfrac{1}{1+(\cos\theta)^x}$ and $S = \sum_{\theta=1^\circ}^{89^\circ} f(\theta)$, then the value of $\sqrt{2S-8}$ is

ANSWER KEY

EXERCISE - 1

1. B **2.** B **3.** D **4.** A **5.** B **6.** A **7.** A **8.** B **9.** A **10.** C **11.** D **12.** C **13.** B
14. A **15.** D **16.** A **17.** B **18.** D **19.** B **20.** C

EXERCISE - 2 : PART # I

1. BD **2.** CD **3.** ABC **4.** AB **5.** AB **6.** ABCD **7.** ACD **8.** AD **9.** ABD **10.** BC **11.** BD
12. ABCD **13.** BD **14.** BD **15.** BD

PART - II

1. C **2.** A **3.** B **4.** D **5.** A **6.** D **7.** B **8.** A

EXERCISE - 3 : PART # I

1. **A** $\rightarrow$ s **B** $\rightarrow$ q **C** $\rightarrow$ r **D** $\rightarrow$ s **2.** **A** $\rightarrow$ q,s **B** $\rightarrow$ p **C** $\rightarrow$ q **D** $\rightarrow$ p **3.** **A** $\rightarrow$ q **B** $\rightarrow$ r **C** $\rightarrow$ q **D** $\rightarrow$ p

PART - II

Comprehension # 1 : 1. A **2.** B **3.** C **Comprehension # 2 : 1.** A,B **2.** C **3.** D **4.** B
Comprehension # 3 : 1. D **2.** A **3.** D

EXERCISE - 5 : PART # I

1. 2 **2.** 1 **3.** 4 **4.** 1 **5.** 3 **6.** 2 **7.** 4 **8.** 2 **9.** 3 **10.** 3 **11.** 2 **12.** 1 **13.** 1
14. 2 **15.** 1 **16.** 2 **17.** 2 **18.** 4 **19.** 4 **20.** 3 **21.** 3

PART - II

1. B **2.** $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$ **3.** **B** **4.** **D** **5.** **B** **6.** **A** **7.** **C** **8.** **A** **9.** 2
10. n = 7 **11.** 3 **12.** D **13.** A,C,D

MOCK TEST

1. A **2.** C **3.** B **4.** A **5.** C **6.** B **7.** A **8.** C **9.** D
10. D **11.** A, D **12.** A, B **13.** A,B,C,D **14.** AB **15.** ABCD **16.** A **17.** C **18.** D
19. D **20.** D **21.** A $\rightarrow$ r B $\rightarrow$ p C $\rightarrow$ p D $\rightarrow$ q **22.** A $\rightarrow$ r,s B $\rightarrow$ p,t C $\rightarrow$ p,q
23. **1.** B **2.** C **3.** A **24.** **1.** D **2.** A **3.** A **25.** **1.** C **2.** B **3.** C
26. 4 **27.** 4 **28.** **29.** 6 **30.** 9