POLYNOMIALS

Class: IX, Mathematics

BOARD INTEGRATED IIT

[SOLUTIONS]

TEACHING TASKS

01 $x^2+2xy+y^2-z^2$

$=(x+y)^2-z^2$

$=(x+y+z)(x+y-z)$

Ans: B

02 $f(x)=x^3-px^2+6x-p$

Remainder $f(p)=p^3-p\cdot p^2+6p-p=5p$ Ans: C

03 $f(x)=px^2+5x+r$

$f(2)=p(2)^2+5(2)+r=0$

$4p+r+10=0 \rightarrow (1)$

$f\left(\frac{1}{2}\right)=p\left(\frac{1}{2}\right)^2+5\left(\frac{1}{2}\right)+r=0$

$\frac{p}{4}+\frac{5}{2}+r=0$

$p+4r+10=0 \rightarrow (2)$

$\Rightarrow$

Solving (1) & (2)

	p	r	1
1	10	4	1
4	10	1	4

$\therefore \frac{p}{10-40}=\frac{r}{10-40}=\frac{1}{16-1}$

$\Rightarrow \frac{p}{-30}=\frac{r}{-30}=\frac{1}{15}$

$\therefore p=-2$

$r=-2$

$\therefore p^2-r^2=0$

Ans: A

04 $a^{1/3} + b^{1/3} + c^{1/3} = 0$

$a+b+c=0 \iff a^3+b^3+c^3 = 3abc$ (2)

$\therefore a+b+c = 3 \cdot a^{1/3} \cdot b^{1/3} \cdot c^{1/3}$

$(a+b+c)^3 = 27abc$

Ans: C

05 $a^{12}x^4 - a^4x^{12}$

$= (a^6x^2)^2 - (a^2x^6)^2$

$= (a^6x^2 + a^2x^6)(a^6x^2 - a^2x^6)$

$= a^2x^2(a^4+x^4)(a^3x - ax^3)(a^3x + ax^3)$

05 $a^{12}x^4 - a^4x^{12}$

$= a^4x^4(a^8 - x^8)$

$= a^4x^4(a^4+x^4)(a^4 - x^4)$

$= a^4x^4(a^4+x^4)(a^2+x^2)(a^2-x^2)$

$= a^4x^4(a^4+x^4)(a^2+x^2)(a+x)(a-x)$

Ans: C

06 opt: A $l = 7y-3$, $b = 5y+4$

$A = (7y-3)(5y+4)$

$= 35y^2 + 13y - 12$

Ans: A

07 $(x-a)^3 + (x-b)^3 + (x-c)^3$

$a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

$(x-a)+(x-b)+(x-c) = 3x - (a+b+c) = 0$.

Hence $3(x-a)(x-b)(x-c)$

Ans: A

08 (3)

$$= \frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)}$$

$$= \frac{(a+b)(a-b)(b+c)(b-c)(c+a)(c-a)}{(a-b)(b-c)(c-a)}$$

$$= (a+b)(b+c)(c+a)$$

Ans: B

09

$$f(x) = x^{140} + 2.x^{151} + K$$

$$f(-1) = (-1)^{140} + 2(-1)^{151} + K = 0$$

$$\Rightarrow 1 - 2 + K = 0$$

$$\Rightarrow K = 1$$

Ans: A

10

$$xy + yz + zx + y^2$$

$$= x(y+z) + y(z+y) = (z+y)(x+y)$$

Now, $x^2 + xy + yz + zx$

$$= x(x+y) + z(y+x)$$

$$= (x+y)(x+z)$$

L.C.M $= (x+y)(y+z)(z+x)$

11

$$P(x) = px^2 + 5x + r$$

$$P(-2) = p(-2)^2 + 5(-2) + r$$

$$4p - 10 + r = 0 \rightarrow (1)$$

$$P\left(-\frac{1}{2}\right) = p\left(-\frac{1}{2}\right)^2 + 5\left(-\frac{1}{2}\right) + r = 0$$

$$p + 4r + 10 = 0 \rightarrow (2)$$

Solving (1) & (2) $p = r = -2$

Ans: A, B

11) $p(x) = px^2 + 5x + r$ (4)

$p(-2) = p(-2)^2 + 5(-2) + r = 0$

$\Rightarrow 4p - 10 + r = 0$

$\Rightarrow 4p + r = 10 \rightarrow (1)$

$p\left(-\frac{1}{2}\right) = p\left(-\frac{1}{2}\right)^2 + 5\left(-\frac{1}{2}\right) + r = 0$

$\frac{p}{4} - \frac{5}{2} + r = 0$

$\Rightarrow p - 10 + 4r = 0$

$\Rightarrow 4r + p = 10 \rightarrow (2)$

from (1) & (2) $r = p$

Ans. A, C

12 Conceptual

Ans: A, D

13 $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f(1) = a + b + c + d + e = 0$

$f(-1) = a - b + c - d + e = 0$

$a + c + e = b + d$

Ans: A, C

14 Statement I: $(3x-1)^7 = a_7x^7 + a_6x^6 + \cdots + a_1x + a_0$

put $x = 1$

$(2)^7 = a_7 + a_6 + \cdots + a_1 + a_0$ (True)

$\therefore 128 = a_7 + a_6 + \cdots + a_1 + a_0$

Statement II: By substituting $x = 1$, we get the sum of the coefficients (False)

Ans: C

15 $(x-\frac{1}{2})(x-\sqrt{2})$

$p(x) = x^2 - (\sqrt{2}+\frac{1}{2})x + \frac{1}{2}\sqrt{2}$

$= x^2 - (\sqrt{2}+\frac{1}{2})x + \frac{1}{\sqrt{2}}$

Ans: C

16 $p(x) = (x-\frac{m}{n})(x+\frac{m}{n}) = 0$

$= x^2 - \frac{m^2}{n^2} = 0$

$p(x) \Rightarrow n^2x^2 - m^2$

Ans: C

17 $p(x) = (x-\sqrt{2})(x-\sqrt{3})$

$p(x) = x^2 - (\sqrt{2}+\sqrt{3})x + \sqrt{6}$

product of the roots $= \frac{c}{a} = \sqrt{6}$

Ans: A

18 $x - y + z = 0$

$\Rightarrow x + z = y$

$\Rightarrow x^3 + z^3 + 3xz(x+z) = y^3$

$\Rightarrow x^3 + z^3 + 3xz(y) = y^3$

$\Rightarrow x^3 + z^3 = y^3 - 3xyz$

Ans: D

19 $x^{1/3} + y^{1/3} + z^{1/3} = 0$

$\boxed{a+b+c=0 \iff a^3+b^3+c^3 = 3abc}$

$x + y + z = 3 \cdot x^{1/3} \cdot y^{1/3} \cdot z^{1/3}$

$\therefore (x+y+z)^3 = 27xyz$

Ans: B

20 $l-m-n=0$

$\Rightarrow l=m+n$

$\Rightarrow l^3 = m^3+n^3+3mn(m+n)$

$\Rightarrow l^3 = m^3+n^3+3mnl$

Ans: B

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21 $\sum_{x,y,z} x^2(y^2-z^2) = x^2(y^2-z^2)+y^2(z^2-x^2)+x^2(y^2-z^2)$

$=0$

Ans: 0

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22 a) $f(x) = x^3-px^2+6x-p$

$f(p) = p^3-p\cdot p^2+6\cdot p-p = 5p$

b) $f(x) = x^3-px^2+5x-p$

$f(-p) = (-p)^3-p(-p)^2+5(-p)-p$

$= -p^3-p^3-5p-p$

$= -2p^3-6p$

$= -2p(p^2+3)$

c) $f(x) = x^3-x^2\cdot p+p^2x-1$

$f(1) = 1-p+p^2-1 = p^2-p$

d) $f(x) = p^3x^3-3x^2p+3px+2$

$f(-1) = -p^3-3p-3p+2$

$= -p^3-6p+2$

Ans: r, t, q, p

LEARNERS TASK

CUQ's

01. opt A: $x^3 - 2x + \sqrt{3}$

opt D: $\frac{1}{x^{-2}} + 5 = x^2 + 5$

Ans: A, D

02 Conceptual

Ans: B

03 $x^{-2}\left(x^5 + \frac{3}{x^{-4}} + x^{11}\right)$

$= x^3 + x^{-2} \cdot x^4 \cdot 3 + x^{-2} \cdot x^{11}$

$= x^3 + 3x^2 + x^9$

Degree = 9

Ans: C

04 $x^3 + 2x^4y^2 - x^5y^3z + 13$

Degree $= 5 + 3 + 1 = 9$

Ans: D

05 $A = 1, B = -1 \Rightarrow \frac{A}{B} = -1$

Ans: B

06 $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 - 3abc$

Ans: A

07 $3 \times 4 = 12$

Ans: D

08 Conceptual

Ans: C

09 $2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

Answer

10 Conceptual

Ans: B

JEE MAIN LEVEL QUESTIONS

01 $3x^2 - 5x = 0$

$\Rightarrow x(3x-5) = 0$

$\Rightarrow x = 0$ or $x = \frac{5}{3}$

Ans: B

02 Conceptual

Ans: A

03 $f(1) = a+b+c+d$

Ans: C

04 $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (9)^2 = a^2+b^2+c^2+2(26)$

$\Rightarrow a^2+b^2+c^2 = 29$

Ans: C

05 $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4$

$= \left(5x - \frac{1}{x} + 2\right)^2$

$= \left(\frac{5x^2+2x-1}{x}\right)^2$

Ans: A

06 Option verification method

Opt A: $(1+a+b)(1-a+b)$

$= (1+b+a)(1+b-a)$

$= (1+b)^2 - a^2$

$= 1+b^2+2b-a^2$

Ans: A

07 $(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc$

$a^3-b^3+1+3ab = a^3+(-b)^3+(1)^3-3(a)(-b)(1)$

$= (a-b+1)(\text{————})$

Ans: B

08 option: C $30^3 + 20^3 - 50^3$ (9)

$\Rightarrow 30+20-50=0$

Ans: C

09 $a(-2)^3 + 3(-2)^2 - 13 = 2(-2)^2 - 5(-2) + a$

$\Rightarrow a = \frac{5}{9}$

Ans: D

10 $(x+y)(1-z) - (y+z)(1-x)$

$= (x - xz + y - yz) - (y - yx + z - zx)$

$= x - xz + y - yz - y + yx - z + zx$

$= x - yz + yx - z$

$= x(1+y) - z(y+1) \Rightarrow$

$= (1+y)(x-z)$

Ans: D

11 $p(a)=0, \quad p(b)=0$

Ans: A, C, D

12 $f(x) = 2x^3 - 5x^2 + ax + b$

$f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 6$

$2a + b = 4 \rightarrow (1)$

$f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0$

$\boxed{b=0} \rightarrow (2)$

$\therefore \boxed{a=2}$

Ans: A, B

13 Statement-I: $f(x) = x^3 + ax^2 + bx + 6$

$f(2) = 8 + 4a + 2b + 6 = 0 \rightarrow (1)$

$f(3) = 27 + 9a + 3b + 6 = 3 \rightarrow (2)$

Solving & substituting we get $a^2 + b^2 = 10$ (True)

Statement-II: Conceptual (True)

Ans: A

14 Statement I: $p(x) = x^3 - 6x^2 + 11x - 6$ (10)

$p(1) = 1 - 6 + 11 - 6 = 0$

$p(2) = 8 - 24 + 22 - 6 = 0$

$p(3) = 27 - 54 + 33 - 6 = 0$ (True)

Statement II: Conceptual (True) Ans: A

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15 $p\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^2 + a\left(\frac{1}{3}\right) = 0$

$\Rightarrow 1 + 3a = 0 \Rightarrow a = \frac{-1}{3}$ Ans: C

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16 $p(1) = 0$

& $p(-1) = -1 - 3 - 3 - 1 = -8$

$\therefore p(1) + p(-1) = -8$ Ans: C

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17 $p(1) = a(1) + b = 0 \Rightarrow a + b = 0$ Ans: C

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18 $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow 8^2 = a^2 + b^2 + c^2 + 2(26)$

$\Rightarrow a^2 + b^2 + c^2 = 12$

$\Rightarrow a^2 + b^2 + c^2 - 20 = -8$ Ans: -8

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19 $f(1) = 1 - a + b - 10 = 0$

$\Rightarrow -a + b = 9 \rightarrow (1)$

$f(-2) = -8 - 4a - 2b - 10 = 0$

$-4a - 2b = 18 \rightarrow (2)$

Solving eqn (1) & eqn (2) $\frac{a}{b} = \frac{0}{9} = 0$

Ans: 0

20 $f(x) = x^3 - 23x^2 + 142x - 120$

$f(1) = 1 - 23 + 142 - 120 = 0$

$\therefore 1$ is a zero

21 A) $x^2y^2 + 2xyz^2 - x^2z^2 - y^2z^2$

$= x^2(y^2 - z^2) + yz^2(2x - yz)$

$= x^2(y^2 - z^2) + yz^2(2x - y)$

B) $x^3 + y^3 - z^3 + 3xyz$

$= x^3 + y^3 + (-z)^3 - 3xy(-z)$

$= (x + y - z)(x^2 + y^2 + z^2 - xy + yz + zx)$

C) $(y - z)x^3 + (z - x)y^3 + (x - y)z^3$

$= (y - z)x^3 + zy^2 - xy^3 + xz^3 - yz^3$

$= x^2(y - z) + y^2(x - z) + z^2(x - y) + xyz$

$= x^2(y - z) + y^2x - y^2z + z^2x - z^2y + xyz$

$= x^2(y - z) + (y^2 + z^2 + yz)x - y^2z - z^2y$

Ans: A, B, C, D

ADDITIONAL PRACTICE QUESTIONS

01. $x^{12} - y^{12} = (x^6 + y^6)(x^6 - y^6)$

$= (x^6 + y^6)(x^3 + y^3)(x^3 - y^3)$

$= (x^6 + y^6)(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$

$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$

Total: 6

Ans: C

02 $p^3(q-r)^3 + q^3(r-p)^3 + r^3(p-q)^3$ (12)

$= [p(q-r)]^3 + [q(r-p)]^3 + [r(p-q)]^3$

$p(q-r) + q(r-p) + r(p-q) = 0$.

$a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

$= 3pqr(p-q)(q-r)(r-p)$

Ans : D

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03 $f(1) = 1-2+3-a-b = 6$

$\Rightarrow a+b = -4$

Ans : C

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04 $(3 \cdot 1 - 1)^7 = a_7 + a_6 + a_5 + \cdots + a_1 + a_0$

$\therefore$ Ans : $2^7 = 128$

Ans : C

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05 $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$

$(a+c+b)(a+c-b) = (a+c)^2 - b^2$

$= (a+c)^2 - ac$

$= a^2 + c^2 + ac$

Ans : B

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06 $x+3 \overline{)} x^4 + x^3 ($

$\longrightarrow$ Degree $= 0$

Since $f(x) = x^4 + x^3$

$f(-3) = (-3)^4 + (-3)^3 =$ Constant.

Ans : B

07 $x^3+3x^2+3x+1=(x+1)^3$

$\therefore f(x)=(x+1)^3$

$f(\pi)=(\pi+1)^3=\left(\frac{22}{7}+1\right)^3=\left(\frac{29}{7}\right)^3$ Ans: C

08 $3x^2+7x-6\overline{)3x^3+x^2-2x+9}\,(x-2$

$3x^3 \pm 7x^2 \mp 6x$

$-6x^2+4x+9$

$-6x^2-14x+12$

$18x-3$ Ans: A

09 Conceptual Ans: D

10 $c^2x+d^2=0$

$\Rightarrow x^2=\frac{-d^2}{c^2}=-\left(\frac{d}{c}\right)^2$

$\Rightarrow x=$

10 $c^2x+d^2=0$

$x=\frac{-d^2}{c^2}=-\left[\frac{-d}{c}\right]^2$ Ans: B

$\Rightarrow$ THE END $\Leftarrow$