



NUMBER SYSTEM



Learning Objectives :

- * Number system and types of number system
- * Properties of number system and Approximations
- * Number line
- * Divisibility Rules
- * Fractions and Decimal
- * Factors & Multiples
- * Prime factorization and HCF and LCM

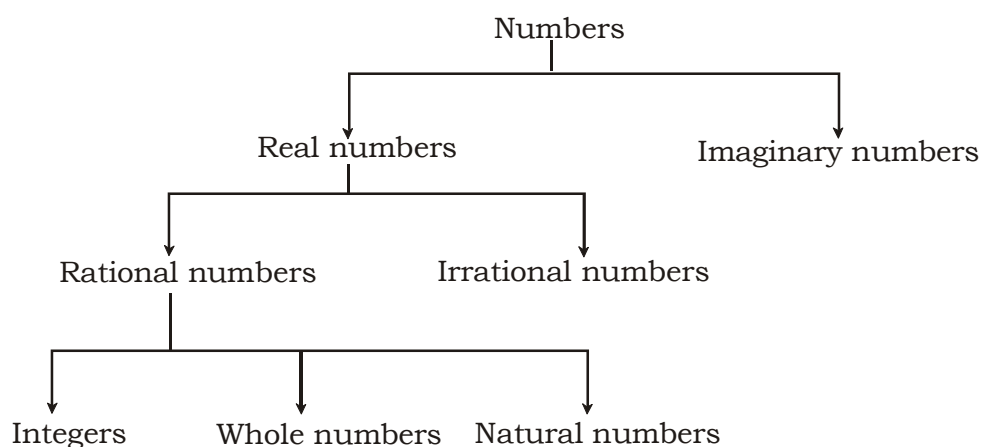
Real time applications :

- * In our daily life we interact with numbers such as making calculations, currency, stock exchange, sports etc.
- * Conversion of one system of units to other such as meter to centimeters kgs to pounds etc.
- * Computers understands the binary system of numbers to process data
- * most popular usage of numbers are Hindu-arabic , roman numbers

Definition of number system :

The system of numeration employing the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 is known as number system.

Explanation of number system by **tree method**



§§ Digits :

The figures 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 that are used to express numbers are called digits. Out of these, 0 is called **insignificant** digit where as the others are called **significant** digits

§§ Numerals : A group of digits denoting a number is called a numeral.

¶¶ Face value and place value:

Face - value of a digit in a number is the digit it self.

Place - value of a digit in a number = face value x value of the place it occupies in the place - value chart.

Ex. In the number 75629

The face - value of 6 is 6

The place - value of 6 is $6 \times 100 = 600$.

§§ Successor & Predecessor:

The **successor** of a number is 1 more than the number and **predecessor** of a number is 1 less than then number.

Ex. The successor of the number 6 is $6 + 1 = 7$.

The predecessor of the number 6 is $6 - 1 = 5$.

§§ Types of Numbers

1. Natural numbers : Numbers which we use for counting the objects are known as natural numbers. They are denoted by 'N'

$$N = \{ 1, 2, 3, 4, 5, \dots \}$$

¶¶ Properties of Natural numbers:

- 1 is a natural number.

Every natural number n has a unique successor $n^* = n + 1$. For example $5^* = 6$.

- 1 not a successor of any element.

Different natural numbers have different successors.

2. Whole numbers : All counting numbers together with zero form the set of whole numbers.

Thus.

(i) 0 is the only whole number which is not a natural number.

(ii) Every natural number is a whose number.

Whole numbers are denoted by 'W'

$$W = \{ 0, 1, 2, 3, 4, 5, \dots \}$$

3. Integers : All natural numbers, 0 and negatives of counting numbers

i.e., $\{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$ together form the set of integers.

(i) Positive Integers : $\{ 1, 2, 3, 4, \dots \}$ is the set of all positive integers.

(ii) Negative Integers : $\{ -1, -2, -3, \dots \}$ is the set of all negative integers.

(iii) Non-Positive and Non-Negative Integers : 0 is neither positive nor negative.

So $\{0, 1, 2, 3, \dots\}$ represents the set of non-negative integers, while $\{0, -1, -2, -3, \dots\}$ represents the set of non-positive integers.

Properties of integers:

(i) For addition ,

(ii) For multiplication ,

i) Let a, b, c are integers. For **addition**, the following properties hold good.

1. Closure property: The sum of two integers is always an integer.

If a, b are integers, then $a + b$ is an integer.

2. Commutative law : $a + b = b + a$

3. Associative law : $a + (b + c) = (a + b) + c$

4. Existence of identity : $a + 0 = 0 + a = a$. **0** is called additive identity.

5. Additive inverse : For any integer a , we have $a + (-a) = (-a) + a = 0$
 $-a$ is called additive inverse of a .

ii) Let a, b, c are integers. For **multiplication**, the following properties hold good.

1. Closure property : The product of two integers is always an integer.

If a, b are integers, then $a \times b$ is an integer.

2. Commutative law : $a \times b = b \times a$

3. Associative law : $a \times (b \times c) = (a \times b) \times c$

4. Existence of identity : $a \times 1 = 1 \times a = a$. **1** is called multiplicative identity.

5. Distributive law : $a \times (b + c) = a \times b + a \times c$. [Multiplication distributes over addition]
 $(a + b) \times c = a \times c + b \times c$.

§§ Factor : A factor of a number is an exact divisor of the number.

Ex. 1, 2, 3, 6 divides '6' exactly

\therefore 1, 2, 3, 6 are called factors or divisors of the number '6'

§§ Multiple : The products we get when a number is multiplied by the numbers 1, 2, 3, 4,..... are called the multiples of the given number.

Ex. The numbers 3, 6, 9, 12, 15, 18,..... are called the multiples of '3'.

Properties of Factors and multiple :

i) '1' is a factor of every number.

ii) Every **non-zero** number is a factor of **itself**.

iii) The factor of a non-zero number is always less than or equal to the number. The **greatest** factor of a **non-zero** number is the number **itself**.

- iv) The number of factors of a number are finite.
- v) Every number is a multiple of itself.
- vi) Every number is a multiple of 1.
- vii) Every multiple of the number is either greater than or equal to that number.
- viii) The multiples of a number are infinite.
- ix) If 'a' is a factor of b and 'b' is a factor of 'a', then $a = b$.

§§ **Prime number :** A number other than '1' is called a prime number if it is divisible only by 1 and itself. (or)

A number greater than one is called prime number if it has exactly two factors, namely 1 and it self.

Ex : 2, 3, 5, 7, 11, 13, 17, 19,.....

§§ **Composite numbers :** A number other than '1', which is not a prime number is called a composite number.

Ex : 4, 6, 8, 9, 12, 15,

Note : '2' is only even prime number

§§ **Even Number :** The number which is divisible by 2 is known as an even number.

Ex : 2, 4, 8, 12, 24,.....

It is also of the form $2n$ (where n = whole number)

§§ **Odd number :** The number which is not divisible by 2 is known as an odd number.

Ex : 3, 9, 11, 17,.....

§§ **Consecutive numbers :** A series of numbers in which each is greater than that which precedes it by 1, are called consecutive numbers.

Ex : 4, 5, 6 or 13, 14, 15 or 202, 203, 204

Ex: 6 and 8, 10 and 12, 24 and 26 etc. are the consecutive even numbers.

Ex: 5 and 7, 11 and 13, 29 and 31 etc. are the consecutive odd numbers.

§§ **Twin Primes :** A pair of prime numbers that differ by 2 are known as Twin primes.

Pairs of twin primes between 1 and 100 are.

(3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61) and (71, 73)

§§ **Prime Triplet :** A set of three consecutive prime numbers differing by 2 is called a prime triplet.

The only prime triplet so far known is (3, 5, 7)

§§ **Co - primes :** Two numbers are said to be co-prime if they do not have a common factor other than 1. [(or) If H.C.F of two numbers is 1, then they are co-prime or relatively prime].

For example : (2, 3), (3, 4), (4, 5), (4, 9), (8, 15) are pairs of co-primes.

Note :

- i) Number 1 is neither prime nor composite.
- ii) 2 is the only even prime number
- iii) 2 is the smallest prime number
- iv) Two prime numbers are always co - prime.
- v) Two co-primes need not be both prime numbers.
- vi) Composite numbers need not be even. 9 is the smallest odd composite number.
- vii) 90, 91, 92, 93, 94, 95, 96 are seven consecutive numbers, less than 100, which are all composite, having no prime number between them.
- viii) There is no number which has no factors.

❖ **Tests for divisibility of numbers**

1. Divisibility by 2

Rule : Any number, the last digit(unit digit) of which is either even or 0, is divisible by 2.

2. Divisibility by 3

Rule : If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

3. Divisibility by 4

Rule : If the last two digits of a number is divisible by 4, the number is divisible by 4.

4. Divisibility by 5

Rule : If a number ends in 5 or 0, the number is divisible by 5.

5. Divisibility by 6

Rule : If a number is divisible by both 3 and 2, the number is also divisible by 6.

6. Divisibility by 8

Rule : If the last three digits of a number is divisible by 8, the number is also divisible by 8.

7. Divisibility by 9

Rule : If the sum of all the digits of a number is divisible by 9, then the number is divisible by 9.

8. Divisibility by 10

Rule : Any number which ends with zero is divisible by 10.

9. Divisibility by 11

Rule : If the sums of digits at odd and even places are equal or differ by a number divisible by 11, then the number is divisible by 11.

10. Divisibility by 12

Rule : Any number which is divisible by both 4 and 3, is also divisible by 12.

§§ **Prime factorisation :**

When a number is expressed as a product of prime numbers, we call it as the prime factorisation of that number. **or**

A factorisation in which every factor is prime, is called the prime factorization of the number.

Ex : The prime factorisation of the number 30 is

$$30 = 2 \times 3 \times 5$$

§§ Fundamental theorem of Arithmetic or Prime factorisation property :

Every number greater than '1' has exactly one prime factorisation.

¶¶ To find the number of different divisors of a composite number :

Rule : Find the prime factors of the number and increase the index of each factor by 1. The continued product of increased indices will give the result including unity and the number itself.

Ex : The number of divisors of 40, except unity, is $40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$

$$\text{Total number of divisors} = (3 + 1)(1 + 1) = 8$$

$$\therefore \text{Number of divisors excluding unity} = 8 - 1 = 7.$$

§§ Common factor : A natural number is said to be a common factor of two or more numbers, if it is a factor of each of them.

§§ Common multiple : A natural number is said to be a common multiple of two or more numbers, if it is a multiple of each of them.

§§ Highest Common Factor (HCF) : The largest of the common factors of two or more numbers is called their highest common factor (H.C.F.).

Ex : Common factors of 12 and 16 are 1, 2, 4

Largest (Highest) common factor of 12 and 16 is 4.

¶¶ To find the HCF of more than two numbers :

Rule : Find the HCF of any two of the numbers and then find the HCF of this HCF and the third number and so on. The last HCF will be the required HCF.

§§ Least common multiple (LCM) : The smallest of the common multiples of two or more numbers is called their lowest common multiple (L.C.M.).

Ex : Multiples of 12 : 12, 24, 36, 48, 60, 72, 84, 96,.....

Multiples of 16 : 16, 32, 48, 64, 80, 96, 112,.....

Common multiples of 12 and 16 : 48, 96,.....

Smallest common multiple of 12 and 16 is 48

¶¶ To find the LCM of two or more given numbers :

Rule : Resolve the given numbers into their prime factors and then find the product of the highest power of all the factors that occur in the given numbers. This product will be the LCM.

¶¶ Relationship between HCF and FCM

Let us consider the two numbers 32 and 48.

LCM of 32 and 48

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\therefore \text{LCM} = 96$$

HCF of 32 and 48 = 16

$$\text{Now LCM} \times \text{HCF} = 16 \times 96 = 1536$$

and 1st number \times 2nd number

$$= 32 \times 48 = 1536$$

Therefore, We can say that $\text{LCM} \times \text{HCF} = 1^{\text{st}} \text{ number} \times 2^{\text{nd}} \text{ number}$

§§ Division algorithm :

Dividend = (Divisor \times Quotient) + Remainder.

From the division Algorithm, we get the following relationships also.

(i) Divisor = (Dividend – Remainder) \div Quotient

(ii) Quotient = (Dividend – Remainder) \div Divisor

Note :

1. 1 is the only whole number which divided by itself gives a quotient equal to itself.
2. Division is repeated subtraction of the same number.
3. Division by zero is not defined.
4. Any whole number divided by 1 gives the quotient as the number itself.
5. Any non-zero whole number, divided by itself gives the quotient 1.
6. Zero divided by any non-zero whole number gives the quotient zero.

§§ Fractions : The numbers of the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$, are known as fractions. (or) A quantity which expresses a part of the whole numbers is called a fraction.

§§ Decimal Fraction : A fraction whose denominator is 10, 100, 1000 etc., is called a decimal fraction.

Ex : $\frac{3}{10}, \frac{7}{100}, \frac{13}{1000}$, etc.

§§ Vulgar Fraction : A fraction whose denominator is a whole number, other than 10, 100, 1000 etc., is called a vulgar fraction.

Ex : $\frac{3}{4}, \frac{21}{109}$, etc.

§§ Proper Fraction : If the numerator of a fraction is less than its denominator, then such a fraction is called a proper fraction.

§§ Improper Fraction : If the numerator of a fraction is greater than or equal to its denominator, then such a fraction is called an improper fraction.

§§ Mixed Numbers : A number which can be expressed as the sum of a natural number and a proper fraction is called a mixed number.

Ex : $1\frac{2}{3}, 7\frac{4}{11}$

Note :

(i) A fraction $\frac{a}{b}$ is said to be irreducible (or in lowest terms) if H.C.F. of a and b is 1.

(ii) Reciprocal of a non-zero fraction $\frac{a}{b}$ is $\frac{b}{a}$

§§ Equivalent Fractions :

When we multiply the numerator and the denominator of a given fraction by the same number, we get its equivalent fraction.

¶ An important Property of Equivalent Fractions :

If two fractions are equivalent, the product of numerator of the first and denominator of the second is equal to the product of the denominator of the first and numerator of the second.

Note :

i) When the numerator and the denominator of the fractions increase by a constant value, the last fraction is the biggest.

ii) The fraction whose numerator after cross - multiplication gives the greater value is greater.

iii) Hence, a fraction is said to be in its lowest terms or in simplest form if the H.C.F its numerator and denominator is 1.

§§ Rational number : A number which can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number.

Ex : $2, 3, \frac{4}{3}$ etc .

Note : If x and y are two distinct rational numbers then $\frac{x+y}{2}$ is a rational number between them.

§§ Recurring Decimals :

A decimal in which a figure or set of figures is repeated continuously is called a recurring or periodic or circulating decimal. The repeated figures or set of figures is called the period of the decimal.

For example : $\frac{1}{3} = 0.333...$

§§ Pure Recurring Decimal : A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal. For example, $0.\overline{142857}$ is a pure recurring decimal.

§§ Mixed Recurring Decimal : A decimal fraction in which some figures do not recur, is called a mixed recurring decimal. For example: $0.29\overline{54}$ is a mixed recurring decimal.

§§ Perfect number : A number 'n' is said to be perfect if the sum of all its factors is equal to '2n'. (or) A number 'n' is said to be perfect if the sum of all its factors (Excluding 'n' itself) is equal to 'n'.

Ex : 6, 28

The factors of 6 are 1, 2, 3 and 6

$$1 + 2 + 3 = 6 \text{ (or) } 1 + 2 + 3 + 6 = 12 = 2 \times 6$$

EXAMPLES

✓ Example 1 :

Simplify (i) 567958×99999

Solution :

We know that the distributive law

$$(i) a \times (b + c) = (a \times b) + (a \times c)$$

$$(ii) a \times (b - c) = (a \times b) - (a \times c), a, b, c \in Z$$

$$\begin{aligned} 567958 \times 99999 &= 567958 \times (100000 - 1) \\ &= (567958 \times 100000) - (567958 \times 1) \\ &= 56795800000 - 567958 \\ &= 56795232042 \end{aligned}$$

ANS : 56795232042

✓ Example 2 :

Which digits should come in place of * and ¥ if the number $62674 * ¥$ is divisible by both 8 and 5 ?

Solution :

Since the given number is divisible by 5,

So 0 or 5 must come in place of ¥

But a number ending with 5 is never divisible by 8. So, 0 will replace ¥.

Now the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4 or 0.

Hence, the digit * is either 0 or 4 and the digit ¥ is 0.

ANS : * = 0 or 4, ¥ = 0

✓

Example 3 :

Find the number of proper factors of the composite number 84.

Solution :

$$84 = 2 \times 42$$

$$= 2 \times 2 \times 21$$

$$= 2 \times 2 \times 3 \times 7 = 2^2 \times 3^1 \times 7^1$$

Total number of factors

$$(2+1)(1+1)(1+1)$$

$$= 3 \times 2 \times 2$$

$$= 12$$

$$\therefore \text{The number of proper factors is } = 12 - 1 = 11$$

ANS : 11

✓

Example 4 :

Given $x * y = \frac{2x+y}{y}$, find

(i) $3 * 4$

(ii) P such that $P * 2 = 7$

(iii) Is * commutative. Give reasons

Solution :

Given $x * y = \frac{2x+y}{y}$

(i) $3 * 4 = \frac{2 \times 3 + 4}{4} = \frac{6 + 4}{4} = \frac{10}{4} = \frac{5}{2}$

(ii) $P * 2 = 7$

$$\Rightarrow \frac{2 \times P + 2}{2} = 7$$

$$\Rightarrow 2P + 2 = 14$$

$$\Rightarrow 2P = 12$$

$$\Rightarrow P = 6$$

$$\therefore P = 6$$

$$(iii) \quad x * y = \frac{2x+y}{y}, \quad y * x = \frac{2y+x}{x}$$

$$x * y \neq y * x, \text{ for all } x \neq y$$

$\therefore x * y$ is not commutative

ANS : i) 5/2 ; ii) 6 III) NO

Example 5 :

Show that $\frac{1}{2} \times \left(-\frac{1}{3} + \frac{1}{2}\right)$ is a rational number between $-\frac{1}{3}$ and $\frac{1}{2}$.

Solution :

$$\frac{1}{2} \times \left(-\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{2} \times \left(\frac{-2+3}{6}\right) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}, \text{ which is a rational number.}$$

Let us now arrange $-\frac{1}{3}, \frac{1}{2}$ and $\frac{1}{12}$ in ascending order of magnitude.

The LCM of 3, 2 and 12 is 12

So, we can write

$$-\frac{1}{3} = \frac{-1 \times 4}{3 \times 4} = \frac{-4}{12}, \quad \frac{1}{2} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12}$$

and the third number = $\frac{1}{12}$

$$\text{clearly } \frac{-4}{12} < \frac{1}{12} < \frac{6}{12} \quad \text{So, } -\frac{1}{3} < \frac{1}{12} < \frac{1}{2}$$

This shows that $\frac{1}{12}$ lies between $-\frac{1}{3}$ and $\frac{1}{2}$.

ANS : Proved

Example 6 :

If the symbol \oplus stands for "half the first number and add the double of the second number"

find the value of (i) $\frac{4}{7} \oplus \frac{3}{4}$ (ii) $\frac{6}{11} \oplus \left(\frac{8}{15} \oplus \frac{5}{6}\right)$ and $\left(\frac{6}{11} \oplus \frac{8}{15}\right) \oplus \frac{5}{6}$

Hence, state whether \oplus is associative

Solution :

Given the symbol \oplus stands for "half the first number and add the double of the second number"

$$(i) \quad \frac{4}{7} \oplus \frac{3}{4} = \frac{1}{2} \times \frac{4}{7} + 2 \times \frac{3}{4} = \frac{4}{14} + \frac{6}{4} = \frac{25}{14}$$

$$(ii) \quad \left(\frac{8}{15} \oplus \frac{5}{6}\right) = \frac{1}{2} \times \frac{8}{15} + 2 \times \frac{5}{6} = \frac{4}{15} + \frac{5}{3} = \frac{4+25}{15} = \frac{29}{15}$$

$$\begin{aligned}\text{Now } \frac{6}{11} \oplus \left(\frac{8}{15} \oplus \frac{5}{6} \right) &= \frac{6}{11} \oplus \frac{29}{15} = \frac{1}{2} \times \frac{6}{11} + 2 \times \frac{29}{15} = \frac{3}{11} + \frac{58}{15} \\ &= \frac{45 + 638}{165} = \frac{683}{165} \text{-----(1)}\end{aligned}$$

$$\text{Now } \frac{6}{11} \oplus \frac{8}{15} = \frac{1}{2} \times \frac{6}{11} + 2 \times \frac{8}{15} = \frac{3}{11} + \frac{16}{15} = \frac{45 + 176}{165} = \frac{221}{165} = \frac{11}{15}$$

$$\begin{aligned}\text{Now } \left(\frac{6}{11} \oplus \frac{8}{15} \right) \oplus \frac{5}{6} &= \frac{11}{15} \oplus \frac{5}{6} = \frac{1}{2} \times \frac{11}{15} + 2 \times \frac{5}{6} = \frac{11}{30} + \frac{5}{3} = \frac{11 + 50}{30} \\ \Rightarrow \left(\frac{6}{11} \oplus \frac{8}{15} \right) \oplus \frac{5}{6} &= \frac{61}{30} \text{-----(2)}\end{aligned}$$

From (1) and (2)

$$\frac{6}{11} \oplus \left(\frac{8}{15} \oplus \frac{5}{6} \right) \neq \left(\frac{6}{11} \oplus \frac{8}{15} \right) \oplus \frac{5}{6}$$

ANS : i) 25/14 ; ii) NO

$\therefore \oplus$ is not associative

✓ **Example 7 :**

Find the number which is nearest to 2009 and is exactly divisible by 6.

Solution :

On dividing 2009 by '6', we get 5 as remainder.

\therefore Number to be added to 2009 = (6-5) = 1

Hence, required number = 2009 + 1 = 2010

ANS :2010

✓ **Example 8 :**

Complete the following magic square by supplying the missing numbers such that the sum of the numbers along any of the horizontal lines, the vertical lines and the diagonals is the same number.

1		15	4
	11	10	
12	7		
13			16

Solution :

Sum of the numbers along one diagonal $13 + 7 + 10 + 4 = 34$

Therefore, sum of the numbers along any of the horizontal lines, vertical lines and the diagonals should be 34.

Now, sum of the numbers along 1st vertical line = 1 + missing number + 12 + 13 = 34

\Rightarrow Missing number = $34 - 26 = 8$

1		15	4
8	11	10	
12	7		
13			16

Sum of the numbers along 1st horizontal line

$$= 1 + \text{missing number} + 15 + 4 = 34$$

$$\Rightarrow \text{Missing number} = 34 - 20 = 14$$

1	14	15	4
8	11	10	
12	7		
13			16

Sum of the numbers along 2nd horizontal line

$$= 8 + 11 + 10 + \text{missing number} = 34$$

$$\Rightarrow \text{Missing number} = 34 - 29 = 5$$

1	14	15	4
8	11	10	5
12	7		
13			16

Sum of the numbers along 4th vertical line

$$= 4 + 5 + \text{missing number} + 16 = 34$$

$$\Rightarrow \text{Missing number} = 34 - 25 = 9$$

1	14	15	4
8	11	10	5
12	7		9
13			16

Sum of the numbers along 3rd horizontal line

$$= 12 + 7 + \text{missing number} + 9 = 34$$

$$\Rightarrow \text{Missing number} = 34 - 28 = 6$$

1	14	15	4
8	11	10	5
12	7	6	9
13			16

Sum of the numbers along 3rd vertical line

$$= 15 + 10 + 6 + \text{missing number} = 34$$

$$\Rightarrow \text{Missing number} = 34 - 31 = 3$$

1	14	15	4
8	11	10	5
12	7	6	9
13		3	16

Sum of the numbers along 4th horizontal line

$$= 13 + \text{missing number} + 3 + 16 = 34$$

⇒ Missing number = $34 - 32 = 2$

Hence, we have the following magic square :

1	14	15	4
8	11	10	5
12	7	6	9
13	2	3	16

Finally, we see that, sum of the numbers along other diagonal
 $= 1 + 11 + 6 + 16 = 34$.

ANS : 34

✓

Example 9 :

Find the L.C.M and H.C.F of 72, 108 and 210.

Solution :

$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

$$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$$

$$210 = 2 \times 3 \times 5 \times 7 = 2 \times 3 \times 5 \times 7$$

The prime numbers common to the given numbers are 2, 3

H.C.F = Product of least powers of 2 and 3

$$= 2 \times 3$$

$$= 6$$

∴ H.C.F of given numbers is 6

All the different prime factors of the given numbers are 2, 3, 5 and 7.

L.C.M = Product of highest powers of 2, 3, 5 and 7

$$= 2^3 \times 3^3 \times 5 \times 7$$

$$= 8 \times 27 \times 5 \times 7$$

$$= 7560$$

ANS : H.C.F = 6 , L.C .M =7560

∴ L.C.M of the given numbers is 7560

✓

Example 10 :

Arrange the fractions $\frac{17}{18}, \frac{31}{36}, \frac{43}{45}, \frac{59}{60}$ in the ascending order.

Solution :

L.C.M of 18, 36, 45 and 60 = 180

$$\text{Now } \frac{17}{18} = \frac{17 \times 10}{18 \times 10} = \frac{170}{180} ; \frac{31}{36} = \frac{31 \times 5}{36 \times 5} = \frac{155}{180} ; \frac{43}{45} = \frac{43 \times 4}{45 \times 4} = \frac{172}{180} \text{ and}$$

$$\frac{59}{60} = \frac{59 \times 3}{60 \times 3} = \frac{177}{180}$$

We have, $155 < 170 < 172 < 177$

$$\text{So, } \frac{155}{180} < \frac{170}{180} < \frac{172}{180} < \frac{177}{180}$$

$$\Rightarrow \frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$$

$$\text{ANS : } \frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$$

✓

Example 11 :

Find the smallest 4 digit number which is exactly divisible by 20, 24, 36 and 40.

Solution :

- Steps**
1. Find L.C.M. of 20, 24, 36 and 40
 2. Divide the smallest 4 digit number by the L.C.M of the given numbers.
 3. Subtract the remainder from the L.C.M
 4. Add the difference to the smallest 4 digit number 5 digits

(1) L.C.M

2	20, 24, 36, 40
2	10, 12, 18, 20
2	5, 6, 9, 10
3	5, 3, 9, 5
5	5, 1, 3, 5,
3	1, 1, 3, 1
	1, 1, 1,

$$\therefore L.C.M = 2 \times 2 \times 2 \times 3 \times 5 \times 3 = 360$$

(2)

360	1000	(2
	720	
	280	

(3) $360 - 280 = 80$

(4) $1000 + 80 = 1080$

\therefore The smallest 4 digit number which is exactly divisible by 20, 24, 36 and 40 is 1080.

✓

Example 12 :

Find the smallest number greater than 63,90 and 135 which when divided by 63,90 and 135 leaves 11 as remainder.

Solution :

Find the L.C.M of 63, 90 and 135

2	63, 90, 135
3	63, 45, 135
3	21, 15, 45
5	7, 5, 15
7	7, 1, 3
3	1, 1, 3
	1, 1, 1

$$\therefore \text{L.C.M} = 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 1890$$

ANS : 1901

\therefore The smallest number which when divided by 63, 90 and 135 leaves '6' as the remainder
 $= 1890 + 11 = 1901$

Note : If the number is not greater than 63, 90 and 135, then the smallest number is 11.

✓ **Example 13 :**

Find the greatest number which divides 110 and 125 leaving the remainders 2 and 5 respectively.

Solution :

The number leaves remainders 2 and 5 when it divides 110 and 125

So, it must divide $110 - 2$, $125 - 5$ i.e., 108, 120

Therefore, required number must be the H.C.F of 108 and 120.

$$108 = 2 \times 2 \times 3 \times 3 \times 3$$

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$\text{H.C.F of 108 and 120} = 2 \times 2 \times 3 = 12$$

Hence, the required number is 12.

ANS : 12

✓ **Example 14 :**

A number when divided by 899 gives a remainder 63. What remainder will be obtained by dividing the same number by 29 ?

Solution :

We have Dividend = (Divisor x Quotient) + Remainder

$$\text{Number} = 899 \times \text{Quotient} + 63$$

$$= 29 \times 31 \times \text{Quotient} + (2 \times 29) + 5$$

$$= 29[31 \times \text{Quotient} + 2] + 5$$

$$= 29 \times (\text{some Quotient}) + 5$$

ANS : 5

Therefore, the remainder obtained by dividing the number by 29 is clearly 5.

✓ **Example 15 :**

The product of two numbers is 7168 and their H C F is 16, find the possible pairs of numbers.

Solution :

The numbers must be multiples of their HCF. (\because The LCM of any two numbers is divisible by its HCF)

So, let the numbers be $16a$ and $16b$

Where a and b are two numbers prime to each other.

$$\therefore 16a \times 16b = 7168 \text{ or } ab = \frac{7168}{16 \times 16} = 28 \Rightarrow ab = 28$$

Now, the pairs of numbers whose product is 28 are 28,1 ; 14, 2 ; 7,4

Here 14 and 2 which are not prime to each other should be rejected.

Hence, the required numbers are

(i) $28 \times 16, 1 \times 16$ or 448, 16

(ii) $7 \times 16, 4 \times 16$ or 112, 64

\therefore The required numbers are 448, 16; or 112, 64

ANS : 448,16 ; 112,64

✓ **Example 16 :**

How many numbers between 200 and 400 are divisible by 3, 4 and 5 together?

Solution :

LCM of 3, 4 and 5 = 60

Number of numbers up to 200 which are divisible by 60 = $\frac{120}{60} = 3 + \frac{1}{3}$, i.e., 3

\therefore 3 numbers are divisible by 60 upto 200.

Number of numbers up to 400 which are divisible by 60 = $\frac{140}{60} = 6 + \frac{1}{3}$, i.e., 6

\therefore 6 numbers are divisible by 60 upto 400.

\therefore The required numbers = $6 - 3 = 3$.

ANS : 3

✓ **Example 17 :**

a) Find the least number which, when divided by 13, 15 and 19, leaves the remainders 2, 4 and 8 respectively. Can we find the specific solution ?

(b) Find the least number which when divided by 13, 15 and 19, leaves the remainders 1, 2 and 3 respectively. Can we find the solution ?

Solution :

a) Yes

This question can be solved because

$$13 - 2 = 15 - 4 = 19 - 8 = 11$$

Now, LCM of 13, 15, 19 = 3705

∴ The required least number = $3705 - 11 = 3694$

b) No. But why ?

Because $13 - 1 \neq 15 - 2 \neq 19 - 3$

ANS : a) Yes, 3694 ; b) NO

✓

Example 18 :

Find the greatest number which divides 62, 132 and 237 to leave the same remainder in each case.

Solution :

Required number = H.C.F of $(132 - 62)$, $(237 - 132)$ and $(237 - 62)$

= H.C.F of 70, 105 and 175 = 35

ANS : 35

∴ The greatest number which divides 62, 132 and 237 to leave the same remainder in each case is 35

✓

Example 19 :

Find the sum of the numbers between 300 and 400 such that when they are divided by 6, 9, and 12.

a) They leave no remainder, and

b) To leave remainder as 4 in each case.

Solution :

The L C M of 6, 9 and 12 = 36

a) Multiples of 36 which lie between 300 and 400 are 324, 360 and 396.

∴ The required sum = $324 + 360 + 396 = 1080$.

b) Here the remainder is '4' in each case

So, the numbers are $324 + 4$, $360 + 4$, $396 + 4$

328, 364, 400

The number 400 does not lie between 300 and 400,

So it is not acceptable.

ANS : a) 1080 ; b) 692

∴ The required sum = $328 + 364 = 692$.

✓

Example 20 :

Last year, Kirthi's age was a multiple of 7. This year, Kirthi's age is a multiple of 5. In how many years will Kirthi be 26 years old?

Solution:

Let 'x' be the present age of Kirthi.

As per the data, x is a multiple of 5, less than 26 and leaves a remainder 1 when divided by 7.

Let us consider all the multiples of 5 less than 26.

They are 5, 10, 15, 20, 25.

ANS : 11

Among them only 15 has the property that it leaves the remainder when divided by 7

Hence, the present age of Kirthi is 15 years. So she will become 26 in 11 years.

✓ **Example 21 :**

John and Mary play a two-person game in which the winner gains 2 points and the loser loses 1 point. If John won exactly 3 games and Mary had a final score of 7 points, how many games did they play?

Solution:

Let John and Mary play 'n' games.

Then Mary won $n-3$ games and lost 3 games.

So, the points gained by Mary is $=2(n-3) - 3 = 2n-9$.

Given that, $2n-9=7$. Hence, $2n = 7 + 9 = 16$

Therefore, $n = 8$. So John and Mary played 8 games.

ANS : 08

✓ **Example 22 :**

The number 315 can be written as the product of two odd integers each greater than 1. In how many ways can this be done?

Solution:

The prime factorization of $315 = 3 \times 3 \times 5 \times 7$.

We have to express this as product of two odd integers, each greater than 1.

This can be done in the following ways:

1. $315 = (3) \times (3 \times 5 \times 7) = 3 \times 105$

2. $315 = (5) \times (3 \times 3 \times 7) = 5 \times 63$

3. $315 = (7) \times (3 \times 3 \times 5) = 7 \times 45$

4. $315 = (3 \times 3) \times (5 \times 7) = 9 \times 35$

5. $315 = (3 \times 5) \times (3 \times 7) = 15 \times 21$

Hence, in 5 ways 315 can be expressed as product of two odd integers each greater than 1.

ANS : 05

✓ **Example 23:**

The sum of three consecutive integers is 90. What is the largest of the three integers?

Solution:

Let n be the largest of the three consecutive integers.

Then the three consecutive numbers are $n-2$, $n-1$ and n .

Given that, $(n-2) + (n-1) + n = 90$

$$3n - 3 = 90$$

$$3n = 90 + 3 = 93$$

Therefore $n = 31$. Hence the largest of the given numbers is 31.

ANS : 31

✓ **Example 24 :**

Each of the integers 226 and 318 have digits whose product is 24. How many three-digit positive integers have digits whose product is 24?

Solution:

We can write 24 as product of 3 digits in the following way:

1. $24 = 1 \times 3 \times 8$. This will give 6 numbers 138, 183, 318, 381, 813, 831.
2. $24 = 1 \times 4 \times 6$ This will give 6 numbers 146, 164, 416, 461, 614, 641.
3. $24 = 2 \times 3 \times 4$ This will give 6 numbers 234, 243, 324, 342, 423, 432.
4. $24 = 2 \times 2 \times 6$ This will give 3 numbers 226, 262, 622.

Hence we can find 21 three digit numbers whose product of the digits is 24.

ANS : 21

✓ **Example 25 :**

In the addition of three - digit numbers shown, the letters x and y represent different digits.

$$\begin{array}{r} 3 \ x \ y \\ + \ y \ x \ 3 \\ \hline 1 \ x \ 1 \ x \end{array}$$

Find the value of $y - x$.

Solution:

Observe the addition process in the ten's digit.

Carry from units place $+ x + x = 1 +$ carry to hundredths place.

As the result in the tens place is 1, x must be either 0 or 5 and the carry from the units place must be 1.

This means that $y + 3 = 10 + x$. As y is a digit, the largest value of $y + 3 = 9 + 3 = 12$.

Hence, $x = 0$ and so $y = 7$.

Therefore, $y - x = 7$.

The addition process in the given problem is:

$$\begin{array}{r} 307 \\ + 703 \\ \hline 1010 \end{array}$$

ANS : 7

✓ **Example 26 :**

Let N be the smallest four digit number such that the three digit number obtained by removing the leftmost digit is one ninth of the original number. What is N ?

Solution:

Let $N = abcd = 1000a + 100b + 10c + d$ where a, b, c, d are digits of N .

Given that $N = 9(bcd)$

$$1000a + 100b + 10c + d = 9(100b + 10c + d)$$

$$1000a = 8(100b + 10c + d) = 8(bcd)$$

As we need N to be least, a should be as small as possible.

Notice that, $1000(1) = 8(125)$ gives the least value for a

$$(a = 1)$$

Hence, $a = 1$ and $bcd = 125$.

Therefore the given number is $N = 1125$

ANS : 1125

✓

Example 27:

In a certain month, three of the Sundays have dates that are even numbers. What is the tenth day of this Month?

Solution:

The month may have 30 or 31 days.

That is, 4 weeks and 2 or 3 days.

So the month contain 5 Sundays, if there is a Sunday on 1st or 2nd for 30 day month and a Sunday on 1st or 2nd or 3rd for 31 day month.

Given that there are 3 Sundays on even number dates.

This means that 2nd must be Sunday.

Hence, the 10th day is Monday as the 9th is again a Sunday.

ANS : Monday

TEACHING TASK

I) MCQ's with single correct answer type :

1. What integer is closest in value to $7 \times \frac{3}{4}$?
a) 5 b) 7 c) 8 d) 3
2. The value of the expression $5^2 - 4^2 + 3^2$ is
a) 16 b) 18 c) 17 d) 19
3. The value of $2^5 + 5$ is ____
a) 34 b) 35 c) 36 d) 37
4. How many prime numbers are there between 20 and 30?
a) 2 b) 1 c) 3 d) none
5. What number should be placed in the box to make $\frac{6 + \square}{20} = \frac{1}{2}$?

- a) 8 b) 3 c) 4 d) 5
6. Four friends equally shared $\frac{3}{4}$ of a pizza, which was left over after a party. What fraction of a whole pizza did each friend get ?
a) $\frac{2}{15}$ b) $\frac{3}{16}$ c) $\frac{2}{9}$ d) $\frac{3}{14}$
7. If $x * y = x + y^2$, then $2 * 3$ equals____
a) 11 b) 10 c) 12 d) 9
8. If $x * y = x^2 + y^2 - xy$, then find the value of $9 * 11$.
a) 100 b) 101 c) 102 d) 103
9. If $a * b = \frac{ab}{a+b}$, find the value of $3 * (3 * 1)$.
a) $\frac{2}{3}$ b) $\frac{3}{4}$ c) $\frac{1}{5}$ d) $\frac{3}{5}$
10. If $a * b = 2a - 3b + ab$, then find the value of $(3 * 5) + (5 * 3)$.
a) 11 b) 22 c) 33 d) 1

II) MCQ's with multi correct answer type :

- ◆ This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options
11. Simplify $186 \times 186 + 159 \times 159 - 2 \times 186 \times 159$.
a) 629 b) 529 c) 1258 d) 1058
12. If a machine produces 150 items in one minute, how many would it produce in 10 seconds?
a) 52 b) 25 c) 5 multiple d) 26
13. Two numbers have a sum of 32. If one of the numbers is -36, what is the other number?
a) 58 b) $\frac{136}{2}$ c) 24 d) 68
14. A number is placed in the box to make the following statement true :
 $8 + \frac{7}{\square} + \frac{3}{1000} = 8.073$. What is this number ?
a) 100 b) 10 multiple c) 10^2 d) none
15. Write all common factors of 18, 30 and 45.
a) 1 b) 3 c) 5 d) 2

III) Matrix matching type:

- ◆ This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column-I** have to be matched with statements (p, q, r, s) in **Column-II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4×4 matrix should be as follows:

Column - I	Column - II
16. i) Additive identity	p) 3
ii) Multiplicative identity of +2	q) 0
iii) successor of additive identity	r) 2
iv) successor of Multiplicative identity	s) 1
A) i-q, ii-p, iii-q, iv-s	B) i-q, ii-s, iii-s, iv-r
C) i-r, ii-p, iii-s, iv-q	D) i-q, ii-q, iii-s, iv-s

III) **Solve the following :**

1. The value of $\frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}$ is ____

2. Find the value of $4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}}$.

3. How many even whole numbers lie between 3^2 and 3^3 ?

4. If $P = 1000$ and $Q = 0.01$, which of the following calculations gives the largest result ?

- a) $P + Q$ b) $P \times Q$ c) $\frac{P}{Q}$ d) $\frac{Q}{P}$

5. Find the number of divisors of the numbers (i) 66 (ii) 100 (iii) 150

6. Find the number of proper factors of the numbers (i) 50 (ii) 102 (iii) 72

7. Which of the following numbers is divisible by 3

(i) 541326 (ii) 5967013.

8. What least value must be assigned to the digit * so that the number 197^*5462 is divisible by 9 ?

9. Which of the following numbers is divisible by 4 ? (i) 67920594 (ii) 618404572

10. How many of the following numbers are divisible by 3 but not by 9 ?

2133, 2343, 3474, 4131, 5286, 5340, 6336, 7347, 8115, 9276.

11. Show that 4832718 is divisible by 11.

12. 5^*2 is a three digit number with * as a missing digit. If the number is divisible by 6, then find all the possible values of the missing digit.

13. If the number 357^*25^* is divisible by both 3 and 5, then find the missing digits in the units

14. What least value must be given to the digit * so that the number 91876*2 is divisible by 8 ?
15. If the number $654x3210$ is divisible by '6', then find all possible values of 'x'.
16. Find the sum of the prime factors of the number 108.
17. Find the number of factors of the number 10.
18. Find the smallest number having five different prime factors.
19. Is whole numbers satisfies the closure, commutative and associative properties under subtraction ? Give an example to each one ?
20. Find the smallest fraction among the fractions $\frac{6}{7}$, $\frac{5}{14}$ and $\frac{10}{21}$.
21. Arrange the following in ascending order
 - a) $\frac{5}{8}$, $\frac{7}{12}$, $\frac{11}{16}$
 - b) $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{12}$, $\frac{11}{24}$
22. Arrange the following in descending order
 - a) $\frac{5}{6}$, $\frac{7}{12}$, $\frac{11}{16}$
 - d) $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{12}$, $\frac{11}{24}$
23. Find the difference between largest and smallest of the fractions $\frac{11}{16}$, $\frac{6}{11}$ and $\frac{3}{8}$.
24. Write any four rational numbers which does not lie in between the rational numbers $\frac{3}{5}$ and $\frac{4}{3}$.
25. Write any four rational numbers which are in between $\frac{3}{5}$ and $\frac{4}{3}$.
26. Ramu bought bananas for Rs. $5\frac{7}{8}$, oranges for Rs. $3\frac{1}{2}$ and apples for Rs. $3\frac{1}{16}$. Find how much did she spend ?
27. Which is greater, the sum of $\frac{1}{2}$ and $\frac{1}{4}$ or the difference of 1 and $\frac{1}{2}$?
28. On dividing 46,570 by 321, we get a remainder 25. Find the quotient.

KEY

I) 1) a 2) b 3) d 4) a 5) c 6) b 7) a 8) d 9) d 10) b
II) 11) a,c 12) b,c 13) b,d 14) a,b 15) a,b III) 16) b

- IV) 1) $3/5$ 2) 9 3) c 4) 8, 9, 12 5) 6, 8, 12 7) 2 9) 5
 10) (2,5,8) 12) (2, 0), (5,0), (0,5), (3, 5), (6, 5), (9, 5) 13) 3
 14) 0, 3, 6, 9 15) 13 16) 4 17) 2310 19) $\frac{5}{14}$
 20) (i) $\frac{7}{12}, \frac{11}{16}, \frac{5}{8}$ (ii) $\frac{11}{24}, \frac{5}{9}, \frac{7}{12}, \frac{2}{3}$ 21) $\frac{95}{176}$ 22) $\frac{6}{15}, \frac{7}{15}, \frac{8}{15}, \frac{21}{15}$
 23) $\frac{10}{15}, \frac{11}{15}, \frac{12}{15}, \frac{13}{15}$ 24) $12\frac{7}{16}$ Rs 26) 145

LEARNER'S TASK



BEGINNERS (Level - I)



I) MCQ'S with single correct answer type :

- Lalita jogs seven blocks the first day of her training program. She increases her distance by two blocks each day. On the last day, she jogs 25 blocks. How many days was she in training?
 a) 10 b) 12 c) 13 d) 15
- Tuesday's high temperature was 4°C warmer than that of Monday's. Wednesday's high temperature was 6°C cooler than that of Monday's. If Tuesday's high temperature was 22°C , what was Wednesday's high temperature?
 a) 18 b) 20 c) 22 d) 24
- John and Mary play a two-person game in which the winner gains 2 points and the loser loses 1 point. If John won exactly 3 games and Mary had a final score of 7 points, how many games did they play?
 a) 6 b) 8 c) 10 d) 12
- When a pitcher is $\frac{1}{2}$ full it contains exactly enough water to three identical glasses. How full would the pitcher be if it had exactly enough water to fill four of the same glasses ?
 a) $3/2$ b) $1/3$ c) $2/3$ d) $3/4$
- The weight limit for an elevator is 1500 kilograms. The average weight of the people in the elevator is 80 kilograms. If the combined weight of the people is 100 kilograms over the limit, how many people are in the elevator ?
 a) 10 b) 12 c) 15 d) 20

**ACHIEVERS (Level - II)****Solve the following :**

1. How many times is the HCF of 48, 36, 72 and 24 contained in their LCM ?
2. Find the number which is nearest to 457 and is exactly divisible by 11.
3. If the L.C.M of two numbers and their product is 12, 24 respectively. Find H.C.F of that numbers.
4. Find the greatest three digit number which is exactly divisible by 7, 24 and 21.
5. Find the H.C.F by division method
 - a) 288, 480
 - b) 65, 95, 135
6. Find the H.C.F by prime factor method
 - a) 150, 225
 - b) 48, 72, 108
 - c) 64, 252, 324
7. Find the L.C.M by prime factor method.
 - a) 96, 256
 - b) 48, 72, 108
8. Find the L.C.M by division method.
 - a) 48, 64, 72, 80
 - b) 24, 42, 60, 90
 - c) 75, 100, 175, 250
9. Find the smallest four digit number which is exactly divisible by 4, 6, 8 and 10.
10. Find the greatest four digit number which is exactly divisible by 6, 8, 10 and 12.
11. The length and breadth of a rectangular field are 525m and 375m respectively. Find the length of longest stick that can measure the length and breadth of the field exactly.
12. Find the smallest number which, when divided by 15, 20, 35 and 40 leaves the remainder 9.
13. HCF and LCM of two numbers are 18 and 1782 respectively. One number is 162. Find the other.
14. Find the greatest 4-digit number which is exactly divisible by 16, 24 and 40.
15. Four bells toll together at 10.00 am. They toll at the interval of 5, 10, 12 and 20 seconds respectively. When again they will toll together ?
16. Find the length of the longest rod that can measure exactly the lengths of 45 cm, 135 cm and 1.05 m.
17. Find the smallest 5-digit number which is exactly divisible by 20, 24, 36 and 40.
18. The length and breadth of a room is 325 cm and 2.25m respectively. Find the largest size of the square tile in meters which can cover the floor of the room exactly.
19. Write the greatest 6-digit number having atleast three different digits.
20. Write the smallest 7-digit number having atleast three different digits.
21. A milk man has 20 litres of milk in one container and 30000 millilitres of milk in another container. Determine the capacity of the biggest container which the milk man can use to measure milk from either container an exact number of times.
22. Find the greatest number which divides 171 and 251 leaving remainder 3 and 6 respectively.
23. Find the greatest number which divides 245 and 1029 leaving the remainder 5 in each case.

24. Find the largest prime number required to test as a divisor to determine the number 117 is a prime number.
25. Which of the following numbers are prime numbers ?
i) 203 ii) 159 iii) 193
26. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find the longest tape which can measure the three dimensions of room exactly.
27. Find the nearest integer to 1829 which is exactly divisible by 12.
28. Find the number of divisors of 37800, excluding unity.
29. The HCF and LCM of two numbers are 44 and 264 respectively. If the first number is divide by 2, the quotient is 44. What is the other number ?
30. The product of two numbers is 2160 and their HCF is 12. Find the possible pairs of numbers.
31. How many natural numbers up to 200 are divisible by 4 and 3 together ?
32. In a division sum, the divisor is 4 times the quotient and 3 times the remainder. What is the dividend if the remainder is 4 ?
33. What smallest fraction should be added to $3\frac{2}{3} + 6\frac{7}{12} + 4\frac{9}{36} + 5 + 7\frac{1}{12}$ to make the sum a whole number ?
34. Find the least number which on adding 9 to it becomes exactly divisible by 15, 25, 30 and 45.
35. Four bells ring at intervals of 6, 8, 12 and 20 minutes. They ring simultaneously at 8 am. At what time they will ring together.
36. What is the least number that must be subtracted from 2,345 to make it exactly divisible by 7 ?
37. What is the least number which, when divided by 52, leaves 33 as the remainder, and when divided by 78 leaves 59, and when divided by 117 leaves 98 as the respective remainders.
38. What least number must be subtracted from 1936 so that the remainder when divided by 9, 10, 15 will leave in each case the same remainder 7 ?
39. Three bells commence tolling together and they toll after 0.25, 0.1 and 0.125 seconds. After what interval will they again toll together ?
40. What is the largest number which divides 77, 147 and 252 to leave the same remainder in each case ?

◆ ■ ■ ◆ **EXPLORERS (LEVEL - II)** ◆ ■ ■ ◆

I) MCQ's with Multiple correct answer type :

◆ This section contains multiple choice questions. Each question has 4 choices (A), (B), (C), (D), out of which **ONE or MORE** is correct. Choose the correct options

- The following is a factor of 'Ramanjan number'
A) 19 B) 17 C) 13 D) 7
- Fractional form of 0.056
A) $\frac{56}{100}$ B) $\frac{28}{500}$ C) $\frac{7}{125}$ D) $\frac{56}{1000}$
- Which of the following are in descending order.
A) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ B) $\frac{2013}{5}, \frac{2014}{5}, \frac{2015}{5}, \frac{2016}{5}, \frac{2017}{5}$
C) $\frac{2017}{7}, \frac{2016}{7}, \frac{2015}{7}, \frac{2014}{7}, \frac{2013}{7}$ D) $\frac{1}{5}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{4}$
- Which of the following is sum of two odd primes
A) 35 B) 42 C) 84 D) 100
- The number '11111111111' is always divisible by.....
A) 3 B) 7 C) 13 D) 11
- The highest and lowest common factor of 36 and 84 is.....
A) 7 B) 1 C) 4 D) 12
- Five digit number a679b is multiple of 72 then the value of a+b
A) 3 B) 5 C) 6 D) 7
- The H.C.F and L.C.M of two numbers are 21 and 84 respectively. If the ratio of the two numbers is 1 : 4, then the larger of the two numbers is.....
A) 12 B) 48 C) 84 D) 108
- The H.C.F of $2^2 \times 3^3 \times 5^5$, $2^3 \times 3^2 \times 5^2 \times 7$ and $2^4 \times 3^4 \times 5 \times 7^2 \times 11$ is
A) $2^2 \times 3^2 \times 5$ B) $2^2 \times 3^2 \times 5 \times 7 \times 11$ C) $4 \times 9 \times 5$ D) $2^4 \times 3^4 \times 5^5$
- Which of following are factors of 2016?
A) 8 B) 16 C) 32 D) 64

III) Matrix matching type:

◆ This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column-I** have to be matched with statements (p, q, r, s) in **Column-II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4×4 matrix should be as follows:

Column - I	Column - II
11. i) factors or divisors of a number are	p) co-primes
ii) a number and successor are	q) finite
iii) a number having only two factors is	r) (3,5,7)
iv) prime triplet is	s) 1
v) number of even primes is	t) prime
A) i-q, ii-p, iii-t, iv-s, v-r	B) i-q, ii-t, iii-p, iv-s, v-r
C) i-q, ii-p, iii-t, iv-r, v-s	D) i-q, ii-t, iii-p, iv-r, v-s



LEARNER'S TASK :

- ☐ **BEGINNERS :** 1-a 2-a 3-b 4-c 5-d
- ☐ **ACHIEVERS :** 9) 1080 10) 9960 11) 75 m 12) 849 13) 9072
 14) after 2 seconds 15) 15 cm 16) 10,080
 17) 0.25 m 18) 999987 19) 1000002 20) 10
 25) 193 26) 75 cm 27) 1824 28) 95 29) 132
 30) (12, 180) and (36, 60) 31) 16 32) 40
 33) $\frac{15}{36}$ 34) 441 35) 10 a.m 36) 449 37) 39
 38) 0.5 sec 39) 35 40) 7
- ☐ **EXPLORERS :**
 1) A,D 2) B,C,D 3) A,C 4) B,C,D 5) A,D
 6) B,D 7) B 8) C 9) A,C 10) A,B,C 11. C