CYCLICITY AND CONGRUENCE (01)

MODULO

Class: VIII, Mathematics (F^+)

SOLUTIONS

TEACHING TASK

01. $64^{99} = \underline{\quad\quad}\boxed{4}$

Since $4^n = \begin{cases} 4 & \text{if } n \text{ is odd} \\ 6 & \text{if } n \text{ is even} \end{cases}$

Ans: D

02. $64^{98} = \underline{\quad\quad}\boxed{6}$

98 is even

Ans: D

03. $(246)^{1009} = \underline{\quad\quad}\boxed{6}$

Ans: 6

04. $(1293)^{255} = (1293)^{4\times 63+3} = \underline{\quad\quad}\boxed{7}$

$= (1293)^{4n+3} = \underline{\quad\quad}\boxed{7}$

Ans: B

05. $(62)^{54} \times (896)^{98} = (62)^{4\times 13+2} \times (896)^{98}$

$\underline{\quad\quad}\boxed{4} \times \underline{\quad\quad}\boxed{6} = \underline{\quad\quad}\boxed{4}$

Ans: A

06. $(292)^{1000} \times (626)^{96}$

(292)

06 $(292)^{1000} \times (626)^{96}$

$= (292)^{4\times 250} \times (626)^{96}$

$= \underline{\qquad\qquad}\boxed{6} \times \underline{\qquad\qquad}\boxed{6} = \underline{\qquad\qquad}\boxed{6}$

Ans: D

07. $4^{5022} = 4^{\cancel{4\times1255}+2} = \underline{\qquad\qquad}\boxed{6}$ Since 5022 is even

08 $5^{1000} = \underline{\qquad\qquad}\boxed{5}$ Ans: D

09 $4^{2023} = \underline{\qquad\qquad}\boxed{4}$ Since 2023 is odd

10. $9^{2022} \times 9^{2023}$

$= 9^{4045} = \underline{\qquad\qquad}\boxed{9}$ Since 4045 is odd

11. Statement I: $17 \equiv 12 \pmod 5$

$\Rightarrow 17-12 = 5 \times 1$ (True)

Statement II: $12 \equiv 17$ (mod)

$\Rightarrow 12-17 = -5 = 5\times(-1)$ True)

Ans: A, B

12 Conceptual A, B, C, D are true

13. Statement I: $2^{2024} = 2^{4k+4} = \underline{\qquad\qquad}\boxed{6}$

(True)

Statement II: Conceptual (True)

Ans: A

14. Statement I:

We have $8 \equiv 1 \pmod 7$

$\Rightarrow 8^{2023} \equiv 1^{2023} \pmod 7$

$\Rightarrow 8^{2023} \equiv 1 \pmod 7$ (True)

Statement II: Conceptual (True) Ans: A

15. $4^{2023} \times 4^{2022}$

$= 4^{4045} = \underline{\hspace{3cm}\boxed{4}}$ since 4045 is odd

16. $4^{1999} + 4^{2000}$

$= \underline{\hspace{2cm}\boxed{4}} + \underline{\hspace{3cm}\boxed{6}}$

$= \underline{\hspace{4cm}\boxed{0}}$ Ans: A

17. Conceptual. Ans: D

18 Conceptual Ans: D

19. $33 \equiv x \pmod{10}$

$\Rightarrow 33 \equiv 3 \pmod{10}$

$\Rightarrow 33 - 3 = 30$ divided by 10 Ans: 3

20. $\cancel{100 \equiv 0 \pmod{14}}$

$\cancel{2^{100} \equiv 0^{100} \pmod 4}$ Re

$\cancel{\Rightarrow 2^{100} \equiv 1 \pmod 4}$

20 We know $2^1 = 2 \pmod{15}$

$2^2 = 4 \pmod{15}$

$2^3 = 8 \pmod{15}$

$2^4 = 16 \pmod{15}$

so, the cycle repeats every 4 terms

: 2, 4, 8, 1.

$2^{100} =$ $=$

Now $\frac{100}{4} = 25$, remainder 0

Since, the remainder is 0, we take the 4th term in the cycle, ~~18~~ which is 1

21 a) $1^{2023} =$ ______ $\boxed{1}$

b) $5^{2024} =$ ______ $\boxed{5}$

c) $6^{2025} =$ ______ $\boxed{6}$

d) $10^{2025} =$ ______ $\boxed{0}$ Ans: s, p, q, r

22 a) $8 \equiv 1 \pmod 7$

$8^{50} = 1^{50} \pmod 7$

$8^{50} = 1 \pmod 7$ $\therefore$ Remainder = 1

b) 3

b) $3^1 = 3 \pmod{13}$

$3^2 = 9 \pmod{13}$

$3^3 = 1 \pmod{13}$

here cycle length is 3

$3344 = 2 \pmod{13}$

$\therefore 3^{3344} = 3^2 \equiv 9 \pmod{13}$

Hence remainder 9

c) $12 \equiv -1 \pmod{13}$

$12^{200} \equiv (-1)^{200} \pmod{13}$

$12^{200} \equiv 1 \pmod{13}$ $\therefore$ remainder $= 1$

d) $5\overline{)17}(3$

15

$\boxed{2} \longrightarrow$ Remainder $= 2$

Ans: S, P, S, Q

Learner's Task

CUQ's

01	Conceptual	Ans: A
02	Conceptual	Ans: C
03	Conceptual	Ans: C
04.	Conceptual	Ans: A
05	Conceptual	Ans: B
06	Conceptual	Ans: D
07	Conceptual	Ans: B

08	Conceptual	Ans: D
09	Conceptual	Ans: D
10	Conceptual	Ans: A

JEE MAINS LEVEL

01. $1453^{71} = 1453^{4\times17+3} = \underline{\qquad\qquad}\boxed{7}$

Ans: B

02. $444^{444} = \underline{\qquad\qquad}\boxed{6}$

Since 444 is even

Ans: B

03. $2016^{2015} - 2015^{2016}$

$= \underline{\qquad\qquad}\boxed{6} - \underline{\qquad\qquad}\boxed{5} = \underline{\qquad\qquad}\boxed{1}$

Ans: A

04. $3211^{124} \times 646^{94}$

$= \underline{\qquad\qquad}\boxed{1} \times \underline{\qquad\qquad}\boxed{6} = \underline{\qquad\qquad}\boxed{6}$

Ans: D

05. $5^{2023} - 8^{2042}$

$= 5^{2023} - 8^{4\times510+2}$

$= \underline{\qquad\qquad}\boxed{5} - \underline{\qquad\qquad}\boxed{4} = \underline{\qquad\qquad}\boxed{1}$

Ans: C

06. We know $2^{69} \equiv 64 \pmod{127}$

We have to find the smallest positive integer K such that $64+K$ is divisible by 127

clearly $K = 63$

07 We know $123 \equiv 3 \pmod 5$ (07)

$$\Rightarrow 123^{456} \equiv 3^{456} \pmod 5$$

We know $3^1 \equiv 3 \pmod 5$

$$3^2 \equiv 4 \pmod 5$$

$$3^3 \equiv 2 \pmod 5$$

$$3^4 = 1 \pmod 5$$

So, the cycle repeats every 4 terms: 3, 4, 2, 1

Now $\frac{456}{4} = 114$, remainder 0.

Since, the remainder 0, we take the 4th term in the cycle, which is 1.

$\therefore$ $123^{456} \equiv 1 \pmod 5$

$\therefore$ The remainder when 123^{456} is divided by 5 is 1

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08 We know $2020 \equiv 2 \pmod{21}$

$$2^{2020} \equiv 2^{2020} \pmod{21}$$

Now, $2^1 \equiv 2 \pmod{21}$

$$2^2 = 4 \pmod{21}$$

$$2^3 = 8 \pmod{21}$$

$$2^4 = 16 \pmod{21}$$

$$2^5 = 11 \pmod{21}$$

$$2^6 = 1 \pmod{21}$$

21) 64 (3
63
1

21) 32 (1
21
11

116(

: The cycle repeates every 6 terms: 2, 4, 8, 16, 11, 1.

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$$\frac{2020}{6} = 336 \text{ remainder } 4.$$

Since the remainder is 4, we take the 4th term in the cycle, which is 16.

$\therefore$ The remainder $= 16$

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09. Conceptual — Ans: D

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10. Conceptual — Ans: D

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11. Conceptual — Ans: A, B

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12. We know $19 \equiv 5 \pmod 7$

$$\Rightarrow 19^{77} \equiv 5^{77} \pmod 7$$

We have
$$5^1 \equiv 5 \pmod 7$$
$$5^2 \equiv 4 \pmod 7$$
$$5^3 \equiv 16 \pmod 7$$
$$5^4 \equiv 2 \pmod 7$$
$$5^5 \equiv 3 \pmod 7$$
$$5^6 \equiv 1 \pmod 7$$

$7)\,25\,(3$
$\underline{21}$
4

$7)\,125\,(17$
$\underline{7}$
55
$\underline{49}$
6

So, the cycle repeats every 6 term. 5, 4, 6, 2, 3, 1.

Now $\frac{77}{6} = 12$ (remainder 5)

We take 5th term in the cycle which is 3

$\therefore$ Remainder is 3

13	<u>Statement I:</u> $5^{2023} = \underline{\qquad\qquad} \boxed{5}$ (True)
	<u>Statement II:</u> Conceptual (True) Ans: A
14.	<u>Statement I:</u> Conceptual (True)
	<u>Statement II:</u> Conceptual (True) Ans: A
15	Conceptual Ans: D
16	$4 \equiv 2 \pmod 2$
	$\Rightarrow 4^2 \equiv 2^2 \pmod 2$
	$\Rightarrow 16 \equiv 4 \pmod 2$ Ans: A
17.	$8^{8097} = 8^{4(2024)+1} = \underline{\qquad\qquad} \boxed{8}$
	Ans: D
18.	<u>option</u> <u>verification</u>:
	option: C $8^{4047} = 8^{4(1011)+3} = \underline{\qquad\qquad} \boxed{2}$
	Ans: C
19.	We have $17 \equiv -1 \pmod{18}$
	$17^{200} \equiv (-1)^{200} \pmod{18}$
	$17^{200} \equiv 1 \pmod{18}$
	$\therefore$ Remainder is $= 1$ Ans: 1

09

10

20. $10^{2023} + 5^{2023} + 36^{2023}$

$= \underline{\qquad\qquad}\boxed{0} + \underline{\qquad\qquad}\boxed{5} + \underline{\qquad\qquad}\boxed{6}$

$= \underline{\qquad\qquad}\boxed{1}$

Ans : 1

21. (i) $10^{25} + 1^{26}$

$= \underline{\qquad\qquad}\boxed{0} + \underline{\qquad\qquad}\boxed{1}$

$= \underline{\qquad\qquad}\boxed{1}$

Ans : 1

(ii) $25^{6} + 36^{27}$

$= \underline{\qquad\qquad}\boxed{5} + \underline{\qquad\qquad}\boxed{6}$

$= \underline{\qquad\qquad}\boxed{1}$

(iii) 2^{8007}

$= 2^{4(2001)+3} = \underline{\qquad\qquad}\boxed{8}$

(iv) $9^{4048} = \underline{\qquad\qquad}\boxed{1}$

Ans: t, t, p, t

22. (i) Conceptual (t)

(ii) Conceptual (q)

(iii) Conceptual (p)

(iv) Conceptual (r)

Ans: t, q, p, r

$\Longrightarrow$ THE END $\Longleftarrow$