JEE - Mains Level questions

Teaching Task

1] The correct option is "Both 5 and 13"

$9^{2n} - 4^{2n} \Rightarrow [9^2]^n - [4^2]^n$

$= 81^n - 16^n$

This is in the form of $a^n - b^n$

$a^n - b^n$ is always divisible by $(a-b)$

$81 - 16 = 65$

$\therefore$ So factors are both 5 and 13.

2] 'a' is a perfect cube.

Any natural number can be represented as $3P$, $3P+1$, $3P+2$.

For $N = 3P$

$\Rightarrow N^3 = (3P)^3 = 27P^3 = 9(3P^3)$

when N^3 is divisible by 9, remainder(a) = 0

For $N = 3P+1$

$\Rightarrow N^3 = (3P+1)^3 \quad \{\because (a+b)^3 = a^3+b^3+3ab(a+b)\}$

$\Rightarrow N^3 = (3P)^3 + 1^3 + 3 \times 3P \times 1(3P+1)$

$N^3 = 27P^3 + 1 + 9P(3P+1)$

$N^3 = 27P^3 + 1 + 27P^2 + 9P$

$N^3 = 27P^3 + 27P^2 + 9P + 1$

$N^3 = 9P(3P^2 + 3P + 1) + 1$

when N^3 is divided by 9, remainder(a) = 1

For $N = 3P+2$

$N^3 = (3P+2)^3$

$N^3 = (3P)^3 + 2^3 + 3 \times 3P \times 2(3P+2)$

$N^3 = 27P^3 + 8 + 18P(3P+2)$

$N^3 = 27P^3 + 8 + 54P^2 + 36P$

$N^3 = 27P^3 + 54P^2 + 36P + 8$

$N^3 = 9P(3P^2 + 6P + 4) + 8$

when N^3 is divisible by 9, remainder = 8

$\therefore$ 0 is a perfect square and perfect cube.

1 is a perfect square and perfect cube.

8 is a perfect cube only

$\therefore$ "a is a perfect cube."

3] H C F of 4052 and 12576 is '4'.

By continued division method.

4052	12576	3
	-12156	
420	4052	9
	-3780	
272	420	1
	-272	
148	272	1
	148	
124	148	1
	124	
24	124	5
	120	
H.C.F ← 4	24	6
	24	
remainder ← 0		

∴ H C F is '4'.

4] H.C.F of 210 and 55 is in the form of $210 \times 5 + 55x$.

Let us find the H.C.F of 210 and 55 by continued division method.

$$
\begin{array}{r|l|l}
55 & 210 & 3 \\
 & 165 & \\
\hline
45 & 55 & 1 \\
 & 45 & \\
\hline
10 & 45 & 4 \\
 & 40 & \\
\hline
5 & 10 & 2 \\
 & 10 & \\
\hline
 & 0 &
\end{array}
$$

$\therefore$ H.C.F of 210 and 55 is '<u>5</u>'.

Now

$$210 \times 5 + 55x = 5$$

$$1050 + 55x = 5$$

$$55x = 5 - 1050$$

$$55x = -1045$$

$$x = \frac{-\cancel{1045}^{19}}{\cancel{55}}$$

$$\boxed{x = -19}$$

5] H.C.F of 65 and 117 is in the form of $65x + 117y$. $(x, y) = ?$

Let us find H.C.F of 65 and 117.

$$65 \overline{)\,117\,}\, 1$$
$$\underline{65}$$
$$52 \overline{)\,65\,}\, 1$$
$$\underline{52}$$
$$13 \overline{)\,52\,}\, 4$$
$$\underline{52}$$
$$0$$

$\therefore$ H.C.F of 65 and 117 = 13

Now Given the expression $65x + 117y$.

we can write the H.C.F of 65 and 117.

$$13 = 65 - 52 \times 1$$
$$\Rightarrow 13 = 65 - [117 - 65 \times 1]$$
$$13 = 65 - 117 + 65 \times 1$$
$$13 = 65 \times 2 - 117 \times 1$$
$$13 = 65 \times 2 + 117 \times (-1)$$

Compare to $65x + 117y$

$$x = 2, \; y = -1$$

#

6] Deducting the remainders from numbers we get,

$398 - 7 = 391$

$436 - 11 = 425$

$542 - 15 = 527$

H.C.F of these new numbers is the largest possible number that divides 398, 436, 542 leaving respective remainders

$391 = 17 \times 23$

$425 = 17 \times 25$

$527 = 17 \times 31$

$\therefore$ H.C.F of 391, 425, 527 is '17' #

7] 44 boys and 32 girls

Find H.C.F of 44 and 32.

$$
\begin{array}{r|l|l}
32 & 44 & 1 \\
 & 32 & \\
\hline
12 & 32 & 2 \\
 & 24 & \\
\hline
8 & 12 & 1 \\
 & 8 & \\
\hline
4 & 8 & 2 \\
 & 8 & \\
\hline
 & 0 &
\end{array}
$$

$\therefore$ H.C.F is $= 4$

So there will be 4 rows each

For boys $= \dfrac{\cancel{44}^{11}}{\cancel{4}}$

$= 11$ rows

For girls $= \dfrac{\cancel{32}\ 8}{\cancel{4}_1}$

$= 8$ rows

$\therefore$ Total rows $= 11 + 8$

$= 19.$

#

8] Reminders are 8 and 4. for 1659 and 2036

so 1659 − 8 = 1651 and 2036 − 4 = 2032

Required largest number is H.C.F of 1651, 2032

1651) 2032 (1
1651
―――――
381) 1651 (4
1524
―――――
127) 381 (3
381
―――――
(0)

∴ largest number = 127.

9) Find L.C.M of 24, 15, 36

3 | 24, 15, 36
2 | 8, 5, 12
2 | 4, 5, 6
2 | 2, 5, 3
5 | 1, 5, 3
  | 1, 1, 3

$= 3 \times 2 \times 2 \times 2 \times 5 \times 3$

$= 360$ (L.C.M)

Now greatest 6-digit number

$= 999999$

Divide with 360

$= \dfrac{999999}{360} = 2777.77$

Now multiply 2777 with L.C.M 360.

$= 2777 \times 360 = 999720$

checking out for the numbers

$999720/24 = 41655$ ✓

$999720/15 = 66648$ ✓

$999720/36 = 27770$ ✓

∴ The greatest number = 999720 #

10] We find the L.C.M of 144, 216

$\Rightarrow 2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 1 \times 3$

$= 432$

$$\begin{array}{r|l} 2 & 144, 216 \\ \hline 2 & 72, 108 \\ \hline 3 & 36, 54 \\ \hline 3 & 12, 18 \\ \hline 2 & 4, 6 \\ \hline 2 & 2, 3 \\ \hline & 1, 3 \end{array}$$

Now divide with 144 and 216

$\Rightarrow \dfrac{\cancel{432}^{\,3}}{\cancel{144}} = 3$

$\Rightarrow \dfrac{\cancel{432}^{\,2}}{\cancel{216}} = 2$

$\therefore$ So $2+3=5$ containers #

JEE - Advanced Level Questions

11) $7 \times 11 + 13 + 13$ is a composite and even number

$7 * 11 * 13 + 13$

$13(7 \times 11 + 1) = 13[77 + 1] = 13[78]$

Hence its a composite number.

$13 \times 78 = 1014$ → and also even number.

12) Given $a = 2^3 \times 3^2 \times 5^2 \times 7$

$= 8 \times 9 \times 25 \times 7$

$= 12,600$

How many zero[0]s are here that is the answer

∴ so Given number has '2' consecutive zeroes

13) Here, number $= 12^n$ wher n is any natural number

Now $12^n = [2^2 \times 3]^n$

Now, For 12^n to end with 0, it should have 2 as well as 5 in it's prime factors to end with '0', Also end with 5, it requires atleast a single multiple of 5 in it's prime factors, so 12^n cannot end with the digit '0' or '5'.

16] Ans- D

solution- 65 and 117 HCF is 13

13 we can write as

$$13 = 65 - 52 \times 1$$

$$13 = 65 - (117 - 65 \times 1)$$

$$13 = 65 \times 2 + 117 \times (-1)$$

$\therefore$ $65m + 117n \Rightarrow m = 2, n = -1$

17] Ans - A

solution $\rightarrow$ 4^n, n is natural number

$$4^1 = 4$$

$$4^2 = 16$$

$$4^3 = 64$$

$$4^4 = 256$$

$\therefore$ end with 4, 6 Not with '0'.

18] Ans - 20

solution - Numbers are in ratio

3 : 4 : 5.

Let the numbers 3a, 4a, 5a

i) $\frac{3a}{4a} = \frac{3}{4}$

$\frac{L.C.M_1}{3^{rd} \text{ number}} = \frac{12a}{5a} = \frac{12}{5}$

Final LCM = 60a

60a = 1200 (∵ Given)

So $a = \frac{\cancel{1200}^{20}}{\cancel{60}}$

Now numbers are 3×20, 4×20, 5×20

= 60, 80, 100

H.C.F ⇒ $\frac{60}{80} = \frac{3}{4}$

⇒ 20

Final H.C.F $= \frac{20}{100} = \frac{1}{5}$

So $\boxed{H.C.F = 20}$

19] Ans - 30 days

solution -

circumference of field = 360 km

cyclist 1 can cover 48 km per 1 day

$\therefore$ For covering 360 km $= \frac{360}{48} = 7.5$ days

cyclist 2 can cover 60 km per 1 day

$\therefore$ For 360 km $= \frac{360}{60} = 6$ days

cyclist 3 can cover 72 km per day

$\therefore$ For 360 km $= \frac{360}{72} = 5$ days

7.5 days $= 7.5 \times 24 = 180$ hours

6 days $= 6 \times 24 = 144$ hours

5 days $= 5 \times 24 = 120$ hours

L.C.M of 180, 144, 120 is 720

Now $= \frac{720}{24} = 30$ days

$\therefore$ The cyclist will meet after 30 days

21] i] Ans - 13

solution :-

odd number are $n = 1, 3, 5, 7, 9, \dots$

Given $3^{2n} + 2^{2n}$

If $n = 1 \Rightarrow 3^2 + 2^2 = 13$

if $n = 3 \Rightarrow 3^6 + 2^6 = 729 + 64$

$= 793$

if $n = 5 \Rightarrow 3^{10} + 2^{10} = 60,073$

similarly all numbers are

13, 793, 60,073 is

divisible by 13.

---

ii] Ans - 1

solution :-

Given $2^x \times 5^x$

if $x = 1 \Rightarrow 2^2 \times 5$

$= 4 \times 5 = 20$

$\therefore$ 20 divisible by '5'.

21] iii] Ans $\rightarrow xy$

$\therefore$ L.C.M of two co-primes of x and y is their product

21] iv] Ans - 1

$\therefore$ co-primes are no common factors except '1'

Student task

1] **Answer – D**

Solution : Greatest natural number we can't determine.

2] **Answer – B**

Solution : whole numbers start with '0'.

3] **Answer – A**

Solution : Additive identity is '0'

Ex) $2 + (-2) = 0$ (or) $2 + 0 = 2$

4] **Answer – B**

Solution : According of Fundemental theorms of arithemetic.

5] **Answer – A**

Solution : two consecutive odd integers

let 3, 5

$3 \,\underline{|\, 3}$
$\quad 1$

$5 \,\underline{|\, 5}$
$\quad 1$

$3 = 3 \times 1$

$5 = 5 \times 1$

$\therefore$ G.C.D of 3, 5 is $= 1$.

6] Answer - A

solution : co-primes having no common factor. so G.C.D is 1.

7] Answer - A

solution : Let 3,5 any natural numbers

3,5 of H.C.F is = 1

3,5 of L.C.M is = 15

$\therefore$ Product of 3,5 = 15 #

8] Answer - B

$\therefore$ L.C.M fraction = $\dfrac{\text{L.C.M of numerator}}{\text{H.C.F of denominator}}$

9] Answer - B

$\therefore$ H.C.F of fraction = $\dfrac{\text{H.C.F of numerator}}{\text{L.C.M of denominator}}$

10] Answer - D

solution : L.C.M of fraction

$= \dfrac{\text{L.C.M of numerators}}{\text{H.C.F of denominators}}$

Given $\left(\frac{2}{3}, \frac{5}{2}, \frac{1}{4}\right) \Rightarrow$ L.C.M of 2, 5, 1 = 10

H.C.F of 3, 2, 4 = 1

$\therefore$ L.C.M $= \frac{10}{1} = 10$ #

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1] **Answer:** A

Solution:

G.C.D $(a,b)=1 \Rightarrow$ G.C.D $(an,b)=1$ then we have

GCD (a,b) = gcd(an,b) = GCD(b,an) = GCD$(bn)=1$

Given GCD $(a,b)=1$, their exist integers

x,y such that $ax+by=1$

$\therefore$ GCD $[\cancel{x}a, an+b]=1$

2] **Answer** - D

Solution: options A, B, C are not having any common factor.

3] Answer – A

Solution :- ∴ $GCD(a, b) = GCD(a, b-a)$

4] Answer – B

Solution :- L.C.M of two numbers = 1200

So, 500 Can't divide with 1200.

$\boxed{L.C.M \times H.C.F = \text{Product of two numbers}}$

5] Answer – B

Solution :- Given L.C.M = 225, H.C.F = 15

$$x = \frac{LCM}{H.C.F} = 15$$

So, x can be written as product of 3, 5 and also 1, 15 which are relatively Prime

Thus, there exists two such pairs where L.C.M = 225 and H.C.F = 15.

6] Answer – D

solution : $2^{100} - 1$

$\Rightarrow (2^{20})^5 - 1^5$

since, $a^n - b^n$ is divisible by $a-b$ therefor,

$2^{100} - 1$ is divisible by $2^{20} - 1$. #

$\Rightarrow 2^{120} - 1 \Rightarrow (2^{20})^6 - 1^6$

Since $a^n - b^n$ is divisible by $a-b$, so

$2^{120} - 1$ is divisible by $2^{20} - 1$.

$\therefore$ Highest common factor $= 2^{20} - 1$ #

7] Answer – D

solution : Given $4m+1$, $4m+3$, $4m-2$

~~Let~~ m is any inter.

$m=1 \Rightarrow 4(1)+1 = 5$ (odd integer)

$m=1 \Rightarrow 4(1)+3 = 7$ (odd integer)

$m=1 \Rightarrow 4\cancel{m}(1)-2 = 2$ (Not odd number)

$\therefore$ Positive odd integer is in the form of $4m+1$ and $4m+3$

Answer - A

8] Solution: we have to find the H.C.F of

420 , 130

$$
\begin{array}{l}
130)\,420\,(3 \\
\quad\ \ 390 \\
\quad\ \ \overline{\ \ 30)\,130\,(4} \\
\qquad\qquad 120 \\
\text{H.C.F} \leftarrow \overline{10)\,30\,(3} \\
\qquad\qquad\quad 30 \\
\qquad\qquad\quad \overline{(0)}
\end{array}
$$

$\therefore$ solution is $= 10$ #

9] Answer - C

Solution: H.C.F of 616 and 32 is '8'.

10] Answer - A

Solution: Given $3240 = 2^3 \times 3^4 \times 5$

$3600 = 2^4 \times 3^2 \times 5^2$

$\therefore$ H.C.F $= 2^3 \times 3^2$; L.C.M $= 2^4 \times 3^5 \times 5^2 \times 7^2$

H.C.F = Product of lowest power of common factor $\therefore 2^2 \times 3^2$

L.CM = Product of higest power of common Prime factors $\therefore 3^2 \times 7^2$

$\therefore$ Hence 3rd number $= 2^2 \times 3^2 \times 7^2 \times 3^3$

$= 2^2 \times 3^5 \times 7^2$ #

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11) **Answer - C**

Solution: Let's look at product of 6.

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90

The pairs whose sums equal to 90

$6 + 84 = 90$ $\quad$ $30 + 60 = 90$

$12 + 78 = 90$ $\quad$ $36 + 54 = 90$

$18 + 72 = 90$ $\quad$ $48 + 42 = 90$

$24 + 66 = 90$

#

12) **Answer - A, B, C**

Solution: $9m, 9m+1, 9m+8, 9m+7$

Let the cube numbers we take

$2^3 = 8,\ 3^3 = 27,\ 4^3 = 64,\ 5^3 = 125$

$8 \Rightarrow 9 \times 0 + 8$ {∴ m any positive integer}

$27 \rightarrow 9 \times 3 + 0 = 9m$ [m = 3 any positive integer]

$64 \rightarrow 9 \times 7 + 1 \Rightarrow 9m + 1$ {m = 7 any positive integer]

$125 \rightarrow 9 \times 13 + 8 \Rightarrow 9m + 8$ [m = 13 any positive integer]

∴ we can't write in the form of $9m + 7$ #

13] Answer - B

solution:

statement (i): H.C.F of 657, 963 is 9

$9 = 657x - (963 \times 15)$

$9 = 657 \times 22 - (963 \times 15)$

$x = 22.$

statement - (ii): every positive integer is in the form of $2m$, where every positive odd integer is in the form of $2m+1$.

14] Answer: C

solution: i) H.C.F of 210 and ~~55~~ 55 is expressed in $210 \times 5 + 55y$

$\therefore$ H.C.F of 210 and 55 $= 5$

$5 = 210 \times 5 + 55y$

$1\cancel{55}y = \cancel{-1045}\ 19$

$\boxed{y = -19}$

statement - ii: H.C.F of $65, 117 = 13$

$13 = 65 - 52 \times 1$

$13 = 65 - (117 - 65 \times 1)$

$13 = 65 - 117 + 65 \times 1$

$13 = 65 \times 2 + 117 \times -1$

$x = 2,\ y = -1 \quad (x, y) = (2, -1)$ #

15] Answer = B

solution: H.C.F of 96 and 404

96) 404 (4
384

20) 96 (4
80

16) 20 (1
16

H.C.F ← 4) 16 (4
16

0

16] Answer → D

solution: L.C.M of 96 and 404

L.C.M × H.C.F = product of 96 and 404

$$L.C.M \times 4 = 96 \times 404$$

$$L.C.M = \frac{96 \times \cancel{404}^{101}}{\cancel{4}_{1}} = 96 \times 101$$

$$= 9676. \ \#$$

17] 32760 Answer = D

solution:

$$2^3 \times 3^6 \times 5^1 \times 1 = \text{Factors}$$
#

2	32760
2	16380
2	8190
5	4095
3	819
3	273
3	81
3	27
3	9
3	3
	1

18]

Answer - B

Solution: Given 4^n; $n \in N$

$n = 1 \Rightarrow 4^1 = 4$

$n = 2 \Rightarrow 4^2 = 16$

$n = 3 \Rightarrow 4^3 = 64$

$n = 4 \Rightarrow 4^4 = 256$

$\therefore$ never ends with '5'.

19]

Answer - 657

Solution: formula as follows

$$H.C.F(306, x) \times L.C.M(306, x) = 306 \times x \quad \text{--- (1)}$$

Given that $H.C.F(306, x) = 9$ and $LCM(306, x) = 22338$, we can plug this values in (1)

$$9 \times 22338 = 306 \times x$$

$$x = \frac{22338 \times 9}{306} = 657 \#$$

20]

Answer - 420

Solution: Given 12, 15, 21

3	12, 15, 21
2	4, 5, 7
5	2, 5, 7
7	2, 1, 7
	2, 1, 1

$$L.C.M = 3 \times 2 \times 5 \times 7 \times 2 \times 1$$

$$= 420. \#$$

21]

i] Answer - t

∴ every positive even integer is in the form of $2m$.

ii] Answer - p

∴ positive integer square is $3m$, $3m+1$

Ex: $2^2 = 4 = 3(1)+1 = 3m+1$

$3^2 = 9 = 3(3) = 3m$

$4^2 = 16 = 3(5)+1 = 3m+1$

iii] Answer - q

∴ positive odd integer should be $= 2m+1$ (or) $2n+1$

iv] Answer - r

cube of positive integer $= 9m, 9m+1, 9m+8$

22] i] Answer - s

solution: Given 140

$$\begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$\Rightarrow 2^2 \times 5 \times 7$.

ii] Answer: p

solution: Given 3825

$$\begin{array}{r|l} 5 & 3825 \\ \hline 5 & 765 \\ \hline 3 & 153 \\ \hline 3 & 51 \\ \hline & 17 \end{array}$$

$\Rightarrow 5^2 \times 3^2 \times 17$ #

iii] Answer - t

solution: Given 4095

$$\begin{array}{r|l} 5 & 4095 \\ \hline 3 & 819 \\ \hline 3 & 273 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$\Rightarrow 3^6 \times 5$ #

iv] Answer - q

solution: Given 1224

$$\begin{array}{r|l} 2 & 1224 \\ \hline 2 & 612 \\ \hline 2 & 306 \\ \hline 3 & 153 \\ \hline 3 & 51 \\ \hline & 17 \end{array}$$

$\Rightarrow 2^3 \times 3^2 \times 17$

#