

PAIR OF STRAIGHT LINES ①

Class: IX. Mathematics

(F⁺)

SOLUTIONS

Teaching Task

01. Given eqn of pair of lines

$$x^2 + xy - 2y^2 - 5x + 2y - 4 = 0$$

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$a=1 \quad \left| \quad h=\frac{1}{2} \quad \right| \quad b=-2 \quad \left| \quad g=-\frac{5}{2} \quad \right| \quad f=1 \quad \left| \quad c=-4 \right.$$

$$P.F = \left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right)$$

$$= \left(\frac{\frac{1}{2} \cdot 1 + 2 \cdot \left(-\frac{5}{2}\right)}{1 \cdot (-2) - \left(\frac{1}{2}\right)^2}, \frac{\left(-\frac{5}{2}\right)\left(\frac{1}{2}\right) - 1 \cdot 1}{1 \cdot (-2) - \left(\frac{1}{2}\right)^2} \right)$$

$$= \left(\frac{\frac{1}{2} - 5}{-2 - \frac{1}{4}}, \frac{-\frac{5}{4} - 1}{-2 - \frac{1}{4}} \right)$$

$$= \left(\frac{-\frac{9}{2}}{-\frac{9}{4}}, \frac{-\frac{9}{4}}{-\frac{9}{4}} \right)$$

$$= (2, 1)$$

Ans: D

02. $7x^2 - 5xy + 6y^2 + x - 3y = 0$

$$a=7, \quad b=6, \quad h=-\frac{5}{2}, \quad g=\frac{1}{2}, \quad f=-\frac{3}{2}, \quad c=0$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow \boxed{\lambda = 1}$$

(2)

$$\therefore x^2 - 5xy + 6y^2 + x - 3y = 0$$

$$P.I = \left(\frac{hf - bg}{ab - h^2}, \frac{sh - af}{ab - h^2} \right)$$

$$= (-3, -1)$$

Ans: A

03. The point of Intersection (P.I)

$$ax^2 + 3xy - 2y^2 - 5x + 5y + c = 0$$

$$\text{For lines} \Rightarrow a + b = 0$$

$$a - 2 = 0$$

$$\Rightarrow \boxed{a = 2}$$

$$c = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\boxed{c = -3}$$

$$P.I = \left(\frac{hf - bg}{ab - h^2}, \frac{sh - af}{ab - h^2} \right)$$

$$= \left(-\frac{1}{5}, -\frac{7}{5} \right)$$

Ans: C

04. $11x^2 + 16xy - y^2 = 0$

$$ax^2 + 2hxy + by^2 = 0$$

$$a = 11, h = 8, b = -1$$

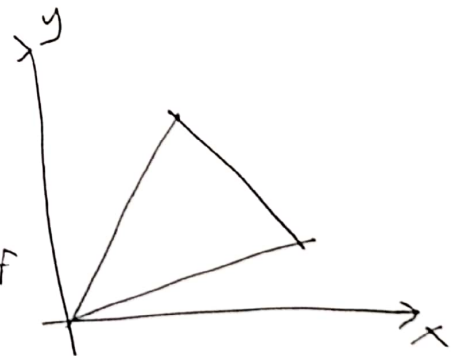
If $lx + my + n = 0$ is one side of an equilateral Δ , then the pair of other two sides are

$$(lx + my)^2 - 3(mx - ly)^2 = 0$$

Given $x - 2y + 13 = 0$ | $l = 1, m = -2, n = 13$

$$lx + my + n = 0$$

$$(x - 2y)^2 - 3(-2x - y)^2 = 0$$



3

$$= (x^2 - 4xy + 4y^2) - 3(4x^2 + 4xy + y^2) = 0$$

$$\Rightarrow x^2 - 4xy + 4y^2 - 12x^2 - 12xy - 3y^2 = 0$$

$$= -11x^2 - 16xy + y^2 = 0$$

$$\Rightarrow 11x^2 + 16xy - y^2 = 0.$$

Which is given, hence the given pair of lines, represent the equilateral Δ .

Ans: A

Q5. $2x^2 + 3xy - 2y^2 = 0$

$a=2, b=-2 \Rightarrow a+b=0.$

Hence, it is a right angled Δ .

Now $2x^2 + 3xy - 2y^2 = 0$

$$\Rightarrow \frac{2x^2 + 4xy}{-xy - 2y^2} = 0$$

$$\Rightarrow 2x(x+2y) - y(x+2y) = 0$$

$$\Rightarrow (x+2y)(2x-y) = 0$$

$$\Rightarrow x+2y=0 \text{ and } 2x-y=0$$

Now Angle b/w $x+2y=0$ and $2x-y=0$

$$\cos \theta = \frac{|a_1 a_2 + b_1 b_2|}{\sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2}}$$

$$= \frac{|1 \cdot 2 + 2 \cdot (-1)|}{\sqrt{1+4} \cdot \sqrt{4+1}} = \frac{0}{\sqrt{5} \cdot \sqrt{5}} = 0$$

$$\therefore \theta = 90^\circ.$$

Hence it is right angled isosceles triangle

Ans: C

06. Slope of $x+3y=1$ is $-\frac{1}{3}$

(4)

Slope of the lines is 3.

Eqn of the line passing through origin having slope 3 is $y=3x$

$$\text{Now } 9x^2 - 12xy + ky^2 = 0$$

$$9x^2 - 12x(3x) + k(3x)^2 = 0$$

$$\Rightarrow k = 3$$

Ans: A

07. $(-2, 3), x^2 - y^2 + 2x + 1 = 0$

$$2h=1 \\ h=\frac{1}{2}$$

$$\begin{aligned} \text{product of lines} &= \frac{|(-2)^2 - 3^2 + 2(-2) + 1|}{\sqrt{(1+1)^2 + 4\left(\frac{1}{2}\right)^2}} \\ &= \frac{|4 - 9 - 4 + 1|}{\sqrt{4+1}} = \frac{8}{\sqrt{5}} \end{aligned}$$

Ans: -

08. $6x^2 - 5xy + 6y^2 + 14x + 16y + 14 = 0$

$$d = \frac{|c|}{\sqrt{(a-b)^2 + 4b^2}}$$

$$= \frac{|14|}{\sqrt{(6-6)^2 + 4\left(-\frac{5}{2}\right)^2}} = \frac{14}{\sqrt{25}} = \frac{14}{5}$$

Ans: B

09. $(f, -g), hxy + gx + fy + c = 0$

$$\begin{aligned} d &= \frac{|h \cdot f(-g) + g \cdot f + f(-g) + c|}{\sqrt{(0-0)^2 + 4\left(\frac{h}{2}\right)^2}} = \frac{|-fgh + c|}{|h|} \\ &= \left| \frac{fgh - c}{h} \right| \end{aligned}$$

Ans: D

10.

$$d = \frac{|2|}{\sqrt{(12-12)^2 + 4\left(\frac{25}{2}\right)^2}} = \frac{2}{25}$$

Ans. B

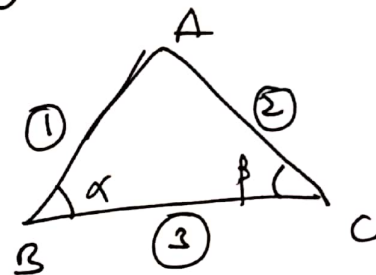
(5)

11.

$$\begin{aligned} x^2 - y^2 &= 0 \\ (x+y)(x-y) &= 0 \\ x+y &= 0 \quad \& \quad x-y=0 \\ \downarrow & \quad \downarrow \\ \textcircled{1} & \quad \textcircled{2} \end{aligned}$$

$$kx + 2y - 1 = 0$$

↓
③



Angle b/w ① & ③

$$\cos \alpha = \frac{|1 \cdot k + 1 \cdot 2|}{\sqrt{1+1} \cdot \sqrt{k^2+1}} = \frac{|k+2|}{\sqrt{2} \cdot \sqrt{k^2+1}}$$

Angle b/w ② & ③

$$\cos \beta = \frac{|1 \cdot k - 1 \cdot 2|}{\sqrt{1+1} \cdot \sqrt{k^2+1}} = \frac{|k-2|}{\sqrt{2} \cdot \sqrt{k^2+1}}$$

We have

$$\frac{|k+2|}{\sqrt{2} \cdot \sqrt{k^2+1}} = \frac{|k-2|}{\sqrt{2} \cdot \sqrt{k^2+1}}$$

Possible only when $k=0$

Ans: D

12

$$I = \frac{|a+2h+b|}{\sqrt{(a-b)^2 + 4h^2}}$$

S.O.B.S

$$(a-b)^2 + 4h^2 = (a+2h+b)^2$$

$$\Rightarrow a^2 - 2ab + b^2 + 4h^2 = a^2 + 4h^2 + b^2 + 2(a+b)2h$$

$$\Rightarrow (a-b)^2 + 4h^2 = (a+b)^2 + 4h^2 + 2(a+b)2h$$

$$\Rightarrow -4ab = 4(a+b)h \Rightarrow h(a+b) + ab = 0$$

Ans: C

13. Statement I $x^2 - y^2 - 2x + 2y = 0$

~~clearly~~ ~~(1,1)~~.

$$\begin{array}{l|l} a=1 & h=0 \\ b=-1 & g=-1 \\ c=0 & f=-1 \end{array}$$

$$\therefore P.I = \left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right) = (1, 1).$$

Given $2x^2 - 5xy + 2y^2 + x + y - 1 = 0$
 $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$$\begin{array}{l|l} a=2 & h=-\frac{5}{2} \\ b=2 & g=\frac{1}{2} \\ c=-1 & f=\frac{1}{2} \end{array}$$

$$P.I = \left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right)$$

$$= \left(\frac{-\frac{5}{2} \cdot \frac{1}{2} - 2 \cdot \frac{1}{2}}{2 \cdot 2 - \left(-\frac{5}{2}\right)^2}, \frac{\frac{1}{2} \left(-\frac{5}{2}\right) - 2 \cdot \frac{1}{2}}{2 \cdot 2 - \left(-\frac{5}{2}\right)^2} \right)$$

$$= \left(\frac{-\frac{5}{4} - 1}{4 - \frac{25}{4}}, \frac{-\frac{5}{4} - 1}{4 - \frac{25}{4}} \right)$$

$$= \left(\frac{-9}{-9}, \frac{-9}{-9} \right) = (1, 1)$$

Hence, the distance b/w the points of intersection is zero

~~Ans. A~~

Statement II: Conceptual (True)

Ans: A

14. Statement I:

$$2h=4 \\ h=2$$

$$d = \frac{|1|}{\sqrt{(2+6)^2 + 4\left(\frac{2}{8}\right)^2}} = \frac{1}{\sqrt{64+16}} \\ = \frac{1}{\sqrt{80}} \quad (\text{True})$$

Statement II: Conceptual (True)

Ans: A

15

Formula: $\frac{c(a+b) - f^2 - g^2}{ab - h^2}$

$$= \frac{1(5+2) - (-1)^2 - (-1)^2}{5 \cdot 2 - 1^2}$$
$$= \frac{7 - 1 - 1}{10 - 1} = \frac{5}{9}$$

Ans: C

16

Formula = $\frac{c(a+b) - f^2 - g^2}{ab - h^2}$

$$= \frac{5(3+1) - 2^2 - 2^2}{3 \cdot 1 - 1^2}$$
$$= \frac{20 - 4 - 4}{3 - 1} = \frac{12}{2} = 6$$

Ans: A

17.

$$D = \frac{|2(-1)^2 - 5(-1)(2) + 2(2)^2|}{\sqrt{(2-2)^2 + 4\left(-\frac{5}{2}\right)^2}}$$
$$= \frac{|2 + 10 + 8|}{5} = \frac{20}{5} = 4$$

Ans: A



* 18.
$$d = \frac{|1^2 - 1 \cdot 1 + 2 \cdot 1^2|}{\sqrt{(1-2)^2 + 4(-\frac{1}{2})^2}} = \frac{|1-1+2|}{\sqrt{1+1}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$
 (8)
Ans: —

19.
$$d = \frac{|2|}{\sqrt{(12-12)^2 + 4(\frac{25}{2})^2}} = \frac{2}{25} = k$$

Ans: 0
: $25k - 2 = 0$

20. $(x+2a)^2 - 3y^2 = 0$ & $x=a$
These lines form an equilateral triangle Ans: 60°
Since $ax^2 + 2hxy + by^2 = (lx+my)^2 - 3(mx-ly)^2$
Condition satisfies. Ans: 60°

21 a) $x^2 + 3xy + 2y^2 - 8x - 11y + 15 = 0$

$$\begin{array}{l|l} a=1 & g=-4 \\ b=2 & f=-\frac{11}{2} \\ c=15 & h=\frac{3}{2} \end{array}$$

$$P.I = \left(\frac{hf-bg}{ab-h^2}, \frac{sh-af}{ab-h^2} \right) = (1, 2)$$

b)
$$d = \frac{|15|}{\sqrt{(1-2)^2 + 4(\frac{3}{2})^2}} = \frac{15}{\sqrt{1+9}} = \frac{15}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{2}$$

c)
$$d = \frac{|1^2 + 3 \cdot 1 \cdot 1 + 2 \cdot 1^2 - 8 \cdot 1 - 11 \cdot 1 + 15|}{\sqrt{(1-2)^2 + 4(\frac{3}{2})^2}} = \frac{1}{\sqrt{10}}$$



d) formula = $\frac{c(a+b) - f^2 - g^2}{ab - h^2}$

$$= \frac{15(1+2) - \left(-\frac{11}{2}\right)^2 - (-4)^2}{1 \cdot 2 - \left(\frac{3}{2}\right)^2}$$

$$= \frac{45 - \frac{121}{4} - 16}{2 - \frac{9}{4}}$$

= 5

Ans: t, p, q, —

22 a) formula = $\frac{c(a+b) - f^2 - g^2}{ab - h^2}$

$$= \frac{-1(1-1) - (1)^2 - 0^2}{1 \cdot (-1) - 0^2}$$

$$= \frac{-1}{-1} = 1$$

b) $3x^2 - 11xy + 10y^2 - 7x + 13y + k = 0$

$a=3$ $h=-\frac{11}{2}$
 $b=10$ $g=-\frac{7}{2}$
 $c=k$ $f=\frac{13}{2}$

P.T = $\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right) = (3, 1)$

c) conceptual (0/0)

d) Conceptual $k=3$

Ans: x, p, s, q

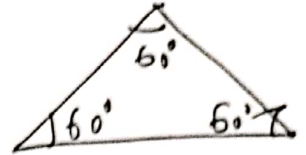


LEARNERS TASK

10

Ques

01. $Ax^2 + 3Hxy + By^2 = 0$



$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\Rightarrow \sqrt{3} = \frac{2\sqrt{h^2 - ab}}{a+b} \quad \text{since } \theta = 60^\circ$$

S.O.B.S

$$\Rightarrow 4(h^2 - ab) = 3(a+b)^2$$

$$4(h^2 - ab) = 3(a^2 + b^2 + 2ab)$$

$$\Rightarrow 4h^2 - 4ab = 3a^2 + 3b^2 + 6ab$$

$$\Rightarrow 4h^2 = 3a^2 + 3b^2 + 10ab$$

$$\Rightarrow 4h^2 = (a+3b)(3a+b)$$

$$\therefore 4H^2 = (A+3B)(B+3A)$$

Ans: D

02. original eqn. $ax^2 + 2hxy + by^2 = 0$

origin is shifted to (α, β)

transformed eqn is $a(x-\alpha)^2 + 2h(x-\alpha)(y-\beta) + b(y-\beta)^2 = 0$

$\therefore$ Point of intersection (α, β)

Ans: B

*03.

$$P = \frac{|h \cdot f(-g) + g \cdot f + f(-g) + c|}{\sqrt{(0-0)^2 + 4\left(\frac{h}{2}\right)^2}}$$

$$= \frac{|-hfg + c|}{h} = \left| \frac{fgh - c}{h} \right|$$

Ans: D



$$\frac{|1 \cdot x + 1 \cdot 2|}{\sqrt{1^2+1^2} \cdot \sqrt{x^2+2^2}} = \frac{|1 \cdot x - 1 \cdot 2|}{\sqrt{1^2+1^2} \cdot \sqrt{x^2+2^2}}$$

(12)

$$\Rightarrow |x+2| = |x-2|$$

$$\Rightarrow x = 0$$

Ans: D

$$d = \frac{|2|}{\sqrt{(12-12)^2 + 4\left(\frac{25}{2}\right)^2}} = \frac{2}{25}$$

Ans: B

$$\begin{aligned} \text{formula} &= \frac{c(a+b) - f^2 - g^2}{ab - h^2} \\ &= \frac{2(12+12) - \left(\frac{11}{2}\right)^2 - (5)^2}{12 \cdot 12 - \left(\frac{25}{2}\right)^2} \\ &= \frac{2}{13} \end{aligned}$$

Ans: D

$$14. \quad xy - x - y + 1 = 0$$

$$\Rightarrow x(y-1) - 1(y-1) = 0$$

$$\Rightarrow (x-1)(y-1) = 0$$

$$\Rightarrow (x, y) = (1, 1)$$

$$\text{Now } a(1) + 2(1) - 3 = 0$$

$$\Rightarrow a = 1$$

Ans: C

$$15. \quad x^2 + px^2 + y - y^2 = 0$$

$$\perp \text{ lines } \Rightarrow a+b=0$$

$$\Rightarrow 1+p=0$$

$$\Rightarrow p=-1$$

$$\therefore x^2 - y^2 + y - y^2 = 0$$

$$a=1 \quad f=\frac{1}{2}$$

$$b=-1 \quad g=0$$

$$c=-2 \quad h=0$$

$$\text{Point Intersect} = \left(0, \frac{1}{2}\right)$$



04. Conceptual (Tone)

Ans: D (11)

05. Conceptual $k=3$

Ans: C

06. Conceptual

Ans: D

07. formula = $\frac{c(a+b) - f^2 - g^2}{ab - h^2}$

Since lines are $\perp \Rightarrow a+b=0$

$$= \frac{-f^2 - g^2}{ab - h^2}$$

$$= \frac{f^2 + g^2}{h^2 - ab}$$

$$= \frac{f^2 + g^2}{h^2 - a(-a)} = \frac{f^2 + g^2}{h^2 + a^2} \quad (\text{or}) \quad \frac{f^2 + g^2}{b^2 + b^2}$$

Ans: D

08. Conceptual

Ans: D

09. Conceptual (Theorem).

Ans: A

10. Conceptual (Theorem)

Ans: B

JEE MAINS LEVEL

01.

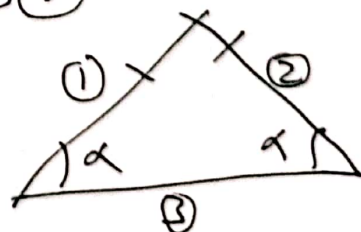
$$x^2 - y^2 = 0$$

$$x + y = 0 \rightarrow \textcircled{1}$$

$$x - y = 0 \rightarrow \textcircled{2}$$

$$\cos \alpha = \frac{|a_1 a_2 + b_1 b_2|}{\sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2}}$$

$$2x + 2y - 1 = 0 \rightarrow \textcircled{3}$$



$$\frac{|1 \cdot x + 1 \cdot 2|}{\sqrt{1^2 + 1^2} \cdot \sqrt{x^2 + 2^2}} = \frac{|1 \cdot x - 1 \cdot 2|}{\sqrt{1^2 + 1^2} \cdot \sqrt{x^2 + 2^2}}$$

$$\Rightarrow |x+2| = |x-2|$$

$$\Rightarrow x = 0$$

Ans: D

02

$$d = \frac{|2|}{\sqrt{(12-12)^2 + 4\left(\frac{25}{2}\right)^2}} = \frac{2}{25}$$

Ans: B

03

$$\text{formula} = \frac{c(a+b) - f^2 - g^2}{ab - h^2}$$

$$= \frac{2(12+12) - \left(\frac{11}{2}\right)^2 - (5)^2}{12 \cdot 12 - \left(\frac{25}{2}\right)^2}$$

$$= \frac{2}{13}$$

Ans: D

04.

$$xy - x - y + 1 = 0$$

$$\Rightarrow x(y-1) - 1(y-1) = 0$$

$$\Rightarrow (x-1)(y-1) = 0$$

$$\Rightarrow (x, y) = (1, 1)$$

$$\text{Now } a(1) + 2(1) - 3 = 0$$

$$\Rightarrow a = 1$$

Ans: C

05.

$$x^2 + py^2 + y - q^2 = 0$$

$$\perp \text{ lines} \Rightarrow a+b=0$$

$$\Rightarrow 1+p=0$$

$$\Rightarrow p = -1$$

$$\therefore x^2 - y^2 + y - q^2 = 0$$

$$a=1 \quad f=\frac{1}{2}$$

$$b=-1 \quad g=0$$

$$c=-q \quad h=0$$

$$\text{Point Intersect} = \left(0, \frac{1}{2}\right)$$



Now $x^2 - y^2 + y - q^2 = 0$

$(0, \frac{1}{2}) \Rightarrow 0^2 - (\frac{1}{2})^2 + (\frac{1}{2}) - q^2 = 0$

$\Rightarrow -\frac{1}{4} + \frac{1}{2} - q^2 = 0$

$\Rightarrow \frac{1}{4} - q^2 = 0$

$\Rightarrow q^2 = \frac{1}{4} \Rightarrow q = \pm \frac{1}{2}$

$\therefore$ point of intersection is $(0, q)$ Ans: C

06. $(y^2 - 4xy - x^2)(x + y + 1) = 0$

Consider $y^2 - 4xy - x^2 = 0$

$\Rightarrow x^2 + 4xy - y^2 = 0$

$ax^2 + 2hxy + by^2 = 0$

$a = 1, b = -1 \Rightarrow a + b = 0.$

hence, the Δ is right angled Δ Ans: B

07. $x^2 - 3y^2 = 0.$

$x = 4 \Rightarrow x - 4 = 0$

$\Rightarrow lx + my + n = 0$

$x^2 - 3y^2 = (x - 4)^2 - 3(0 \cdot x - 1 \cdot y)^2$

$= (x - 4)^2 - 3(4 - y)^2$

$= x^2 - 8x$

07. Given pair of lines $x^2 - 3y^2 = 0$

Given line $x - 4 = 0$ } $l = 1, m = 0, n = -4$
 $lx + my + n = 0$

Now, $x^2 - 3y^2 = (lx + my)^2 - 3(mx - ly)^2$
 $= (1 \cdot x + 0 \cdot y)^2 - 3(0 \cdot x - 1 \cdot y)^2$
 $= x^2 - 3y^2$

Hence it is an equilateral Δ Ans: D

08

$$d = \frac{|2 \cdot 0^2 - 5 \cdot 0 \cdot 1 + 1|^2}{\sqrt{(2-1)^2 + 4 \left(-\frac{5}{2}\right)^2}} = \frac{1}{\sqrt{26}}$$

Ans: B

09.

$$d = \frac{|1^2 - 4 \cdot 1 \cdot K + 10^2|}{\sqrt{(1-1)^2 + 4(-2)^2}} = \frac{3}{2}$$

$$\Rightarrow K = 5$$

Ans: B

10. $3x^2 - 11xy + 10y^2 - 7x + 13y + 10 = 0$

$$\begin{array}{l|l} a=3 & h=-\frac{11}{2} \\ b=10 & g=-\frac{7}{2} \\ c=10 & f=\frac{13}{2} \end{array}$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

point of Intersection = $\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right)$

$$= \left(\frac{-\frac{11}{2} \cdot \frac{13}{2} - 10 \cdot \left(-\frac{7}{2}\right)}{3 \cdot 10 - \left(-\frac{11}{2}\right)^2}, \frac{\left(-\frac{7}{2}\right) \left(-\frac{11}{2}\right) - 3 \left(\frac{13}{2}\right)}{3 \cdot 10 - \left(-\frac{11}{2}\right)^2} \right)$$

$$= (3, 1)$$

Ans: B

11. point of intersection of $x^2 - y^2 - 2x + 2y = 0$ is (1, 1)
 point of Intersection of $2x^2 - 5xy + 2y^2 + x + y = 0$ is (1, 1)
 ... Hence distance 0
 Ans: B

12. P.I = $\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right) = (2, 1)$

Ans: B

13. Statement I: $x^2 - 5xy + 4y^2 + 8x - 11y + 7 = 0$ (15)
 $P \cdot I = (1, 2)$ (True)

Ans: A

Statement II: Conceptual (True)

14. Statement I:

$$\frac{(k^2 + 4 \cdot k \cdot k + 3k^2)}{\sqrt{(1-3)^2 + 4(2)^2}} = \frac{4}{\sqrt{5}}$$

$$\Rightarrow k = \pm 1$$

Statement II: Conceptual (True)

Ans: A

15. $3x^2 + 7xy + 2y^2 + 5x - 5y + 2 = 0$

$$P \cdot I = \left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right)$$

$$= \left(-\frac{3}{5}, -\frac{1}{5} \right)$$

Ans: B

16. $2x^2 + 3xy - 2y^2 + 3x - 9y - 9 = 0$

$$P \cdot I = \left(\frac{3}{5}, -\frac{9}{5} \right)$$

Ans: A

17. $d = \frac{|11|}{\sqrt{(1-1)^2 + 4(-2)^2}} = \frac{1}{4}$

Ans: D

18. $\frac{|1k|}{\sqrt{(1+3)^2 + 4(3)^2}} = \frac{1}{\sqrt{5-2}}$

$$\therefore k = \pm 1$$

Ans: D



19.

$$x^2 + 3xy + 2y^2 - 3x - 4y + 2 = 0$$

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$\begin{matrix} a=2 \\ b=2 \\ c=2 \end{matrix} \left| \begin{matrix} h=\frac{3}{2} \\ g=-\frac{3}{2} \\ f=-2 \end{matrix} \right. \quad P.I = \left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2} \right)$$

$$= \left(\frac{\frac{3}{2}(-2) - 2(-\frac{3}{2})}{2 \cdot 2 - (\frac{3}{2})^2}, \frac{(-\frac{3}{2})(\frac{3}{2}) - 2(-2)}{2 \cdot 2 - (\frac{3}{2})^2} \right)$$

$$= \left(\frac{-3 + 3}{4 - \frac{9}{4}}, \frac{-\frac{9}{4} + 4}{4 - \frac{9}{4}} \right)$$

$$= \left(0, \frac{7}{7} \right) = (0, 1) \rightarrow (\alpha, \beta)$$

$$2\alpha + \beta = 2 \cdot 0 + 1 = 1$$

Ans: 1

20.

$$\text{formula} = \frac{c(a+b) - f^2 - g^2}{ab - h^2}$$

$$= \frac{3(1-1) - 2^2 - (-\frac{3}{2})^2}{1 \cdot (-1) - (-3)^2} =$$

$$= \frac{-4 - \frac{9}{4}}{-1 - 9} = \frac{-25}{4 \times -10} = \frac{5}{8}$$

$$\therefore \frac{3}{2\sqrt{k}} = \frac{5}{8} \Rightarrow \sqrt{k} = \frac{12}{5}$$

$$\Rightarrow k = \frac{144}{25}$$

Ans: —

19.

$$x^2 + 3xy + 2y^2 - 3x - 4y + 2 = 0$$

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$\begin{aligned} a=2 & \quad h=\frac{3}{2} \\ b=2 & \quad g=-\frac{3}{2} \\ c=2 & \quad f=-2 \end{aligned} \quad P.I = \left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right)$$

$$= \left(\frac{\frac{3}{2} \cdot (-2) - 2 \cdot (-\frac{3}{2})}{2 \cdot 2 - (\frac{3}{2})^2}, \frac{(-\frac{3}{2}) \left(\frac{3}{2}\right) - 2 \cdot (-2)}{2 \cdot 2 - (\frac{3}{2})^2} \right)$$

$$= \left(\frac{-3 + 3}{4 - \frac{9}{4}}, \frac{-\frac{9}{4} + 4}{4 - \frac{9}{4}} \right)$$

$$= \left(0, \frac{7}{7} \right) = (0, 1) \rightarrow (\alpha, \beta)$$

$$2\alpha + \beta = 2 \cdot 0 + 1 = 1$$

Ans: 1

20.

$$\text{formula} = \frac{c(a+b) - f^2 - g^2}{ab - h^2}$$

$$= \frac{3(1-1) - 2^2 - (-\frac{3}{2})^2}{1 \cdot (-1) - (-3)^2} =$$

$$= \frac{-4 - \frac{9}{4}}{-1 - 9} = \frac{-25}{4 \times -10} = \frac{5}{8}$$

$$\therefore \frac{3}{2\sqrt{k}} = \frac{5}{84} \Rightarrow \sqrt{k} = \frac{12}{5}$$

$$\Rightarrow k = \frac{144}{25}$$

Ans: —

(17)

- 21
- a) $3x^2 - 2xy + 3y^2 = 0$ $P \cdot I \rightarrow (0, 0)$
 - b) $x^2 - y^2 - 2x + 2y = 0$ $P \cdot I \rightarrow (1, 1)$
 - c) $x^2 - y^2 + 2y - 1 = 0$ $P \cdot I \rightarrow (0, 1)$
 - d) $x^2 - y^2 - 2x + 1 = 0$ $P \cdot I \rightarrow (1, 0)$

Ans: P, Q, R

22 a) $x^2 - 2xy + 3y^2 - 2x + y + 1 = 0$

perpendicular distance from $(0, 0)$

$$d = \frac{|1|}{\sqrt{(1-3)^2 + 4(-1)^2}} = \frac{1}{\sqrt{4+4}} = \frac{1}{2\sqrt{2}}$$

b) $\perp$ distance $= \frac{3}{2\sqrt{2}}$

c) $\perp$ distance $= 3$

d) $\perp$ distance $= \frac{|1^2 + 2^2 - 2 \cdot 1 + 3 \cdot 2 - 1|}{\sqrt{(1-1)^2 + 4(0)^2}}$

$$= \frac{|finite value|}{0} = \text{not defined}$$

Ans: S, P, Q, R