SURDS

§§ **Surd :** Let '*a*' be a positive rational number, and '*n*' be a positive integer. If the n^{th} root of a, (i.e., $\sqrt[n]{a}$) is not a rational number, then $\sqrt[n]{a}$ is called a ***surd*** of order *n*.

Ex : $\sqrt{2},\sqrt{3},\sqrt{8/7}$ are 2^{nd} order surds (quadratic surds)

$\sqrt[3]{2},\sqrt[3]{3},$ ...are 3^{rd} order surds (cubic surds)

$\sqrt[4]{2},\sqrt[4]{3}$,... are 4^{th} order surds (biquadratic surds)

$\sqrt{4},\sqrt{9},\sqrt[3]{8}$,... are not surds

Note : $\sqrt[n]{a}$ is always positive

§§ **Different kinds of surds :**

1. Pure surd : (Entire surd)

If a surd is written entirely inside the n^{th} root, it is called a ***pure surd***.

Ex : 1) $2\sqrt{3}=\sqrt{12},\sqrt{12}$ is called a pure surd.

2) $a\sqrt{b}$ is a pure surd, if a = 1 and $\sqrt{b}$ is irrational

2. Simple surd :

A surd which is expressed in the form of $a\sqrt{b}$ where '*b*' is the least positive rational number, is called a ***simple surd***.

It consists of a single term, hence it is also called a ***monomial surd***.

Ex : The simplest form of the surd $\sqrt{12}=2\sqrt{3}$

The simplest form of the surd $\sqrt[4]{405}=3\sqrt[4]{5}$

3. Mixed surds :

If '*a*' is a rational number and $\sqrt[n]{b}$ is a surd, then $a\pm\sqrt[n]{b}$ and $a\sqrt[n]{b}$ are called ***mixed surds***.

Ex : $3\pm\sqrt{2},4\pm3\sqrt{5},6\sqrt{3}$

4. Compound surd :

The sum or difference of a rational number and one or more dissimilar surds is called a compound surd.

Ex : $3+\sqrt{2},2-\sqrt{5},2+\sqrt{3}-\sqrt{5},\sqrt{3}+\sqrt{2}-\sqrt{5}$

5. Binomial surd :

A compound surd consisting of two terms is called a ***binomial surd***.

6. Trinomial surd :

A compound surd consisting of three terms is called a ***trinomial surd***.

7. Like surds, unlike surds :

If the quotient of two surds is a rational number, then they are said to be ***like*** (***similar***) ***surds***; Otherwise, they are ***unlike*** (***dissimilar***) **surds**.

Ex. $3\sqrt{5}, 4\sqrt{5}, 5\sqrt{5}$ are like surds.

$3\sqrt{5}, 4\sqrt{3}, 6\sqrt{2}$ are dissimilar surds (unlike)

Note :

1) If $\sqrt{a}$ and $\sqrt{b}$ are dissimilar surds then there exists no surd of the form $\sqrt{c}$ such that $\sqrt{a} + \sqrt{b} = \sqrt{c}$.

2) If $\sqrt{a}$ and $\sqrt{c}$ are surds, then there is no rational number $b(\neq 0)$ satisfying $\sqrt{a} = b + \sqrt{c}$

3) If *a, b, c* and *d* are rational numbers and $\sqrt{b}, \sqrt{d}$ are surds such that $a + \sqrt{b} = c + \sqrt{d}$,then *a* = *c* and *b* = *d*.

8. Conjugate surds :

If the sum and product of two binomial surds are rational numbers, then they are called ***conjugate surds***.

Ex : Two surds $a + \sqrt{b}, a - \sqrt{b}$ are conjugate to each other $5 + \sqrt{6}, 5 - \sqrt{6}$ are conjugate surds. $\sqrt{2} + \sqrt{3}, \sqrt{2} - \sqrt{3}$ are not conjugate surds.(Since their sum is not rational number)

9. Rationalising factors :

If the product of two surds is a rational number, each surd is called a ***rationalising factor*** of the other.

Ex. 1) $\sqrt{x} + \sqrt{y}$ and $\sqrt{x} - \sqrt{y}$ are rationalising factors of each other.

2) $5 - \sqrt{3}$ and $5 + \sqrt{3}$ are rationalising factors of each other.

3) $\left(\sqrt[3]{3} - \sqrt[3]{2}\right)\left(\sqrt[3]{9} + \sqrt[3]{6} + \sqrt[3]{4}\right) = \sqrt[3]{27} - \sqrt[3]{8} = 1$

Hence $\sqrt[3]{9} + \sqrt[3]{6} + \sqrt[3]{4}$ is a rationalising factor of $\sqrt[3]{3} - \sqrt[3]{2}$

4) $6\sqrt{6}.\sqrt{6} = 36$ $\quad\therefore \sqrt{6}$ is a rationalising factor of $6\sqrt{6}$

5) $5\sqrt{2} \times 3\sqrt{2} = 30$ and $5\sqrt{2} \times 2\sqrt{2} = 20$

$3\sqrt{2}$ and $2\sqrt{2}$ are rationalizing factors of $5\sqrt{2}$

Note : A surd may have any number of rationalising factors. Hence rationalising factor of a surd is not unique.

§§ Square roots of Surds :

Procedure to find square root of $a+\sqrt{b}$

Let $\sqrt{a+\sqrt{b}} = \sqrt{x}+\sqrt{y}$, $x \ge y$

Then $a+\sqrt{b} = (\sqrt{x}+\sqrt{y})^2$

$\Rightarrow a+\sqrt{b} = x+y+2\sqrt{xy}$

Compare rational and irrational parts on both sides,

$\Rightarrow x+y=a$ --------- (A) , $2\sqrt{xy} = \sqrt{b}$

we have $(x-y)^2 = (x+y)^2 - 4xy$

$= a^2 - b$

$\Rightarrow x-y = \sqrt{a^2-b}$ ---------(B)

From (A) and (B) we get $x = \dfrac{a+\sqrt{a^2-b}}{2}, y = \dfrac{a-\sqrt{a^2-b}}{2}$

Note : if $\sqrt{a-\sqrt{b}} = \sqrt{x}-\sqrt{y}$, then $x = \dfrac{a+\sqrt{a^2-b}}{2}, y = \dfrac{a-\sqrt{a^2-b}}{2}$

$\sqrt{a \pm \sqrt{b}} = \sqrt{x}+\sqrt{y}$ where $x = \dfrac{a+\sqrt{a^2-b}}{2}, y = \dfrac{a-\sqrt{a^2-b}}{2}$

$\sqrt{a+b+c+2\sqrt{ab}+2\sqrt{bc}+2\sqrt{ca}} = \sqrt{a}+\sqrt{b}+\sqrt{c}$

$\sqrt{a+\sqrt{b}+\sqrt{c}+\sqrt{d}} = \sqrt{x}+\sqrt{y}+\sqrt{z}$ where x= $\sqrt{\dfrac{bd}{4c}}$; y= $\sqrt{\dfrac{bc}{4d}}$, z= $\sqrt{\dfrac{cd}{4b}}$

EXAMPLE

√ **Example 1 :** Find the positive square root of $5+\sqrt{24}$.

Sol : Let $\sqrt{5+\sqrt{24}} = \sqrt{x}+\sqrt{y}, x \ge y$

Then $5+\sqrt{24}=(\sqrt{x}+\sqrt{y})^2$

$\Rightarrow\ 5+\sqrt{24}=x+y+2.\sqrt{xy}$

compare rational and irrational parts on both sides.

$\Rightarrow\ x+y=5$ ------------(1) , $2.\sqrt{xy}=\sqrt{24}$

$\Rightarrow\ 4xy=24$ ----------(2)

We have $(x-y)^2 = (x+y)^2-4xy$

$= 5^2-24$ $(\because (2))$

= 25 – 24

= 1

$\Rightarrow\ x-y=1$ -----------(3)

Solving (1) and (3),

we get $x=3,\ y=2$

$\therefore \sqrt{5+\sqrt{24}}=\sqrt{3}+\sqrt{2}$

Aliter - I

Let $\sqrt{5+\sqrt{24}}=\sqrt{x}+\sqrt{y},\ x\geq y$

Here a = 5, b= 24

we have $x=\dfrac{a+\sqrt{a^2-b}}{2},\ y=\dfrac{a-\sqrt{a^2-b}}{2}$

$\Rightarrow\ x=\dfrac{5+\sqrt{25-24}}{2},\ y=\dfrac{5-\sqrt{25-24}}{2}$

$\Rightarrow\ x=\dfrac{5+1}{2},\ y=\dfrac{5-1}{2}$

$\Rightarrow\ x=3,\ y=2$

$\therefore \sqrt{5+\sqrt{24}}=\sqrt{3}+\sqrt{2}$

Aliter - II

$\sqrt{5+\sqrt{24}} = \sqrt{5+2\sqrt{6}}$

$= \sqrt{(3+2)+2\sqrt{3\times 2}}$

$=\sqrt{(\sqrt{3}+\sqrt{2})^2}$ $(\because a+b+2.\sqrt{ab}=(\sqrt{a}+\sqrt{b})^2)$

$= \sqrt{3}+\sqrt{2}$

$\therefore \sqrt{5+\sqrt{24}} = \sqrt{3}+\sqrt{2}$

√ **Example 2 :**

Show that $\sqrt{6+2\sqrt{3}+2\sqrt{6}+2\sqrt{2}} - \frac{1}{\sqrt{5-2\sqrt{6}}}$ is a rational number.

Solution :

We have,

$$6+2\sqrt{3}+2\sqrt{6}+2\sqrt{2}$$
$$= 3+2+1+2\sqrt{3\times 2}+2\sqrt{2\times 1}+2\sqrt{1\times 3}$$
$$= \left(\sqrt{3}+\sqrt{2}+1\right)^2$$

$$\therefore \sqrt{6+2\sqrt{3}+2\sqrt{6}+2\sqrt{2}} = \sqrt{3}+\sqrt{2}+1$$

Hence,

$$\sqrt{6+2\sqrt{3}+2\sqrt{6}+2\sqrt{2}} - \frac{1}{\sqrt{5-2\sqrt{6}}}$$
$$= \sqrt{3}+\sqrt{2}+1 - \frac{1}{\sqrt{3}-\sqrt{2}}$$
$$= \sqrt{3}+\sqrt{2}+1 - \left(\sqrt{3}+\sqrt{2}\right)$$
$$= 1 \quad \text{which is a rational number.}$$

√ **Example 3:** Find $\sqrt{5-\sqrt{10}-\sqrt{15}+\sqrt{6}}$.

Sol : Let $\sqrt{5-\sqrt{10}-\sqrt{15}+\sqrt{6}} = \left|\sqrt{x}-\sqrt{y}+\sqrt{z}\right|$

Then $5-\sqrt{10}-\sqrt{15}+\sqrt{6} = (\sqrt{x}-\sqrt{y}+\sqrt{z})^2$

$= x+y+z-2\sqrt{xy}-2\sqrt{yz}+2\sqrt{zx}$

$\therefore x+y+z=5, \quad 4xy=15, \quad 4yz=10, \; 4zx=6$

$\frac{(4xy)(4zx)}{4yx} = \frac{15\times 6}{10} = 9 \;\Rightarrow\; 4x^2 = 9 \;\Rightarrow x = \frac{3}{2}$

Similarly, $\frac{(4yz)(4xy)}{4zx} = \frac{15\times 10}{6} = 25 \Rightarrow y = \frac{5}{2}$

and z = 1 $\therefore \sqrt{5-\sqrt{10}-\sqrt{15}+\sqrt{6}} = \sqrt{\frac{3}{2}} - \sqrt{\frac{5}{2}} + 1$.

√ **Example 4:** If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}, y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then fin the value x^2+xy+y^2.

Sol : Given $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 5-2\sqrt{6}$, $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 5+2\sqrt{6}$

$x^2 = 25+24-20\sqrt{6}, \quad y^2 = 25+24+20\sqrt{6}, xy = 1$

now $x^2+y^2+xy = 99$

√ **Example 5:** If $\frac{12}{3+\sqrt{5}+2\sqrt{2}} = a+\sqrt{5}+\sqrt{2}-\sqrt{10}$, then find the value of a.

Sol : $\frac{12}{3+\sqrt{5}+2\sqrt{2}} = \frac{12(3+\sqrt{5}-2\sqrt{2})}{(3+\sqrt{5}+2\sqrt{2})(3+\sqrt{5}-2\sqrt{2})}$

$= \frac{12(3+\sqrt{5}-2\sqrt{2})}{\left(3+\sqrt{5}\right)^2-\left(2\sqrt{2}\right)^2} = \frac{12(3+\sqrt{5}-2\sqrt{2})}{\left(9+5+6\sqrt{5}\right)-8}$

$= \frac{12(3+\sqrt{5}-2\sqrt{2})}{6(\sqrt{5}+1)}$

$= \frac{12(3+\sqrt{5}-2\sqrt{2})(\sqrt{5}-1)}{6(\sqrt{5}+1)(\sqrt{5}-1)}$

$= \frac{12(3+\sqrt{5}-2\sqrt{2})(\sqrt{5}-1)}{6(5-1)}$

$= \frac{12}{24}(3\sqrt{5}+5-2\sqrt{10}-3-\sqrt{5}+2\sqrt{2})$

$= \frac{1}{2}\left(2+2\sqrt{2}+2\sqrt{5}-2\sqrt{10}\right)$

$= 1+\sqrt{2}+\sqrt{5}-\sqrt{10}$

$\therefore\ a = 1$

√ **Example 6 :** Prove that $\dfrac{3}{\sqrt{11-4\cdot\sqrt{7}}} - \dfrac{1}{\sqrt{7+4\cdot\sqrt{3}}} - \dfrac{4}{\sqrt{10-\sqrt{84}}} = 0$

Sol : $\sqrt{11-4\sqrt{7}} = \sqrt{11-2\sqrt{7\times 4}} = \sqrt{7}-\sqrt{4} = \sqrt{7}-2$

$\sqrt{7-4\sqrt{3}} = \sqrt{7+2\sqrt{4\times 3}} = \sqrt{4}+\sqrt{3} = 2+\sqrt{3}$

$\sqrt{10-\sqrt{84}} = \sqrt{10-2\sqrt{21}} = \sqrt{10-2\sqrt{7\times 3}} = \sqrt{7}-\sqrt{3}$

$\therefore$ LHS $= \dfrac{3}{\sqrt{7}-2} - \dfrac{1}{2+\sqrt{3}} - \dfrac{4}{\sqrt{7}-\sqrt{3}}$

$= \dfrac{3(\sqrt{7}+2)}{7-4} - \dfrac{2-\sqrt{3}}{4-3} - \dfrac{4(\sqrt{7}+\sqrt{3})}{7-3}$

$= (\sqrt{7}+2)-(2-\sqrt{3})-(\sqrt{7}+\sqrt{3})$

$= 0$

√ **Example 7:** If $x > 2$, find $\sqrt{2x-3+2\sqrt{x^2-3x+2}}$.

Sol : $\sqrt{2x-3+2\sqrt{x^2-3x+2}}$

$= \sqrt{2x-3+2\sqrt{(x-1)(x-2)}}$

$= \sqrt{(x-1)+(x-2)+2\sqrt{(x-1)(x-2)}}$

$= \sqrt{(\sqrt{x-1}+\sqrt{x-2})^2}$ as $x > 2$

$= \sqrt{x-1}+\sqrt{x-2}$

√ **Example 8:** If $x = \dfrac{1}{2}\left(\sqrt{a}+\dfrac{1}{\sqrt{a}}\right)$, then show that $\dfrac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}} = \dfrac{a-1}{2}$.

Sol : $x = \dfrac{1}{2}\left(\sqrt{a}+\dfrac{1}{\sqrt{a}}\right)$

$x^2-1 = \dfrac{1}{4}\left(\sqrt{a}+\dfrac{1}{\sqrt{a}}\right)^2 - 1$

$$=\frac{1}{4}\left(a+\frac{1}{a}+2\right)-1$$

$$=\frac{1}{4}\left(a+\frac{1}{a}-2\right)=\frac{1}{4}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)^2$$

$$\therefore \sqrt{x^2-1}=\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)$$

$$\therefore \frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}=\frac{\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)}{\frac{1}{2}\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)-\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)}$$

$$=\frac{\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)}{\frac{1}{\sqrt{a}}}=\frac{1}{2}(a-1)$$

√ **Example 9:** Show that $\sqrt[3]{1-\sqrt{2}}\cdot\sqrt[6]{2\sqrt{2}+3}=-1$.

Sol : $2\sqrt{2}+3=2+1+2\sqrt{2\times1}=\left(\sqrt{2}+\sqrt{1}\right)^2$

$\sqrt[6]{2\sqrt{2}+3}=\sqrt[3]{\sqrt{2}+1}$

$\therefore L.H.S=\sqrt[3]{1-\sqrt{2}}.\sqrt[3]{1+\sqrt{2}}=\sqrt[3]{(1+\sqrt{2})(1-\sqrt{2})}$

$=\sqrt[3]{-1}$

$=-1$

√ **Example 10:** Solve $\left(4+\sqrt{15}\right)^{3/2}+\left(4-\sqrt{15}\right)^{3/2}=x\sqrt{10}$.

Sol : $\left(4+\sqrt{15}\right)^{3/2}+\left(4-\sqrt{15}\right)^{3/2}=x\sqrt{10}$ -----------(1)

$4+\sqrt{15}=\frac{1}{2}(8+2\sqrt{15})=\frac{1}{2}(\sqrt{5}+\sqrt{3})^2$

$\therefore (4+\sqrt{15})^{3/2}=\frac{1}{\sqrt[2]{8}}(\sqrt{5}+\sqrt{3})^3$

Similarly $(4-\sqrt{15})^{3/2} = \frac{1}{\sqrt[2]{8}}\left(\sqrt{5}-\sqrt{3}\right)^3$

(1) $\Rightarrow \frac{1}{2\sqrt{2}}\left[(\sqrt{5}+\sqrt{3})^3+\left(\sqrt{5}-\sqrt{3}\right)^3\right] = x\sqrt{10}$ --------(2)

$\left(a+b\right)^3+\left(a-b\right)^3 = (a^3+3a^2b+3ab^2+b^3)+\left(a^3-3a^2b+3ab^2-b^3\right)$

$= 2(a^3+3ab^2)$

$\therefore$(2) $\Rightarrow \frac{1}{2\sqrt{2}}.2\left(5\sqrt{5}+9\sqrt{5}\right) = x\sqrt{10}$

$\Rightarrow \frac{14\sqrt{5}}{\sqrt{2}} = x\sqrt{10}$

$\therefore x = \frac{14\sqrt{5}}{\sqrt{20}} = \frac{14\sqrt{5}}{2\sqrt{5}} = 7$

$\therefore x = 7$

§§ Cube roots of Surds.

$\sqrt[3]{a+b\sqrt{c}} = \sqrt{\frac{b-c}{3}}+\sqrt{c} = x+\sqrt{y}$ Where $x^3+3xy=a$

Note: A)The above formula can be applied only if $x^3+3xy=a$

B) $\sqrt[3]{(x+3y)\sqrt{x}+(3x+y)\sqrt{y}} = \sqrt{x}+\sqrt{y}$

√ **Example 11:** Find $\sqrt[3]{38+17\sqrt{5}}$.

Sol : Let $\sqrt[3]{38+17\sqrt{5}} = x+\sqrt{y}$ ------ (1)

Then $\sqrt[3]{38-17\sqrt{5}} = x-\sqrt{y}$ ------ (2)

(1) $\Rightarrow 38+17\sqrt{5} = (x+\sqrt{y})^3 = (x^3+3xy)+(3x^2+y)\sqrt{y}$

$\therefore x^3+3xy = 38$ -------(3)

(1), (2) $\Rightarrow \sqrt[3]{(38+17\sqrt{5})(38-17\sqrt{5})} = (x+\sqrt{y})(x-\sqrt{y})$

$$\sqrt{38^2 - 5.17^2} = x^2 - y$$

$$\sqrt[3]{1444 - 1445} = x^2 - y$$

$$x^2 - y = \sqrt[3]{-1} = -1 \qquad \text{-------} \quad (4)$$

$$y = x^2 + 1$$

$\therefore$ (3) $\Rightarrow$ $x^3 + 3x(x^2 + 1) = 38$

$$4x^3 + 3x = 38$$

$\therefore x = 2$ (by inspection)

$$\therefore y = x^2 + 1 = 5$$

$$\therefore \sqrt[3]{38 + 17\sqrt{5}} = 2 + \sqrt{5}$$

♥ ***TIPS AND TRICKS***

A) R.F of $\sqrt{a} \pm \sqrt{b}$ is $\sqrt{a} \mp \sqrt{b}$

B) R.F of $\sqrt[3]{a} + \sqrt[3]{b}$ is $\sqrt[3]{a}^{2} - \sqrt[3]{ab} + \sqrt[3]{b}^{2}$

C) R.F of $\sqrt[3]{a} - \sqrt[3]{b}$ is $\sqrt[3]{a}^{2} + \sqrt[3]{ab} + \sqrt[3]{b}^{2}$

D) R.F of $\sqrt[4]{a} + \sqrt[4]{b}$ is $(\sqrt[4]{a} - \sqrt[4]{b})(\sqrt{a} + \sqrt{b})$

5) R.F of $\sqrt[4]{a} - \sqrt[4]{b}$ is $(\sqrt[4]{a} + \sqrt[4]{b})(\sqrt{a} + \sqrt{b})$

6) R.F of $\sqrt[6]{a} \pm \sqrt[6]{b}$ is $(\sqrt[6]{a} \mp \sqrt[6]{b})(\sqrt[3]{a}^{2} + \sqrt[3]{ab} \pm \sqrt[3]{b}^{B})$

7) $\sqrt[5]{a} \pm \sqrt[5]{b}$ is $\sqrt[5]{a^4} \mp \sqrt[5]{a^3b} + \sqrt[5]{a^2b^2} \mp \sqrt[5]{ab^3} + \sqrt[5]{b^4}$

8) $\sqrt[8]{a} + \sqrt[8]{b}$ is $(\sqrt[8]{a} - \sqrt[8]{b})(\sqrt[4]{a} + \sqrt[4]{b})(\sqrt{a} + \sqrt{b})$

♣ **IMPORTANT RESULTS**

1) If $\sqrt{a}, \sqrt{b}$ are dissimillar surds , then there exists no surd of the form $\sqrt{c}$, surd that $\sqrt{a} \pm \sqrt{b} = \sqrt{c}$

2) If a,b,c,d are all rational numbers and b,d are not perfect squares a+ $\sqrt{b}$ =c+ $\sqrt{d}$ then a=c and $\sqrt{b} = \sqrt{d}$

3) $$\frac{1}{\sqrt[3]{a} + \sqrt[3]{b}} = \frac{\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}}{a + b}$$

4) $\frac{1}{\sqrt[3]{a}-\sqrt[3]{b}} = \frac{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}{a-b}$

5) If $(a+\sqrt{b})^{x^2-k} + (a-\sqrt{b})^{x^2-k}$ =2a and a^2-b=1 then $x^2-k = \pm 1$

6) If $(a+\sqrt{b})^{x^2-k} + (a-\sqrt{b})^{x^2-k}$ =2(a^2+b) and a^2-b = 1 then $x^2-k = \pm 2$

7) $\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+----\infty}}}} = \frac{1+\sqrt{1+4a}}{2}$ (a>o)

8) $\sqrt{a-\sqrt{a-\sqrt{a------\infty}}} = \frac{\sqrt{4a+1}-1}{2}\ (a>0)$

9) $\sqrt{a\sqrt{a\sqrt{a..........\infty}}} = a \quad (a>0)$

10) $\sqrt{a\sqrt{a\sqrt{a..........\ ....n\ times}}} = a^{\frac{2^n-1}{2^n}} \quad (a>0)$

TEACHING TASK

I) MCQ's with single correct answer:

1. If $\sqrt{10+2\sqrt{21}} = \sqrt{7}+\sqrt{x}$ then x= ....

A) 3 B) 6 C) 5 D) 2

2. If $(4+\sqrt{15})^{3/2} + (4-\sqrt{15})^{\frac{1}{3}} = x\sqrt{10}$ then x = ...

A) 8 B) 7 C) 6 D) 5

3. $\sqrt{6\sqrt{6\sqrt{6...\infty}}}$ = ...

A) 8 B) 7 C) 6 D) 5

4. $\sqrt[3]{16+8\sqrt{5}} + \sqrt[3]{16-8\sqrt{5}}$ = ...

A) 3 B) 6 C) 5 D) 2

5. $\sqrt{\frac{6+2\sqrt{3}}{33-19\sqrt{3}}} = a + b\sqrt{3}$ then a + b = ...

A) 8 B) 6 C) 5 D)7

6. The square root of $49+20\sqrt{6}$ is

A) $3+3\sqrt{5}$ B) $5+3\sqrt{6}$ C) $5+2\sqrt{6}$ D) $2+5\sqrt{6}$

7. $\sqrt{27 - 6\sqrt{6} - 12\sqrt{3} + 6\sqrt{2}}$ =

A) $3\sqrt{2} + \sqrt{6} - \sqrt{3}$ B) $3\sqrt{2} - \sqrt{6} - \sqrt{3}$

C) $3\sqrt{2} - \sqrt{6} + \sqrt{3}$ D) $3\sqrt{2} + \sqrt{6} + \sqrt{3}$

8. $\sqrt{39 - 4\sqrt{21} - 4\sqrt{35} + 8\sqrt{15}}$ =

A) $3\sqrt{2} - 2\sqrt{3} + \sqrt{5}$ B) $2\sqrt{5} + 2\sqrt{3} - \sqrt{7}$

C) $2\sqrt{2} - 2\sqrt{3} - \sqrt{5}$ D) $2\sqrt{7} + 2\sqrt{3} - \sqrt{3}$

9. $\sqrt{7 - 3\sqrt{5}} + \sqrt{3 + \sqrt{5}}$ =

A) $3\sqrt{3}$ B) $2\sqrt{2}$ C) $2\sqrt{5}$ D) $3\sqrt{5}$

10. If $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$ then $\sqrt{1 - Sin2\theta} + \sqrt{1 + Sin2\theta} =$

A) $2Sin\theta$ B) $-2Sin\theta$ C) $2Cos\theta$ D) $-2Cos\theta$

11. $\sqrt{6 + 2\sqrt{5 - \sqrt{(13 + \sqrt{48})}}} =$

A) $\sqrt{2} + 1$ B) $\sqrt{5} + 1$ C) $\sqrt{3} + 1$ D) $\sqrt{6} + \sqrt{2}$

12. $\sqrt{21 - 4\sqrt{5} + 8\sqrt{3} - 4\sqrt{15}} =$

A) 2 + 2 $\sqrt{3} - \sqrt{5}$ B) 2-3 $\sqrt{2} + \sqrt{5}$ C) 2-2 $\sqrt{3} + \sqrt{5}$ D) $\sqrt{2} + \sqrt{3} - 2\sqrt{5}$

13. Which one is true

A) $\sqrt[10]{3} > \sqrt[4]{2} > \sqrt[5]{5}$ B) $\sqrt[10]{3} < \sqrt[4]{2} < \sqrt[5]{5}$

C) $\sqrt[10]{3} < \sqrt[4]{2} > \sqrt[5]{5}$ D) $\sqrt[10]{3} > \sqrt[4]{2} < \sqrt[5]{5}$

14. Rationalising factor of $a+b^{1/3}$ is

A) $a^2 - ab^{1/3} + b^{2/3}$ B) $a^3 - ab^{1/3} + b$ C) $a - ab^{1/3} + b$ D) $(a^{1/3} + b)$

15. Rationalising factor of $\sqrt[3]{25} + \sqrt[3]{5} + 1$ is

A) $\sqrt[3]{5} + 1$ B) $\sqrt[5]{3} + 1$ C) $\sqrt[3]{5} - 1$ D) $\sqrt[3]{25} - \sqrt[3]{5} - 1$

16. Rationalising factor of $\sqrt[5]{3} + 1$ is

A) $\left(3^{\frac{4}{5}} + 3^{\frac{3}{5}} - 3^{\frac{2}{5}} + 1\right)$ B) $\left(3^{\frac{4}{5}} + 3^{\frac{3}{5}} - 3^{\frac{2}{5}} + 3^{\frac{1}{5}} + 1\right)$

C) $\left(3^{\frac{4}{5}}+3^{\frac{3}{5}}+3^{\frac{2}{5}}+3^{\frac{1}{5}}+1\right)$ D) $\left(3^{\frac{4}{5}}-3^{\frac{3}{5}}+3^{\frac{2}{5}}-3^{\frac{1}{5}}+1\right)$

17. $\sqrt{5}\,\sqrt[4]{5}\sqrt[8]{5}$..........$\infty =$

A) 5 B) $\log_{10}^{5}$ C) 1 D) $\sqrt{5}$

18. $\left(x+\sqrt{x^2-bc}\right)\left(y+\sqrt{y^2-ca}\right)\left(z+\sqrt{z^2-ab}\right)$

$=\left(x-\sqrt{x^2-bc}\right)\left(y-\sqrt{y^2-ca}\right)\left(z-\sqrt{z^2-ab}\right) = k \Rightarrow k =$

A) $\pm a^2b^2c^2$ B) $\pm a^3b^3c^3$ C) $\pm a^4b^4c^4$ D) $\pm abc$

19. $\sqrt{16+2\sqrt{55}} =$

A) $3+\sqrt{7}$ B) $\sqrt{11}+\sqrt{5}$ C) $\sqrt{12}+\sqrt{4}$ D) $\sqrt{13}+\sqrt{3}$

20. $\sqrt{11\sqrt{7}+28} =$

A) $\sqrt[4]{7}\left(\sqrt{7}+2\right)$ B) $\sqrt[4]{7}\left(\sqrt{6}+\sqrt{5}\right)$ C) $\sqrt[4]{7}\left(\sqrt{8}+\sqrt{3}\right)$ D) $\left(\sqrt{8}-\sqrt{3}\right)$

21. $\sqrt{23\sqrt{2}-12\sqrt{5}} =$

A) $\sqrt[4]{5}\left(\sqrt{2\sqrt{3}-\sqrt{5}}\right)$ B) $\sqrt[4]{5}\left(\sqrt{3\sqrt{2}-\sqrt{5}}\right)$

C) $\sqrt[4]{2}\left(3\sqrt{2}-\sqrt{5}\right)$ D) $\sqrt[4]{2}\left(3\sqrt{2}-\sqrt{5}\right)$

22. $\sqrt{(a+b+c)+2\sqrt{ac+bc}} =$

A) $\sqrt{a}+\sqrt{b+c}$ B) $\sqrt{a}+\sqrt{b}+\sqrt{c}$

C) $\sqrt{a+b}+\sqrt{c}$ D) $\sqrt{a}-\sqrt{b}+\sqrt{c}$

23. $\sqrt{(2x-1)+2\sqrt{x^2-x-6}} =$

A) $\sqrt{x+2}+\sqrt{x+3}$ B) $\sqrt{x-2}+\sqrt{x-3}$

C) $\sqrt{x-3}+\sqrt{x+2}$ D) $\sqrt{x-2}+\sqrt{x+3}$

24. $\sqrt{a+b+2x+2\sqrt{ab+(a+b)x+x^2}} =$

A) $\sqrt{a+b+2x}$ B) $\sqrt{a}+\sqrt{b}+\sqrt{x}$

C) $\sqrt{a+x}+\sqrt{b+x}$ D) $\sqrt{a+b+x}+\sqrt{x}$

25. $\sqrt[3]{37-30\sqrt{3}} =$

A) $2-\sqrt{3}$ B) $3-\sqrt{3}$ C) $1-2\sqrt{3}$ D) $2-2\sqrt{3}$

26. $\sqrt[3]{54\sqrt{3}+41\sqrt{5}} =$

A) $\sqrt{3}+\sqrt{5}$ B) $2\sqrt{3}+\sqrt{5}$ C) $\sqrt{3}+2\sqrt{5}$ D) $3\sqrt{3}+\sqrt{5}$

27. $\sqrt{2+\sqrt{5}}\left(17\sqrt{5}-38\right)^{1/6} =$

A) 2 B) 4 C) 5 D) 1

28. x = $\sqrt{17}-\sqrt{16}, y=\sqrt{15}-\sqrt{14} \Rightarrow$

A) x = y B) x > y C) x < y D) xy = 1

29. Greatest among x=$6\left(\sqrt[3]{5}\right), y=8\left(\sqrt[3]{2}\right), z=2\left(\sqrt[3]{130}\right), w=\sqrt[3]{900}$

A) x B) y C) z D) w

30. $\sqrt[6]{2},\sqrt[3]{2},\sqrt[4]{2}$ satisfies the following order

A) $\sqrt[6]{2}<\sqrt[4]{2}<\sqrt[3]{2}$ B) $\sqrt[6]{2}>\sqrt[4]{2}>\sqrt[3]{2}$

C) $\sqrt[6]{2}<\sqrt[4]{2}>\sqrt[3]{2}$ D) $\sqrt[6]{2}>\sqrt[4]{2}<\sqrt[3]{2}$

31. $\frac{1}{\sqrt{15+4\sqrt{14}}}+\frac{2}{\sqrt{12-2\sqrt{35}}}-\frac{3}{\sqrt{13-4\sqrt{10}}} =$

A) 6 B) 1 C) 0 D) -1

32. Rationalising factor of $2^{1/3} - 2^{-1/3}$ is

A) $2^{1/3} - 2^{-1/3}$ B) $2^{2/3} + 2^{-2/3}$ C) $2^{2/3} + 2^{-2/3} +1$ D) $2^{2/3} + 2^{-2/3} - 1$

33. $\frac{a-b}{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}-\frac{a+b}{\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}} =$

A) $2a^{1/3}$ B) $-2a^{1/3}$ C) $-2b^{1/3}$ D) $2b^{1/3}$

34. Rationalising factor of $\left(27^{\frac{1}{9}}-4^{\frac{1}{6}}\right)$ is

A) $27^{\frac{2}{9}}-27^{\frac{1}{9}}.4^{\frac{1}{3}}+4^{\frac{1}{2}}$ B) $3^{\frac{2}{3}}+3^{\frac{1}{3}}.2^{\frac{1}{3}}+2^{\frac{2}{3}}$

C) $3^{\frac{2}{3}}-3^{\frac{1}{3}}.2^{\frac{1}{3}}+2^{\frac{2}{3}}$ D) $27^{\frac{2}{9}}+27^{\frac{1}{9}}.2^{\frac{1}{3}}+4^{\frac{1}{2}}$

35. $\sqrt{6+\sqrt{6+\sqrt{6+.........\alpha}}}$ =

A) 6 B) 3 C) 2 D) ∞

36. $\sqrt{x}+\sqrt{x-\sqrt{1-x}}=1 \Rightarrow x=$

A) $\frac{4}{25}$ B) $\frac{12}{25}$ C) $\frac{16}{25}$ D) $\frac{2}{25}$

37. x = $\sqrt{7+4\sqrt{3}} \Rightarrow x+\frac{1}{x}=$

A) 2 B) 4 C) $2\sqrt{3}$ D) $\sqrt{3}$

38. x = $\sqrt{7}+\sqrt{3}, xy=4 \Rightarrow x^4+y^4=$

A) 400 B) 368 C) 352 D) 200

II) **MCQ with one or more than one correct answers :**

- *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options*

1. $\sqrt{\sqrt{3}-\sqrt{(4+\sqrt{5})+\sqrt{17-4\sqrt{15}}}}=$

A) i B) $\sqrt{-1}$ C) $-i^3$ D) $-i^2$

2. If $(7+4\sqrt{3})^{x^2-8}+(7-4\sqrt{3})^{x^2-8}=14$ then x =

a) ±3 b) ±7 c) $\pm\sqrt{3}$ d) $\pm\sqrt{7}$

3. If y= $\sqrt{2+\sqrt{2}}$, z= $\sqrt{2-\sqrt{2}}$ and x=y+z, then x= ...

A) $\sqrt{2+\sqrt{2}}$ B) $\sqrt{\sqrt{2}\sqrt{2+\sqrt{2}}}$ C) $\sqrt{2}\sqrt{2+\sqrt{2}}$ D) $\sqrt{4+2\sqrt{2}}$

4. $\frac{\sqrt[3]{2} \times \sqrt[4]{3}}{\sqrt[12]{27}}$ = ...

A) $2\sqrt[4]{\frac{1}{2}}$ B) $\sqrt[12]{16}$ C) $\sqrt[3]{2}$ D) $\sqrt[4]{8}$

5. If a= $\sqrt[4]{6}$, b= $\sqrt[12]{125}$, c= $\sqrt[3]{4}$ and d= $\sqrt[6]{10}$ then .... [NTSE-2013]

A) $d>c, b>c$ B) $a<b, b>c, d<c$ C) $c>a>b>d$ D) $d>c$

6. $\frac{1}{2\sqrt{1}+1\sqrt{2}}+\frac{1}{3\sqrt{2}+2\sqrt{3}}+\frac{1}{4\sqrt{3}+3\sqrt{4}}+.......+\frac{1}{25\sqrt{24}+24\sqrt{25}}$ = ... [AMTI-2008]

A) $\frac{9}{10}$ B) $\frac{4}{5}$ C) $\frac{14}{15}$ D) $\frac{7}{15}$

7. If $\frac{12}{3+\sqrt{5}+2\sqrt{2}}$ = a+ $\sqrt{5}+\sqrt{2}-\sqrt{10}$ then a= .... [IIT-1980]

A) 1 B) 2 C) 3 D) 4

8. Which of the following is greatest .... [NTSE-2013]

A) $\sqrt{11}-\sqrt{10}$ B) $\sqrt{9}-\sqrt{8}$ C) $\sqrt{7}-\sqrt{5}$ D) $\sqrt{12}-\sqrt{11}$

9. The smallest positive number from the numbers below .. [NTSE-2014]

A) 10-3$\sqrt{11}$ B) 3$\sqrt{11}$-10 C) 51-10$\sqrt{26}$ D) 18-5$\sqrt{13}$

10. if $(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}$ = 10 then x= .... [IIT-85,EAMCET-12]

A) 2 B) -2 C) $\sqrt{2}$ D) -$\sqrt{2}$

11. If $\sqrt{\frac{6+2\sqrt{3}}{33-19\sqrt{3}}}$ = a+b$\sqrt{3}$ then a+b =.......... [IIT-1975]

12. $\sqrt[3]{16+8\sqrt{5}}$ + $\sqrt[3]{16-8\sqrt{5}}$ = ... [AMTI-2010]

13. The value of $\sqrt{1+2008\sqrt{1+2009\sqrt{1+2010\sqrt{1+2011.2013}}}}$ IS ....

[AMTI-2009]

14. If x= $\frac{2ab}{b^2+1}$ and $-1\le b\le 1$, a.0 then $\frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$ = ... [IIT-1979]

A) a B) 2a C) b D) 2b

III) ***Assertion and Reasoning type questions :***

- *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of* which ***ONLY ONE*** *is correct Choose the correct option.*

A) Both A and R are correct and R is correct explanation of A

B) Both A and R are correct and R is not correct explanation of n

C) A is correct and R is incorrect

D) A is incorrect and R is correct

1. **A :** The surd $\sqrt[7]{49}$ is a pure surd

R : The surd $a.\sqrt[n]{b}$ where a = 1 is a pure surd

2. **A :** The surd $4\sqrt[3]{81}$ is in simplest form

R : The simplest form of surd $a.\sqrt[n]{b}$ where b is least positive rational number

3. **A :** $\sqrt[4]{256}$ is a biquadratic surd

R : The simples form of $\sqrt[5]{320}$ is $2\sqrt[5]{10}$

LEARNER'S TASK

BEGINNERS (Level - I)

Single Correct answer type :

1. The square root of $8+2\sqrt{15}$ is

A) $\sqrt{5}+\sqrt{3}$ B) $\sqrt{5}-\sqrt{3}$ C) $\sqrt{5}+3$ D) $5+\sqrt{3}$

2. The positive square root of $12\sqrt{5}+2\sqrt{55}$ is

A) $\sqrt[4]{3}\left(\sqrt{13}+1\right)$ B) $\sqrt[4]{5}\left(\sqrt{11}+1\right)$ C) $\sqrt[4]{2}\left(\sqrt{14}+1\right)$ D) $\sqrt[4]{5}\left(\sqrt{12}+1\right)$

3. The positive square root of $\sqrt{32}-\sqrt{24}$ is

A) $\sqrt{2}\left(\sqrt{3}-1\right)$ B) $\sqrt{2}\left(\sqrt{2}-1\right)$ C) $\sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)$ D) $\sqrt[4]{2}\left(\sqrt{2}-\sqrt{3}\right)$

4. The positive square root of the $5\sqrt{6}+12$ is

A) $\sqrt{3}+\sqrt{2}$ B) $\sqrt[4]{6}\left(\sqrt{3}+\sqrt{2}\right)$ C) $\sqrt[4]{2}\left(\sqrt{3}+\sqrt{2}\right)$ D) $\sqrt[4]{3}\left(\sqrt{3}+\sqrt{2}\right)$

5. The positive square root of the $11\sqrt{7}+28$ is

A) $\sqrt[4]{7}\left(\sqrt{5}+2\right)$ B) $\sqrt[4]{7}\left(\sqrt{7}+2\right)$ C) $\sqrt[4]{7}\left(\sqrt{3}+2\right)$ D) $\sqrt[4]{7}\left(\sqrt{6}+2\right)$

6. The positive square root of the $7\sqrt{3}-12$ is

A) $2-\sqrt{3}$ B) $\sqrt{3}\left(2-\sqrt{3}\right)$ C) $\sqrt[4]{3}\left(2-\sqrt{3}\right)$ D) $\sqrt[4]{3}\left(3-\sqrt{2}\right)$

7. The positive square root of the $14\sqrt{5}-30$ is

A) $\sqrt[4]{2}\left(\sqrt{3}-2\right)$ B) $\sqrt[4]{5}\left(\sqrt{3}-5\right)$ C) $\sqrt[4]{5}\left(\sqrt{3}-\sqrt{2}\right)$ D)None

8. $\sqrt{4+\sqrt{5}+\sqrt{17-4\sqrt{15}}}$

A) $\sqrt{2}+1$ B) $\sqrt{3}+2$ C) $\sqrt{3}+1$ D) $\sqrt{2}-1$

9. $\sqrt{12-2\sqrt{20}}$

A) $\sqrt{10}-2$ B) $\sqrt{10}-\sqrt{2}$ C) $\sqrt{10}+\sqrt{2}$ D) $\sqrt{10}+2$

10. $\sqrt{\sqrt{3}-\sqrt{4+\sqrt{5}+\sqrt{17-4\sqrt{15}}}}$ =

A) 1 B) -1 C) i D) -i

11. $\sqrt{28+10\sqrt{3}}-\sqrt{7+4\sqrt{3}}$ =

A) 1 B) 2 C) 3 D) 4

12. $\sqrt{7-3\sqrt{5}}=$

A) $\frac{3-\sqrt{5}}{\sqrt{2}}$ B) $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{2}}$ C) $\frac{2-\sqrt{3}}{\sqrt{2}}$ D) $\frac{3-\sqrt{2}}{\sqrt{2}}$

13. $\sqrt{6-\sqrt{7}+\sqrt{27+4\sqrt{35}}}=$

A) $\sqrt{5}$ -1 B) $\sqrt{5}$ - $\sqrt{2}$ C) $\sqrt{5}$ +1 D) 2 + $\sqrt{5}$

14. $\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}=$

A) 2 B) 3 C) $\sqrt{5}$ D) $\sqrt{3}$

15. $\sqrt[4]{193+132\sqrt{2}}=$

A) 2 + $\sqrt{3}$ B) 5 + $\sqrt{3}$ C) $3+\sqrt{2}$ D) $7+\sqrt{2}$

16. $\sqrt{\frac{15}{2}+2\sqrt{3}+2\sqrt{6}+4\sqrt{2}}=$

A) $2+\sqrt{2}+\sqrt{\frac{3}{2}}$ B) $\sqrt{3}+\sqrt{2}+\sqrt{\frac{5}{2}}$ C) $\sqrt{5}+\sqrt{2}+\frac{1}{\sqrt{2}}$ D) $\sqrt{5}+\sqrt{2}-1$

17. $\sqrt{x^2-\sqrt{x^4-4}}=$

A) $\dfrac{\sqrt{x+2}-\sqrt{x-2}}{\sqrt{2}}$ B) $\dfrac{\sqrt{x^2+2}-\sqrt{x^2-2}}{\sqrt{2}}$

C) $\dfrac{\sqrt{x^4+2}+\sqrt{x^4-2}}{2}$ D) $\dfrac{\sqrt{x^4+2}-\sqrt{x^4-2}}{2}$

18. $\sqrt{a+3b+4+4\sqrt{a}-4\sqrt{3b}-2\sqrt{3ab}}=$

A) $2+\sqrt{a}-\sqrt{3b}$ B) $\sqrt{2a}-\sqrt{b}+2$

C) $\sqrt{a}+2-\sqrt{b}$ D) $\sqrt{a}-2-\sqrt{b}$

19. $\sqrt[3]{45+29\sqrt{2}}=$

A) $3+\sqrt{2}$ B) $2+\sqrt{2}$ C) $3+2\sqrt{2}$ D) $1+2\sqrt{2}$

20. $\sqrt[3]{14\sqrt{5}-18\sqrt{3}}=$

A) $\sqrt{2}-\sqrt{6}$ B) $\sqrt{3}-\sqrt{5}$ C) $\sqrt{5}-\sqrt{3}$ D) $\sqrt{6}-\sqrt{2}$

21. If $7+\sqrt{15}>10$ then

A) $7-\sqrt{15}>3.4$ B) $7-\sqrt{15}<3.4$

C) $7-\sqrt{15}=3.4$ D) Cannot be determined

22. $\dfrac{1}{1+\sqrt{2}+\sqrt{3}}=$

A) $\dfrac{\sqrt{2}(1-\sqrt{2}-\sqrt{3})}{4}$ B) $\dfrac{\sqrt{2}(1+\sqrt{2}-\sqrt{3})}{4}$

C) $\dfrac{\sqrt{2}(1+\sqrt{2}-\sqrt{3})}{2}$ D) $\dfrac{\sqrt{2}(1-\sqrt{2}-\sqrt{3})}{2}$

23. The rationalizing factor of $\left(\sqrt[8]{3}-\sqrt[8]{2}\right)\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\left(\sqrt{3}+\sqrt{2}\right)$ is

A) $\left(\sqrt[8]{3}+\sqrt[8]{2}\right)$ B) $\left(\sqrt[8]{3}-\sqrt[8]{2}\right)$ C) $\left(\sqrt[8]{4}-\sqrt[8]{2}\right)$ D) $\sqrt{3}-\sqrt{2}$

24. $\dfrac{1}{\sqrt{2}+1}+\dfrac{1}{\sqrt{3}+\sqrt{2}}+\dfrac{1}{\sqrt{4}+\sqrt{3}}+....+\dfrac{1}{\sqrt{100}+\sqrt{99}}=$

A) 1 B) 2 C) 9 D) 10

9. x = $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \Rightarrow$ $x^2 - 8x + 1 =$

A) 5 B) 3 C) 4 D) 0

10. $\sqrt[4]{7-4\sqrt{3}}.\sqrt{2+\sqrt{3}} =$

A) 3 B) 1 C) 2 D) 4

ACHIEVERS (Level - II)

Solve the following

1. Find the value of $\sqrt{\frac{9}{2}-2\sqrt{5}}+\sqrt{\frac{9}{2}+2\sqrt{5}}$

2. Find the value of $\sqrt{\frac{23}{2}-4\sqrt{7}}+\sqrt{\frac{23}{2}+4\sqrt{7}}$

3. with out extraacting the roots ,determine which is greater

$\sqrt{11}-\sqrt{5} or \sqrt{19}-\sqrt{13}$?

4. with out extracting the roots ,determine which is greater

$\sqrt{12}-\sqrt{7}$ or $\sqrt{13}-\sqrt{8}$?

5. Find the square root of a) $\sqrt{12+4\sqrt{5}}$ b) $\sqrt{11-4\sqrt{7}}$.

6. Find the square root of i) $\sqrt{11-4\sqrt{7}}$ ii) $9-4\sqrt{5}$

7. It x =$2+2^{2/3}+2^{-2/3}$ then find x^3-6x^2+9x-2

8. Find the value of $\sqrt{12+\sqrt{12+\sqrt{12+................\infty}}}$

9. Find the value of $\sqrt{2+\sqrt{2+\sqrt{2+................\infty}}}$

10. $(5+2\sqrt{12})^{x^2-8}+(5-2\sqrt{12})^{x^2-8}=10$ then find x.

11. Prove $\frac{(a+2)^2(a-3)+(a^2-4)\sqrt{a^2-9}}{(a-2)^2(a+3)+(a^2-4)\sqrt{a^2-9}}=\frac{(a+2)(\sqrt{a-3})}{(a-2)(\sqrt{a+3})}$

12. $\sqrt{6-\sqrt{6-\sqrt{6-..........\ ..\infty}}}$

13. Find the square root of

i) $8+2\sqrt{15}$ ii) $5+2\sqrt{6}$ iii) $9-4\sqrt{5}$

14. Which is greater $x=\sqrt{11}-\sqrt{10}$; $y=\sqrt{12}-\sqrt{11}$?

15. If $x=\frac{1}{2}\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)$ then find $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}$

EXPLORERS (Level - III)

I) MCQ's with More than one answer type :

♦ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options*

1. Which of the following have conjugate surds?

A) $\sqrt{5}-\sqrt{3}$ B) $\sqrt{5}+\sqrt{3}$ C) $5+\sqrt{3}$ D) $3-\sqrt{5}$

2. Which of the following are in ascending order ?

A) $\sqrt[4]{5}, \sqrt[3]{4}, \sqrt[4]{7}$ B) $\sqrt[3]{4}, \sqrt[4]{5}, \sqrt[4]{7}$

C) $\sqrt[3]{4}, \sqrt[4]{7}, \sqrt[3]{8}$ D) done

3. $\sqrt[3]{20+14\sqrt{2}}-\sqrt[3]{-20+14\sqrt{2}}=$

A) 2 B) $\sqrt[3]{64}$ C) 4 D) $2\sqrt{2}$

II) Comprehension questions

♦ *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE is** correct. Choose the correct option.*

The product of two surds is rational number then each of two surds is called Ratinalising factor of other

The R.F. of $\sqrt[m]{a^n}$ is $\sqrt[m]{a^{m-n}}$ since m>n

The R.F. of $(\sqrt{a}\pm\sqrt{b})$ is $(\sqrt{a}\mp\sqrt{b})$

The R.F of $(\sqrt[3]{a}\pm\sqrt[3]{b})$ is $(\sqrt[3]{a^2}\mp\sqrt[3]{ab}+\sqrt[3]{b^2})$

The R.F's of monomial surds are infinite.

1. If $x = 5 + 2\sqrt{6}$ then $x + \frac{1}{x}$ = ...

A) 12 B) 10 C) 24 D) $10\sqrt{6}$

2. The R.F of $\sqrt[3]{4} + \sqrt[3]{2}$ +1 is ....

A) $-1+\sqrt[3]{2}$ B) $1+\sqrt[3]{2}$ C) $\sqrt[3]{4}+2$ D) $\sqrt[3]{4}-2$

3. The R.F of $2^{1/3} + 2^{-1/3}$ is ....

A) $2^{2/3} + 1 + 2^{-2/3}$ B) $2^{1/3} - 1 + 2^{1/3}$ C) $2^{2/3} - 1 + 2^{-2/3}$ D) $2^{1/3} + 1 + 2^{-1/3}$

KEY

ΦΦ TEACHING TASK :

I)) 1.A 2.B 3.C 4.D 5.A 6.C 7.B 8.B 9.B 10.A
11.C 12.A 13.B 14.A 15.C 16.D 17.A 18.D
19.B 20.A 21.C 22.C 23.C 24.B 25.C 26.B 27.D 28.C
29.A 30.A 31.C 32.C 33.C 34.B 35.B 36.C 37.B 38.B

II) 1.A,B,C 2.A,D 3.C,D 4.B,C 5.3 6.2 7.1 8.3
9.3 10.1,2,3,4 11.8 12.2 13.2009 14.3

III) 1.A 2.A 3.B

ΦΦ LEARNER'STASK :

☐ **BEGINNERS :**

I) 1.A 2.B 3.D 4.B 5.B 6.C 7.D 8.C 9.B 10.B
11.C 12.A 13.D 14.A 15.C 16.A 17.B 18.A 19.A 20.C 21.B
22..B 23.A 24..C 25.D 26..B

☐ **EXPLORERS:**

I) 1.C,D 2.A,C 3.B,C

II) 1.B 2.A 3.C