

EXONENTS AND POWERS

Learning Objectives:

- ◆ What is base and exponent
- ◆ Laws of exponents
- ◆ How to write the numbers in standard form
- ◆ Exponential equations
- ◆ Comparison of different exponents.

Real life Applications:

- Φ Exponents are used in computer Games, physics, pH and Richter measuring scales, science, Engineering, Economics, Accounting, Finance and many other disciplines.
- Φ Exponential Growth is a critically important aspect of Finance, Demographics, Biology, Economics, Resources, Electronics and many other areas.
- Φ Exponential Decay is associated with light, sound, sporting fixtures, dangerous chemicals and radioactive waste.
- Φ A large distances exponents are used to measure large distances. For example, the distance from earth to the moon is 3.85×10^5 km (approx).
- Φ People who use exponents are economists, bankers, financial advisors, insurance risk assessors, biologists, engineers, computer programmers, chemists, physicists, geographers, sound engineers, statisticians, mathematicians, geologists and many other professions.

Exponent :

For any rational number 'a' and natural number 'n' multiplication of 'a' n times symbolically written as a^n (read as a to the power of n)

$$\text{i.e } a.a.a.....(n \text{ times}) = a^n$$

Here a is called base and 'n' is called exponent of a (or) power of a.

Note: $a.a.a.....(n \text{ times})$ is called product form and a^n is called its exponential form.

Exponential Equation :

An equation which has an un-known quantity as an exponent is called an exponential equation.

Ex : i) $5^x = 625$ ii) $3^{x-5} = 1$

$\Psi a^{\frac{1}{n}}$ is called the nth root of a and may be written as $\sqrt[n]{a}$.

$\Psi \sqrt[n]{a}$ is called a radical of Index 'n' and 'a' is called the radicand.

Laws of exponents :

If 'a' and 'b' are any rational numbers different from zero and if m, n are any rational numbers then,

i) **Multiplication Property:**

$$a^m \times a^n = a^{m+n}$$

ii) **Power of a power property:**

$$(a^m)^n = a^{mn}$$

iii) **Power of a product property :**

$$(a \cdot b)^m = a^m \cdot b^m$$

iv) **Division Property :**

$$\frac{a^m}{a^n} = a^{m-n} \quad \text{if } m > n$$

$$= \frac{1}{a^{n-m}} \quad \text{if } m < n$$

$$= 1 \quad \text{if } m = n$$

v) **Power of a Quotient property :**

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

vi) **Power with exponent zero law :**

$$a^0 = 1 \quad (a \neq 0)$$

vii) **Power with exponent being negative integer :**

$$a^{-m} = \frac{1}{a^m} \quad \text{or} \quad \frac{1}{a^{-m}} = a^m$$

Note :

φ The product of 5 and n is written as $5 \times n$. It is also written as $5.n$ (or) $5n$

φ “ $5n$ ” means “5 times n ”

φ $a = 1.a = a$

$$2a = 2.a = a+a$$

$$3a = 3.a = a+a+a$$

$$ma = m.a = a+a+a+\dots\dots+a \quad (\text{m times})$$

φ The coefficient of x is 1

φ The coefficient of $-x$ is -1

§§ Powers of (-1) :

$$\text{We have } (-1)^1 = -1$$

$$(-1)^2 = (-1)(-1) = 1$$

$$(-1)^3 = (-1)(-1)(-1) = -1$$

$$(-1)^4 = (-1)(-1)(-1)(-1) = 1 \text{ and so-on}$$

consequently we have

$$(-1)^{\text{odd natural number}} = -1$$

$$(-1)^{\text{even natural number}} = 1$$

§§ Scientific notation of the numbers:

Very large numbers or very small numbers can be expressed in simplest form by using scientific notation.

Ex : 39087.8 can be expressed as $3.90878 \times 10^4 \approx 3.9 \times 10^4$

1 light year = 9.46×10^{15} m

TEACHING TASK

I) MCQs with one option is correct:

1. $a.a.a.a = \dots$
 - A) a
 - B) a^2
 - C) a^4
 - D) a^3
2. $p=2, q=4$ then $2pq = \dots$
 - A) 32
 - B) 16
 - C) 8
 - D) 2
3. $\frac{1}{10000} =$
 - A) -10^{-4}
 - B) $16a^4$
 - C) 10^{-4}
 - D) 10^4
4. $3pq^2r^2 \div 9pqr =$
 - A) $3pqr$
 - B) $\frac{1}{3}pq$
 - C) $\frac{1}{3}qr$
 - D) $\frac{1}{3}pqr$
5. $(a^3 - b^3) \div (a - b) =$
 - A) $(a+b)^2 - ab$
 - B) $(a+b)^2 + ab$
 - C) $4ab$
 - D) $a^2 - b^2$
6. $(4+9+1+3)^3 =$
 - A) 4913
 - B) 4319
 - C) 4931
 - D) 4139
7. The simplified form of $\left(\frac{-2}{5}\right)^{-3}$ is
 - A) $\frac{-127}{8}$
 - B) $\frac{-125}{8}$
 - C) $\frac{125}{4}$
 - D) $\frac{125}{8}$
8. If $\frac{5x-3}{19x+2} = \frac{3}{5}$ then $x =$
 - A) $-\frac{21}{32}$
 - B) $-\frac{32}{21}$
 - C) $\frac{21}{32}$
 - D) $\frac{32}{21}$
9. If $x+y=5$ and $x^2+y^2=10$ then $xy=$
 - A) $7\frac{1}{2}$
 - B) $7\frac{2}{3}$
 - C) $2\frac{1}{7}$
 - D) $7\frac{2}{7}$
10. $(50.5)^2 - (49.5)^2 = \dots$
 - A) 1000
 - B) 100
 - C) 10
 - D) 1
11. $(6^{-1} - 8^{-1})^{-1} = \dots$
 - A) $-\frac{1}{2}$
 - B) -2
 - C) $\frac{1}{24}$
 - D) 24

12. $2^{-3} + 2^{-2} + 2^0 + 2 + 2^2 = \dots$

- A) 8 B) $\frac{1}{8}$ C) $\frac{59}{8}$ D) $\frac{1}{4}$

13. $(-n^2)^2 (-n^2)^3 = \dots$

- A) $-n^{10}$ B) n^{10} C) n^9 D) n^{12}

14. If $x = (a^m)^n : y = (a^n)^m$ then the relation between x and y is \dots

- A) $x=y$ B) $xy=1$ C) $x+y=1$ D) $x>y$

15. $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}} \right)^3 \right]^{\frac{1}{4}} = \dots$

- A) 1 B) 5 C) 3 D) 9

16. what is the value of $2^{0.64} \times 2^{0.36}$

- A) 1 B) 2 C) 16 D) 32

17. A boy was asked to write $2^5 \times 9^2$ but he wrote 2592. what is the numerical difference between the two ?

- A) 0 B) 1 C) 2 D) 3

18. If $x = 2$ and $y = 3$ then find the value of $\left[\frac{1}{x^x} + \frac{1}{y^y} \right]$

- A) $-\frac{31}{108}$ B) $\frac{31}{108}$ C) $\frac{125}{171}$ D) $\frac{153}{222}$

II) MCQs with more than one option is correct:

◆ This section contains multiple choice questions. Each question has 4 choices (A), (B), (C), (D), out of which ONE or MORE is correct. Choose the correct options

1. $8^3 \times 8^2 \times 8^{-5}$ is equal to

- A) 8^0 B) 1 C) 8 D) 1^8

2. If $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, then the value of n is

- A) 2 B) 3 C) 9 D) $\sqrt[4]{81}$

3. The square root of $3^0 + 3^1 + 3^2 + 3^3 + 3^4$

- A) 1 B) 11 C) $\sqrt{11^2}$ D) 11^2

4. The value of $\sqrt[4]{3\sqrt{x^2}}$

- A) x B) x^2 C) $x^{1/6}$ D) $x^{2/3 \times 1/4}$

5. If $\frac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$ then the value of m-n is

- A) 1 B) 3^0 C) 3 D) $3 \cdot 3^{-1}$

III) Match the following:

◆ This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column-I** have to be matched with statements (p, q, r, s) in **Column-II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-r, B-r, C-p, C-q and D-s, then the correct bubbled 4*4 matrix should be as follows:

Column -I	Column -II
1. 5^{-5}	() (A) 6^{12}
2. $6^{-6} \times 6^6$	() (B) 1
3. If $x^3 = 7^3$, then x	() (C) 2
4. $2^{1^{2^3}}$	() (D) 7
	(E) $\frac{1}{5^5}$

IV) Solve the following:

1. Find the value of each of the following

$$\text{a) } \left(-\frac{2}{3}\right)^5 \quad \text{b) } \left(-\frac{1}{10}\right)^6$$

- 2.** Express the following in exponential form (scientific notation).

a)96326000000 b)0.000939478

3. Write each of the following as single notation.

a) $3^2 \times 3^5 \times 3^4$

$$\text{c)} \left(-\frac{6}{5}\right)^2 \times \left(-\frac{6}{5}\right)^3 \times \left(-\frac{6}{5}\right)^2 \times \left(-\frac{6}{5}\right)^8 \times \left(-\frac{6}{5}\right)^3$$

$$\text{d}) \left(-\frac{1}{3}\right)^2 \times \left(-\frac{1}{3}\right)^3 \times \left(-\frac{1}{3}\right)^5 \times \left(-\frac{1}{3}\right)^4 \times \left(-\frac{1}{3}\right)^8$$

- #### 4. Simplify

a) $27^4 \div 3^4$ b) $\frac{2^3 x 2^{-2}}{4^{-4} x 4^2}$

- ## 5. Simplify

$$\text{a)} \left(\frac{81}{16} \right)^{-\frac{1}{2}} \quad \text{b)} \sqrt{361} - \sqrt[4]{256}$$

- 6.** Simplify $\left\{(-8)^{-4} \div 2^{-8}\right\}^2$

7. Which is greater in each of the following.

a) $2^{12}, 3^8$ b) $7^8, 9^6$

8. Find the value of x when

a) $(2^3)^4 = (2)^{2x}$ b) $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^8 = \left(\frac{2}{3}\right)^{5x}$ c) $3^x \cdot 4^x = 1$

9. Solve the following.

a) $5^{3x+1} = 25^{x+2}$ b) $27^{x+2} = 9^{x+3} = 3^4$

10. By what number 3^{-4} should multiply to get 729.

11. Simplify $\left[\left\{ \left(\frac{2}{3} \right)^{-2} \right\} \div \left(\frac{3}{4} \right)^{-3} \right] \times \left(\frac{2}{3} \right)^{-2}$

12. Simplify the following.

a) $\left(\frac{-2p^2q}{3pq^2} \right)^{-2} \times \left(\frac{6p^2}{q^4} \right)^2$ b) $\frac{(5pqr)^{-2}}{25p^2q^2r}$

13. $\left(\frac{x^m}{x^n} \right)^p \left(\frac{x^n}{x^p} \right)^m \left(\frac{x^p}{x^m} \right)^n = ?$

14. Find the value of y when $\frac{x^{2y-3} \times (x^2)^{y+1}}{(x^4)^{-3}} = (x^3)^3 \div (x^6)^{-3}$

15. Show that $\frac{a^{p+q} \cdot a^{q+r} \cdot a^{r+p}}{(a^p \cdot a^q \cdot a^r)^2} = 1$

16. Show that $(a^{x+y})^{x-y} \times (a^{y+z})^{y-z} \times (a^{z+x})^{z-x} = 1$

17. Find the value of $\frac{1}{1+x^{-m}} + \frac{1}{1+x^m}$

18. Show that value of $\left(\frac{m^p}{m^{-q}} \right)^{p-q} \times \left(\frac{m^q}{m^{-r}} \right)^{q-r} \times \left(\frac{m^r}{m^{-p}} \right)^{r-p} = 1$.

19. Find the value of $\left[\frac{x^{2-3n} \cdot x^{4+3n}}{x^3} \right]^2$ ($x \neq 0$)

20. Find the value of $\frac{(x^{a+b})^3 \cdot (x^{b+c})^3 \cdot (x^{c+a})^3}{(x^a \cdot x^b \cdot x^c)^6}$ ($x \neq 0$)

21. Find the value of $\frac{3^a \cdot 4^{a-2} \cdot 25^{a+1}}{9^{a-1} \cdot 2^{a+1} \cdot 5^{a-2}}$

22. If m=2, n=-3, p=-2 than

- | | |
|------------------------|----------------------------|
| a)mnp | b)3mn-2np |
| c) $(m+n)^2 - (n+p)^2$ | d) $m^2 + n^2 + p^2 - mnp$ |

23. Find the value of x when $\frac{3^{2x} \cdot 9^{4x+1} \cdot 27}{(81)^{2x}} = 1$

24. Which is greater 10^{600} or 5^{900} .

25. Find the value of $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$

26. Simplify $\frac{a^{2x+4} \cdot a^{(x+2)(x+3)}}{(a^2)^{x+1} \cdot a^{x(2x+1)}}$

27. Simplify the following $(64)^{2/3} + \sqrt[2]{100} + 5^0 + (81)^{-1/4} \times \left(\frac{25}{16}\right)^{-1/2}$

28. Find the value of x when

$$\left(-\frac{2}{7}\right)^{x+3} \div \left(-\frac{2}{7}\right)^{-2x+3} = \left(-\frac{2}{7}\right)^{2x+5} \times \left(\left(-\frac{2}{7}\right)^{-2}\right)^{x+4}$$

29. If $\frac{1}{2}x(3^x - 3^{x-1}) = 81$, then x=?

30. Find the value of $(x^{m+n})^{m-n} \cdot (x^{n+p})^{n-p} \cdot (x^{n+m})^{n-m}$

31. Find the value of $a^{x-y} \times b^{y-z} \times a^{y-x} \times b^{z-y} \times 2^5$

32. $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$ then the value of n is ?



• ■ ■ • **BEGINNERS (Level - I)** • ■ ■ •

I) **MCQs with one option is correct:**

1. Which of the following is constant?
A) 1 B) z C) p D) x
2. The coefficient of x in $-5x$ is
A) 5 B) -5 C) x D) -x
3. A symbol which takes various numerical values is called
A) Constant B) Variable C) Term D) Coefficient
4. The product of 20 and fourth power of $(x - y)$ is
A) $20(x - y)^4$ B) $20 + (x - y)^4$ C) $20(x + y)^4$ D) $4(x - y)^{20}$
5. The exponential form of $2010 \times 2010 \times \dots \times 2010$ (2010 times)
A) 2×2010^0 B) $(2010)^{2010}$ C) $(2010)^0$ D) 2010
6. In $5x^2$ index of the base x is
A) 5 B) 2 C) x^2 D) 1

7. The value of $(7^0 - 3^0) \times 6^0$ is
 A) 1 B) $(7^0 - 3^0)$ C) 6 D) 0
8. The product of the two powers of the same base is a power of the same base with the index equal to the.....
 A) Sum of bases B) Product of indices
 C) Sum of the indices D) Product of the bases
9. $(x^{-1} + y^{-1})^{-1} = \dots$
 A) xy B) $x+y$ C) $\frac{xy}{x+y}$ D) $\frac{x+y}{xy}$
10. $\left(\frac{32}{243}\right)^{-\frac{3}{5}} = \dots$
 A) $\frac{27}{8}$ B) $\frac{8}{27}$ C) $\frac{-8}{27}$ D) $\frac{-27}{8}$
11. $(-1)^{2014} = \dots$
 A) 1 B) 0 C) -1 D) 2014
12. $3^0 \times (1 - 3^{-3}) =$
 A) 1 B) 0 C) $\frac{26}{27}$ D) $\frac{27}{26}$
13. The value of $\frac{a^{10}}{a^{12}} =$
 A) a^2 B) a^{22} C) a^{-2} D) $a^{\frac{5}{6}}$
14. If $a = 5$ and $b = 3$, then $a^3 - b^5 =$
 A) -120 B) 120 C) 108 D) -118
15. If $2^n = 64$, then $2^{n-3} =$
 A) 6 B) 8 C) 12 D) 10
16. $1^0 \cdot 2^0 \cdot 3^0 \cdot 4^0 \cdots - 100^0 = \dots$
 A) 0 B) 1 C) 100 D) 500
17. $4^{3^2} - 4^8 =$
 A) $(3 \times 2)^{16}$ B) 3×2^{16} C) 2×3^{16} D) 6^{16}
18. If $2^n = 64$, then $2^{n+2} = \dots$
 A) 2^{12} B) 576 C) 2^8 D) 2^{10}
19. If $3^{x-1} = 9$ and $4^{y+2} = 64$ then $\frac{x}{y} = \dots$
 A) 0 B) 3 C) 1 D) -3
20. If $2^x = 1$ then $x = \dots$
 A) 0 B) 1 C) 2 D) $\frac{1}{2}$

- 21.** $x^6 \div x^{-3} =$
- A) x^{-9} B) x^3 C) x^9 D) x^{-3}
- 22.** $8^x \times 2^2 = 2^6$ then $x =$
- A) $\frac{3}{4}$ B) $\frac{4}{3}$ C) 0 D) 4
- 23.** If $8^{x-1} = 2^{x+3}$, then $x =$
- A) 1 B) 2 C) 3 D) 4
- 24.** If $27^{x+1} = 9^{x+3}$, then $x =$
- A) 1 B) 2 C) 3 D) 4
- 25.** $(-1)^1 + (-1)^2 + (-1)^3 + \dots + (-1)^{2014}$
- A) 2014 B) 2015 C) 1 D) 0
- 26.** 0.000019 can be expressed as.....
- A) 1.9×10^{-5} B) 1.9×10^{-8} C) 0.19×10^{-7} D) 1.9×10^5
- 27.** If $x^{m-n} \cdot x^{n-p} \cdot x^{p-m} =$
- A) 0 B) 1 C) x^{p-m-n} D) x
- 28.** If $x = (a^m)^n$ and $y = (a^n)^m$, then
- A) $x = y$ B) $x \neq y$ C) $xy = 1$ D) $xy = 0$
- 29.** If $a \neq 0$ and $b \neq 0$, then $\frac{a^{15}b^4}{a^6b^2} =$
- A) a^2b^9 B) a^9b^2 C) $a^{-9}b^2$ D) a^9b^{-2}
- 30.** $(a^{4n}a^{3n}a^{2n})^2 =$
- A) a^{28n} B) a^{18n^3} C) a^{18n} D) a^{28n^3}
- 31.** If $x \neq 0$, then $\left[\frac{x^{2-3n} \cdot x^{4-3n}}{x^3} \right]^3 =$
- A) x^2 B) 1 C) x^3 D) x^9
- 32.** The value of $\left(\frac{-5}{6} \right)^5 \div \left(\frac{-5}{6} \right)^7$ is
- A) $-\frac{36}{25}$ B) $-\frac{30}{25}$ C) $\frac{36}{25}$ D) $-\frac{25}{36}$
- 33.** The value of $\left\{ \left(\frac{1}{3} \right)^{-3} - \left(\frac{1}{2} \right)^{-3} \right\} \div \left(\frac{1}{4} \right)^{-3}$ is
- A) $\frac{19}{64}$ B) $\frac{64}{19}$ C) $\frac{27}{16}$ D) $\frac{5}{16}$
- 34.** The value of $\left\{ \left(\frac{3}{2} \right)^{-1} \div \left(\frac{-2}{5} \right)^{-1} \right\}$ is
- A) $\frac{3}{5}$ B) $-\frac{3}{5}$ C) $\frac{2}{5}$ D) $-\frac{4}{15}$

35. $\left[\left(\frac{1}{4} \right)^2 - \left(\frac{1}{4} \right)^3 \right] \times 2^6 =$
 A) 1 B) 2 C) 3 D) 4
36. If $4 \times 8^m = 2^5$, then $\left[\frac{1}{m^{-1}} \right]^{-1} =$
 A) 1 B) 2 C) 2 D) 4
37. The value of x such that $(\sqrt[3]{4})^{2x+\frac{1}{2}} = \frac{1}{32}$ is
 A) -4 B) 4 C) 5 D) -5
38. $(x^{a+b})^{a-b} \times (x^{b-c})^{b+c} \times (x^{c+a})^{c-a} =$
 A) x^{a+b+c} B) $x^{2a+2b+2c}$ C) 0 D) 1
39. $\frac{6a^9 b^4 c^5}{2ab^2 c^3}$ is simplified then the answer is
 A) $3a^8 b^2 c$ B) $3a^5 b^2 c^2$ C) $3a^8 b^3 c^2$ D) $3a^8 b^2 c^2$

• ■ ■ • ACHIEVERS (Level - II) • ■ ■ •

I) **Solve the following :**

1. Find the value of each of the following
 a) 2^4 b) 4^4

2. Find the value of each of the following

a) $\left(\frac{2}{3}\right)^4$ b) $\left(\frac{3}{5}\right)^3$

c) $\left(-\frac{3}{4}\right)^6$ d) $\left(\frac{-2}{5}\right)^6$

3. Write each of the following in exponential form

a) $5 \times 5 \times 5 \times 5 \times 5 \times 5$ b) $\left(\frac{-6}{5}\right) \times \left(\frac{-6}{5}\right) \times \left(\frac{-6}{5}\right) \times \left(\frac{-6}{5}\right)$

4. Express the following in exponential form

a) 625 b) 169 c) $\frac{32}{243}$

5. Express the following in exponential form (scientific notation)

a) 59370000000 b) 0.06398521478

6. Write each of the following as single notation

a) $4^4 \times 4^6$ b) $6^2 \times 6^{-3}$
 c) $(-2)^2 \times (-2)^3 \times (-2)^4 \times (-2)^6$ d) $\left(-\frac{1}{5}\right)^3 \cdot \left(-\frac{1}{5}\right)^2 \cdot \left(-\frac{1}{5}\right)^6$

7. Write each of the following with a single exponent
- a) $(2^5)^6$ b) $(4^2)^3$
8. Find the value of each of the following
- a) $\frac{1}{4^{-2}}$ b) 6^{-3} c) $64^3 \div 4^8$
9. Simplify the following
- a) $2^2 + 2^0 + 2^1 + 2^{-2}$ b) $(-1)^1 + (-1)^2 + (-1)^3 + (-1)^4 + (-1)^5 + (-1)^6$ c) $\frac{3^{-2} \cdot 5^{-3} \cdot 5^2}{3^{-4} \cdot 5^{-2} \cdot 3^3}$
10. Evaluate the following
- a) $(64)^{1/4}$ b) $\sqrt[3]{125} + \sqrt{25}$
11. Simplify $(32)^{-\frac{2}{5}} \div (125)^{-\frac{2}{3}} = ?$
12. Which is greater in each of the following.
- a) $5^{10}, 3^5$ b) $2^5, 8^3$
13. Find the value of x when
- a) $9x \cdot 3^x = 3^{2x}$ b) $5^{-3} = \frac{1}{5^x}$ c) $((-3)^3)^x \times (-3)^3 = (-3)^{2x}$
14. Solve the following.
- a) $2^x = 32$ b) $8^{x+2} = 2^{4x-3}$
15. By what number 5^{-1} should multiply to get $\frac{1}{15}$.
16. Find the value of x when $2^{2(x+1)} \cdot 4^{x+1} \cdot 8^{2x+2} = 1$
17. If $9^{x+2} = 720 + 9^x$ then $x = ?$
18. Solve $3^{4x+2} = 27^{x+1}$.
19. Find the value of x, when $2^{-5} \times 2^{2x+1} = 2^{13} \div 2^7$
20. If $(81)^{-1} \times (3)^{-5} \times (27)^{5/6} \cdot (81)^2 = 3^{2x}$ then $x =?$
21. If $a^{mn} = a^m \cdot a^n$ then find the value of $n(n-2) + n(m-2)$ in terms of n?
22. The population of Tamilnadu as per 2010 census is 6,50,00,000. Write in standard form?
23. $x^{p-q} \cdot x^{q-r} \cdot x^{r-p} = ?$
24. Simplify $a^{x+y-z} \times a^{y+z-x} \times a^{z+x-y}$
25. Simplify $(x^m)^{n-p} \cdot (x^n)^{p-m} \cdot (x^p)^{m-n}$
26.
$$\frac{x^{m+n} \times x^{m+\ell} \times x^{\ell+m}}{(x^m \times x^n \times x^\ell)^2} = ?$$
27. $(x^{a-b})^{a+b} \cdot (x^{b-c})^{b+c} \cdot (x^{c+a})^{c-a} = ?$

28. $\left[\frac{x^{3n+1} \cdot x^{3n-1}}{x^{2n+1}} \right]^2 \quad (x \neq 0) = ?$

29. Simplify $\frac{(3^{-2})^5 \times (5^{-3})^4 \times (7^{-4})^5}{(2^{-1})^2 \times (7^2)^{-10} \times (3^4)^{-2} \times (5^2)^{-3}}$

30. Simplify the following.

a) $3pq^2r^2 \div 9rp^2q$ b) $\frac{6a^9b^4c^5}{2ab^2c^3}$

31. Simplify $(xy)^{b-c} \cdot (yz)^{c-a} \cdot (zx)^{a-b}$

32. Simplify $\frac{x^{4n+2} \cdot x^{2(2n+3)}}{x^{2n+2} \cdot x^{4n}}$

33. Simplify $\frac{p^{-4} \cdot q^3 \cdot r^{-2} \cdot s^6}{r^3 p^{-2} s^0 \cdot q^{-4}}$

34. Simplify $x^{m-n} \cdot x^{n-p} \cdot x^{p-m}$

EXPLORERS (Level - III)

I) **Multiple choice questions more than one option :**

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C), (D), out of which **ONE or MORE** is correct. Choose the correct options

1. $2^{100} \cdot 2^{-100} = \dots$

- A) 0 B) 1 C) 2^0 D) 2^1

2. $(2^3)^3 \times (2^7)^2 =$

- A) $2^9 \times 2^{14}$ B) 2^{23} C) $(2^7)^2$ D) 2^{14+9}

3. Which is the value of $8^{\frac{2}{3}}$

- A) 2^2 B) $\sqrt[3]{64}$ C) 4 D) 2^3

4. $(-2)^6 = \dots$

- A) -64 B) 8^2 C) $\sqrt{4096}$ D) 64

5. If $x^5 = (5^5)^5$ then $x =$

- A) 5^5 B) 5 C) 3125 D) 625

6. Fourth power of '2a' can be written as

- A) $2a^4$ B) $16a^4$ C) 4^2a D) $(2a)^4$

II) Match the following :

◆ This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column-I** have to be matched with statements (p, q, r, s) in **Column-II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-r, B-q, C-p, C-q and D-s, then the correct bubbled 4*4 matrix should be as follows:

Column - I**Column - II**

- | | | |
|-----------|-----------------------------|---|
| A. | 1. $\frac{2^{10}}{2^{-10}}$ | () (A) 1 |
| | 2. 6^{18} | () (B) 0 |
| | 3. If $2^x = 16$, then x | () (C) $(6^{10})^8$ |
| | 4. $5^0 - 6^0$ | () (D) 2^{20}
(E) 4
(F) $2^{18} \times 3^{18}$ |

Column - I**Column - II**

- | | | |
|-----------|---|-----------------------------------|
| B. | 1. $100^{2^{30/10}}$ | () (A) 0 |
| | 2. $(12^0 - 11^0)(3^1 - 1^3)$ | () (B) 12 |
| | 3. $(128)^{\frac{5}{7}}$ | () (C) 32 |
| | 4. If $5^x = 25^y = 625$, then $x^2 - y^2$ | () (D) 1
(E) 100^2
(F) 1 |

III) Fill in the blanks:

1. For $n \in \mathbb{N}$, $(-1)^{4n} = \dots$
2. $3^6 = \dots$
3. $(3^4)^2 = \dots$
4. $\left(-\frac{2}{3}\right)^5 = \dots$
5. The exponential notation of $(-6)(-6)(-6)(-6)(-6) = \dots$
6. The exponential notation of $25 = \dots$
7. $\left(\frac{1}{6}\right)^5 \cdot \left(\frac{1}{6}\right)^4 \cdot \left(\frac{1}{6}\right)^{-5} = \dots$
8. $\left(\frac{5}{2}\right)^{-5} \cdot \left(\frac{2}{5}\right)^{-5} = \dots$
9. $2^7 - 7^2 = \dots$

10. $\frac{1}{2^{-3}} = \dots$
11. $x^x \cdot x^x \cdot x^{2x} = \dots$
12. If $5^x = 50$ then $5^{x+1} = \dots$
13. If $3^x = \frac{1}{27}$ then $x = \dots$
14. $(1000)^0 = \dots$
15. If $3^{x+2} = 18$ then $3^x = \dots$
16. If $\left(x^{\frac{2}{3}}\right)^a = 1$ then $a = \dots$
17. Exponential form of 5,97,80,069 = \dots

IV) State whether the following statements are true or false :

1. $(6^0 - 7^0) = 0.$
2. $x^{2017} \times \frac{1}{x^{2016}} = 1.$
3. If $a^0 = 1$ then a is any real number.
4. If $2^x = 4^2$, then $x=2.$
5. If $\frac{1}{3^x} = 3^2$, then $x=2.$
6. $(-1)^{2016} = -1.$
7. If $x^y = x^{2^3}$, then $y=6$ ($x \neq 0, x \neq 1$)
8. If $5^x = 7^x$, then $x=2.$
9. The value of $3(x^2 - 3x + 2)$, when $x=2$, is 0.


RESEARCHERS (Level - IV)

I) MCQs with one option is correct:

1. If $4^x - 4^{x-1} = 24$ then $(2x)^x$ equals [IMO - 2008]
 A) $5\sqrt{5}$ B) $\sqrt{5}$ C) $25\sqrt{5}$ D) 125
2. $(256)^{0.16} \times (256)^{0.09} = \dots$ [Ssc- 2004]
 A) 4 B) 16 C) 64 D) 256.25
3. Find the value of $\left[\left(\frac{1}{4} \right)^2 - \left(\frac{1}{4} \right)^3 \right] \times 2^6$ [SLSTSE-2013]

- A) 1 B) 2 C) 3 D) 4
4. Convert 400 grams per liter into milligrams per liter in proper scientific notation
[KWEST - 2012]
A) 4×10^5 B) 4×10^2 C) 4×10^{-5} D) 4×10^3
5. The value of the expression $\frac{-2^5 \cdot 6^3}{(-2)^7 \cdot (-3)^2}$ is equal to
[KWEST- 2012]
A) 6 B) -6 C) 8 D) -8
6. Express the number 9587.3 in proper scientific notation with the correct amount of significant figures:
[KWEST - 2012]
A) 9.5873×10^3 B) 95.873×10^3 C) 958.73×10^3 D) 9.5873×10^4
7. If $m = (-1)^{2000}$ and $n = (-1)^{2002}$, then the value of $\frac{m}{n}$ is
[SLSTSE-2016]
A) -1 B) 1 C) 2000 D) 2002
8. The value of $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$ is
[SLSTSE-2016]
A) 0 B) -1 C) 1 D) x^{a+b}
9. If $3^x = 6561$, then 3^{x-3} is
[SLSTSE-2017]
A) 81 B) 243 C) 729 D) 27
10. If $3^{x+5} = 81^{x-1}$ then $x = ?$
[NIMO-2018]
A) 8 B) 0 C) 12 D) 3
11. $4^3 + 4^3 + 4^3 + 4^3$ is not equal to
[NIMO-2019]
A) 4^4 B) $4^2 \times 4^2$ C) 256 D) 4^6


KEY
ΦΦ TEACHING TASK :

- | | | | | | | |
|------|-------------------------|---------------------------------|-------------------------------------|-------------------------------------|-----------------------------|------------------------|
| I) | 1) C
7) B
13) B | 2) B
8) A
14) A | 3) C
9) A
15) B | 4) C
10) B
16) B | 5) A
11) D
17) A | 6) A
12) C
18) B |
| II) | 1) A,B,D | 2) B,D | 3) B,C | 4) C,D | 5) A,B,D | |
| III) | 1) E | 2) B | 3) D | 4) C | | |
| IV) | 1) a) $-\frac{32}{243}$ | b) $\frac{1}{1000000}$ | 2) | a) 9.6326×10^{10} | b) 9.39478×10^{-4} | |
| | 3) a) 3^{11} | b) $\left(\frac{1}{2}\right)^7$ | c) $\left(-\frac{6}{5}\right)^{18}$ | d) $\left(-\frac{1}{3}\right)^{22}$ | 4) a) 3^8 | b) 2^5 |

- 5) a) $\frac{4}{9}$ b) 15 6) 2^{-8} 7) a) 3^8 b) 7^8
- 8) a) $x = 6$ b) $x = 2$ c) $x = 0$ 9) a) $x = 3$ b) $x = 1, \frac{2}{3}$ 10) 3^{10}
- 11) $3^7 \cdot 2^{-10}$ 12) a) $3^4 \cdot p^2 q^{-6}$ b) $\frac{1}{625} p^{-4} \cdot q^{-4} \cdot r^{-3}$ 13) 1 14) $y = 4$
- 17) 1 19) x^6 20) 1 21) $2^{a-5} \cdot 3^{-a+2} \cdot 5^{a+4}$
- 22) a) 12 b) -30 c) -24 d) 5 23) $x = -\frac{5}{2}$ 24) 5^{900} 25) 1
- 26) a^{-x^2+4x+8} 27) $\frac{409}{15}$ 28) $x = -1$ 29) $x = 5$ 30) $x^{n^2-p^2}$
- 31) 32 32) $n = 3$

LEARNER'S TASK :**BEGINNERS :**

- | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|
| 1) A | 2) B | 3) B | 4) A | 5) B | 6) B | 7) D |
| 8) C | 9) C | 10) A | 11) A | 12) C | 13) C | 14) D |
| 15) B | 16) B | 17) B | 18) C | 19) B | 20) A | 21) C |
| 22) B | 23) C | 24) C | 25) D | 26) A | 27) B | 28) A |
| 29) B | 30) C | 31) D | 32) C | 33) A | 34) D | 35) C |
| 36) A | 37) A | 38) D | 39) D | | | |

ACHIEVERS :

1. a) 16 b) 256 2. a) $\frac{16}{81}$ b) $\frac{27}{125}$ c) $\frac{729}{4096}$ d) $\frac{64}{15625}$ 3. a) 5^6 b) $\left(-\frac{6}{5}\right)^4$
4. a) 5^4 b) 13^2 c) $\left(\frac{2}{3}\right)^5$ 5. a) 5.937×10^{10} b) $6.398521478 \times 10^{-2}$
6. a) 4^{10} (or) 2^{20} b) 6^{-1} (or) $\frac{1}{6}$ c) $(-2)^{15}$ d) $\left(-\frac{1}{5}\right)^{11}$ 7. a) 2^{30} b) 4^6
8. a) 16 b) $\frac{1}{216}$ c) 4 9. a) $\frac{29}{4}$ b) 0 c) $\frac{5}{3}$ 10. a) $\sqrt{8}$ (or) $2\sqrt{2}$ b) 10
11. $\frac{25}{4}$ 12. a) 5^{10} b) 8^3 13. a) $x = 2$ b) $x = 3$ c) $x = 3$
14. a) $x = 5$ b) $x = 9$ 15. $\frac{1}{3}$ 16. $x = -1$ 17. $x = 1$ 18. $x = 1$
19. $x = \frac{3}{4}$ 20. $x = 5$ 21. $n^3 - 4n^2 + 4n$ (or) $n(n^2 - 4n + 4)$
22. 6.5×10^7 23. 1 24. a^{x+y+z} 25. 1 26. x^{m-n} 27. 1
28. x^{8n-2} 29. $2^2 \times 3^{-2} \times 5^{-6}$ 30. a) $\frac{qr}{3p}$ b) $3a^8 b^2 c^2$ 31. $x^{a-c} \cdot y^{b-a} \cdot z^{c-b}$ 32. x^{2n+6}

33. $p^{-2}q^7r^{-5}s^6$

34. 1

□ EXPLORERS :

I) 1. B,C	2. A,B,D	3. A,B,C	4. B,C,D	5. A,C	6. B,D
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II) (<u>A</u> : 1. D	2. F	3. E	4. B)	(<u>B</u> : 1.E	2. A	3.C	4. B)
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III) 1. 1	2. 729	3. 6561	4. $-\frac{32}{243}$	5. $(-6)^5$	6. 5^2	7. $\left(\frac{1}{6}\right)^4$
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8. 1	9. 79	10. 8	11. x^{4x}	12. 250	13. -3	14. 1
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15. 2	16. 0	17. 5.9780069×10^7
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IV) 1.T	2. F	3. T	4. F	5. F	6. F	7. F
8. F	9. T					

□ RESEARCHERS :

1. C	2. A	3. C	4. A	5. B	6. A	7. B
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8. C	9. B	10. D	11. A, B, C			
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