

HINTS & SOLUTIONS

EXERCISE - 1

Single Choice

9. Sum of interior angles of a n sided polygon

$$= (n-2) \times 180$$

$$= \frac{n}{2} [240 + (n-1)5]$$

$$\Rightarrow n=9, 16$$

$n=16$ is to be rejected.

$$(T_{16}=120^\circ + 15 \times 5^\circ = 195^\circ > 180^\circ)$$

$$11. \frac{S_{kx}}{S_x} = \frac{\frac{kx}{2} [2a + (kx-1)d]}{\frac{x}{2} [2a + (x-1)d]} = k$$

$$\left[\frac{2a + (kx-1)d}{2a + (x-1)d} \right]$$

If $2a-d=0$, then $\frac{S_{kx}}{S_x}$ is independent of x

So $d=2a$

- 12.
- $a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_n = c$

$$\frac{a_1 + a_2 + a_3 + \dots + 2a_n}{n} \geq (a_1 a_2 a_3 \dots 2a_n)^{1/n} \geq (2c)^{1/n}$$

$$\Rightarrow a_1 + a_2 + a_3 + \dots + 2a_n \geq n(2c)^{1/n}$$

17. Horizontal
- $1 + \frac{1}{4} + \frac{1}{16} + \dots = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$

$$\text{Vertical } \frac{1}{2} + \frac{-1}{8} + \frac{1}{32} + \dots = \frac{\frac{1}{2}}{1 + \frac{1}{4}} = \frac{2}{5}$$

- 21.
- $\alpha + \beta = 3$
- ,
- $\alpha\beta = a$

$$\gamma + \delta = 12, \gamma\delta = b$$

$\alpha, \beta, \gamma, \delta$ are in G.P

Let r be the common ratio

$$\text{So } \alpha(1+r) = 3$$

$$\alpha r^2(1+r) = 12 \Rightarrow r^2 = 4$$

$$r = 2$$

$$\text{So } \alpha = 1$$

$$\text{So } a = 2, b = 32$$

22. We have
- $b^2 = ac$
- (i)

$$\text{and } 2\log\left(\frac{3b}{5c}\right) = \log\left(\frac{5c}{a}\right) + \log\left(\frac{a}{3b}\right)$$

$$= \log\left(\frac{5c}{a} \cdot \frac{a}{3b}\right) = \log\left(\frac{5c}{3b}\right) = -\log\left(\frac{3b}{5c}\right)$$

$$\Rightarrow 3\log\left(\frac{3b}{5c}\right) = 0 \Rightarrow b = \frac{5}{3}c \quad \dots \dots \text{(ii)}$$

$$\text{From (i) \& (ii), we have } a = \frac{b^2}{c} = \frac{25c}{9}$$

$$\text{Now, we have } b+c = \frac{5c}{3} + c = \frac{8c}{3} < \frac{25c}{9} = a$$

Hence a, b, c cannot form the sides of a triangle.

- 25.
- $a + b + c + d = 2$

$$\Rightarrow a, b, c, d > 0$$

$$\frac{(a+b)+(c+d)}{2} \geq \sqrt{(a+b)(c+d)}$$

$$1 \geq \sqrt{(a+b)(c+d)} \geq 0$$

$$\Rightarrow 0 \leq (a+b)(c+d) \leq 1$$

$$0 \leq M \leq 1$$

$$29. \Rightarrow \frac{a+rd}{a+nd} = \frac{a+nd}{a+md}$$

$$\Rightarrow \frac{1+r(d/a)}{1+n(d/a)} = \frac{1+n(d/a)}{1+m(d/a)} \quad \text{Let } \frac{d}{a} = x$$

$$\Rightarrow (1+nx)^2 = (1+rx)(1+mx)$$

$$\Rightarrow (n^2-mr)x^2 + (2n-r-m)x = 0 \Rightarrow x=0$$

$$\text{or } x = -\left(\frac{2n-r-m}{n^2-mr}\right) = \left(\frac{2n-r-m}{\frac{n(m+r)}{2}-n^2}\right) = \frac{-2}{n}$$

$$(m, n, r \text{ are in H.P.} \Rightarrow mr = \frac{n(m+r)}{2})$$

EXERCISE - 2

Part # I : Multiple Choice

1. $\alpha + \beta + \gamma = -3a$, $\alpha\beta\gamma = -c$
 $\alpha\beta + \beta\gamma + \gamma\alpha = 3b \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{3b}{\alpha\beta\gamma}$
 $\Rightarrow \frac{2}{\beta} + \frac{1}{\beta} = \frac{3b}{-c} \quad (\because \alpha, \beta, \gamma \text{ in H.P.})$
 $\Rightarrow \frac{1}{\beta} = -\frac{b}{c} \Rightarrow \beta = \frac{-c}{b}$
2. $x_1 + x_2 = 3$ $x_3 + x_4 = 12$ $x_1 x_2 = A$
 $x_3 x_4 = B$
 let $x_1 = ar$, $x_2 = ar^2$, $x_3 = ar^3$, $x_4 = ar^4$
 $a + ar = 3$ $ar^2(1+r) = 12$
 $a^2 r = A$ $r^2(3) = 12$
 $1, 2, 4, 8$ $r^2 = 4$
 $A = 2$ $x_1 + x_3 = 4$
 $r = 2$ $a = 1$
 $B = 32$ $x_2 + x_4 = 10$
3. Using AM $\geq$ GM,
 $\frac{1 + a_1 + a_1^2}{3} \geq (1 \cdot a_1 \cdot a_1^2)^{1/3} \Rightarrow 1 + a_1 + a_1^2 \geq 3a_1$
 $\Rightarrow 1 + a_2 + a_2^2 \geq 3a_2$
 $\Rightarrow 1 + a_n + a_n^2 \geq 3a_n$
 Multiplying these,
 $(1 + a + a^2) \dots (1 + a_n + a_n^2) \geq 3^n (a_1 a_2 a_3 \dots a_n) = 3^n \cdot 1$
5. $a_1 + a_{10} = a_2 + a_9 = \dots = (a+b)$
 $g_1 g_{10} = g_2 g_9 = \dots = ab$
 $\Rightarrow \frac{5(a+b) + 4(a+b) + 3(a+b) + 2(a+b) + (a+b)}{ab}$
 $= 15 \frac{(a+b)}{ab} = \frac{30}{h} \quad \left(\because h = \frac{2ab}{a+b} \right)$
6. $a_1 + (a_1 + d), (a_1 + 2d), \dots, b_1 + (b_1 + d_1), (b_1 + 2d_1), \dots$
 hence $a_{100} = a_1 + 99d$
 $b_{100} = b_1 + 99d_1$
 add $\frac{a_{100} + b_{100}}{2} = \frac{a_1 + b_1 + 99(d + d_1)}{2}$
 hence $d + d_1 = 0 \Rightarrow d = -d_1 \Rightarrow$ (A)
 (B) and (C) are obviously true.

$$\sum_{r=1}^{100} (a_r + b_r) = \frac{100}{2} [(a_1 + b_1) + (a_{100} + b_{100})] = \frac{100 \times 200}{2}$$

$$= 10^4 \Rightarrow \text{(D) (using } S_n = \frac{n}{2}(a+d) \text{)}$$

$$9. \sum_{r=1}^n (2r-1) = n^2$$

$$\Rightarrow a = 2n-1$$

$$\Rightarrow n = \frac{a+1}{2}$$

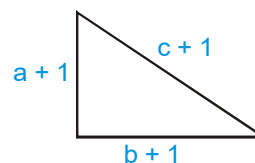
$$\Rightarrow (a+1)^2 + (b+1)^2 = (c+1)^2$$

$$= 8^2 + 6^2 = 10^2 \quad (\because a+b+c=21)$$

$$\Rightarrow a=7 \quad b=5 \quad c=9$$

$$\text{Hence } G=9 \quad L=5$$

$$G-L=4 \quad \& \quad a-b=2$$



$$12. \sum_{r=1}^n r(r+1)(2r+3) = an^4 + bn^3 + cn^2 + dn + e$$

$$\sum_{r=1}^n (r^2+r)(2r+3) = \sum_{r=1}^n (2r^3+5r^2+3r)$$

$$= 2 \cdot \frac{n^2(n+1)^2}{4} + 5 \frac{n(n+1)(2n+1)}{6} + 3 \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2} \left[n(n+1) + \frac{5}{3}(2n+1) + 3 \right]$$

$$= \frac{n(n+1)}{2} \left[\frac{6(n^2+n) + 10(2n+1) + 18}{6} \right]$$

$$= \frac{n(n+1)}{12} [6n^2 + 26n + 28]$$

$$= \frac{1}{12} [6n^4 + 26n^3 + 28n^2 + 6n^3 + 26n^2 + 28n]$$

$$= \frac{1}{12} [6n^4 + 32n^3 + 54n^2 + 28n]$$

$$a = \frac{6}{12}, \quad b = \frac{32}{12}, \quad c = \frac{54}{12}, \quad d = \frac{28}{12} = \frac{7}{3}$$

$$e = 0$$

$$\text{So } a+c = b+d$$

$$b - \frac{2}{3} = \frac{32}{12} - \frac{2}{3} = \frac{24}{12} \quad c-1 = \frac{42}{12}$$

$$\text{so } a, b-\frac{2}{3}, c-1 \text{ are in A.P. \& } \frac{c}{a} = \frac{54}{6} = 9 \text{ is an integer}$$

16. since $x, |x+1|, |x-1|$ are in A.P.

so $2|x+1| = x + |x-1| \quad \dots (i)$

Case-I If $x < -1$, then (i) becomes

$$-2(x+1) = x - (x-1)$$

$$\Rightarrow x = -\frac{3}{2}$$

then series $-\frac{3}{2}, \frac{1}{2}, \frac{5}{2}, \dots$

$$\therefore S_{20} = \frac{20}{2} [-3 + (20-1)2] = 350$$

21. $a + b + c = xb$

Divide by $b, \frac{a}{b} + 1 + \frac{c}{b} = x$

or $\frac{1}{r} + 1 + r = x$ where r is common ratio of G.P.

$$\Rightarrow r^2 + r(1-x) + 1 = 0$$

since r is real & distinct $\Rightarrow D > 0$

$$\therefore (1-x)^2 - 4 > 0 \Rightarrow x^2 - 2x - 3 > 0$$

or $(x+1)(x-3) > 0 \Rightarrow x > 3$ or $x < -1$

Part II : Assertion & Reason

5. $r = 4R \sin\left(\frac{A}{2}\right) \sin\left(\frac{B}{2}\right) \sin\left(\frac{C}{2}\right) \leq 4R \left(\frac{1}{8}\right)$

$$\left(\because \sin\left(\frac{A}{2}\right) \sin\left(\frac{B}{2}\right) \sin\left(\frac{C}{2}\right) \leq \frac{1}{8} \right)$$

$$\Rightarrow R \geq 2r$$

if $R = 12$ and $r = 8$

then, $12 \geq 16$ (impossible)

6. **Statement - I**

$$\text{Let } E = \frac{(\sin x + 1)^3 + (\cos^3 x + 1)}{(\sin x + 1) \cos x}$$

$$\therefore E = \frac{(\sin x + 1)^2}{\cos x} + \frac{1}{\cos x (\sin x + 1)} + \frac{\cos^2 x}{(1 + \sin x)} \geq 3$$

$$\left\{ \frac{(1 + \sin x)^2}{\cos x} \cdot \frac{1}{\cos x (1 + \sin x)} \cdot \frac{\cos^2 x}{(1 + \sin x)} \right\}^{1/3} \geq 3$$

$$\Rightarrow E \geq 3$$

Hence minimum value of given expression is 3.

Statement - II

$$-\sqrt{a^2 + b^2} \leq a \sin \theta + b \cos \theta \leq \sqrt{a^2 + b^2}$$

7. $S = \ell n^2 + 2\ell n^2 + 5\ell n^2 + \dots T_n \dots (i)$

$$S = \ell n^2 + 2\ell n^2 + 5\ell n^2 + \dots T_n \dots (ii)$$

$$(i) - (ii) \Rightarrow T_n = \ell n^2 + \ell n^2 + 3\ell n^2 + \dots n \text{ terms}$$

$$T_n = \ell n^2 + \ell n^2(n-1)^2$$

now successive difference in A.P.

$$\Rightarrow T_n = an^2 + bn + c$$

St.-II is correct.

8. **Given** $(abc)^{1/3} \geq \frac{a+b+c}{3}$

$$\Rightarrow a = b = c \text{ (GM} \geq \text{AM which is possible only if GM = AM)}$$

$$\therefore 3a + 4b + 5c = 12 \Rightarrow a = b = c = 1$$

$$\therefore \frac{1}{a^2} + \frac{1}{b^3} + \frac{1}{c^5} = 3$$

10. In any ΔABC , we have $r_1 + r_2 + r_3 = 4R + r \leq \frac{9R}{2}$

11. **Statement - I**

$$\frac{\frac{1}{b} + \frac{1}{a}}{\frac{2}{b} - \frac{1}{a}} + \frac{\frac{1}{b} + \frac{1}{c}}{\frac{2}{b} - \frac{1}{c}} = \frac{\frac{1}{b} + \frac{1}{a}}{\frac{1}{c}} + \frac{\frac{1}{b} + \frac{1}{c}}{\frac{1}{a}} \left(\because \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \right)$$

$$= \frac{c}{b} + \frac{c}{a} + \frac{a}{b} + \frac{a}{c} = \frac{a+c}{b} + \left(\frac{c}{a} + \frac{a}{c} \right)$$

$$= \frac{(a+c)^2}{2ac} + \left(\frac{c}{a} + \frac{a}{c} \right) = \frac{1}{2} \left(\frac{a}{c} + \frac{c}{a} + 2 \right) + \left(\frac{c}{a} + \frac{a}{c} \right) > 4$$

$$[\because x + \frac{1}{x} > 2 \text{ when } x > 0, x \neq \frac{1}{x}]$$

Statement - II

Is False $\therefore$ Numbers should be positive

12. Let a, b, c in G.P. then $b^2 = ac$

then $a + b, 2b, b + c$ in H.P.

$$\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c} \text{ in AP}$$

$$\frac{2}{2b} = \frac{1}{a+b} + \frac{1}{b+c}$$

$$(a+b)(b+c) = (a+c+2b)b$$

$$ab + b^2 + ac + bc = ab + bc + 2b^2$$

$$\therefore b^2 = ac$$

So statement (1) and (2) is true

15. **Statement - I**

$$(AM)(HM) = (GM)^2$$

True for any 3 numbers in G.P.

Statement II

False if number are not in G.P.

EXERCISE - 3

Part # I : Matrix Match Type

4. (A) $S_n = 4 + 11 + 22 + 37 \dots T_n$
 $S_n = 4 + 11 + 22 + 37 \dots T_n$
 $T_n = 4 + 7 + 11 + 15 + \dots n \text{ terms}$
 $T_n = 4 + \frac{(n-1)}{2}(14 + (n-2)4) = 1 + 2n^2 + n$

(B) $|1^2 - 2^2 + 3^2 - 4^2 \dots 2n \text{ terms}|$
 $= |(1-2)(1+2) + (3-4)(3+4) + (5-6)(5+6)|$
 $= |-3 - 7 - 11 - 15 \dots n \text{ terms}|$
 $= 3 + 7 + 11 + \dots n \text{ terms}$
 $= \left(\frac{n}{2}(6 + (n-1)4) \right) = \frac{n}{2}(4n+2) = (2n^2 + n)$

(C) $3 + 7 + 11 + 15 \dots = 2n^2 + n$

(D) Coefficient of x^n is

$$-2(1+2+3+\dots \text{term}) = -\frac{2n(n+1)}{2} = -(n^2 + n)$$

Part # II : Comprehension

Comprehension-3

$$ax^2 + 2bx + b = 5x^2 - 3bx - a$$

$$\Rightarrow (a-5)x^2 + 5bx + (b+a) = 0$$

If $a \neq 5$, then since $x \in \mathbb{R}$

$$D = 25b^2 - 4(b+a)(a-5) \geq 0 \quad \forall b \in \mathbb{R}$$

$$\Rightarrow 25b^2 - 4(a-5)b - 4a(a-5) \geq 0 \quad \forall b \in \mathbb{R}$$

$$\therefore 16(a-5)^2 + 16(25)a(a-5) \leq 0$$

$$\Rightarrow 16(a-5)(a-5+25a) \leq 0$$

$$\Rightarrow (a-5)(26a-5) \leq 0$$

$$\therefore a \in \left[\frac{5}{26}, 5 \right)$$

If $a = 5$,

$$5bx + (b+5) = 0$$

not satisfied for $b = 0$

$$\therefore a_m \in \{1, 2, 3, 4\}$$

$$t_r = (r-1)(r-2)(r-3)(r-4)$$

$$S_n = \frac{1}{5} \sum_{r=1}^n (r-4)(r-3)(r-2)(r-1)(r-(r-5))$$

$$= \frac{1}{5} \sum_{r=1}^n ((r-4)(r-3)(r-2)(r-1) \\ r - (r-5)(r-4)(r-3)(r-2)(r-1))$$

$$= \frac{1}{5} n(n-1)(n-2)(n-3)(n-4)$$

$$S_n = 0 \Rightarrow n = 1, 2, 3, 4 \quad n = 0 \text{ (rejected)}$$

$$\sum \frac{1}{t_r} = \frac{1}{3} \lim_{n \rightarrow \infty} \sum_{r=5}^n \frac{(r-1) - (r-4)}{(r-4)(r-3)(r-2)(r-1)}$$

$$= \frac{1}{3} \lim_{n \rightarrow \infty} \sum_{r=5}^n \left(\frac{1}{(r-4)(r-3)(r-2)} - \frac{1}{(r-3)(r-2)(r-1)} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{3} \left[\frac{1}{6} - \frac{1}{(n-3)(n-2)(n-1)} \right] = \frac{1}{18}$$

4. $g(n) - g(n-1) = 1^2 + 2^2 + 3^2 + \dots$
 $+ (n-1)^2 + n^2 - (1^2 + 2^2 + 3^2 + \dots + (n-1)^2)$
 $= n^2$

Comprehension 4

1. $g(n) - g(n-1) = 1^2 + 2^2 + 3^2 + \dots$
 $+ (n-1)^2 + n^2 - (1^2 + 2^2 + 3^2 + \dots + (n-1)^2)$
 $= n^2$

2. $g(n) - f(n) = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} = \frac{n(n+1)}{2}$
 $\left(\frac{2n+1}{3} - 1 \right) = \frac{n(n+1)}{2} \cdot \frac{2n-2}{3}$
 $= \frac{n(n+1)(n-1)}{3} = \frac{(n-1)n(n+1)}{3}$

for $n = 2$ $\frac{(n-1)n(n+1)}{3} = \frac{1 \cdot 2 \cdot 3}{3}$ which is divisible

by 2 but not by 2^2

$\therefore$ greatest even integer which divides

$$\frac{(n-1)n(n+1)}{3}, \text{ for every } n \in \mathbb{N}, n \geq 2, \text{ is } 2$$

3. $f(n) + 3g(n) + h(n)$
 $= \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{2} + \left(\frac{n(n+1)}{2} \right)^2$
 $= \frac{n(n+1)}{2} \left(1 + 2n + 1 + \frac{n(n+1)}{2} \right) = (1 + 2 + 3 + \dots + n)$
 $\left(2n + 2 + \frac{n(n+1)}{2} \right)$

$\Rightarrow$ for all $n \in \mathbb{N}$

Comprehension 5

$$a + b + c = 5 \quad (a, b, c > 0) \quad x^2 y^3 = 243 = 3^5$$

$$1. \quad \frac{a + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + c}{5} \geq \left(\frac{ab^3c}{27} \right)^{\frac{1}{5}} \quad (\because AM \geq GM)$$

$$\Rightarrow 1^5 \geq \frac{ab^3c}{27} \Rightarrow ab^3c \leq 27$$

2. Using $AM \geq HM$

$$\frac{a + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + c}{5} \geq \frac{5}{\frac{1}{a} + \frac{3}{b} + \frac{3}{b} + \frac{3}{b} + \frac{1}{c}}$$

$$\Rightarrow \frac{1}{5} \geq \frac{1}{\frac{1}{a} + \frac{9}{b} + \frac{1}{c}}$$

3. Using $AM \geq GM$

$$\frac{x^2 + y + y + y + 1}{5} \geq (x^2 \cdot y \cdot y \cdot y \cdot 1)^{\frac{1}{5}}$$

$$\Rightarrow x^2 + 3y + 1 \geq 5 \cdot (243)^{\frac{1}{5}} \geq 15$$

But $x^2, y \neq 1$, hence $x^2 + 3y + 1 > 15$

$$4. \quad \frac{x + x + y + y + y}{5} \geq (x^2 y^3)^{\frac{1}{5}} \geq \frac{5}{\frac{2}{x} + \frac{3}{y}}$$

$$\Rightarrow \frac{2x + 3y}{5} \geq 3 \geq \frac{5xy}{3x + 2y}$$

EXERCISE - 4

Subjective Type

$$1. \quad a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} \dots \dots \dots$$

$$= \frac{1}{a_1 + a_n} \left[\frac{a_1 + a_n}{a_1 a_n} + \frac{a_2 + a_{n-1}}{a_2 a_{n-1}} + \frac{a_3 + a_{n-2}}{a_3 a_{n-2}} + \dots + \frac{a_1 + a_n}{a_n a_1} \right]$$

$$= \frac{1}{a_1 + a_n} \left[\frac{1}{a_n} + \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_{n-1}} + \dots + \frac{1}{a_1} + \frac{1}{a_n} \right]$$

$$= \frac{2}{a_1 + a_n} \left[\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_n} \right]$$

$$2. \quad T_r = (2r + 1) 2^r$$

$$S_n = 3 \cdot 2^1 + 5 \cdot 2^2 + 7 \cdot 2^3 + \dots + (2n + 1) 2^n$$

Sub

$$2S_n = 3 \cdot 2^2 + 5 \cdot 2^3 + \dots + (2n - 1) 2^n + (2n + 1) 2^{n+1}$$

$$-S_n = 3 \cdot 2^1 + 2^3 + 2^4 + \dots + 2^{n+1} - (2n + 1) 2^{n+1}$$

$$\therefore -S_n = 2 + 2^2 + 2^3 + 2^4 + \dots + 2^{n+1} - (2n + 1) 2^{n+1}$$

$$\therefore -S_n = \frac{2[2^{n+1} - 1]}{2 - 1} - (2n + 1) 2^{n+1}$$

$$S_n = (2n + 1) 2^{n+1} - (2^{n+2} - 2)$$

$$n \cdot 2^{n+2} + \underbrace{2^{n+1} - 2^{n+2}}_{-2^{n+1}} + 2$$

$$n \cdot 2^{n+2} - 2^{n+1} + 2$$

$$4n \cdot 2^n - 2 \cdot 2^n + 2$$

$$R = 4; S = -2; T = 2$$

$$\therefore R + S + T = 4$$

$$3. \quad 160$$

$$6. \quad n \cdot 2^{n+2} - 2^{n+1} + 2.$$

$$7. \quad LHS = 9 + 8 \times 10 + 8 \times 10^2 + \dots + 8 \times 10^{n-1} + 4 \times 10^n + 4 \times 10^{n+1} + \dots + 4 \times 10^{n+n-1}$$

$$= 9 + 8 \times 10^1 \left(\frac{10^{n-1} - 1}{9} \right) + 4 \times 10^n \left[\frac{10^n - 1}{9} \right]$$

$$= \frac{81 + 8 \times 10^n - 80 + 4 \times 10^{2n} - 4 \times 10^n}{9}$$

$$= \frac{1 + 4 \times 10^n + 4 \times 10^{2n}}{9}$$

$$= \frac{3 + 6 \times 10^n}{9} = \left(\frac{1 + 2 \times 10^n}{3} \right)^2$$

$$RHS = 7 + 6 \times 10^1 \frac{(10^{n-1} - 1)}{9} = \frac{63 + 6 \times 10^n - 60}{9}$$

$$= \frac{3 + 6 \times 10^n}{9} = \frac{1}{3} (1 + 2 \times 10^n)$$

$$8. \quad \frac{1}{201}$$

$$9. \quad n^2$$

10. $(a-3d)(a-d)(a+d)(a+3d)+16d^4$
 $= (a^2-9d^2)(a^2-d^2)+16d^4$
 $= a^4-a^2d^2-9a^2d^2+9d^4+16d^4$
 $= a^4-10a^2d^2+25d^4 = [a^2-5d^2]^2$
 $= (a^2-d^2-4d^2)^2 = ((a-d)(a+d)-(2d)^2)^2$
 $\therefore (a-d), (a+d), 2d$ are integers.

Hence Proved

11. $a=1, b=9$ or $b=1, a=9$

12. (i) Let $b=ar$
 $c=ar^2$ and $d=ar^3$
 So $a^2(1-r^2), a^2(r^2)(1-r^2), a^2r^4(1-r^2)$ these are in G.P.
 So $(a^2-b^2), (b^2-c^2), (c^2-d^2)$ are in G.P.

- (ii) $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}$
 $= \frac{1}{a^2(1+r^2)}, \frac{1}{a^2r^2(1+r^2)}, \frac{1}{a^2r^4(1+r^2)}$ are in G.P.

13. 6, 3

14. 3, 7, 11 or 12, 7, 2

15. $\frac{T_q}{T_p} = \frac{T_r}{T_q}$ = common ratio

$$\frac{a+(q-1)d}{a+(p-1)d} = \frac{a+(r-1)d}{a+(q-1)d}$$

using dividendo

$$\frac{(q-p)}{a+(p-1)d} = \frac{(r-q)}{a+(q-1)d}$$

$$\Rightarrow \frac{T_q}{T_p} = \frac{r-q}{q-p} = \frac{q-r}{p-q}$$

16. $[n(n+1)(n+2)]/6$

EXERCISE - 5

Part # I : AIEEE/JEE-MAIN

2. $\frac{a}{1-r} = 20 \dots (i) \quad \frac{a^2}{1-r^2} = 100 \dots (ii)$

from (i) and (ii)

$$\frac{a}{1+r} = 5 \quad (\because a = 20(1-r) \text{ by (i)})$$

$$\Rightarrow \frac{20(1-r)}{1+r} = 5 \Rightarrow 5r = 3 \Rightarrow r = 3/5$$

6. Given that A.M. = 9 and G.M. = 4
 If α, β are roots of quadratic equations then quadratic equation is
 $x^2 - x(\alpha + \beta) + \alpha\beta = 0 \dots (1)$

$$\text{A.M.} = \frac{\alpha + \beta}{2} = 9$$

$$\Rightarrow \alpha + \beta = 18 \dots (2)$$

$$\text{G.M.} = \sqrt{\alpha\beta} = 4$$

$$\Rightarrow \alpha\beta = 16 \dots (3)$$

so the required equation will be

$$x^2 - 18x + 16 = 0$$

9. $\therefore \frac{(S_m)_{\text{IAP}}}{(S_n)_{\text{IAP}}} = \frac{m^2}{n^2}$ then $\frac{T_m}{T_n} = \frac{2m-1}{2n-1}$

$$\text{so } \frac{a_6}{a_{21}} = \frac{2 \times 6 - 1}{2 \times 21 - 1} = \frac{11}{41}$$

12. $a + ar = 12 \dots (1)$

$$ar^2 + ar^3 = 48 \dots (2)$$

$$ar^2(a + ar) = 48 \dots (3)$$

$$\text{so } r^2 = 4 \quad a(1+r) = 12$$

$$r = 2 \quad a(3) = 12$$

because +ve G.P. $\boxed{a=4}$

13. $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots \infty$

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots \infty$$

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \frac{4}{3^5} + \dots \infty$$

$$= \frac{4}{3} + \frac{4/9}{1 - \frac{1}{3}} = \frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2 \Rightarrow S = 3$$

14. $4500 = 150 \times 10 + \{148 + 146 + \dots \text{ upto } n \text{ terms}\}$

$$= 1500 + \frac{n}{2} \{296 + (n-1)(-2)\}$$

$$\Rightarrow n^2 - 149n + 3000 = 0$$

$$\Rightarrow (n-24)(n-125) = 0$$

$$\Rightarrow n = 24 \quad \because n \neq 125$$

$$\text{So total time taken} = 10 + 24 = 34 \text{ min.}$$

15. Saving after first 3 month = 600

$$600 + \left\{ \begin{array}{l} 240 + 280 + \dots \\ \text{let } n \text{ month} \end{array} \right\} = 11040$$

$$|240 + 280 + \dots n \text{ terms}| = 10440$$

$$n/2 [480 + (n-1)40] = 10440$$

$$n \{440 + 40n\} = 20880$$

$$n^2 + 11n - 522 = 0$$

$$n = 18, -29 \quad (-29 \text{ rejected})$$

$$\text{Total months} = n + 3$$

$$18 + 3 = 21 \text{ Months}$$

16. $\sum_{r=1}^{100} a_{2r} = a_2 + a_4 + a_6 + \dots + a_{200} = \alpha$

$$= (a+d) + (a+3d) + \dots + (a+199d) = \alpha$$

$$\sum_{r=1}^{100} a_{2r-1} = \beta = a_1 + a_3 + \dots + a_{199} = \beta$$

$$= a + (a+2d) + \dots + (a+198d) = \beta$$

$$\frac{100}{2} [a+d+a+199d] = \alpha$$

$$\Rightarrow 50(2a+200d) = \alpha \quad \dots (i)$$

$$\frac{100}{2} [a+a+198d] = \beta$$

$$\Rightarrow 50(2a+198d) = \beta \quad \dots (ii)$$

$$(1)-(2)$$

$$\alpha - \beta = 50(2d)$$

$$= d = \frac{\alpha - \beta}{100}$$

17. Statement-I :

$$(1^3-0^3) + (2^3-1^3) + (3^3-2^3) + \dots + (20^3-19^3) = 20^3 = 8000$$

Statement-I is true.

Statement-II :

$$\sum_{k=1}^n k^3 - (k-1)^3 = (1^3-0^3) + (2^3-1^3) + (3^3-2^3) + \dots + n^3 + (n-1)^3 = n^3.$$

Statement-2 is true and Statement-2 is a correct explanation of Statement-1.

18. $100T_{100} = 50T_{50}$
 $100(a+99d) = 50(a+4d)$
 $a+149d = 0$
 $T_{150} = a+149d = 0$

19. $S = \frac{7}{10} + \frac{77}{100} + \frac{777}{1000} + \dots$

$$S = \frac{7}{9} \left\{ \frac{10-1}{10} + \frac{100-1}{100} + \frac{1000-1}{1000} + \dots \right\}$$

$$= \frac{7}{9} \left\{ 20 - \frac{1}{10} \left(\frac{1-10^{-20}}{9/10} \right) \right\}$$

$$= \frac{7}{9} \left\{ 20 - \frac{1}{9} (1-10^{-20}) \right\} = \frac{7}{81} (179 + 10^{-20})$$

25. $a+d, a+4d, a+8d$ are in G.P.

$$(a+4d)^2 = (a+d)(a+8d)$$

$$\Rightarrow a^2 + 8ad + 16d^2 = a^2 + 9ad + 8d^2$$

$$\Rightarrow 8d^2 = ad \quad \Rightarrow \frac{a}{d} = 8$$

$$\therefore \text{Common ratio} = \frac{a+4d}{a+d} = \frac{8+4}{4+1} = \frac{4}{3}$$

26. $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \dots = \frac{16}{5} m$

$$\Rightarrow \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 + \dots 10 \text{ terms} = \frac{16}{5} m$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 [2^2 + 3^2 + 4^2 + 5^2 + \dots 10 \text{ terms}] = \frac{16}{5} m$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 [2^2 + 3^2 + 4^2 + \dots + 11^2] = \frac{16}{5} m$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 [1^2 + 2^2 + 3^2 + \dots + 11^2 - 1^2] = \frac{16}{5} m$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 \left[\frac{11 \cdot 12 \cdot 23}{6} - 1 \right] = \frac{16}{5} m \quad (\text{given})$$

$$\Rightarrow \frac{16}{25} [22 \cdot 23 - 1] = \frac{16}{5} m \quad \Rightarrow \frac{1}{5} (505) = m$$

$$\Rightarrow m = 101$$

Part # II : IIT-JEE ADVANCED

$$6. \frac{x}{1-r} = 5 \Rightarrow 5 - 5r = x \Rightarrow r = 1 - \frac{x}{5}$$

$$\text{As } |r| < 1 \quad \text{i.e. } \left| 1 - \frac{x}{5} \right| < 1; -1 < 1 - \frac{x}{5} < 1$$

$$-5 < 5 - x < 5 = -10 < -x < 0 = 10 > x > 0$$

$$\text{i.e. } 0 < x < 10$$

$$13. A_1 = \frac{a+b}{2}; \quad G_1 = \sqrt{ab}; \quad H_1 = \frac{2ab}{a+b}$$

$$A_n = \frac{A_{n-1} + H_{n-1}}{2}; \quad G_n = \sqrt{A_{n-1} H_{n-1}}$$

$$H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1} + H_{n-1}}$$

We know that

$$G^2 = AH$$

So clearly

$$G_1 = G_2 = G_3 = \dots G_n = \sqrt{ab}$$

$$14. A_2 \text{ is AM of } A_1, H_1 \text{ and } A_1 > H_1$$

$$\Rightarrow A_1 > A_2 > H_1$$

$$A_3 \text{ is A.M. of } A_2, H_2$$

$$\Rightarrow A_2 > A_3 > H_2$$

$$\therefore A_1 > A_2 > A_3 \dots$$

$$15. \text{ as above}$$

$$A_1 > H_2 > H_1$$

$$A_2 > H_3 > H_2$$

$$\text{so } \dots H_3 > H_2 > H_1$$

$$\Rightarrow H_1 < H_2 < H_3 \dots$$

$$16. \text{ Let } a_1 = a, a_2 = ar, a_3 = ar^2, a_4 = ar^3$$

$$\text{Now } b_1 = a, b_2 = a + ar, b_3 = a + ar + ar^2$$

$$b_4 = a + ar + ar^2 + ar^3$$

so b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P. & nor in H.P.

so **S(I)** is true & **S(II)** is False.

$$17. S_n = cn^2$$

$$S_{n-1} = c(n-1)^2$$

$$T_n = S_n - S_{n-1} = c(2n-1)$$

$$T'_n = T_n^2 = c^2(4n^2 - 4n + 1)$$

$$\sum T'_n = nc^2 \left(\frac{4n^2 - 1}{3} \right)$$

$$18. S_k = \frac{k-1}{1 - \frac{1}{k}} = \frac{1}{(k-1)!} \text{ for } k \geq 2, S_1 = 0$$

$$\text{Now } \frac{100^2}{100!} + \sum_{k=2}^{100} \left((k^2 - 3k + 1) \cdot \frac{1}{(k-1)!} \right) + S_1$$

$$\frac{100^2}{100!} + \frac{1}{1!} + \sum_{k=3}^{100} \left(\frac{1}{(k-3)!} - \frac{1}{(k-1)!} \right) + 0$$

$$\left(\because S_2 = \frac{1}{1!} \right)$$

$$= \frac{100^2}{100!} + 1 + \left| \frac{1}{0!} - \frac{1}{2!} \right| + \left| \frac{1}{1!} - \frac{1}{3!} \right| + \left| \frac{1}{2!} - \frac{1}{4!} \right| + \dots$$

$$+ \left| \frac{1}{97!} - \frac{1}{99!} \right|$$

$$= \frac{100^2}{100!} + 3 - \frac{1}{98!} - \frac{1}{99!}$$

$$= \frac{100^2}{100!} + 3 - \frac{100}{99!} = 3$$

$$19. a_1 = 15$$

$$27 - 2a_2 > 0$$

$$a_k = 2a_{k-1} - a_{k-2} \quad \forall k = 3, 4, 5 \dots 11$$

$$\frac{a_1^2 + a_2^2 + \dots + a_{11}^2}{11} = 90$$

$$a_{k-1} = \frac{a_k + a_{k-2}}{2}$$

all $a_i (i = 1, 2, \dots, 11)$ are in A.P.

Let the numbers are

$$(a_6 + 5d), (a_6 + 4d), \dots, a_6, \dots, (a_6 - 4d), (a_6 - 5d)$$

$$11a_6^2 + 110d^2 = 990$$

$$a_6 = 15 - 5d$$

$$a_6^2 + 10d^2 = 90$$

$$(15 - 5d)^2 + 10d^2 = 90 \Rightarrow 7d^2 - 30d + 27 = 0$$

$$\Rightarrow d = 3, \frac{9}{7}$$

$$\text{for } d = 3 \Rightarrow a_2 = 12 \quad (\text{possible})$$

$$\text{for } d = 9/7 \Rightarrow a_2 = 13.7$$

(not possible since $a_2 < 13.5$)

$$a_6 = 0 \Rightarrow \frac{a_1 + a_2 + \dots + a_{11}}{11} = a_6 = 0$$

20. As
- $a > 0$

and all the given terms are positive

hence considering A.M. $\geq$ G.M. for given numbers :

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + a^8 + a^{10}}{7}$$

$$\geq (a^{-5} \cdot a^{-4} \cdot a^{-3} \cdot a^{-3} \cdot a^{-3} \cdot a^8 \cdot a^{10})^{\frac{1}{7}}$$

$$\Rightarrow \frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + a^8 + a^{10}}{7} \geq 1$$

$$\Rightarrow (a^{-5} + a^{-4} + 3a^{-3} + a^8 + a^{10})_{\min} = 7$$

where $a^{-5} = a^{-4} = a^{-3} = a^8 = a^{10}$ i.e. $a = 1$

$$\Rightarrow (a^{-5} + a^{-4} + 3a^{-3} + a^8 + a^{10} + 1)_{\min} = 8$$

when $a = 1$

21. Consider
- $d \neq 0$
- the solution is

 $a_1, a_2, a_3, \dots, a_{100} \rightarrow AP$

$$a_1 = 3 ; S_p = \sum_{i=1}^p a_i \quad 1 \leq n \leq 20$$

$$m = 5n$$

$$\frac{S_m}{S_n} = \frac{\frac{m}{2}[2a_1 + (m-1)d]}{\frac{n}{2}[2a_1 + (n-1)d]}$$

$$\frac{S_m}{S_n} = \frac{5[(2a_1 - d) + 5nd]}{[(2a_1 - d) + nd]}$$

for $\frac{S_m}{S_n}$ to be independent of n

$$\therefore 2a_1 - d = 0 \Rightarrow d = 2a_1$$

$$\Rightarrow d = 6 \Rightarrow a_2 = 9$$

$$\text{If } d = 0 \Rightarrow a_2 = a_1 = 3$$

- 22.
- $a_1, a_2, a_3, \dots$
- be in H.P.

$$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots \text{ be in A.P.}$$

$$\text{in A.P. } T_1 = \frac{1}{a_1} = \frac{1}{5} \text{ and } T_{20} = \frac{1}{a_{20}} = \frac{1}{25}$$

$$\therefore T_{20} = T_1 + 19d$$

$$\frac{1}{25} = \frac{1}{5} + 19d \Rightarrow d = -\frac{4}{19 \times 25}$$

$$T_n = T_1 + (n-1)d < 0$$

$$\Rightarrow \frac{1}{5} - \frac{(n-1)4}{19 \times 25} < 0 \Rightarrow \frac{1}{5} < \frac{4(n-1)}{25 \times 19}$$

$$\Rightarrow \frac{5 \times 19}{4} + 1 < n \Rightarrow \frac{99}{4} < n$$

$$\Rightarrow \text{least positive integer } n \text{ is } 25.$$

- 23.
- $S_n = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 \dots$

$$S_n = (3^2 - 1^2) + (4^2 - 2^2) + \dots$$

$$S_n = 2(1 + 2 + 3 + \dots + 4n)$$

$$= \frac{2(4n)(4n+1)}{2}$$

$$S_n = 4n(4n+1)$$

$$S_n = 4n(4n+1) = 1056 \text{ is possible when } n = 8$$

$$4n(4n+1) = 1088 \text{ not possible}$$

$$4n(4n+1) = 1120 \text{ not possible}$$

$$4n(4n+1) = 1332 \text{ possible when } n = 9.$$

24. When 1 and 2 are removed from numbers 1 to
- n
- then we get maximum possible sum of remaining numbers and when
- $n-1, n$
- are removed then we get minimum possible sum of remaining numbers.

$$\Rightarrow \frac{n(n+1)}{2} - (2n-1) \leq 1224 \leq \frac{n(n+1)}{2} - 3$$

$$\Rightarrow \begin{cases} n^2 + n - 2454 \geq 0 \\ n^2 - 3n - 2446 \leq 0 \end{cases}$$

$$\Rightarrow \begin{cases} n \geq 50 \\ n \leq 50 \end{cases} \Rightarrow n = 50$$

Now let x and $x+1$ be two consecutive numbers

$$\Rightarrow \frac{50(50+1)}{2} - x - x - 1 = 1224$$

$$\Rightarrow x = 25$$

$$\Rightarrow 25^{\text{th}} \text{ and } 26^{\text{th}} \text{ cards are removed from pack}$$

$$\Rightarrow k = 25 \Rightarrow k - 20 = 5$$

- 27.
- $\log_e b_1, \log_e b_2, \log_e b_3, \dots, \log_e b_{101}$
- are in A.P.

$$b_1, b_2, b_3, \dots, b_{101}$$

$$\text{Given } \log_e(b_2) - \log_e(b_1) = \log_e(2) \Rightarrow \frac{b_2}{b_1} = 2 = r$$

$$a_1, a_2, a_3, \dots, a$$

$$a_1 = b_1 = a$$

$$b_1 + b_2 + b_3 + \dots + b_{51} = t,$$

$$S = a_1 + a_2 + \dots + a_{51}$$

$$t = \text{sum of 51 terms of G.P.} = b_1 \frac{(r^{51} - 1)}{r - 1} = \frac{a(2^{51} - 1)}{2 - 1} = a(2^{51} - 1)$$

$$s = \text{sum of 51 terms of A.P.} = \frac{51}{2} [2a_1 + (n-1)d] = \frac{51}{2} (2a + 50d)$$

Given $a_{51} = b_{51}$
 $a + 50d = a(2)^{50}$
 $50d = a(2^{50} - 1)$

Hence, $s = a \frac{51}{2} [2^{50} + 1] \Rightarrow s = a \left(51 \cdot 2^{49} + \frac{51}{2} \right)$

$$s = 2 \left(4 \cdot 2^{49} + 47 \cdot 2^{49} + \frac{51}{2} \right)$$

$$\Rightarrow s = a \left((2^{51} - 1) + 47 \cdot 2^{49} + \frac{53}{2} \right)$$

$$s - t = a \left(47 \cdot 2^{49} + \frac{53}{2} \right)$$

Clearly : $s > t$

$$a_{101} = a_1 + 100d = a + 2a \cdot 2^{50} - 2a = a(2^{51} - 1)$$

$$b_{101} = b_1 r^{100} = a \cdot 2^{100}$$

Hence, $b^{101} > a^{101}$

MOCK TEST

1. If $1^2 + 2^2 + 3^2 + \dots + 2003^2 = (2003)(4007)(334)$
 $(1)(2003) + (2)(2002) + (3)(2001) + \dots + (2003)(1)$
 $= (2003)(334)(x)$

$$\Rightarrow \sum_{r=1}^{2003} r(2003 - r + 1) = (2003)(334)(x)$$

$$\Rightarrow 2004 \cdot \sum_{r=1}^{2003} r - \sum_{r=1}^{2003} r^2 = (2003)(334)(x)$$

$$\Rightarrow 2004 \cdot \left(\frac{2003 \cdot 2004}{2} \right) - 2003 \cdot (4007) \cdot 334 = (2003)$$

$$(334)(x)$$

$$\Rightarrow x = 2005$$

2. (B)

$$S = \frac{a(1-r^n)}{1-r}, P = a^n \cdot r^{\frac{n(n-1)}{2}}$$

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} \dots \text{to } n \text{ terms} = \frac{1-r^n}{a(1-r)r^{n-1}}$$

$$S^n = R^n P^k \Rightarrow \left(\frac{S}{R} \right)^n = P^k \Rightarrow (a^2 r^{n-1})^n = P^k$$

$$\Rightarrow P^2 = P^k \Rightarrow k = 2$$

3. $a + 6d = 9$

$$T_1 T_2 T_7 = a(a+d)(a+6d)$$

$$= 9a(a+d) = 9(9-6d)(9-5d)$$

$$\therefore T_7 = a + 6d = 9$$

Let $A = T_1 T_2 T_7$

$$\frac{dA}{d(d)} = 9[-45 - 54 + 60d] = 0$$

$$60d = 99$$

$$d = \frac{33}{20}$$

4. (D)

$$a_1 + a_3 + a_5 + \dots + a_{199} = \beta$$

$$a_2 + a_4 + a_6 + \dots + a_{200} = \alpha$$

$$a_2 - a_1 + a_4 - a_3 + a_6 - a_5 \dots a_{200} - a_{199} = \alpha - \beta$$

$$d + d + d \dots d = \alpha - \beta$$

$$d = \frac{\alpha - \beta}{100}$$

5. $S_1 + S_2 + S_3 + \dots + S_p$

$$\Rightarrow S_1 = \frac{n}{2} [2 \cdot 1 + (n-1)1]$$

$$S_2 = \frac{n}{2} [2 \cdot 2 + (n-1)3]$$

$$S_3 = \frac{n}{2} [2 \cdot 3 + (n-1)5]$$

$$S_p = \frac{n}{2} [2 \cdot p + (n-1)(2p-1)]$$

So $S_1 + S_2 + \dots + S_p = \frac{n}{2} [2(1+2+\dots+p) + (n-1)(1+3+5+\dots+(2p-1))]$

$$= \frac{n}{2} \left[2 \cdot \frac{p(p+1)}{2} + (n-1)p^2 \right] = \frac{np}{2} (np+1)$$

6. (B)

$$S = 1 + 4x + 7x^2 + 10x^3 + \dots$$

$$x.S = x + 4x^2 + 7x^3 + \dots$$

Subtract

$$S(1-x) = 1 + 3x + 3x^2 + 3x^3 + \dots$$

$$S(1-x) = 1 + 3x \left(\frac{1}{1-x} \right) \quad |x| < 1$$

$$S = \frac{1+2x}{(1-x)^2}$$

Given $\frac{1+2x}{(1-x)^2} = \frac{35}{16}$

$$\Rightarrow 16 + 32x = 35 + 35x^2 - 70x \Rightarrow 35x^2 - 102x + 19 = 0$$

$$\Rightarrow 35x^2 - 7x - 95x + 19 = 0$$

$$\Rightarrow 7x(5x-1) - 19(5x-1) = 0$$

$$\Rightarrow (5x-1)(7x-19) = 0 \Rightarrow x = \frac{1}{5}, \frac{19}{7}$$

But $|x| < 1$ $\therefore x = \frac{1}{5}$

7. $a = A + (p-1)d \Rightarrow d = \frac{a-b}{p-q}$

$$b = A + (q-1)d$$

$$S = \frac{p+q}{2} [2A + (p+q-1)d] = \frac{p+q}{2} [A + (p-1)d$$

$$+ A + (q-1)d + d] = \frac{p+q}{2} [a + b + d]$$

$$= \frac{p+q}{2} \left[a + b + \frac{a-b}{p-q} \right]$$

8. (A)

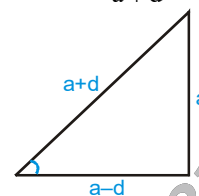
Let the sides be $a-d, a, a+d$

where $a > d > 0$

we have

$$(a+d)^2 = (a-d)^2 + a^2$$

$$\Rightarrow d = \frac{a}{4} \text{ we have } \sin \theta = \frac{a}{a+d}$$



$$\Rightarrow \cos \theta = \frac{3}{5}, \quad \sin \theta = \frac{4}{5}$$

9. $t_n = S_n - S_{n-1}$

$$= \frac{n}{6} (2n^2 + 9n + 13) - \frac{(n-1)}{6} \{2(n-1)^2 + 9(n-1) + 13\}$$

$$= \frac{1}{6} [2n^3 + 9n^2 + 13n - 2(n-1)^3 - 9(n-1)^2 - 13]$$

$$= (n+1)^2 \quad (n-1)]$$

$$\sum_{r=1}^{\infty} \frac{1}{r \cdot (r+1)} = \sum_{r=1}^{\infty} \left(\frac{1}{r} - \frac{1}{r+1} \right) = 1$$

10. (C)

S_1 : To get the maximum value of $P = a_1 a_2 + a_2 a_3 + \dots + a_7 a_8$ maximize one term and minimize other.

$$\therefore a_3 = a_4 = \dots = a_8 = 0$$

$$\therefore P = a_1 a_2$$

$$\therefore \left(\frac{a_1 + a_2}{2} \right)^2 \geq a_1 a_2$$

$$\therefore \left(\frac{16}{2} \right)^2 \geq a_1 a_2$$

$$\Rightarrow a_1 a_2 \leq 64$$

$$\therefore P_{\max} = 64$$

$$S_{II}: \frac{x^2 + r^2}{2} \geq \sqrt{x^2 r^2} \text{ and } \frac{y^2 + s^2}{2} \geq \sqrt{y^2 s^2}$$

$$\therefore x^2 + r^2 \geq 2xr \text{ and } y^2 + s^2 \geq 2ys$$

$$\therefore 2(xr + ys) \leq x^2 + y^2 + r^2 + s^2$$

$$\Rightarrow 2(xr + ys) \leq 2 \Rightarrow (xr + ys) \leq 1$$

$$\therefore (xr + ys)_{\max} = 1$$

S_{III} : Let α and β be two positive real numbers such that

$$A = \frac{\alpha + \beta}{2} \quad G^2 = \alpha\beta$$

$$\text{so } \alpha = \frac{\alpha + \beta}{2} + \frac{\alpha - \beta}{2} \quad \text{and} \quad \frac{\alpha - \beta}{2} = \sqrt{\frac{(\alpha + \beta)^2 - 4\alpha\beta}{4}} = \sqrt{\left(\frac{\alpha + \beta}{2}\right)^2 - \alpha\beta} = \sqrt{A^2 - G^2}$$

$$\text{so } \alpha = A + \sqrt{A^2 - G^2}$$

$$\text{and } \beta = \frac{\alpha + \beta}{2} - \left(\frac{\alpha - \beta}{2}\right) = A - \sqrt{A^2 - G^2}$$

$$\text{S}_4 : q = \frac{p+r}{2} \Rightarrow q^3 = \frac{(p+r)^3}{8}$$

$$\Rightarrow 8q^3 = p^3 + r^3 + 3pr(p+r)$$

$$\Rightarrow 8q^3 = p^3 + r^3 + 3pr(2q)$$

$$\Rightarrow p^3 + r^3 = 8q^3 - 6pqr$$

$$\begin{aligned} 11. \sum_{r=1}^n \frac{1}{\sqrt{a+rx} + \sqrt{a+(r-1)x}} \\ &= \sum_{r=1}^n \frac{\sqrt{a+rx} - \sqrt{a+(r-1)x}}{(a+rx) - (a+(r-1)x)} \\ &= \sum_{r=1}^n \frac{\sqrt{a+rx} - \sqrt{a+(r-1)x}}{x} \\ &= \frac{1}{x} [(\sqrt{a+x} - \sqrt{a+0x}) + (\sqrt{a+2x} - \sqrt{a+x}) \\ &\quad + (\sqrt{a+3x} - \sqrt{a+2x}) + \dots + (\sqrt{a+nx} - \sqrt{a+(n-1)x})] \\ &= \frac{1}{x} [\sqrt{a+nx} - \sqrt{a}] = \frac{n}{\sqrt{a} + \sqrt{a+nx}} \end{aligned}$$

12. (A, C)

$$S = 1 + \frac{1}{(1+3)} (1+2)^2 + \frac{1}{(1+3+5)} (1+2+3)^2 + \frac{1}{(1+3+5+7)} (1+2+3+4)^2 + \dots$$

r^{th} term

$$T_r = \frac{1}{r^2} (1+2+\dots+r)^2 = \frac{1}{r^2} \left\{ \frac{r(r+1)}{2} \right\}^2 = \frac{r^2 + 2r + 1}{4}$$

$$\therefore T_7 = 16 \quad \text{and} \quad S_{10} = \sum_{r=1}^{10} T_r = \frac{1}{4}$$

$$\left\{ \frac{(10)(10+1)(20+1)}{6} + (10)(10+1) + 10 \right\} = \frac{505}{4}$$

13. $1, \log_y x, \log_z y, -15 \log_x z$ are in AP.

Let common diff. is d .

$$\log_y x = 1 + d \Rightarrow x = (y)^{1+d}$$

$$\log_z y = 1 + 2d \Rightarrow y = (z)^{1+2d}$$

$$-15 \log_x z = 1 + 3d \Rightarrow z = x^{\left(\frac{1+3d}{-15}\right)}$$

$$\text{So } x = (y)^{1+d} \\ = ((z)^{1+2d})^{1+d}$$

$$x = (x)^{\left(\frac{1+3d}{-15}\right)(1+d)(1+2d)}$$

$$\text{So } (1+d)(1+2d)(1+3d) = -15$$

$$\text{So } d = -2$$

$$x = (y)^{-1}$$

$$y = (z)^{-3}$$

$$z = (x)^{1/3}$$

$$z^3 = x$$

14. (A, B, C)

$$\begin{aligned} \sum_{r=1}^n r^2 + \sum_{r=1}^n r &= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \\ &= \frac{1}{6} n \cdot (n+1) [2n+1+3] = \frac{n(n+1)(n+2)}{3} \\ a &= 0, b = 1, c = 2 \end{aligned}$$

15. (A, B, C, D)

Since a_{912} , a_{951} and a_{480} is divisible by 3 then a_{91} is not prime

$$\begin{aligned} a_{91} &= \frac{10^{91} - 1}{10 - 1} = \frac{10^{91} - 1}{10^7 - 1} \times \frac{10^7 - 1}{10 - 1} \\ &= (1 + 10^7 + \dots + 10^{84})(1 + 10 + \dots + 10^6) \end{aligned}$$

$$\Rightarrow a_{91} \text{ is not prime}$$

16. (A)

$$3(a^2 + b^2 + c^2 + 1) - 2(a + b + c + ab + bc + ca) = 0$$

$$\Rightarrow (a-1)^2 + (b-1)^2 + (c-1)^2 + (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$\Rightarrow a = b = c = 1$$

17. (A)

obviously both the statements are true and statement-II explains statement-I.

19. (A)

Statement -I : Consider 'a' quantities equal to $\frac{1}{a}$, 'b'

quantities equal to $\frac{1}{b}$ & 'c' quantities equal to $\frac{1}{c}$

$$A.M. = \frac{a \cdot \frac{1}{a} + b \cdot \frac{1}{b} + c \cdot \frac{1}{c}}{a + b + c} = \frac{3}{a + b + c}$$

$$G.M. = \left(\frac{1}{a^a} \cdot \frac{1}{b^b} \cdot \frac{1}{c^c} \right)^{\frac{1}{a+b+c}}$$

A.M. $\geq$ G.M.

$$\frac{3}{a + b + c} \geq \frac{1}{\frac{a}{a+b+c} + \frac{b}{a+b+c} + \frac{c}{a+b+c}}$$

$$\Rightarrow \frac{a}{a+b+c} \cdot \frac{b}{a+b+c} \cdot \frac{c}{a+b+c} \geq \frac{a+b+c}{3}$$

Statement-II : $A = \frac{a_1 + a_n}{2}$, $G = \sqrt{a_1 a_n}$

We may assume that

$$a_1 a_n - a_2 a_{n-1} = a_1 \{a_1 + (n-1)d\} - (a_1 + d) \{a_1 + (n-2)d\} \\ = -(n-2)d^2 < 0$$

$$a_1 a_n < a_2 a_{n-1}$$

Similarly $a_1 a_n < a_3 a_{n-2}$

If n is even $(a_1 a_n)^{n/2} < a_1 a_2 a_3 \dots a_n$

If n is odd

$$\Rightarrow n = 2m + 1$$

$$a_1 a_{2m+1} < a_{m+1}^2$$

similarly $a_1 a_{2m+1} < a_m a_{m+2}$

$$(a_1 a_n)^{n/2} < a_1 a_2 \dots a_n$$

$\therefore$ **A.M. $\geq$ G.M.**

$$\left(\frac{a_1 + a_2 + a_3 + \dots + a_n}{n} \right)^n > a_1 a_2 \dots a_n$$

$$\Rightarrow (a_1 a_n)^{n/2} < a_1 a_2 \dots a_n < \left(\frac{a_1 + a_n}{2} \right)^n$$

$$\Rightarrow G^n < a_1 a_2 \dots a_n < A^n$$

20. (B)

Statement-I a, A, b are in A.P.

$$\Rightarrow 2A = a + b \quad \dots (i)$$

a, p, q, b are in G.P.

$$\Rightarrow pq = ab \quad \dots (ii)$$

and let common ratio of G.P. be r

$$\therefore b = ar^3 \Rightarrow r = \left(\frac{b}{a} \right)^{\frac{1}{3}}$$

$$p = ar \Rightarrow p = a \cdot \left(\frac{b}{a} \right)^{\frac{1}{3}}$$

$$\Rightarrow p^3 = a^2 b \quad \dots (iii)$$

$$q = ar^2 \Rightarrow q = a \cdot \left(\frac{b}{a} \right)^{\frac{2}{3}}$$

$$\Rightarrow q^3 = ab^2 \quad \dots (iv)$$

from (i), (ii), (iii) & (iv)

$$p^3 + q^3 = 2A pq$$

statement-II is obviously true

21. (A) $\rightarrow$ (r), (B) $\rightarrow$ (p, t), (C) $\rightarrow$ (s), (D) $\rightarrow$ (q)

$$(A) F(n+1) = \frac{2F(n)+1}{2} = F(n) + \frac{1}{2}$$

$$\therefore F(1), F(2), F(3), \dots$$

is an AP with common difference $\frac{1}{2}$

$$(B) a_1 + 2d + a_1 + 4d + a_1 + 10d + a_1 + 16d + a_1 + 18d \\ = 5a_1 + 50d \\ = 5(a_1 + 10d) = 10 \text{ i.e. } a_1 + 10d = 2$$

$$\text{Now, } \sum_{i=1}^{21} a_i = \frac{21}{2} [2a_1 + 20d] = 21(a_1 + 10d) = 42$$

$$(C) S = 1 + 5 + 13 + 29 + \dots + t_{10}$$

$$S = 1 + 5 + 13 + \dots + t_9 + t_{10}$$

Subtrating

$$t_{10} = 1 + 4 + 8 + 16 + \dots \text{ up to 10 terms}$$

$$= 1 + (4 + 8 + 16 + \dots \text{ up to 9 terms})$$

$$= 2045$$

(D) Sum of all two digit numbers

$$= \frac{90}{2} (10 + 99) = (45) (109)$$

Sum of all two digit numbers is divisible by 2

$$= \frac{45}{2} (10 + 98) = (45) (54)$$

Sum of all two digit numbers is divisible by 3

$$= \frac{30}{2} (12 + 99) = 15 (111)$$

Sum of all two digit numbers divisible by 6

$$= \frac{15}{2} (12 + 96) = 15 (54)$$

The required sum is

$$45(109) + 15(54) - (45)(54) - 15(111) = 1620$$

22. (A) → (q), (B) → (r), (C) → (p), (D) → (t)

(A) $a + b = 12$

$$ab + \frac{6ab}{a+b} = 48$$

$$ab + \frac{ab}{2} = 48$$

$$\therefore ab = 32$$

$$(B) S = \frac{5}{1^2 \cdot 4^2} + \frac{11}{4^2 \cdot 7^2} + \frac{17}{7^2 \cdot 10^2} + \dots$$

$$\Rightarrow 3S = \frac{3 \cdot 5}{1^2 \cdot 4^2} + \frac{3 \cdot 11}{4^2 \cdot 7^2} + \frac{3 \cdot 17}{7^2 \cdot 10^2} + \dots$$

$$\Rightarrow 3S = \frac{(4-1) \cdot (4+1)}{1^2 \cdot 4^2} + \frac{(7-4)(7+4)}{4^2 \cdot 7^2} + \dots$$

$$\frac{(10-7)(10+7)}{7^2 \cdot 10^2} + \dots$$

$$\Rightarrow 3S = \frac{4^2 - 1^2}{1^2 \cdot 4^2} + \frac{7^2 - 4^2}{4^2 \cdot 7^2} + \frac{10^2 - 7^2}{7^2 \cdot 10^2} + \dots$$

$$\Rightarrow 3S = 1 - \frac{1}{4^2} + \frac{1}{4^2} - \frac{1}{7^2} + \frac{1}{7^2} - \frac{1}{10^2} + \dots$$

$$\Rightarrow 3S = 1 \Rightarrow S = \frac{1}{3}$$

$$(C) \text{ H.M of } \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \text{ is } \frac{4}{2+3+4+5} = \frac{2}{7}$$

(D) Since G.M. lies between the numbers

$$GM = -\sqrt{(-4) \times (-9)} = -6$$

23.

1. (B)

$$G_1 G_2 \dots G_n = (\sqrt{1 \times 1024})^n = 2^{5n}$$

$$\therefore 2^{5n} = 2^{45}$$

$$\therefore n = 9$$

2. (B)

$$A_1 + A_2 + A_3 + \dots + A_{m-1} + A_m = 1025 \times 171$$

$$\therefore m \left(\frac{-2 + 1027}{2} \right) = 1025 \times 171$$

$$\therefore m = 342$$

3. (A)

$$\text{Since } n = 9, \therefore r = (1024)^{\frac{1}{9+1}} = 2$$

$$\therefore G_1 = 2, r = 2$$

$$G_1 + G_2 + \dots + G_n = \frac{2 \cdot (2^9 - 1)}{2 - 1} = 1024 - 2$$

$$= 1022$$

24.

1. (C)

2. (B)

3. (C)

Let numbers in set A be $a - D, a, a + D$ and in set B be $b - d, b, b + d$

$$3a = 3b = 15 \Rightarrow a = b = 5$$

$$\text{set A} = \{5 - D, 5, 5 + D\}$$

$$\text{set B} = \{5 - d, 5, 5 + d\}$$

$$\text{where } D = d + 1$$

$$\frac{p}{q} = \frac{5(25 - D^2)}{5(25 - d^2)} = \frac{7}{8}$$

$$25(8 - 7) = 8(d + 1)^2 - 7d^2$$

$$\Rightarrow d = -17, 1 \text{ but } d > 0$$

$$\Rightarrow d = 1$$

So numbers in Set A are 3, 5, 7

number in Set B are 4, 5, 6

$$\text{Now } p = 3 \times 5 \times 7 = 105$$

$$q = 4 \times 5 \times 6 = 120$$

$$\text{value of } D + d = 3$$

25.

1. (C)

2. (B)

3. (C)

Let four integers be $a-d$, a , $a+d$ and $a+2d$ where a and d are integers and $d > 0$.

$$\therefore a+2d = (a-d)^2 + a^2 + (a+d)^2$$

$$\Rightarrow 2d^2 - 2d + 3a^2 - a = 0 \quad \text{.....(i)}$$

$$\therefore d = \frac{1}{2} \left[1 \pm \sqrt{1+2a-6a^2} \right] \quad \text{.....(ii)}$$

Since d is positive integer

$$\therefore 1+2a-6a^2 > 0$$

$$6a^2 - 2a - 1 < 0$$

$$\Rightarrow \frac{1-\sqrt{7}}{6} < a < \frac{1+\sqrt{7}}{6} \quad \therefore a \text{ is an integer}$$

$$\therefore a = 0 \quad \text{Put in (ii)}$$

$$\therefore d = 1 \text{ or } 0 \quad \text{but} \quad \therefore d > 0$$

$$\therefore d = 1$$

$$\therefore \text{The four numbers are : } -1, 0, 1, 2$$

26. (12)

Let the series be $a, ax, ax^2, ax^3, \dots$ given that $|x| < 1$ and $x \neq 0$.

$$\text{Also, } \frac{T_4}{T_2} = \frac{ax^3}{ax} = \frac{1}{16} \Rightarrow x^2 = \frac{1}{16}$$

$$\Rightarrow x = \pm \frac{1}{4}$$

$$\text{But since it is a decreasing G.P.} \Rightarrow x = \frac{1}{4}$$

$$\text{Also, } \frac{T_3}{T_2^2} = \frac{ax^2}{(ax)^2} = \frac{1}{9} \Rightarrow \frac{1}{a} = \frac{1}{9}$$

$$\Rightarrow a = 9$$

$$S_\infty = \frac{a}{1-r} = \frac{9}{1-\frac{1}{4}} = \frac{9 \times 4}{3} = 12$$

27. Let first installment be ' a ' and the common difference of the A.P. be ' d '

$$\text{So } a + (a+d) + (a+2d) + \dots + (a+39d) = 3600$$

$$\Rightarrow \frac{40}{2} [2a + 39d] = 3600$$

$$\Rightarrow 2a + 39d = 180 \quad \text{..... (i)}$$

$$\text{and } \frac{30}{2} [2a + 29d] = 2400$$

$$\Rightarrow 2a + 29d = 160 \quad \text{..... (ii)}$$

By equations (i) & (ii), we get $d = 2$ and $a = 51$

28. (7)

$$(1^2 + 2^2 + \dots + n^2) - (a_1 + a_2 + \dots + a_n) = \frac{1}{3} n(n^2 - 1) \quad \text{.....(i)}$$

Replacing n by $(n-1)$, then

$$(1^2 + 2^2 + \dots + (n-1)^2) - (a_1 + a_2 + \dots + a_{n-1}) = \frac{1}{3} (n-1)((n-1)^2 - 1) \quad \text{.....(ii)}$$

Subtracting (ii) from (i)

$$n^2 - a_n = n^2 - n \Rightarrow a_n = n$$

$$\Rightarrow a_7 = 7$$

$$29. q = \frac{p}{2} [2A + (p-1)d]$$

$$\Rightarrow \frac{2q}{p} = 2A + (p-1)d \quad \text{.....(i)}$$

$$p = \frac{q}{2} [2A + (q-1)d]$$

$$\Rightarrow \frac{2p}{q} = 2A + (q-1)d \quad \text{.....(ii)}$$

on subtracting equation (i) from (ii), we get

$$\frac{2}{pq} (q^2 - p^2) = (p-q)d$$

$$\Rightarrow d = \frac{-2}{pq} (p+q)$$

$\therefore$ Sum of $(p+q)$ terms is

$$= \frac{p+q}{2} [2A + (p+q-1)d]$$

$$= \frac{p+q}{2} [2A + (p-1)d + qd]$$

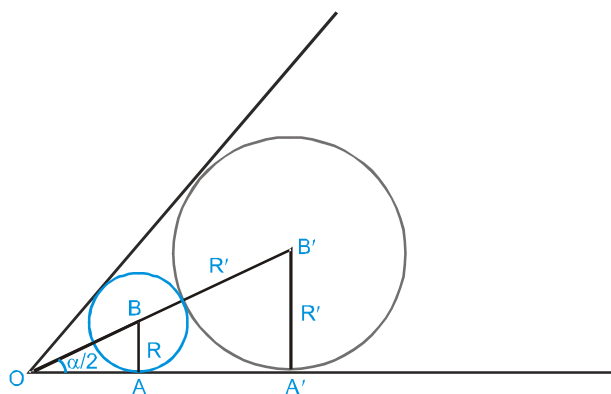
$$= \frac{p+q}{2} \left[\frac{2q}{p} + q \left\{ \frac{-2}{pq} (p+q) \right\} \right]$$

$$= \frac{p+q}{2} \left[\frac{2q}{p} - 2 - \frac{2q}{p} \right] = -(p+q)$$

30. Radius of first circle = R

Let Radius of second circle be = R'

$$\frac{OB}{BA} = \operatorname{cosec} \frac{\alpha}{2}$$



$$\therefore OB = R \operatorname{cosec} \frac{\alpha}{2}$$

Now $\frac{OB'}{A'B'} = \operatorname{cosec} \frac{\alpha}{2}$

$$\Rightarrow OB' = R' \operatorname{cosec} \frac{\alpha}{2} = OB + R + R'$$

$$\Rightarrow R + R \operatorname{cosec} \frac{\alpha}{2} + R' = R' \operatorname{cosec} \frac{\alpha}{2}$$

$$\Rightarrow R(1 + \operatorname{cosec} \alpha/2) = R' (\operatorname{cosec} \alpha/2 - 1)$$

$$\Rightarrow R' = R \left(\frac{1 + \sin \alpha/2}{1 - \sin \alpha/2} \right)$$

if radius of the third circle be R'', then

Similarly $R'' = R \left(\frac{1 + \sin \alpha/2}{1 - \sin \alpha/2} \right)^2$

So $R + R' + R'' + \dots$ n terms

$$= R \left[\frac{\left(\frac{1 + \sin \alpha/2}{1 - \sin \alpha/2} \right)^n - 1}{\left(\frac{1 + \sin \alpha/2}{1 - \sin \alpha/2} \right) - 1} \right]$$

$$= R \left(\frac{1 - \sin \alpha/2}{2 \sin \alpha/2} \right) \left[\left(\frac{1 + \sin \alpha/2}{1 - \sin \alpha/2} \right)^n - 1 \right]$$