

Tasks

① $M_p = \frac{1}{10} M_E$

$v_p = \frac{1}{2} v_E$

$v_E \propto \sqrt{\frac{M}{R}}$

$\Rightarrow \frac{v_p}{v_E} = \sqrt{\frac{M_p}{M_E} \cdot \frac{R_E}{R_p}}$

$\Rightarrow \frac{\frac{1}{2} v_E}{v_E} = \sqrt{\frac{\frac{1}{10} M_E}{M_E} \cdot \frac{R}{R_p}}$

$\Rightarrow \frac{1}{2} = \sqrt{\frac{1}{10} \cdot \frac{R}{R_p}}$

$\Rightarrow \frac{1}{2} = \frac{1}{\sqrt{10}} \sqrt{\frac{R}{R_p}}$

$\Rightarrow R_p = \frac{1}{5} R$

② $\frac{v_1}{v_2} = \frac{1}{2} ; \frac{d_1}{d_2} = \frac{2}{1}$

$\Rightarrow v_E \propto \sqrt{d} R$

$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{d_1}{d_2} \cdot \frac{R_1}{R_2}}$

$\Rightarrow \frac{1}{2} = \sqrt{\frac{2}{1} \cdot \frac{R_1}{R_2}}$

$\Rightarrow \frac{1}{2} = \sqrt{2} \sqrt{\frac{R_1}{R_2}}$

$\Rightarrow \frac{R_1}{R_2} = \frac{1}{2 \cdot 2} = \frac{1}{4}$

③ Given

$M = 5 \times 10^{24} \text{ kg}$

$R = 667 \times 10^3 \text{ m}$

$v_E = \sqrt{\frac{2GM}{R}}$

$v_E = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5 \times 10^{24}}{6670 \times 10^3}}$

$v_E = 10^4 \text{ m/sec}$

$v_E = 10 \text{ km/sec}$

④ $v_E = 12 \text{ km/sec}$

$R' = \frac{1}{4} R$

$v_E \propto R$

$\Rightarrow \frac{v_E}{v_E'} = \frac{R}{R'}$

$\Rightarrow \frac{12}{v_E'} = \frac{R}{\frac{1}{4} R} = 4$

$\Rightarrow \frac{12}{v_E'} = 4$

$\Rightarrow v_E' = 3 \text{ km/sec}$

⑤ $v_E = 11.2 \text{ km/sec}$

$v_p = ? ; R_p' = 2R_E$

$v_E \propto \sqrt{d} R$

$\Rightarrow \frac{v_E}{v_p} = \frac{R_E}{R_p}$

$\Rightarrow \frac{11.2}{v_p} = \frac{R_E}{2R_E}$

$\Rightarrow \frac{11.2}{v_p} = \frac{1}{2} \Rightarrow v_p = 22.4 \text{ km/sec}$

⑥ $G \cdot P \cdot E = - \frac{GMm}{R+h}$

$h = R$

$= - \frac{GMm}{R+R}$

$= - \frac{GMm}{2R}$
 $= - \frac{gR^2 m}{2R}$
 $= - \frac{m g R}{2}$

$g = \frac{GM}{R^2}$

$\Rightarrow GM = gR^2$

⑦ $v_E = 11 \text{ km/s}$

$v_E \propto R$

$\frac{v_E}{v_p} = \frac{R_E}{R_p}$

$\Rightarrow \frac{11}{v_p} = \frac{R}{2R}$

$\Rightarrow v_p = 22 \text{ km/sec}$

⑧ $v_E' = 10 v_E$

$v_E \propto \frac{1}{\sqrt{R}}$

$\frac{v_E}{v_E'} = \sqrt{\frac{R'}{R}}$

$\Rightarrow \frac{v_E}{10 v_E} = \sqrt{\frac{R'}{6400}}$

$\Rightarrow \left(\frac{1}{10}\right)^2 = \frac{R'}{6400}$

$\Rightarrow R' = 64 \text{ km}$

⑨ $L = m v r$

$L = m r^2 \omega$

$\frac{dL}{dt} = \frac{1}{2} m \frac{dA}{dt}$

$= \frac{d}{dt} (m r^2 \omega)$

$\Rightarrow \frac{dA}{dt} \propto \omega r^2$

⑩

$v_E \propto \sqrt{2gR}$

$= \sqrt{2} \sqrt{gR}$

$v_E = \sqrt{2} v_0$

$\Rightarrow \frac{v_E}{v_0} = \sqrt{2}$

$H_{\max} = \frac{v^2}{2g}$

$H_{\max} = \frac{(\sqrt{2gR})^2}{2g} = \frac{2gR}{2g}$

$H_{\max} = R$

LTark

① $R_p = 4R$

$g_p = 9g_e$

$v_e = \sqrt{2gR}$

$\Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e} \cdot \frac{R_p}{R_e}}$

$\Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{9g_e \times 4R_e}{g_e R_e}}$

$\Rightarrow \frac{v_p}{v_e} = \sqrt{36} = 6$

$\Rightarrow v_p = 6v_e$

$\Rightarrow v_p = 67.2 \text{ km/sec}$

⑤ $v_e = 11.2 \text{ km/sec}$

$M_p = \frac{M}{4}$ & $R_p = \frac{R}{2}$

$\Rightarrow v_e \propto \sqrt{\frac{M}{R}}$

$\Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$

$\Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{M}{4} \times \frac{R}{\frac{R}{2}}}$

$\Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{1}{4} \times 2} = \frac{1}{\sqrt{2}}$

$\Rightarrow v_p = \frac{v_e}{\sqrt{2}} = 8 \text{ km}$

⑩ $R_p = R$ & $T_p = 24 \text{ hr}$

$R' = 4R$ & $T' = ?$

$T \propto (R)^{\frac{3}{2}}$

$\Rightarrow \frac{T_p}{T'} = \left(\frac{R_p}{R'} \right)^{\frac{3}{2}}$

$\Rightarrow \frac{T_p}{T'} = \left(\frac{R}{4R} \right)^{\frac{3}{2}} = \left(\frac{1}{4} \right)^{\frac{3}{2}} \Rightarrow T' = 8 \text{ day}$

$\Rightarrow \frac{T_p}{T'} = \frac{1}{2^3} = \frac{1}{8}$

$\Rightarrow T' = 8 T_p$

$\Rightarrow T' = 8 \times 24 \text{ hr}$

$\Rightarrow T' = 8 \text{ day}$

② $v_0 = 7 \text{ km/sec}$

$v_0 = \sqrt{\frac{GM}{R+h}}$

$\frac{v_0}{v_0'} = \sqrt{\frac{R+h_2}{R+h_1}}$

$\frac{v_0}{v_0'} = \sqrt{\frac{R+4R}{R}}$

$\frac{v_0}{v_0'} = \sqrt{\frac{5R}{R}}$

$\frac{v_0'}{v_0} = \frac{v_0}{\sqrt{5}} = \frac{7}{\sqrt{5}}$

$= 1.4 \text{ km/sec}$

③ $U \propto \sqrt{\frac{M}{R}}$

$\Rightarrow \frac{U}{U_p} = \sqrt{\frac{M_e}{M_p} \cdot \frac{R_p}{R_e}}$

$\Rightarrow \frac{U}{U_p} = \sqrt{\frac{M}{8M} \cdot \frac{8R}{R}}$

$\Rightarrow U_p = U$

(Given $M_p = 2M$ & $R_p = 2R$)

④ $v_e = 11.2 \text{ km/sec}$; $R_p = 2R$ & $M_p = 8M$

$v_e \propto \sqrt{\frac{M}{R}}$

$\Rightarrow \frac{v_e}{v_p} = \sqrt{\frac{M_e}{M_p} \cdot \frac{R_p}{R_e}}$

$= \frac{11.2}{v_p} = \sqrt{\frac{M_e}{8M_e} \cdot \frac{2R_e}{R_e}}$

$\Rightarrow \frac{11.2}{v_p} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

$\Rightarrow v_p = 2 \times 11.2 = 22.4 \text{ km/sec}$

⑦ Given $T_A = 8 \text{ hr}$ & $T_B = 1 \text{ hr}$

$T^2 \propto R^3 \Rightarrow T \propto R^{\frac{3}{2}}$

$\Rightarrow T_A^{\frac{2}{3}} \propto R$

$\Rightarrow \frac{R_A}{R_B} = \left(\frac{T_A}{T_B} \right)^{\frac{2}{3}} = \left(\frac{8}{1} \right)^{\frac{2}{3}}$

$\Rightarrow \frac{R_A}{R_B} = (2^3)^{\frac{2}{3}} = 2^2 = 4$

⑮ $\frac{m_1}{m_2} = \frac{2}{3}$ & $\frac{R_1}{R_2} = \frac{3}{2}$

$\frac{v_1}{v_2} = \sqrt{\frac{M_1}{M_2} \cdot \frac{R_2}{R_1}} = \sqrt{\frac{2}{3} \times \frac{2}{3}}$

$\frac{v_1}{v_2} = \frac{2}{3}$

⑯ Given $h = R$ & $\frac{h}{R} = 1$

$v = \sqrt{\frac{GM}{R+h}}$; $v \propto \frac{1}{\sqrt{R+h}}$

$\frac{v_1}{v_2} = \sqrt{\frac{R+h_2}{R+h_1}} \Rightarrow \frac{v_1}{v_2} = \sqrt{2}$

$\frac{v_1}{v_2} = \sqrt{\frac{R+R}{R+0}} \Rightarrow v_2 = \frac{v_1}{\sqrt{2}}$

$\frac{v_1}{v_2} = \sqrt{\frac{2R}{R}}$

⑰ $v_e \propto R$; $R_p = 3R_e$

$\frac{v_e}{v_p} = \frac{R_e}{R_p}$

$\frac{v_e}{v_p} = \frac{R}{3R}$

$\Rightarrow \frac{v_e}{v_p} = \frac{1}{3}$

$\Rightarrow v_p = 3v_e$

$\Rightarrow v_p = 33.6 \text{ km/sec}$

