ALGEBRAIC EXPRESSIONS ①

Class: IX, Mathematics

IIT FOUNDATION

I TEACHING TASK

01. $\frac{x^3 + x^4 - x^6}{x^2} = x + x^2 - x^4$, degree = 4

Ans: D

02 0

Ans: D

03 $(3x - x^2)^2 = 9x^2 - 6x^3 + x^4$

Degree = 4

Ans: C

04 $3x^2y^3 + 5xy^7 - x^6 \rightarrow$ Degree = 8

$3x^5 - 4x^3 + 2 \rightarrow$ Degree = 5

Difference = 8 − 5 = 3

Ans: B

05 $a^4 + 4a^2b^2 + b^4 = a^4 - 8a^2b^2 + b^4 + X$

$\Rightarrow X = 12a^2b^2$

Ans: D

06 $a^4 - 4a^2b^2 + b^4 + X = a^4 + 8a^2b^2 + b^4$

$\Rightarrow X = 12a^2b^2$

Ans: D

07 $x^3 + 3x - 8 + A = 3x^3 + x^2 + 6$ [illegible]

$\Rightarrow A = 2x^3 + x^2 - 3x + 14$

Ans: A

08 $x^3 - 3x^2 + 5x - 1 - A = 2x^3 + x^2 - 4x + 2$

$\Rightarrow A = -x^3 - 4x^2 + 9x - 3$ Ans: D

09. $(3x-4)(5x+7) = 15x^2 - 9x - 28$

$\Rightarrow 15x^2 + 21x - 20x - 28 = 15x^2 - ax - 28$

$\Rightarrow 15x^2 + x - 28 = 15x^2 - ax - 28$

$\Rightarrow a = -1.$ Ans: B

10. $(3x^2 - 5x + 6)(-8x^3)$

$= (3.0^2 - 5.0 + 6)(-8.0^3) = 0$ Ans: D

11. $(x^2y - 1)(3 - 2x^2y)$

$= 3x^2y - 2x^4y^2 - 3 + 2x^2y$

$= 5x^2y - 2x^4y^2 - 3$ Ans: B

12 $\frac{1}{3}(6x^2 + 15y^2)(6x^2 - 15y^2)$

$= \frac{(6x^2)^2 - (15y^2)^2}{3} = \frac{36x^4 - 225y^4}{3}$

$= 12x^4 - 75y^4$ Ans: C

13. $x - \frac{1}{x} = \sqrt{6}$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 \cdot x \cdot \frac{1}{x} = 6$

$\Rightarrow x^2 + \frac{1}{x^2} = 8$ Ans: D

14. $2x + y = 5 \rightarrow (1)$

$(1) \times 2 \Rightarrow 4x + 2y = 10$ Ans: D

15. Dividend = Divisor × quotient + Remainder

$= (2x^2 - 6x + 7)(3x - 2) + (-6x + 5)$

$= 6x^3 - 4x^2 - 18x^2 + 12x + 21x - 14 - 6x + 5$

$= 6x^3 - 22x^2 + 27x - 9$ Ans: C

16. $\frac{x^2 + 2x - 63}{x - 7} = \frac{(x+9)(x-7)}{x-7} = x + 9$ Ans: C

17 $(2xy^2)(-3x^2y) = -6x^3y^3$ Ans: C

18 $X - Y = (3x^3 + 3x^2 + 3x + 3) - (3x^2 - 3x + 3)$

$= 3x^3 + 3x^2 + 3x + 3 - 3x^2 + 3x - 3$

$= 3x^3 + 6x$ Ans: D

19 $(4x^2 - 5y^2 + 3z^2) + (-3x^2 + 4y^2 - 2z^2) + A$

$= x^2 + y^2 + z^2$

$\Rightarrow x^2 - y^2 + z^2 + A = x^2 + y^2 + z^2$

$\Rightarrow A = 2y^2$ Ans: B

20. $(a^3 - 2a^2 + 4a - 5) - (-a^3 - 8a + 2a^2 + 5)$

$= a^3 - 2a^2 + 4a - 5 + a^3 + 8a - 2a^2 - 5$

$= 2a^3 - 4a^2 + 12a - 10$ Ans: C

II

1) $\frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$ (4)

at $n=4$, $\frac{4^4}{4} + \frac{4^3}{2} + \frac{4^2}{4}$

$= 64 + 32 + 4 = 100$

which is also divisible by 2 Ans: A, B

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2. $(-5x^2y)\left(\frac{-2}{3}xy^2z\right)\left(\frac{8}{15}xyz^2\right)\left(-\frac{1}{4}z\right)$

$= -\frac{4}{9}(xyz)^4$ Ans:

$= -\frac{4}{9}x^4y^4z^4 = -\left(\frac{2}{3}x^2y^2z^2\right)^2$ Ans: B, C, D

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03 $\frac{(4ab)^2}{-6a^2} \div \frac{(-a^3b)^2}{2b^2}$

$= \frac{(4ab)^2}{-6a^2} \times \frac{2b^2}{(-a^3b)^2}$

$= \frac{16a^2b^2}{-6a^2} \times \frac{2b^2}{a^6b^2}$

$= \frac{-16b^2}{3a^4}$ or $\frac{-16}{3}\left(\frac{b}{a^2}\right)^2$ Ans: C, D

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04. option verification.

option D: $(4x+2)(4x^2+x+4) = 16x^3+12x^2+18x+8$

Hence Remainder = 0 Ans: A, B, D

III

01. i) $-2x^2 + 5x$

$= -2(-1.5)^2 + 5(-1.5)$

$= -2(2.25) - 7.5$

$= -4.5 - 7.5$

$= -12$

Ans: A

ii) $1.2x^3$

$= 1.2(-1.5)^3$

$= 1.2(-3.375)$

$= -4.05$

Ans: B

(iii) $-5x^2 + 5.2x^2 + 1.5x^2 - 0.7x^2$

$= (-5 + 5.2 + 1.5 - 0.7)x^2$

$= x^2$

$= (1.5)^2$

$= 2.25$

Ans: C

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02 i) $9x^3 - 12x^2 - 6x + 6 \div x - 2$

By Horner's method

$x=2$	9	−12	−6	6
	0	18	12	12
	9	6	6	18

Quotient $= 9x^2 + 6x + 6$, Degree $= 2$

Remainder $= 18$, Ans.

IV. Matrix Matching

1. i) $(x-2)(x+2) = x^2-4$

(ii) $(x^2-x+2)(x+2) = x^3+x^2+4$

(iii) $(x^2+x-2)(x+2) = x^3+3x^2-4$

(iv) $(x^2+x+2)(x+2) = x^3+3x^2+4x+4.$

Ans: S, R, T, P

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2 i) $(x^3+3x^2+4x+4)-(2x-3) = x^3+3x^2+2x+1$

(ii) $(x^3+3x^2+4x+4)-(x^2-x+4)$

$= x^3+2x^2+5x$

(iii) $(x^3+3x^2+4x+4)-(-x^3+4x-3)$

$= x^3+3x^2+4x+4+x^3-4x+3$

$= 2x^3+3x^2+7$

Ans: S, R, P, T

V.

1) $(2008+x+2008)^{2008-2006+2007} = (2008-x-2008)$

$\Rightarrow (2\times 2008+x)^{2009} = (-x)^{2009}$

$\Rightarrow 2\times 2008+x = -x$

$\Rightarrow 2\times 2008 = -2x$

$\Rightarrow x = -2008$

Ans: -2008

02 $7x + 7x + \cdots$ 7 terms

$= 7 \times 7x$

$= 7 \times 7 \times 7$

$= 7^3$

Ans: 7^3

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03 $2007x + 2008 = 0$ | $2007x - 2008 = 0$

$\Rightarrow x = \frac{-2008}{2007}$ | $\Rightarrow x = \frac{2008}{2007}$

$\text{Sum} = \frac{-2008}{2007} + \frac{2008}{2007} = 0$

Ans: 0

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04 $(ax + b) = 0$ | $bx - a = 0$

$\Rightarrow x = -\frac{b}{a}$ | $\Rightarrow x = \frac{a}{b}$

$\text{product} = -\frac{b}{a} \times \frac{a}{b} = -1$

Ans: -1

~~05~~ ~~$x^7 - 2x^6 + 4$~~

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05 Standard form $x^7 - 2x^6 + 4x^5 - 3x^4 - 8x^3 + 3x + 7$

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06 a) $\frac{4}{7}p^4 - 7p^2 - 5 \rightarrow$ polynomial

b) $11x^3 + 4x^2 + 7x^{1/2} + 3 \rightarrow$ not polynomial

c) $2x^3 + \frac{4}{x^2} + 9 \rightarrow$ not polynomial

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07 ~~a) $1 + 2x + x^2 \rightarrow$ degree $= 2$~~

07. a) $1+2x+x^2 \Rightarrow$ Degree $= 2$

b) $xy+yz+zx \Rightarrow$ Degree $= 2$

c) $p^2q^2+2pq^3-p^2q+7 \Rightarrow$ Degree $= 4$

d) $m^4-n^2+8 \Rightarrow$ Degree $= 4$

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08 $ax-b=0$

$\Rightarrow x = \frac{b}{a}$ is ~~the zero polynomial~~

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08 $ax-b$ is a ~~zero~~ polynomial

$\Rightarrow a=b=0$

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09. $a, b, c \rightarrow$ Smallest consecutive non-negative integers

$a=0,\ b=1,\ c=2$

$a^3+b^3+c^3-3abc = 0^3+1^3+2^3-3.0.1.2$

$= 9$ Ans. 9

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10. The third proportional of 6, 6 is 6.

$\therefore 5(6-3)-7(6-6)+3 = 24-3(8-6)$ (True)

$= 18 = 18$ (True)

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11. first four whole numbers $\rightarrow 0, 1, 2, 3$

$x+y = 0\times1\times2\times3$

$\Rightarrow x+y=0$

Any two numbers which satisfy the above equation are solutions. i.e. Infinite solutions

12 x is neither positive nor negative (a)

$\therefore \quad x = 0$

$3x - b = 3(0) - b = -b$, Smallest rational number. We can not define smallest rational number.

13 $x = 0, \quad y = 1, \quad z = -7$

$\frac{4xy}{7} + 7z = \frac{4 \cdot 0 \cdot 1}{7} + 7 \cdot (-7) = -49$

14 (i) $\frac{1}{2} + \frac{2y}{3} + \frac{3}{4} + \frac{4y}{5} - \frac{5}{6} + \frac{6y}{7} = 0$

$= \frac{210 + 280y + 315 + 336y - 350 + 360y}{420} = 0$

$= \frac{976y + 175}{420} = 0$

$\Rightarrow y = \frac{-175}{976}$

(ii) $x^7 - 128 = 0$

$\Rightarrow x^7 = 128$

$\Rightarrow x^7 = 2^7$

$\Rightarrow x = 2$

15 $(x^2)(x^2 + \text{---})(x^n + \text{---}) = (x^6 + \text{---})$

$\therefore 2 + n = 6$

$\Rightarrow n = 4$

LEARNER'S TASK

LEVEL-I

01. Degree = n — Ans: A

02 Degree — Ans: A

03 $p(x) = 5x - \pi$

$p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi$ — Ans: A

04. $cx + d = 0 \Rightarrow x = -\frac{d}{c}$ — Ans: B

05 Additive inverse $-x^2 + x - 2$ — Ans: A

06. Additive identity = 0 — Ans: A

07 Additive inverse $-2x^2 + 3x + 1$ — Ans: A

08 $(a^3 - 2a^2 + 4a - 5) - (-a^3 + 2a^2 - 8a + 5)$

$= a^3 - 2a^2 + 4a - 5 + a^3 - 2a^2 + 8a - 5$

$= 2a^3 - 4a^2 + 12a - 10$ — Ans: A

09 $(-y^3 - 6x^2y - xy^2 + x^3) - (x^3 - xy^2 + 5x^2y - y^3)$

$= -y^3 - 6x^2y - xy^2 + x^3 - x^3 + xy^2 - 5x^2y + y^3$

$= -11x^2y$ — Ans: C

10 $(a+b)(a^2 - ab + b^2) = a^3 + b^3$ — Ans: C

11. $(0.5x - y) \times (0.5y + y)$

$= 0.25x^2 - y^2$

Ans : D

12. $x^4 - 9x^3 - 2x^2 + 6x - 8 \div (x-3)$

By ~~Horner's~~ method

	1	−9	−2	6	−8
$x=3$ |	0	3	−18	−120	−228
	1	−6	−20	−114	

12. $x^4 - 9x^3 - 2x^2 + 6x - 8 \div (x-3)$

	1	−9	−2	6	−8
$x=3$ |	0	3	−18	−60	−162
	1	−6	−20	−54	−170

Remainder = −170

Ans: B

13. $\frac{84a^5x^3}{-12a^4x} = \frac{-7a}{x^2} = -7ax^{-2}$

Ans: A

14. $\frac{5a^3b - 7ab^3}{ab} = \frac{ab(5a^2 - 7b^2)}{ab} = 5a^2 - 7b^2$

Ans. A

LEVEL – II

01. a) Like terms → $-yz, 2zy$

b) Like terms → $(7p, -2p, 3p), (8pq, -5pq)$

c) Like terms → $(m^2n^2, -9m^2n^2), ($

02 a) $1+2x+x^2 \rightarrow$ Degree $= 2$

b) $xy+yz+zx \rightarrow$ Degree $= 2$

c) $m^4-n^2+8 \rightarrow$ Degree $= 4$

03 perimeter $= 2(l+b)$

$= 2(3x+2x)$

$= 10x$

04 a) $\frac{4}{7}p^4-7p^2-5 \rightarrow$ polynomial, since degree $= 4$, which is non-negative integer

b) $11x^3+4x^2+7\sqrt{x}+3 \rightarrow$ not polynomial

c) $a^{-1}+ab^2-1 \rightarrow$ not polynomial

05 a) $p(x)-1.2x^2$

$p(2) = -1.2(2)^2 = -4.8$

$p(3) = -1.2(3)^2 = -10.8$

$p(-1.5) = -1.2(-1.5)^2 = -1.2 \times 2.25 = -2.7$

b) $p(x) = \frac{1}{2}x^3$

$p(2) = \frac{1}{2}(2)^3 = 4$

$p(3) = \frac{1}{2}(3)^3 = \frac{27}{2}$

~~$p(-1) = \frac{1}{2}(-1)^3 = \frac{-1}{2}$~~

$p(-1.5) = \frac{1}{2}(-1.5)^3$

$= -\frac{3.375}{2}$

$= -1.6875$

6 $= \dfrac{7.83 \times 7.83 - 1.17 \times 1.17}{6.66}$

$= \dfrac{(7.83)^2 - (1.17)^2}{6.66}$

$= \dfrac{(7.83 + 1.17)(7.83 - 1.17)}{6.66}$

$= \dfrac{9 \times 6.66}{6.66} = 9.$

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7. $-5x^2 + 5.2x^2 + 1.5x^2 - 0.7x^2$

$= (-5 + 5.2 + 1.5 - 0.7)x^2$

$= x^2$

$= (0.5)^2$

$= 0.25$

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08 $\dfrac{7ab + 8bc - 7ac}{abc + 1}$

$= \dfrac{7(5)(3) + 8(3)(4) - 7(5)(4)}{(5)(3)(4) + 1}$

$= \dfrac{105 + 96 - 140}{60 + 1} = \dfrac{61}{61} = 1$

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09 $x + y + z = a^4 + a^3 + a^2 - 6 + a^2 - 2a^3 - 2 + 3a +$
$8 - 3a - 2a^2 + a^3$

$= a^4$

10. $0-(-3x^3+2x^2-11x+5)$

$= 3x^3-2x^2+11x-5$

11 a) $(2x+3y)(2x-3y)$

$=(2x)^2-(3y)^2$

$=4x^2-9y^2$

b) $(5xy)(2xy+x+y+z)$

$=10x^2y^2+5x^2y+5xy^2+5xyz$

12 a) $(2a+3b)^2+(3a-2b)^2$

$=4a^2+12ab+9b^2+9a^2-12ab+4b^2$

$=13a^2+13b^2$

$=13(a^2+b^2)$

b) $\left(x+\frac{1}{x}\right)^2+\left(x-\frac{1}{x}\right)^2 = 2\left(x^2+\frac{1}{x^2}\right)$

13 $(1-x-x^2)(2x^2-1)+(x-x^2)$

$=(1-6-6^2)(2\cdot 6^2-1)+(6-6^2)$

$=(-41)(71)+(-30)$

$=-2941$

14 (i) $A \times B \times C$

$=(2x+3)\times[(3x-5)\times(x^2+3x-1)]$

$=(2x+3)\times[3x^3+9x^2-3x-5x^2-15x+5]$

$= (2x+3)\left[3x^3+4x^2-18x+5\right]$

$= 6x^4+8x^3-36x+10x+9x^3+12x^2-54x+15$

$= 6x^4+17x^3-100x+15$

$(A \times B) \times C = 6x^4+17x^3-100x+15$.

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15 $(2x-y)(2y+x)$

$= 4xy+2x^2-2y^2-xy$

$= 2x^2+3xy-2y^2$

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16. (i) $(a+b)^2 = (a+b)(a+b)$

$= a^2+ab+ba+b^2$

$= a^2+2ab+b^2$

(ii) $(a+b)(a-b) = a^2-ab+ba-b^2$

$= a^2-b^2$.

17. $y^2-y+6)\ 4y^5+5y^4+11y^3-y^2+y+6\ (4y^3+9y^2+45y-10$

$-4y^5+4y^4-24y^3$

$9y^4+36y^3-y^2$

$-9y^4+9y^3-54y^2$

$45y^3-55y^2+y$

$-45y^3+45y^2-270y$

$-10y^2-269y+6$

$-10y^2+10y-60$

$-279y+66$

(16)

$\therefore$ Quotient $= 4y^3+9y^2+45y-10$

Remainder $= -279y+66$

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18. $3x+4)\ 6x^2+29x+28\ (2x+7$

$-6x^2+8x$

$21x+28$

$-21x+28$

0

Since Remainder is 0, $3x+4$ is a factor of $6x^2+29x+28$

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19 $f(x) = 9x^3 - 12x^2 - 6x + 6$

Remainder $f\left(\frac{-2}{3}\right)$

$f\left(-\frac{2}{3}\right) = 9\left(\frac{-2}{3}\right)^3 - 12\left(\frac{-2}{3}\right)^2 - 6\left(\frac{-2}{3}\right) + 6$

$= \frac{-8}{3} - \frac{16}{9} + 4 + 6$

$= \frac{50}{9}$

EXPLORERS : LEVEL - III

01. $f(x) = x^2 - 2x$

$x^2 - 2x = 0$

$\Rightarrow x(x-2) = 0$

$\Rightarrow x = 0$ or 2

∴ The zeroes are 0 & 2

Ans: A, C

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02 $x + \frac{1}{x}$, $x - \frac{1}{x}$, $x^2 + \frac{1}{x} + 2$ are not polynomials

Ans: A, C, D

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03 x, y, z are variables

Ans: A, B, C

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04 $\left(3x - \frac{4}{5}y^2x\right) \times \frac{1}{2}xy$

$= 3x \times \frac{1}{2}xy - \frac{2}{5}y^3x^2$

$= \frac{3}{2}x^2y - \frac{2}{5}y^3x^2$

$= x^2y\left(\frac{3}{2} - \frac{2}{5}y^2\right)$

Ans: B, C

05 $2x-2)\ 8x^3+6x^2-4x-12\ (4x^2+7x+5$ (18)

$8x^3-8x^2$

$14x^2-4x$

$14x^2-14x$

$10x-12$

$10x-10$

-2

∴ Quotient $= 4x^2+7x+5$

Remainder $= -2$

Ans: C, B

II

1) (i) $7x^2-3x+4 \rightarrow$ Degree $= 2$

(ii) $2x^2+3xy^2+4y^2 \rightarrow$ Degree $= 3$

(iii) $8x^2+9xy+7y^2 \rightarrow$ Degree $= 2$

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2 (i) $4x+2y=10$

$\Rightarrow 2x+y=5$

(ii) $4x+2y=10 \Rightarrow 2x+y=5$

S.O.B.S S.O.B.S

$\Rightarrow 4x^2+4xy+y^2=25$

$\Rightarrow 8x^2+8xy+2y^2=50$

(iii) $16x^2+16xy+4y^2=100$.

III Matrix Matching

1 (i) Additive inverse of $-x^2+x-2$ is x^2-x+2

(ii) $x^2+x-2 \rightarrow -x^2-x+2$

(iii) $-x^2-x+2 \rightarrow x^2+x-2$

(iv) $-x^2-x-2 \rightarrow x^2+x+2$

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2 (i) $x^2-4 \div (x-2)$

$x=2$	1	0	−4
	0	2	4
	1	2	0 → Remainder

(ii) $x^3-4x^2+16 \div (x-2)$

$x=2$	1	−4	0	16
	0	2	−4	−8
	1	−2	−4	8 → Remainder

(iii) $x^2+4x+4 \div (x-2)$

$x=2$	1	4	4
	0	2	12
	1	6	16 → Remainder

(iv)

$x=2$	1	0	−4	4
	0	2	4	0
	1	2	0	4 → Remainder

⇒ THE END ⇐