WS-3. Task

① ~~Given~~ $\frac{P}{Q} = \frac{3}{1}$

① Given P, Q are two forces.

The maximum resultant of P and Q is P+Q

Then minimum resultant of P and Q is P−Q

∴ Acc to given data $\frac{P+Q}{P-Q} = \frac{3}{1}$

$\Rightarrow P+Q = 3(P-Q)$

$\Rightarrow P+Q = 3P-3Q$ [By interchanging P & 3Q]

$\Rightarrow 3Q+Q = 3P-P$

$\Rightarrow \cancel{4}^{2}Q = \cancel{2}P \Rightarrow P = 2Q.$

②

let the two forces be F_1 & F_2 ; Given $F_1 = F_2$

when F_1 & F_2 are mutually Perpendicular (ie $\theta = 90°$)

The resultant = 141.4 N.

Acc to Parallelogram law of vectors

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\Rightarrow 141.4 = \sqrt{F_1^2 + F_1^2 + 2(F_1)(F_1)\cos 90}$$

$$\Rightarrow 141.4 = \sqrt{2F_1^2} \Rightarrow 141.4 = \sqrt{2}F_1$$

$$\Rightarrow F_1 = \frac{141.4}{\sqrt{2}} = 100N$$

If $\theta = 120°$

∴ The resultant force $= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120}$

$$= \sqrt{F_1^2 + F_1^2 + 2F_1F_1(-\tfrac{1}{2})}$$

$$= \sqrt{F_1^2 + \cancel{F_1^2} - \cancel{F_1^2}} = \sqrt{F_1^2} = F_1 = 100N$$

③ Acc to Parallelogram law of vectors, Resultant

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

Given the two forces are $P = 2P$; $Q = \sqrt{2}P$

and $R = \sqrt{10}P$

$$\therefore \sqrt{10}P = \sqrt{(2P)^2+(\sqrt{2}P)^2+2(2P)(\sqrt{2}P)\cos\theta}$$

By doing squaring on both sides

$$\Rightarrow [\sqrt{10}P]^2 = [\sqrt{4P^2+2P^2+4\sqrt{2}P^2\cos\theta}]^2$$

$$\Rightarrow 10P^2 = 6P^2+4\sqrt{2}P^2\cos\theta$$

$$\Rightarrow 10 = 6+4\sqrt{2}\cos\theta \Rightarrow 10-6 = 4\sqrt{2}\cos\theta$$

$$\Rightarrow \cancel{4} = \cancel{4}\sqrt{2}\cos\theta$$

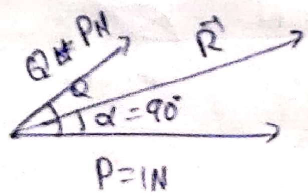
$$\Rightarrow 1 = \sqrt{2}\cos\theta$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

④ ⑥ let the two forces are $P = 1N$; $Q = PN$

Resultant $R = 1$

Graphically



Acc to Parallelogram law of vectors $R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$$1 = \sqrt{1^2+P^2+2(1)(P)\cos\theta}$$

$$\Rightarrow 1 = 1+P^2+2P\cos\theta$$

$$\Rightarrow 1 = 1+P^2+\cancel{2P}\left(-\frac{1}{\cancel{P}}\right)$$

$$\Rightarrow 1 = 1+P^2-2 \Rightarrow P^2 = 2$$

$$\Rightarrow P = \sqrt{2}$$

From

$$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\Rightarrow \tan 90 = \frac{P\sin\theta}{1+P\cos\theta}$$

$$\Rightarrow \infty = \frac{P\sin\theta}{1+P\cos\theta}$$

$$\Rightarrow 1+P\cos\theta = \frac{P\sin\theta}{\infty} = 0$$

$$\Rightarrow P\cos\theta = -1 \Rightarrow \cos\theta = -\frac{1}{P}$$

$$\therefore \cos\theta = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = 135^\circ$$

$$\left[\begin{array}{l}\cos 135^\circ \\ = \cos(90+45^\circ) \\ = -\sin 45^\circ \\ = -\frac{1}{\sqrt{2}}\end{array}\right]$$

Scanned with OKEN Scanner

(7) Given R is the resultant $= Q$

$\therefore$ Acc to Parallelogram law of vectors

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$\Rightarrow Q = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

By squaring on both sides we get

$$\Rightarrow Q^2 = \left[\sqrt{P^2+Q^2+2PQ\cos\theta}\,\right]^2$$

$$\Rightarrow Q^2 = P^2+Q^2+2PQ\cos\theta$$

$$\Rightarrow 0 = P^2+2PQ\cos\theta \Rightarrow 2PQ\cos\theta = -P^2$$

$$\Rightarrow \cos\theta = \frac{-P^2}{2PQ} = \frac{-P}{2Q}$$

If $\vec{Q}$ is doubled i.e $2Q$

The angle made by the resultant with $\vec{P}$ is

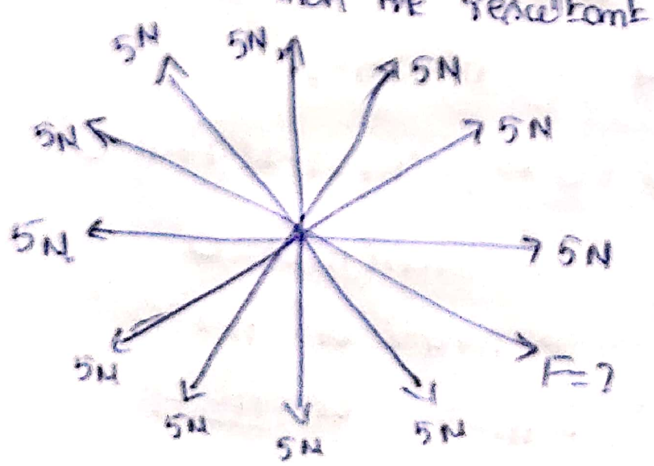
(14) Minimum No. of forces required to make resultant zero is $n = \frac{360}{\theta}$

Here $\theta = 30^\circ$ $\therefore$ $n = \frac{360}{\theta} = \frac{360}{30} = 12$

$\therefore$ No. of forces equal in magnitude is required is $= 12$

But in question 11 forces are there

Then the resultant of all force = The remaining un-balanced force $= 5N$.



(8) let the two forces are P & Q

$P = 4N$; $Q = 3N$

Acc to given data $\left(\sqrt{P^2+Q^2+2PQ\cos\theta}\right)^2 = 12 + \left(\sqrt{P^2+Q^2}\right)^2$

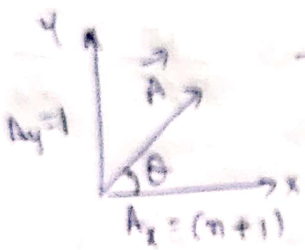
$\Rightarrow \cancel{P^2+Q^2} + 2PQ\cos\theta = 12 + \cancel{P^2+Q^2}$

$\Rightarrow 2PQ\cos\theta = 12 \Rightarrow 2\times4\times3\cos\theta = 12$

$\Rightarrow 2\cos\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

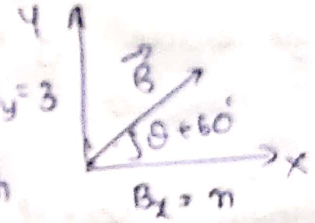
(9) let the vector be $\vec{A}$



$\therefore A_x = (n+1)$

$A_y = 1N$

After the system is turned through an angle 60°, the vector be $\vec{B}$



Here when coordinate system is turned through an angle, No change in magnitude of the vector

(i.e) $|\vec{A}| = |\vec{B}|$

$\Rightarrow \sqrt{A_x^2 + A_y^2} = \sqrt{B_x^2 + B_y^2}$ [By squaring on both sides]

$\Rightarrow \left(\sqrt{(n+1)^2+n^2}\right)^2 = \left(\sqrt{n^2+3^2}\right)^2$

$\Rightarrow (n+1)^2 + \cancel{n^2} = \cancel{n^2} + 9$

$\Rightarrow n^2+2n+1 = 9 \Rightarrow n^2+2n-8 = 0$

$\Rightarrow n^2+4n-2n-8 = 0 \Rightarrow n(n+4)-2(n+4) = 0$

$\Rightarrow (n+4)(n-2) = 0$

$\Rightarrow n+4 = 0$ or $n-2 = 0$

$\Rightarrow n = -4$ or $\boxed{n = 2}$

(10) Given F_1 & F_2 are two forces. Their resultant is

$$F = \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$$

$$\Rightarrow F^2 = F_1^2+F_2^2+2F_1F_2\cos\theta \rightarrow (1)$$

If F_2 is reversed the resultant is

$$F = \sqrt{F_1^2+(-F_2)^2+2F_1(-F_2)\cos\theta}$$

$$\Rightarrow F^2 = F_1^2+F_2^2-2F_1F_2\cos\theta \rightarrow (2)$$

If F_2 is doubled (ie) $2F_2$, then the resultant also doubled $(2F)$

$$\therefore (2F)^2 = \sqrt{F_1^2+(2F_2)^2+2F_1(2F_2)\cos\theta}$$

$$\Rightarrow (2F)^2 = F_1^2+4F_2^2+4F_1F_2\cos\theta$$

$$\Rightarrow 4F^2 = F_1^2+4F_2^2+4F_1F_2\cos\theta \rightarrow (3)$$

By doing (1) − (2) we get

$$F^2-F^2 = \cancel{F_1^2}+\cancel{F_2^2}+2F_1F_2\cos\theta-\cancel{F_1^2}-\cancel{F_2^2}+2F_1F_2\cos\theta$$

$$\Rightarrow \boxed{0 = 4F_1F_2\cos\theta \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°}$$

$\therefore$ From (3) $4F^2 = F_1^2+4F_2^2+0 \Rightarrow F_1^2+4F_2^2 = 4F^2 \rightarrow (4)$

From (1) $F^2 = F_1^2+F_2^2+2F_1F_2(0) \Rightarrow F_1^2+F_2^2 = F^2$

$$3F_2^2 = 3F^2$$

$$\Rightarrow \boxed{F_2 = F}$$

Scanned with OKEN Scanner

(11) Given $\vec{A} = 4\hat{i} - 2\hat{j} + 6\hat{k}$ & $\vec{B} = \hat{i} - 2\hat{j} - 3\hat{k}$

$\therefore \vec{A} + \vec{B} = 4\hat{i} - 2\hat{j} + 6\hat{k} + \hat{i} - 2\hat{j} - 3\hat{k}$

$\vec{A} + \vec{B} = 5\hat{i} - 4\hat{j} + 3\hat{k}$

$|\vec{A} + \vec{B}| = \sqrt{5^2 + (-4)^2 + 3^2} = \sqrt{25+16+9} = \sqrt{50}$

$\Rightarrow \sqrt{2 \times 25} = 5\sqrt{2}$

The angle made by the vector with x-axis is

to $\cos\alpha = \dfrac{(\vec{A}+\vec{B})_x}{|\vec{A}+\vec{B}|}$

$\Rightarrow \cos\alpha = \dfrac{5}{5\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$

(15) let the two vectors be $\vec{P} = 5N$ & $\vec{Q} = 3N$.

$\vec{P}$ & $\vec{Q}$ are in same direction $\theta = 0^\circ$

$\therefore$ The resultant $= \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{5^2 + 3^2 + 2(5)(3)\cos 0}$

$= \sqrt{25 + 9 + 30(1)}$

$= \sqrt{64} = \pm 8$

(16) let the two vectors are $\vec{P} = 20N$; $\vec{Q} = 25N$.

and Resultant $\vec{R} = 39$ N.

$\therefore$ Acc to Parallelogram law of vectors

$\vec{R} = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow 39 = \sqrt{(20)^2 + (25)^2 + 2(20)(25)\cos\theta}$

By squaring on both sides

$\Rightarrow 39^2 = (\sqrt{400 + 625 + 1000\cos\theta})^2$

$\Rightarrow 1521 = 1025 + 1000 \cos\theta$

$\Rightarrow 1521 - 1025 = 1000 \cos\theta$

$\Rightarrow 496 = 1000 \cos\theta$

$\Rightarrow \cos\theta = \frac{496}{1000} \simeq \frac{1}{2}$

$\Rightarrow \theta = 60^\circ$

(5) when three forces are given (or) acting on a body then they keep the body in equilibrium only when the sum of the two forces is greater (or) equal to the 3rd force.

In option 3 we can see that $2N + 6N = 8N > 5N$ satisfying the condition.

(17) Given $R = 1414N$; let the two forces are P & Q; Given $P = Q$; $\theta = 90^\circ$

$\therefore$ Acc to Parallelogram law of vectors

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\Rightarrow 1414 = \sqrt{P^2 + P^2 + 2P(P)\cos 90}$$

$$\Rightarrow 1414 = \sqrt{2P^2 + 2P^2 \cos} \Rightarrow 1414 = \sqrt{2}\,P$$

$$\Rightarrow P = \frac{1414}{\sqrt{2}} = \frac{1414}{1.414}$$

$$\Rightarrow P = 10^3 N = 1 \times 10^3 N.$$

Scanned with OKEN Scanner

(18) let the two forces are $P = 4x$ & $Q = 3x$

The maximum resultant of two forces $= P+Q = 7N$

Minimum resultant of two forces $= P-Q = 1N$

$$\therefore \frac{\cancel{P+Q}}{P-Q} = \frac{7}{1} \Rightarrow \frac{\cancel{4x+3x}}{\cancel{4x-3x}} = \frac{7}{1}$$

$$\Rightarrow \cancel{P+Q = 7P-7Q} \Rightarrow \cancel{\frac{7x}{x}} =$$

$$\Rightarrow 7Q + Q = 7P - P$$

$$\Rightarrow 8Q = 6P$$

$$\therefore 4x + 3x = 7$$

$$\Rightarrow 7x = 7$$

$$\Rightarrow x = 1$$

(19)

(a) Given $\vec{A} - \vec{B} = \vec{C}$

$$\Rightarrow |\vec{A} - \vec{B}| = |\vec{C}|$$

$$\Rightarrow \sqrt{A^2 + B^2 - 2AB\cos\theta} = C$$

and $A - B = C$

$$\Rightarrow (A-B)^2 = C^2$$

$$\Rightarrow A^2 + B^2 - 2AB = C^2 \rightarrow (1)$$

By squaring on both sides we get

$$\Rightarrow A^2 + B^2 - 2AB\cos\theta = C^2 \text{ from } (1)$$

$$\therefore A^2 + \cancel{B^2} - 2AB\cos\theta = A^2 + \cancel{B^2} + 2AB$$

$$\Rightarrow \cancel{2AB} - 2AB\cos\theta = \cancel{0} - 2AB$$

$$\cos\theta = 1 \Rightarrow \theta = 0^\circ$$

(b) $\vec{A} + \vec{B} = \vec{C}$

$$\Rightarrow \sqrt{A^2 + B^2 + 2AB\cos\theta} = C$$

By squaring on both sides

$$\Rightarrow A^2 + B^2 + 2AB\cos\theta = C^2 \rightarrow (1)$$

$A - B = C$

$$(A-B)^2 = C^2$$

$$\Rightarrow A^2 + B^2 - 2AB = C^2 \rightarrow (2)$$

From (1) and (2)

$$\Rightarrow A^2 + \cancel{B^2} + 2AB\cos\theta = \cancel{A^2 + B^2} - 2AB$$

Scanned with OKEN Scanner

Contnu

$$2\cancel{AB}\cos\theta = -2\cancel{AB}$$

$$\Rightarrow \cos\theta = -1$$

$$\Rightarrow \theta = 180^\circ$$

(c) $\vec{A} - \vec{B} = \vec{C}$

$$\Rightarrow |\vec{A} - \vec{B}|^2 = |\vec{C}|^2$$

$$\Rightarrow \left\{\sqrt{A^2+B^2-2AB\cos\theta}\right\}^2 = C^2$$

$$\Rightarrow A^2+B^2-2AB\cos\theta = C^2 \rightarrow (2)$$

$$A^2+B^2 = C^2 \rightarrow (1)$$

From (1) & (2)

$$\Rightarrow A^2+B^2-2AB\cos\theta = \cancel{A^2}+\cancel{B^2}$$

$$\Rightarrow -2AB\cos\theta = 0$$

$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$$

(d) Given $\vec{A} + \vec{B} = \vec{C}$ and $A = B = C$

$$\Rightarrow |\vec{A} + \vec{B}|^2 = |\vec{C}|^2$$

$$\Rightarrow \left\{\sqrt{A^2+B^2+2AB\cos\theta}\right\}^2 = C^2$$

$$\Rightarrow A^2+B^2+2AB\cos\theta = C^2$$

$$\therefore \cancel{C^2}+C^2+2(C)(C)\cos\theta = \cancel{C^2}$$

$$\Rightarrow C^2+2C^2\cos\theta = 0$$

$$\Rightarrow 2\cancel{C^2}\cos\theta = -\cancel{C^2}$$

$$\Rightarrow \cos\theta = -\frac{1}{2}$$

$$\theta = 120^\circ = \frac{2\pi}{3}.$$

<u>L Task</u> <u>SAQ's</u>

(1) Given two vectors let it be P, Q are unit vectors i.e) $|\vec{P}| = |\vec{Q}| = 1$; Here the resultant is also a unit vector i.e) $|\vec{R}| = 1$

Acc to Parallelogram law of vectors

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$\Rightarrow 1 = \sqrt{1^2+1^2+2(1)(1)\cos\theta}$$

$$\Rightarrow 1 = \sqrt{1+1+2\cos\theta}$$

Scanned with OKEN Scanner

By squaring on both sides we get

$$\Rightarrow \left(\sqrt{2+2\cos\theta}\right)^2 = 1^2$$

$$\Rightarrow 2(1+\cos\theta) = 1 \Rightarrow 1+\cos\theta = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \frac{1}{2} - 1$$

$$\Rightarrow \cos\theta = \frac{-1}{2}$$

$$\Rightarrow \theta = 120^\circ$$

$\therefore$ The difference of two unit vectors is

$$R' = \sqrt{P^2+Q^2-2PQ\cos\theta}$$

$$= \sqrt{1^2+1^2-2(1)(1)\left(\frac{-1}{\cancel{2}}\right)}$$

$$R' = \sqrt{1+1+1} = \sqrt{3}$$

(2) Given

$\vec{R} = \vec{P} + \vec{Q}$; $\vec{S} = \vec{P} - \vec{Q}$

$\Rightarrow |\vec{R}| = |\vec{P} + \vec{Q}|$; $|\vec{S}| = |\vec{P} - \vec{Q}|$

$\Rightarrow R = \sqrt{P^2+Q^2+2PQ\cos\theta}$; $S = \sqrt{P^2+Q^2-2PQ\cos\theta}$

By squaring on both sides.

$\Rightarrow R^2 = \left(\sqrt{P^2+Q^2+2PQ\cos\theta}\right)^2$; $S^2 = \left(\sqrt{P^2+Q^2-2PQ\cos\theta}\right)^2$

$\Rightarrow R^2 = P^2+Q^2+2PQ\cos\theta$; $S^2 = P^2+Q^2-2PQ\cos\theta$

$$\therefore R^2+S^2 = P^2+Q^2+\cancel{2PQ\cos\theta}+P^2+Q^2-\cancel{2PQ\cos\theta}$$

$$= 2(P^2+Q^2)$$

(7) let the two forces are $F_1 = 3N$ and $F_2 = 4N$

If the particle is simultaneous acted by two force. then the resultant force is in between F_1+F_2 and $F_1 \sim F_2$ (i.e) $4+7=7N$ and $4-3=1N$.

Scanned with OKEN Scanner

$\Rightarrow \quad 2P = 5Q.$

(3) let the two forces are P and Q.

The greater resultant of P & Q = $P+Q = 7\,N$

The least resultant of P & Q = $P-Q = 3\,N$

$2P = 10N$

$P = 5N$

$\therefore$ we know $P - Q = 3$

$\Rightarrow Q = P - 3 \Rightarrow Q = 2\,N.$

If Each force is increased by 3N. then

$P' = P+3 = 5+3 = 8N;\ Q' = Q+2 = 3+2 = 5N.;\ \theta = 60^\circ$

$\therefore$ The resultant $R' = \sqrt{P'^2 + Q'^2 + 2P'Q'\cos\theta}$

$= \sqrt{(8)^2 + (5)^2 + 2(8)(5)\cos 60^\circ}$

$= \sqrt{64 + 25 + \cancel{2} \times 40 \times \frac{1}{\cancel{2}}} = \sqrt{60+25+40}$

$= \sqrt{125}$

(5) Given $\theta = 0^\circ$ and $\vec{c}$ is the resultant of $\vec{a}$ and $\vec{b}$

$\cos 0^\circ = 1$

$\therefore \quad \vec{c} = \sqrt{\bar{a}^2 + b^2 + 2ab\cos\theta}$

$|\vec{c}| = \sqrt{a^2 + b^2 + 2ab\cos 0^\circ} = \sqrt{a^2 + b^2 + 2ab}$

$|\vec{c}| = (\sqrt{(a+b)^{\cancel{2}}}) \Rightarrow |\vec{c}| = a + b.$

Scanned with OKEN Scanner

(4) let the two forces are F_1 & F_2

Given $F_1 + F_2 = 18 \rightarrow (1)$

when $\theta = 120°$: the resultant is $\sqrt{228}$

Acc to Parallelogram law of vectors

$$R' = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\Rightarrow \sqrt{228} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$$

By doing squaring on both sides we get

$$\Rightarrow (\sqrt{228})^2 = (\sqrt{F_1^2 + F_2^2 + 2F_1F_2(-\tfrac{1}{2})})^2$$

$$\Rightarrow 228 = F_1^2 + F_2^2 - F_1F_2 \rightarrow (2)$$

From (1) $(F_1 + F_2)^2 = 18^2$

$$\Rightarrow \cancel{F_1^2} + \cancel{F_2^2} + 2F_1F_2 = 324 \rightarrow (3)$$

From (2) $\cancel{F_1^2} + \cancel{F_2^2} - F_1F_2 = 228$

$\quad - \quad - \quad + \quad\quad -$

$$\Rightarrow 3F_1F_2 = 96 \Rightarrow F_1F_2 = 32$$

From $(F_1 \sim F_2)^2 = (F_1 + F_2)^2 - 4F_1F_2$

$$= 18^2 - 4 \times 32$$

$$(F_1 \sim F_2)^2 = 324 - 128 = 196 \Rightarrow F_1 \sim F_2 = \sqrt{196}$$

$$= F_1 \sim F_2 = 14 \rightarrow (4)$$

By solving (1) & (4) we get

$$F_1 + \cancel{F_2} = 18$$
$$\Rightarrow F_1 - \cancel{F_2} = 14$$
$$2F_1 = 4$$
$$F_1 = 2N.$$

From (1) $F_1 + F_2 = 18$

$$\Rightarrow F_2 = 18 - F_1$$
$$= 16N.$$

Scanned with OKEN Scanner

(6) Let the two forces are P and Q

The greatest resultant of P and $Q = P+Q = 29$ kgwt

The least resultant of P and $Q = P-Q = 5$ kgwt

$2P = 34$

$\Rightarrow P = 17$ kgwt

From $P-Q = 5$ kgwt

$\Rightarrow Q = P-5 = 17-5 = 12$ kgwt

If each force is increased by 3 kgwt, then

$P' = 17+3 = 20$ kgwt ; $Q' = Q+3 = 12+3 = 15$ kgwt

$\therefore$ The new resultant is $R' = \sqrt{P'^2+Q'^2+2P'Q'\cos\theta}$

Given $\theta = 90°$ $R' = \sqrt{(20)^2+(17)^2+2(20)(17)\cos 90}$

$= \sqrt{400+289+2(340)(0)}$

$= \sqrt{689} \simeq 27$ kgwt

(8) let the two vectors are $P = 8N$ & $Q = 6N$.

$R = 10$ N.

Acc to Parallelogram law of vectors

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$\Rightarrow R^2 = P^2+Q^2+2PQ\cos\theta$$

$$\Rightarrow 10^2 = 8^2+6^2+2(8)(6)\cos\theta$$

$$\Rightarrow 100 = 64+36+96\cos\theta$$

$$\Rightarrow 100 = 100+96\cos\theta \Rightarrow 96\cos\theta = 100-100$$

$$\Rightarrow 96\cos\theta = 0$$

$$\Rightarrow \cos\theta = 0$$

$$\Rightarrow \theta = 90°$$

Scanned with OKEN Scanner

(9) let the vectors be P and Q; Given $P=Q=R$

Acc to Parallelogram law of vectors

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$\Rightarrow R^2 = P^2+Q^2+2PQ\cos\theta$$

$$\Rightarrow \cancel{P^2} = \cancel{P^2}+P^2+2PP\cos\theta$$

$$\Rightarrow 0 = P^2+2P^2\cos\theta$$

$$\Rightarrow -P^2 = 2P^2\cos\theta \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ$$

(10) Given $|\vec{A}| = |\vec{B}|$

The magnitude of $|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$

The magnitude of $|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos\theta}$

$$\therefore |\vec{A}+\vec{B}| = n|\vec{A}-\vec{B}|$$

$$\Rightarrow \sqrt{A^2+B^2+2AB\cos\theta} = n\sqrt{A^2+B^2-2AB\cos\theta}$$

By squaring on both sides & $|\vec{A}|=|\vec{B}|$

$$\Rightarrow [\sqrt{A^2+A^2+2AA\cos\theta}]^2 = n^2[\sqrt{A^2+A^2-2AA\cos\theta}]^2$$

$$\Rightarrow 2A^2+2A^2\cos\theta = n^2(2A^2-2A^2\cos\theta)$$

$$\Rightarrow \cancel{2A^2}[1+\cos\theta] = n^2\cancel{2A^2}(1-\cos\theta)$$

$$\Rightarrow \cancel{2}\cos^2\frac{\theta}{2} = n^2\cancel{2}\sin^2\frac{\theta}{2}$$

$$\Rightarrow \cos\frac{\theta}{2} = n\sin\frac{\theta}{2} \Rightarrow \tan\frac{\theta}{2} = \frac{1}{n}$$

$$\Rightarrow \frac{\theta}{2} = \tan^{-1}\left[\frac{1}{n}\right]$$

$$\Rightarrow \theta = 2\tan^{-1}\left[\frac{1}{n}\right]$$

Scanned with OKEN Scanner

(12) let the two vectors be $\vec{A}$ and $\vec{B}$

Resultant is $\vec{C} = \sqrt{A^2+B^2+2AB\cos\theta}$

$\Rightarrow C^2 = A^2+B^2+2AB\cos\theta \rightarrow (1)$

If $\vec{B}$ is reversed $\vec{D}$ is the resultant

$\vec{D} = \sqrt{A^2+(-B)^2+2A(-B)\cos\theta}$

$\Rightarrow D^2 = A^2+B^2-2AB\cos\theta \rightarrow (2)$

By Adding (1) & (2) $C^2+D^2 = A^2+B^2+\cancel{2AB\cos\theta}+$
$A^2+B^2-\cancel{2AB\cos\theta}$

$\Rightarrow C^2+D^2 = 2(A^2+B^2)$

(13) The resultant of any two forces lies in between

P+Q and P−Q.

For 7N & 5N, the resultant is in b/w

7+5 and 7−5

$\Rightarrow$ 12 and 2, Here 3N is in b/w 12 & 2

(14) Given $F_1 = 8N$; $F_2 = 6N$ & $\theta = 90°$

$\therefore$ the resultant $= \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

$= \sqrt{8^2+6^2+2\times8\times6\times\cos 90°}$

$= \sqrt{64+36+96(0)}$

$= \sqrt{100} = 10N$

Scanned with OKEN Scanner

(15) If $\theta = 120^\circ$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$R = \sqrt{8^2 + 6^2 + 2(8)(6)\cos 120^\circ}$

$= \sqrt{64 + 36 + \cancel{2}(48)(-\frac{1}{\cancel{2}})}$

$= \sqrt{100 - 48} = \sqrt{52} = 7.2\,N.$

(16) The angle made by the resultant with 8N.

$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{6\sin 90^\circ}{8 + Q\cos 90^\circ}$

$\tan\alpha = \frac{6}{8} = \frac{3}{4}$

$\Rightarrow \alpha = 37^\circ$

(17) Given $P = Q = 15\,N$; Resultant $R = 5x$

$\theta = 120^\circ$

The resultant $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow 5x = \sqrt{(15)^2 + (15)^2 + 2(15)(15)\cos 120}$

$\Rightarrow (5x) = \sqrt{(15)^2 + (15)^2 + \cancel{2}(15)^2(-\frac{1}{\cancel{2}})}$

$\Rightarrow 5x = \sqrt{(15)^2 + \cancel{(15)^2} - \cancel{(15)^2}}$

$\Rightarrow 5x = \sqrt{15^{\cancel{2}}} \Rightarrow 5x = 15 \Rightarrow x = 3\,N$

(18) let the two forces are $F_1 = 10N$; $F_2 = 5N$

The minimum resultant of two forces

$= F_1 - F_2 = 10 - 5 = 5\,N$

Scanned with OKEN Scanner

(19) let the two forces are P and Q

$\cancel{P = 2Q}$ $Q = 2P$ & $\theta = \frac{2\pi}{n}$

Given the resultant makes an angle 90° with small force

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\Rightarrow \tan 90^\circ = \frac{Q\sin\theta}{P+Q\cos\theta} \Rightarrow \infty = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\Rightarrow P + Q\cos\theta = 0 \Rightarrow \cos\theta = -\frac{P}{Q}$$

$$\Rightarrow \cos\theta = \frac{-P}{2P} = \frac{-1}{2} \Rightarrow \theta = 120^\circ = \frac{2\pi}{3}$$

$$\Rightarrow \frac{2\pi}{n} = \frac{2\pi}{3}$$

$$\Rightarrow n = 3$$

Scanned with OKEN Scanner