7. HEIGHTS AND DISTANCES

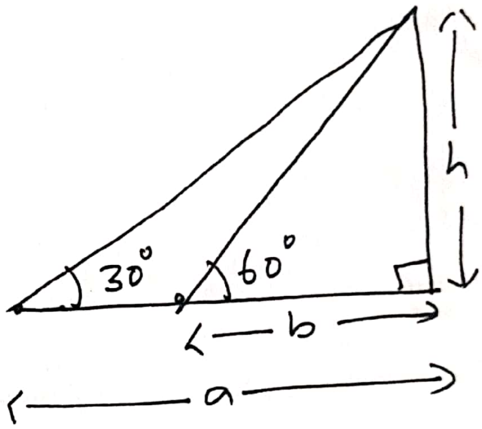
Class: IX MATHEMATICS

FOUNDATION SOLUTIONS

TEACHING TASK 2025/26

JEE MAINS

01. From diagram



$$\tan 30° = \frac{h}{a}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a}$$

$$\Rightarrow h = \frac{a}{\sqrt{3}} \longrightarrow (i)$$

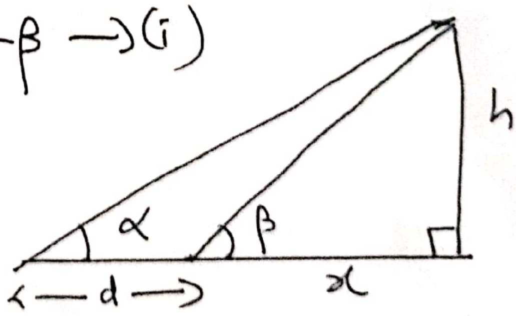
Again, $\tan 60° = \frac{h}{b}$

$$\Rightarrow \sqrt{3} = \frac{h}{b}$$

$$\Rightarrow h = \sqrt{3}\, b \longrightarrow (ii)$$

Now, $(i)\times(ii) \Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$ Ans. B

02. From the diagram



$\tan\beta = \frac{h}{x} \Rightarrow x = h\cot\beta \rightarrow (i)$

Now, $\tan\alpha = \frac{h}{d+x}$

$$\Rightarrow x = h\cot\alpha - d \rightarrow (ii)$$

From (i) and (ii)

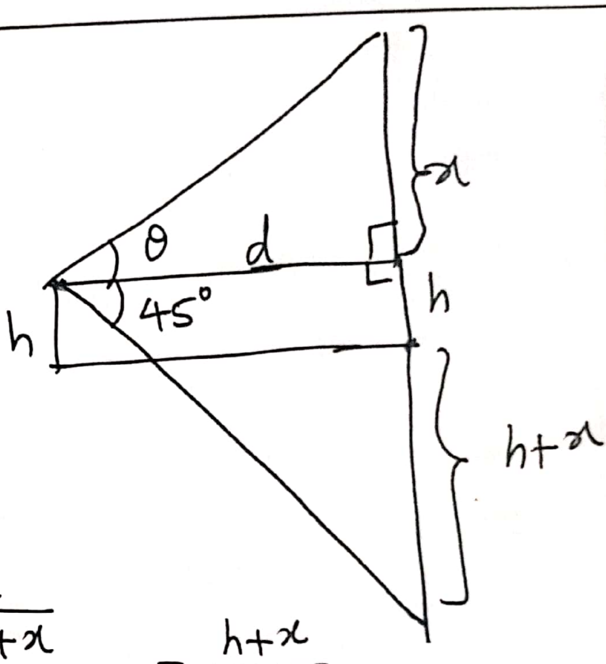
$\Rightarrow h\cot\beta = h\cot\alpha - d$

$\Rightarrow h\cot\alpha - h\cot\beta = d$

$\Rightarrow h = \dfrac{d}{\cot\alpha - \cot\beta}$

Ans: A

03 $\tan\theta = \dfrac{x}{d} \longrightarrow (1)$



$\tan 45^\circ = \dfrac{2h+x}{d}$

$\Rightarrow d = 2h+x \longrightarrow (2)$

Now, $\therefore \tan\theta = \dfrac{x}{2h+x}$

$\tan(45^\circ+\theta) = \dfrac{1+\tan\theta}{1-\tan\theta}$

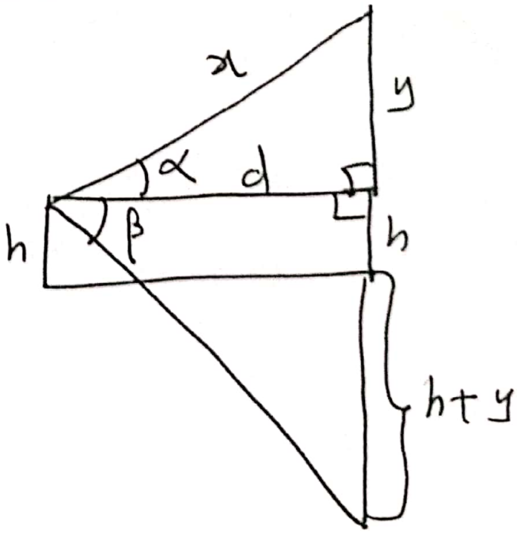
$= \dfrac{1+\dfrac{x}{2h+x}}{1-\dfrac{x}{2h+x}} = \dfrac{h+x}{h}$

$\therefore h+x = h(\tan 45^\circ+\theta)$

$\therefore$ The height of the cloud $= h(\tan 45^\circ+\theta)$

Ans.. A

04 $\tan\alpha = \dfrac{y}{d} \Rightarrow d = \dfrac{y}{\tan\alpha}$



$\tan\beta = \dfrac{2h+y}{d} \Rightarrow d = \dfrac{2h+y}{\tan\beta}$

$\therefore \dfrac{y}{\tan\alpha} = \dfrac{2h+y}{\tan\beta}$

$\Rightarrow \dfrac{y}{\tan\alpha} = \dfrac{2h}{\tan\beta} + \dfrac{y}{\tan\beta}$

$$\Rightarrow \frac{y}{\tan\alpha} - \frac{y}{\tan\beta} = \frac{2h}{\tan\beta}$$

$$\Rightarrow y\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha \cdot \tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$\Rightarrow \frac{y}{\tan\alpha} = \frac{2h}{\tan\beta - \tan\alpha}$$

$$\Rightarrow d = \frac{2h}{\tan\beta - \tan\alpha} \longrightarrow (1)$$

Now, $\sec\alpha = \frac{x}{d}$

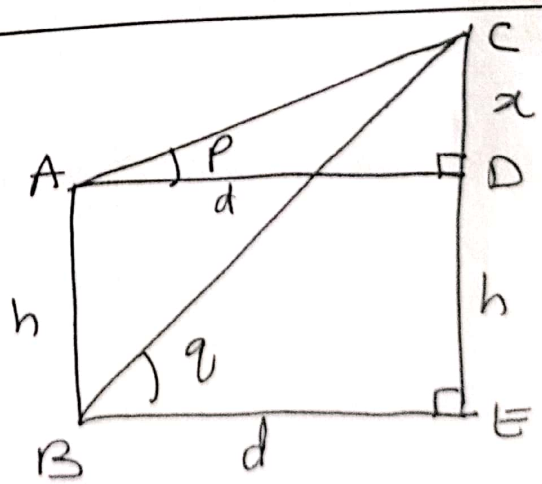
$$\Rightarrow x = d\sec\alpha$$

$$\Rightarrow x = \frac{2h\sec\alpha}{\tan\beta - \tan\alpha} \longrightarrow (\because \text{from } (1))$$

Hence, The distance b/w the cloud and the observer is $\frac{2h\sec\alpha}{\tan\beta - \tan\alpha}$

$\boxed{\text{Ans: D}}$

$\boxed{05}$ From the diagram
AB = Building
CE = Hill
A, B = Position of the observers



Now, $\tan P = \frac{x}{d}$ and

$$\tan q = \frac{x+h}{d}$$

$$\Rightarrow d = x\cot P \quad \text{also} \quad d = (x+h)\cot q$$

$$\therefore x \cot p = (x+h)\cot q$$

$$\Rightarrow x(\cot p - \cot q) = h\cot q$$

$$\Rightarrow x = \frac{h\cot q}{\cot p - \cot q}$$

Now, height of the hill

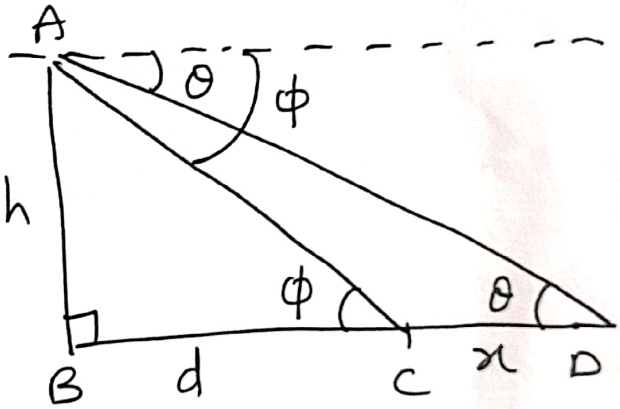
$$x + h = \frac{h\cot q}{\cot p - \cot q} + h$$

$$= \frac{h\cot q + h\cot p - h\cot q}{\cot p - \cot q}$$

$$= \frac{h\cot p}{\cot p - \cot q}$$

Ans: C

06. In the diagram
C, D = Position of the boats
A = Position of the observer
AB = Light house



Now, $\tan\phi = \frac{h}{d} \Rightarrow d = \frac{h}{\tan\phi}$

$$\tan\theta = \frac{h}{d+x} \Rightarrow d + x = \frac{h}{\tan\theta}$$

$$\therefore \frac{h}{\tan\phi} + x = \frac{h}{\tan\theta}$$

$$\Rightarrow x = \frac{h}{\tan\theta} - \frac{h}{\tan\phi}$$

$$\Rightarrow x = h\left(\frac{\tan\phi - \tan\theta}{\tan\theta \cdot \tan\phi}\right)$$

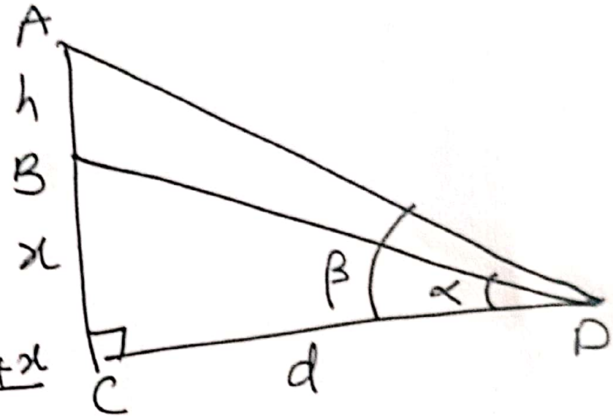
∴ The distance between the two boats is $\dfrac{h(\tan\phi - \tan\theta)}{\tan\theta \cdot \tan\phi}$ (5)

Ans, B

07 D = Position of the observer

AB = Flag staff

BC = Vertical tower



Now, $\tan\alpha = \dfrac{x}{d}$ | $\tan\beta = \dfrac{h+x}{d}$

$\Rightarrow d = \dfrac{x}{\tan\alpha}$ | $\Rightarrow d = \dfrac{h+x}{\tan\beta}$

$$\therefore \frac{x}{\tan\alpha} = \frac{h+x}{\tan\beta}$$

$$\Rightarrow \frac{x}{\tan\alpha} = \frac{h}{\tan\beta} + \frac{x}{\tan\beta}$$

$$\Rightarrow x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = \frac{h}{\tan\beta}$$

$$\Rightarrow x\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha \cdot \tan\beta}\right) = \frac{h}{\tan\beta}$$

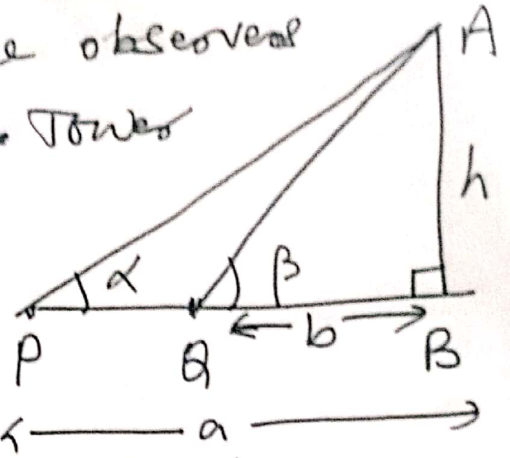
$$\Rightarrow x = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$$

∴ Height of the tower is $\dfrac{h\tan\alpha}{\tan\beta - \tan\alpha}$

Ans: B

08 $P, Q \to$ Position of the observers

$AB =$ ~~Position of the~~ Tower



Given $\alpha + \beta = 90^\circ$

$\Rightarrow \tan(\alpha+\beta) = \tan 90^\circ$

$\Rightarrow \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} =$ not defined

$\Rightarrow 1 - \tan\alpha \cdot \tan\beta = 0$

$\Rightarrow \tan\alpha \cdot \tan\beta = 1$

$\Rightarrow \dfrac{h}{a} \cdot \dfrac{h}{b} = 1$

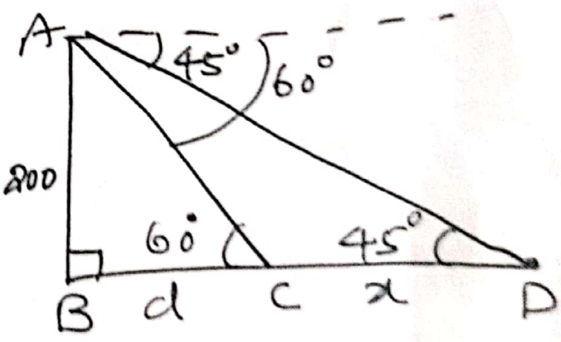
$\Rightarrow h = \sqrt{ab}$

$\therefore$ Height of the tower is $\sqrt{ab}$

Ans. A

09 $AB =$ Altitude of the aeroplane

$C, D =$ opposite sides of the banks of a river



$CD = x =$ width of the river

Now, $\tan 60^\circ = \dfrac{200}{d} \Rightarrow d = \dfrac{200}{\sqrt{3}} \to (1)$

Also, $\tan 45^\circ = \dfrac{200}{d+x}$

$\Rightarrow d + x = 200$

$\Rightarrow \dfrac{200}{\sqrt{3}} + x = 200$ ($\because$ from (1))

$$\Rightarrow x = 200 - \frac{200}{\sqrt{3}}$$

$$\Rightarrow x = 200\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$$

Ans: B

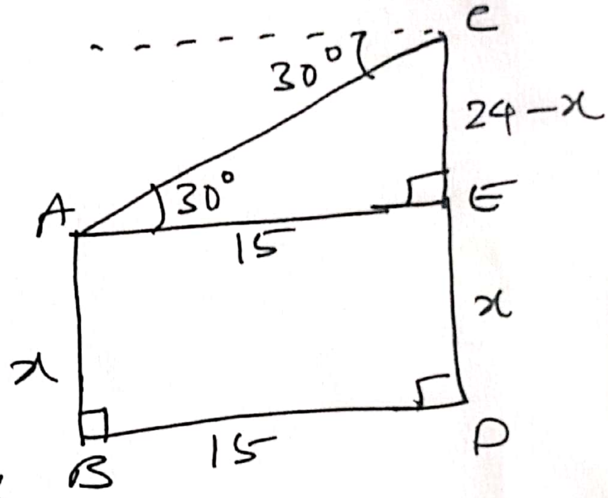
10 AB → First pole

CD → Second pole

BD = 15 mt, CD = 24 mt

Let AB = x mt

$\therefore$ CE = $(24-x)$ mt



In $\triangle AEC$, $\tan 30^\circ = \frac{24-x}{15}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{24-x}{15}$$

$$\Rightarrow 24 - x = \frac{15}{\sqrt{3}} = 5\sqrt{3} = 8.67$$

$$\Rightarrow x = 24 - 8.67$$

$$\Rightarrow x = 15.34 \text{ (Approx)}$$

Ans: C

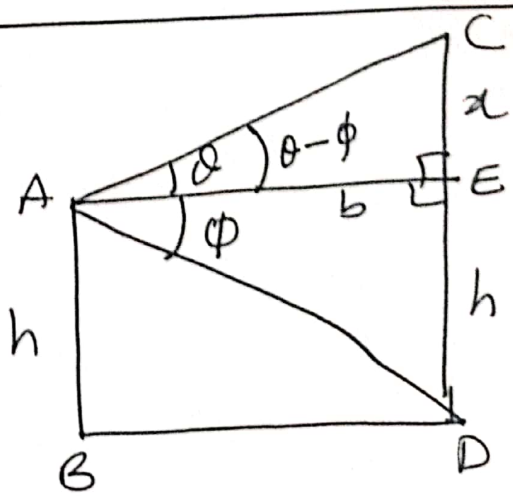
JEE ADVANCED LEVEL

01. AB = Window

CD = Building

BD = Road

AB = h



$$\tan\phi = \frac{h}{b} \rightarrow (1)$$

$$\therefore \tan(\theta-\phi) = \frac{x}{b}$$

$$\Rightarrow \frac{\tan\theta - \tan\phi}{1+\tan\theta\cdot\tan\phi} = \frac{x}{b}$$

$$\Rightarrow \frac{\frac{\sin\theta}{\cos\theta} - \frac{h}{b}}{1+\frac{\sin\theta}{\cos\theta}\cdot\frac{h}{b}} = \frac{x}{b} \quad (\because \text{from } ①)$$

$$\Rightarrow \frac{b\sin\theta - h\cos\theta}{b\cos\theta + h\sin\theta} = \frac{x}{b}$$

$$\Rightarrow x = \frac{b(b\sin\theta - h\cos\theta)}{b\cos\theta + h\sin\theta}$$

Height of the building $= h+x$

$$\therefore h+x = h + \frac{b(b\sin\theta - h\cos\theta)}{b\cos\theta + h\sin\theta}$$

$$= \frac{bh\cos\theta + h^2\sin\theta + b^2\sin\theta - bh\cos\theta}{b\cos\theta + h\sin\theta}$$

$$= \frac{(h^2+b^2)\sin\theta}{b\cos\theta + h\sin\theta}$$

Ans: C

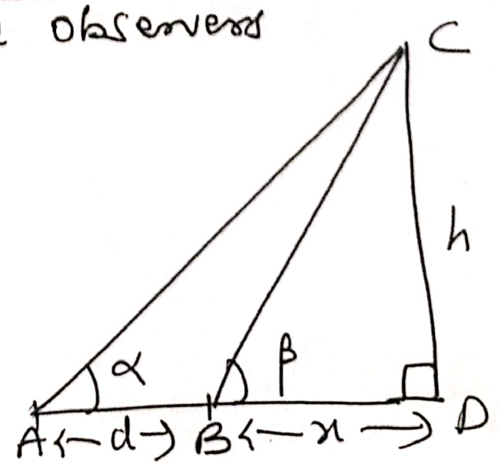
---

02 A, B → Position of the observers

CD = tower

AB = d

Let BD = x, CD = h



Now, $\tan\beta = \frac{h}{x}$ and $\tan\alpha = \frac{h}{d+x}$

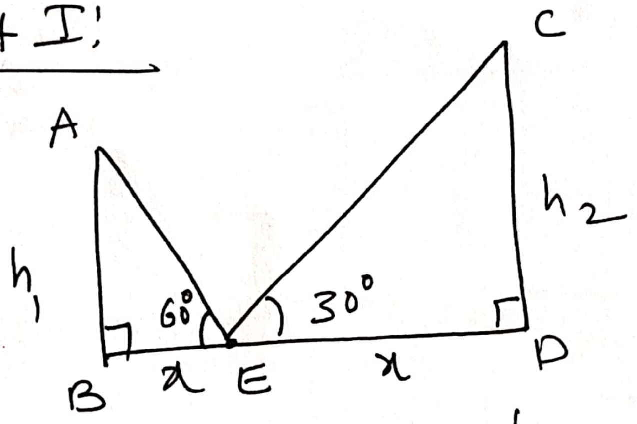
$\Rightarrow x = h\cot\beta$ and $d+x = h\cot\alpha$

$\Rightarrow d + h\cot\beta = h\cot\alpha$

$\Rightarrow h = \frac{d}{\cot\alpha - \cot\beta}$

$\therefore$ Height of the tower $= \frac{d}{\cot\alpha - \cot\beta}$ Ans B

Q3 statement I:



$\tan 60° = \frac{h_1}{x}$

$\Rightarrow x = \frac{h}{\sqrt{3}} \rightarrow (i)$

$\tan 30° = \frac{h_2}{x}$

$\Rightarrow x = \sqrt{3}\, h_2 \rightarrow (ii)$

From (i) and (ii)

$\Rightarrow \frac{h_1}{\sqrt{3}} = \sqrt{3}\, h_2 \Rightarrow h_1 = 3h_2$

$\Rightarrow h_1 : h_2 = 3 : 1$ (True)

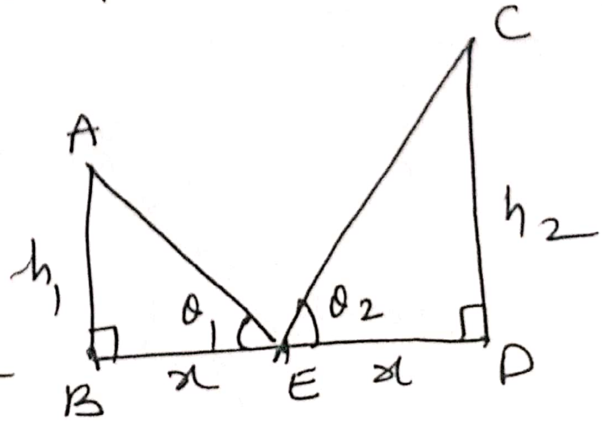
statement II:

$\tan\theta_1 = \frac{h_1}{x}$

$\Rightarrow x = \frac{h_1}{\tan\theta_1}$

$\tan\theta_2 = \frac{h_2}{x}$

$\Rightarrow x = \frac{h_2}{\tan\theta_2}$

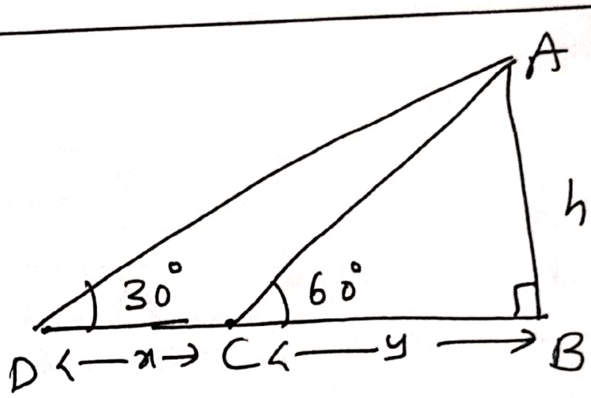


$$\Rightarrow \frac{h_1}{\tan\theta_1} = \frac{h_2}{\tan\theta_2}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

$$\Rightarrow h_1 : h_2 = \tan\theta_1 : \tan\theta_2$$ (True) Ans: A

04. Statement I:



$$\tan 30° = \frac{h}{x+y}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y}$$

$$\Rightarrow x+y = \sqrt{3}h \rightarrow (1)$$

Again, $\tan 60° = \frac{h}{y} \Rightarrow \sqrt{3} = \frac{h}{y}$

$$\Rightarrow y = \frac{h}{\sqrt{3}} \rightarrow (2)$$

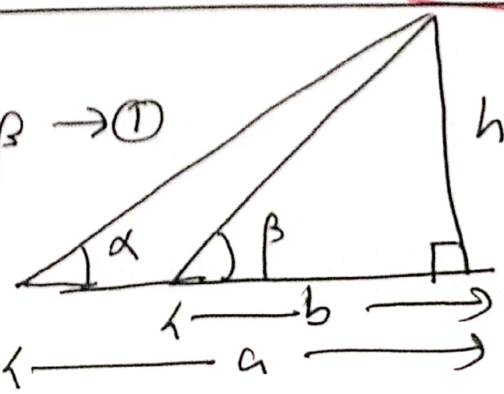
from (1) $\Rightarrow x + \frac{h}{\sqrt{3}} = \sqrt{3}h$

It is not possible to find h value unless the value of x is given. (False)

Statement II: Question Repeated. Please Refere Q No: 2 (Advanced question) (True)

Ans: D

05 Statement I:



$$\tan\beta = \frac{h}{b} \Rightarrow h = b\tan\beta \rightarrow (1)$$

Also, $\tan\alpha = \frac{h}{a}$

$$\Rightarrow h = a\tan\alpha \rightarrow (2)$$

Now, ① × ② $\Rightarrow h^2 = ab\tan\alpha \cdot \tan\beta$

$\Rightarrow h = \sqrt{ab\tan\alpha \cdot \tan\beta}$ (False)

Statement II: $a = 9\,mt$, $b = 4\,mt$

$\alpha = 30^\circ$, $\beta = 60^\circ$

Now, $h = \sqrt{ab\tan\alpha \cdot \tan\beta}$

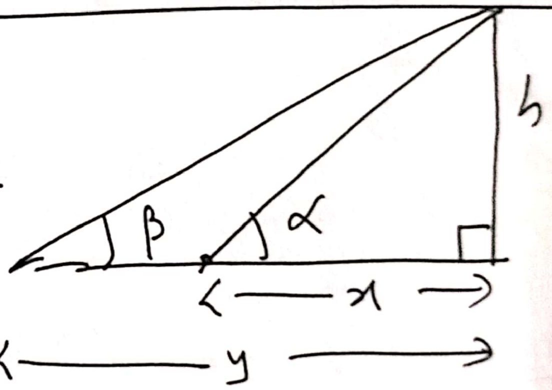
$= \sqrt{9 \cdot 4 \cdot \tan 30^\circ \cdot \tan 60^\circ}$

$= \sqrt{36 \times \frac{1}{\sqrt{3}} \times \sqrt{3}}$

$= 6\,mt$ (True)

Ans: D

06 Statement I:



$\tan\alpha = \frac{h}{x}$ and $\tan\beta = \frac{h}{y}$

$\alpha + \beta = 90^\circ$

$\Rightarrow \tan(\alpha+\beta) = \tan 90^\circ$

$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} =$ Not defined

$\Rightarrow \tan\alpha \cdot \tan\beta = 1$

$\Rightarrow \frac{h}{x} \cdot \frac{h}{y} = 1$

$\Rightarrow h^2 = xy$

$\Rightarrow h = \sqrt{xy}$ (TRUE)

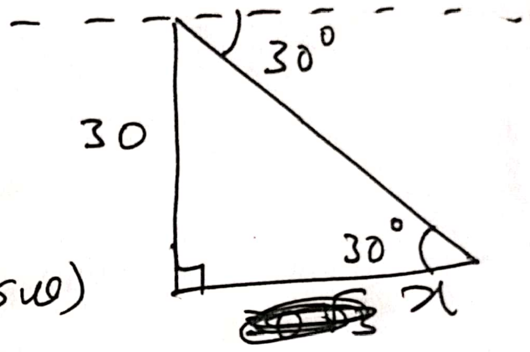
Statement II: Given $x = a^2 + b^2$, $y = 2ab$

We have $h = \sqrt{xy}$

$$h = \sqrt{(a^2+b^2) \cdot 2ab} \neq (a+b) \text{ (False)}$$

$\boxed{\text{Ans. C}}$

07 <u>Assertion:</u>



$$\tan 30 = \frac{30}{x}$$

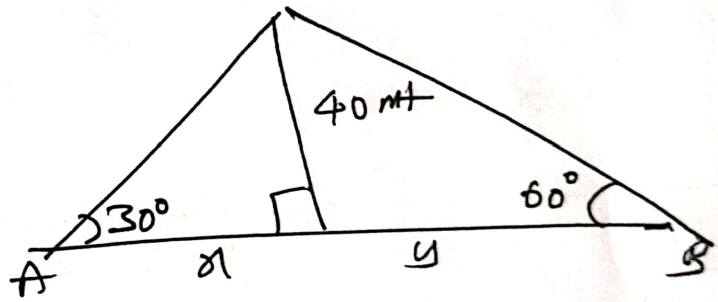
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{30}{x}$$

$$\Rightarrow x = 30\sqrt{3} \text{ mt (True)}$$

Ans. A

<u>Reason:</u> Conceptual (True)

08 <u>Assertion:</u>



$$\tan 30^\circ = \frac{40}{x}$$

$$\Rightarrow x = 40\sqrt{3}$$

$$\tan 60^\circ = \frac{40}{y}$$

$$\Rightarrow y = \frac{40}{\sqrt{3}}$$

Distance $= x + y$

$$= 40\sqrt{3} + \frac{40}{\sqrt{3}} = \frac{160}{\sqrt{3}}$$

$= 92.38$ (Approx) (True)

Ans. A

<u>Reason:</u> Conceptual (True)

09

$$d = \frac{h(\tan\alpha + \tan\beta)}{\tan\alpha \cdot \tan\beta}$$

here $h = 10$, $\alpha = 45^\circ$, $\beta = 30^\circ$

$$d = \frac{10(\tan 45^\circ + \tan 30^\circ)}{\tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{10\left(1+\frac{1}{\sqrt{3}}\right)}{1\cdot\frac{1}{\sqrt{3}}} = 10(\sqrt{3}+1)\,mt$$ Ans. A

10. $d = 20\sqrt{3}$, $\alpha = 15^\circ$, $\beta = 30^\circ$

$$\therefore 20\sqrt{3} = \frac{h(\tan 15^\circ + \tan 30^\circ)}{\tan 15^\circ \cdot \tan 30^\circ}$$

$$\Rightarrow 20\sqrt{3} = \frac{h\left((2-\sqrt{3})+\frac{1}{\sqrt{3}}\right)}{(2-\sqrt{3})\cdot\frac{1}{\sqrt{3}}}$$

$$\Rightarrow h = \frac{10\sqrt{3}(2-\sqrt{3})}{\sqrt{3}-1}$$ Ans. B

11 $\alpha = 45^\circ$, $d = 30\sqrt{2}$ $h = 2\,mt$

We have $$30\sqrt{2} = \frac{2(\tan 45^\circ + \tan\beta)}{\tan 45^\circ \cdot \tan\beta}$$

$$\Rightarrow 30\sqrt{2} = \frac{2(1+\tan\beta)}{\tan\beta}$$

$$\Rightarrow \tan\beta = \frac{2}{30\sqrt{2}-2}$$

$$\Rightarrow \tan\beta = \frac{1}{15\sqrt{2}-1}$$ Ans. B

12 $$H = \frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$$

14

$\alpha = 30^\circ$, $\beta = 60^\circ$

$$H = \frac{h\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)}{\sqrt{3} - \frac{1}{\sqrt{3}}} \Rightarrow H = 2h$$

$h : H = 1 : 2$

Ans.- A

---

13 $$H = \frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$$

$$\Rightarrow h = \frac{H(\tan\beta - \tan\alpha)}{(\tan\beta + \tan\alpha)}$$

$$\Rightarrow h = \frac{H\left(\frac{1}{\cot\beta} - \frac{1}{\cot\alpha}\right)}{\left(\frac{1}{\cot\beta} + \frac{1}{\cot\alpha}\right)}$$

$$\Rightarrow h = \frac{H(\cot\alpha - \cot\beta)}{(\cot\alpha + \cot\beta)}$$

Ans.-A

$\boxed{14}$ $\alpha = 0^\circ$

Now, $$H = \frac{h(\tan\beta + \tan\alpha)}{(\tan\beta - \tan\alpha)}$$

$$= \frac{h(\tan\beta + \tan 0)}{(\tan\beta - \tan 0^\circ)} = h$$

$\Rightarrow H = h$

$\therefore h^2 + H^2 = 2h^2$ or $2H^2$

$\boxed{\text{Ans: —}}$

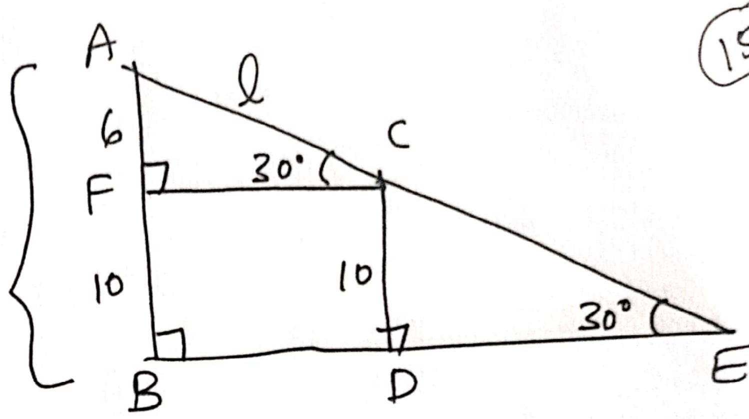
15

AB, CD → Poles

AB = 16 mt

CD = 10 mt

AF = 6 mt



AC = length of wire

In $\triangle AFC$ → $\sin 30^\circ = \frac{6}{l}$

$\Rightarrow \frac{1}{2} = \frac{6}{l} \Rightarrow l = 12$ cm

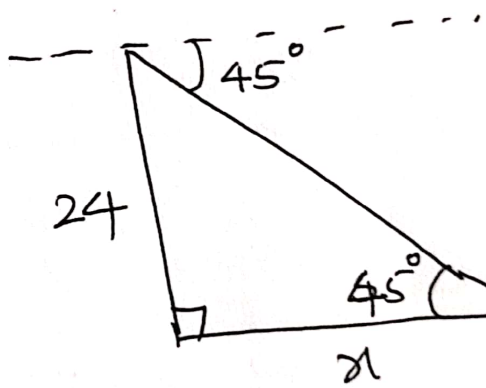
Ans. 12

16

$\tan 45^\circ = \frac{24}{x}$

$\Rightarrow 1 = \frac{24}{x}$

$\Rightarrow x = 24$



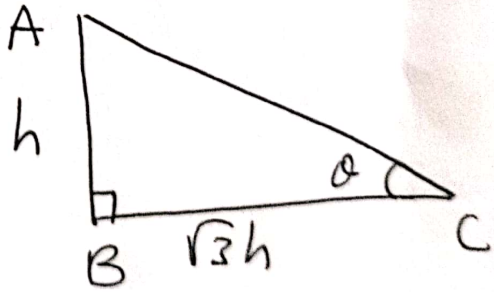
Ans: 24

17 a) AB → Tower

BC → Shadow

$\tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

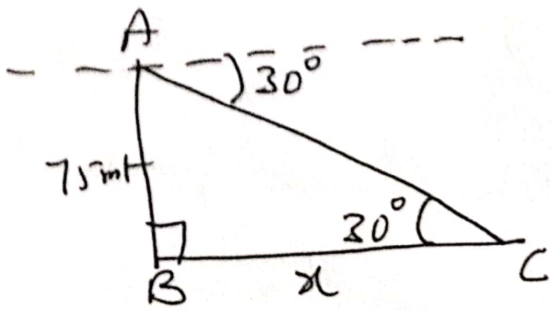
$\Rightarrow \theta = 30^\circ$ (a)



b) A → Position of observer

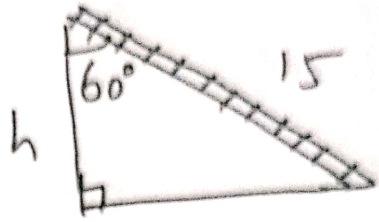
C → position of car

$\tan 30^\circ = \frac{75}{x}$



$\Rightarrow x = 75\sqrt{3}$ (t)

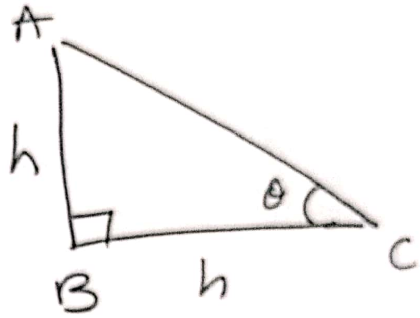
c) $\cos 60^\circ = \frac{h}{15}$



$\Rightarrow \frac{1}{2} = \frac{h}{15}$

$\Rightarrow h = \frac{15}{2}$ (r)

d) AB → Pole



BC → shadow

$\tan\theta = \frac{AB}{BC} = 1$

$\theta = 45^\circ$ (s)

Ans: q, t, r, s

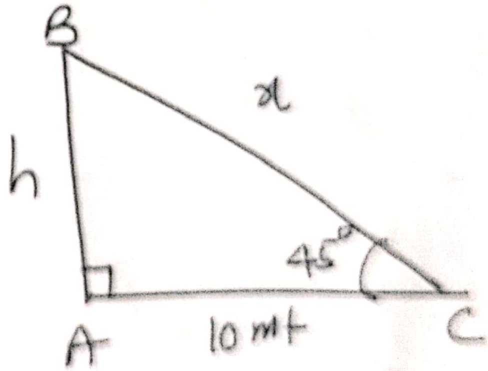
18 a) $\tan 45^\circ = \frac{h}{10}$

$\Rightarrow h = 10$

$\sin 45^\circ = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{10}{x}$

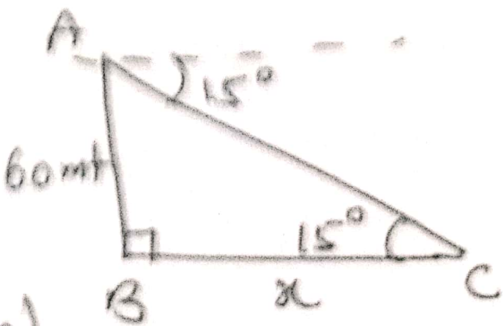
$\Rightarrow x = 10\sqrt{2}$



∴ ~~Len~~ Height of the tree before it was broken $= h + x = 10 + 10\sqrt{2}$

$= 10(\sqrt{2}+1)$ (t)

b) $\tan 15^\circ = \frac{60}{x}$



$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{60}{x}$

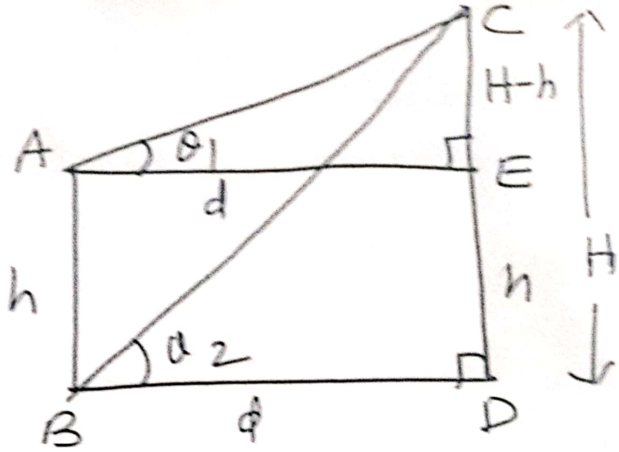
$\Rightarrow x = 60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ (q)

(17)

c) AB → Building

CD → hill

A, B → Position of observers



$$\tan\theta_1 = \frac{H-h}{d} \qquad \tan\theta_2 = \frac{H}{d}$$

$$\Rightarrow d = \frac{H-h}{\tan\theta_1} \rightarrow (1) \qquad \Rightarrow d = \frac{H}{\tan\theta_2} \rightarrow (2)$$

from (1) & (2)

$$\frac{H-h}{\tan\theta_1} = \frac{H}{\tan\theta_2}$$

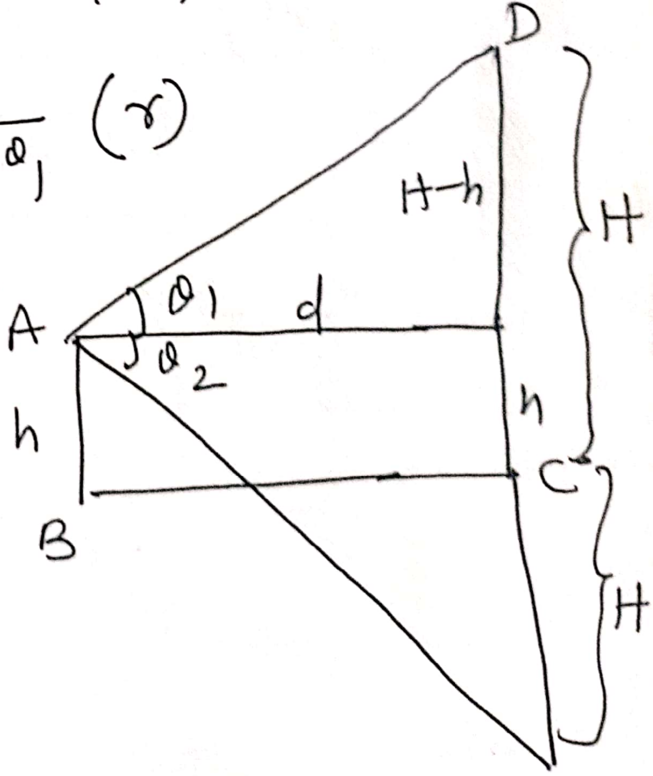
$$\Rightarrow \frac{H}{\tan\theta_1} - \frac{H}{\tan\theta_2} = \frac{h}{\tan\theta_1}$$

$$\Rightarrow H\left(\frac{\tan\theta_2 - \tan\theta_1}{\cancel{\tan\theta_1}\cdot\tan\theta_2}\right) = \frac{h}{\cancel{\tan\theta_1}}$$

$$\Rightarrow H = \frac{h\cdot\tan\theta_2}{\tan\theta_2 - \tan\theta_1} \quad (\gamma)$$

d) $\tan\theta_1 = \frac{H-h}{d}$ | $\tan\theta_2 = \frac{H+h}{d}$

$\Rightarrow d = \frac{H-h}{\tan\theta_1}$ | $\Rightarrow d = \frac{H+h}{\tan\theta_2}$



$$\therefore \frac{H-h}{\tan\theta_1} = \frac{H+h}{\tan\theta_2}$$

$$\Rightarrow \frac{H}{\tan\theta_1} - \frac{H}{\tan\theta_2} = \frac{h}{\tan\theta_2} + \frac{h}{\tan\theta_1}$$

$$\Rightarrow H(\tan\theta_2 - \tan\theta_1) = h(\tan\theta_1 + \tan\theta_2)$$

$$\Rightarrow H = \frac{h(\tan\theta_1 + \tan\theta_2)}{\tan\theta_2 - \tan\theta_1}$$

$$\Rightarrow H = \frac{h\left(\frac{\sin\theta_1}{\cos\theta_1} + \frac{\sin\theta_2}{\cos\theta_2}\right)}{\left(\frac{\sin\theta_2}{\cos\theta_2} - \frac{\sin\theta_1}{\cos\theta_1}\right)}$$

$$\Rightarrow H = \frac{h(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)}{(\sin\theta_2\cos\theta_1 - \cos\theta_2\sin\theta_1)}$$

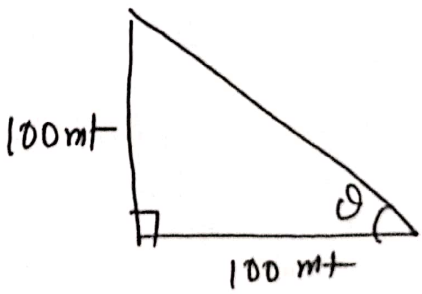
$$\Rightarrow H = \frac{h\sin(\theta_1+\theta_2)}{\sin(\theta_2-\theta_1)} \quad (s)$$

Ans: t, q, r, s

LEARNERS TASK (CUQ's)

01. $\tan\theta = \frac{100}{100} = 1$

$\Rightarrow \theta = 45^\circ$



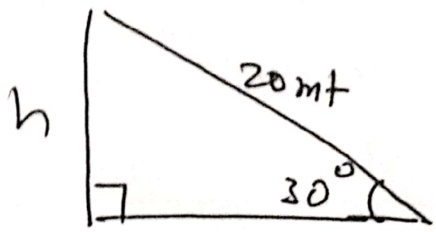
Ans. C

02. ∴ ~~Tan30°=~~

$\sin 30^\circ = \frac{h}{20}$

$\Rightarrow \frac{1}{2} = \frac{h}{20}$

$\Rightarrow h = 10$



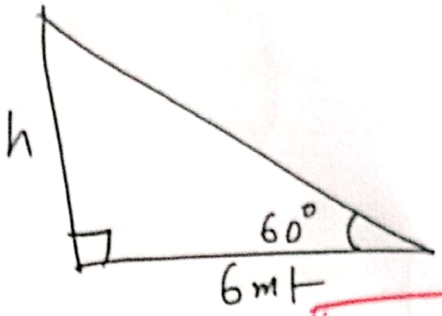
Ans. C

03

$\tan 60^\circ = \frac{h}{6}$

$\Rightarrow \sqrt{3} = \frac{h}{6}$

$\Rightarrow h = 6\sqrt{3}$ mt



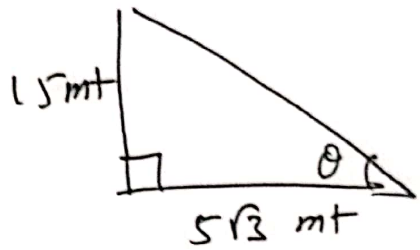
$\boxed{\text{Ans: B}}$

04

$\tan\theta = \frac{15}{5\sqrt{3}} = \frac{3}{\sqrt{3}}$

$= \sqrt{3}$

$\theta = 60^\circ$



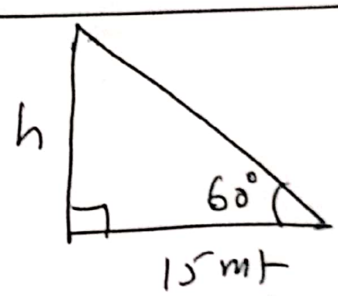
Ans: A

05

$\tan 60^\circ = \frac{h}{15}$

$\Rightarrow \sqrt{3} = \frac{h}{15}$

$\Rightarrow h = 15\sqrt{3}$



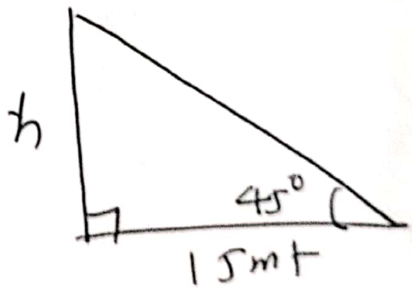
Ans.. A

06

$\tan 45^\circ = \frac{h}{15}$

$\Rightarrow 1 = \frac{h}{15}$

$\Rightarrow h = 15$ mt



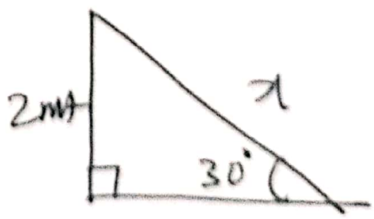
Ans. D

07

$\sin 30^\circ = \frac{2}{x}$

$\Rightarrow \frac{1}{2} = \frac{2}{x}$

$\Rightarrow x = 4$



Ans : C

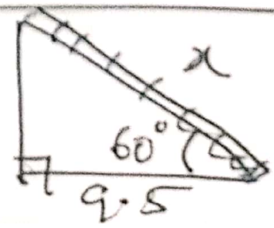
08

$\cos 60^\circ = \frac{9.5}{x}$

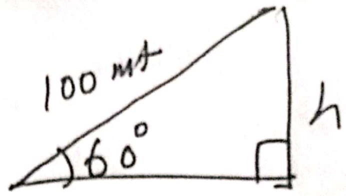
$\Rightarrow \frac{1}{2} = \frac{9.5}{x}$

$x = 19$ mt

Ans : A



09. $\sin 60^\circ = \frac{h}{100}$



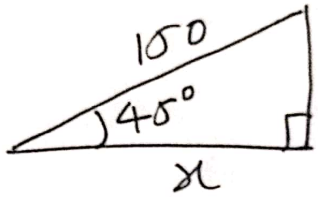
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{100}$

$\Rightarrow h = 50\sqrt{3}$

Ans: C

---

10 $\cos 45^\circ = \frac{x}{150}$



$\Rightarrow \frac{1}{\sqrt{2}} = \frac{x}{150}$

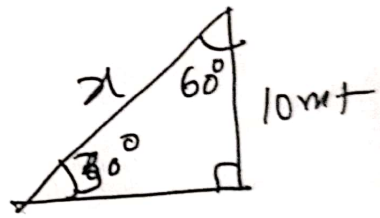
$\Rightarrow x = \frac{150}{\sqrt{2}} = 75\sqrt{2}$

Ans: B

JEE MAINS LEVEL

---

01. $\sin 30^\circ = \frac{10}{x}$



$\Rightarrow \frac{1}{2} = \frac{10}{x}$

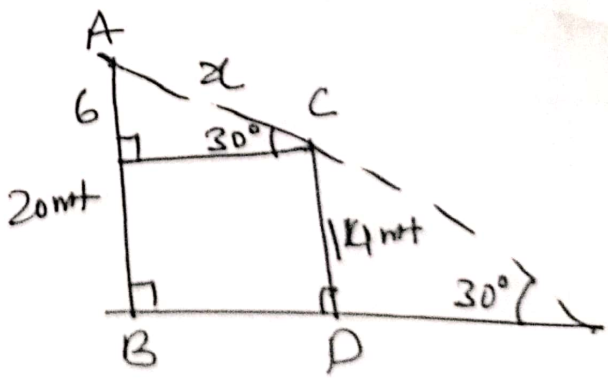
$\Rightarrow x = \frac{20}{}$

Total length of the rope $= 3 \times \frac{20}{} = 60$ mt

Ans: D

---

02 $\sin 30^\circ = \frac{6}{x}$



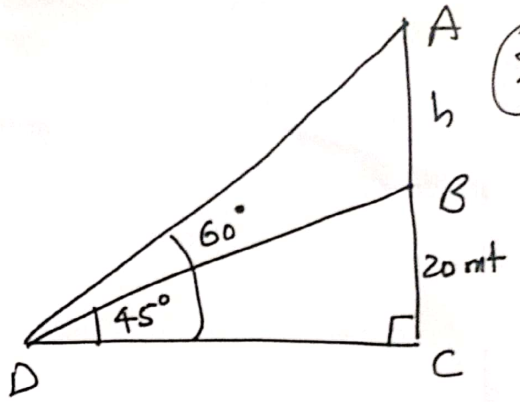
$\Rightarrow \frac{1}{2} = \frac{6}{x}$

$\Rightarrow x = 12$ mt

Ans: A

03

03 In the diagram



AB = tower

BC = Building

$\tan 45^\circ = \frac{BC}{CD}$

$\Rightarrow 1 = \frac{20}{CD}$

$\Rightarrow CD = 20$ mt

Now, $\tan 60^\circ = \frac{AC}{CD}$

$\Rightarrow \sqrt{3} = \frac{20+h}{20}$

$\Rightarrow 20+h = 20\sqrt{3}$

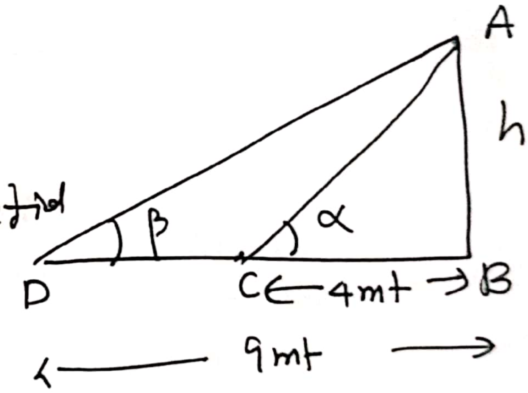
$\Rightarrow h = 20\sqrt{3} - 20$

$\Rightarrow h = 20(\sqrt{3}-1)$

$\Rightarrow h = 14$ mt /Approx.

Ans. D

04 $\alpha + \beta = 90^\circ$



$\Rightarrow \tan(\alpha+\beta) = \tan 90^\circ$

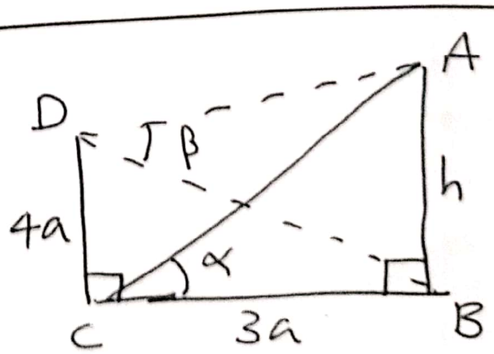
$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} =$ not defind

$\Rightarrow \tan\alpha \cdot \tan\beta = 1$

$\Rightarrow \frac{h}{4} \cdot \frac{h}{9} = 1 \Rightarrow h^2 = 36 \Rightarrow h = 6$ mt

Ans: A

05 In the figure



AB = Tower

BC = First direction

CD = Second direction

Given $BC = 3a$ and $CD = 4a$

Now, In $\triangle BCD$

$BD^2 = BC^2 + CD^2$

$= (3a)^2 + (4a)^2$

$= 9a^2 + 16a^2$

$= 25a^2$

$\therefore BD = 5a$

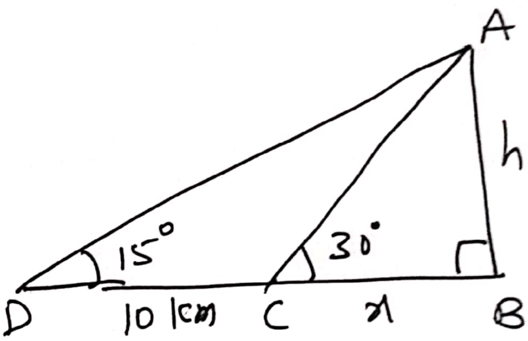
Now, $\tan\alpha = \frac{h}{3a}$, $\tan\beta = \frac{h}{5a}$

$\Rightarrow h = 5a \tan\beta$ also $h = 3a \tan\alpha$

$\boxed{\text{Ans: A, B}}$

06 AB = mountain

Let BC = x Km



Now, $\tan 30° = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow x = \sqrt{3}h$

$\tan 15° = \frac{h}{10+x}$ $\Rightarrow 2-\sqrt{3} = \frac{h}{10+x}$

$\Rightarrow 10+x = \frac{h}{2-\sqrt{3}}$

$\Rightarrow 10+\sqrt{3}h = \frac{h}{2-\sqrt{3}}$

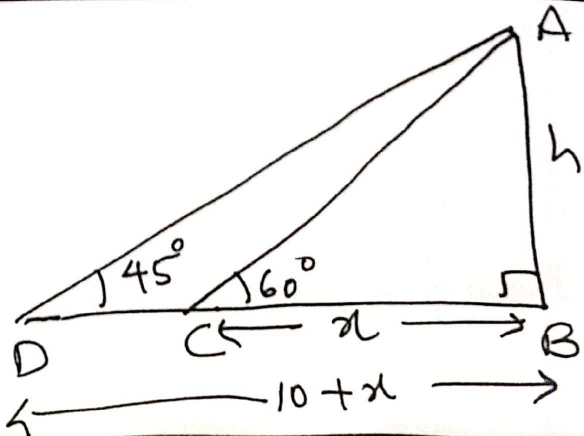
$\Rightarrow h = 5$ mt

Ans. D

07 $\tan 60° = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow x = \frac{h}{\sqrt{3}}$



Also, $\tan 45^\circ = \frac{h}{10+x}$

$\Rightarrow 1 = \frac{h}{10+x}$

$\Rightarrow 10+x = h$

$\Rightarrow 10 + \frac{h}{\sqrt{3}} = h$

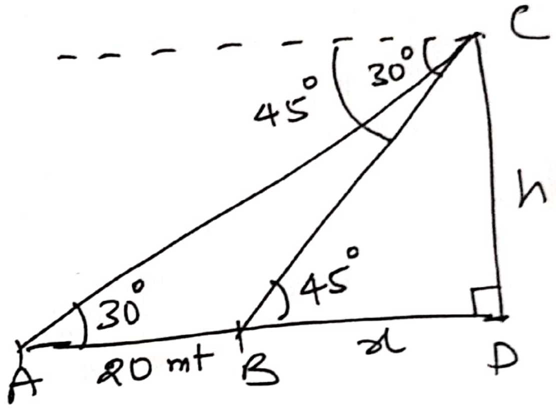
$h = \frac{10\sqrt{3}}{\sqrt{3}-1}$

$= 23.66$ mt

Ans. C

---

08 From figure



$\tan 45^\circ = \frac{h}{x}$

$\Rightarrow 1 = \frac{h}{x}$

$\Rightarrow h = x \rightarrow (i)$

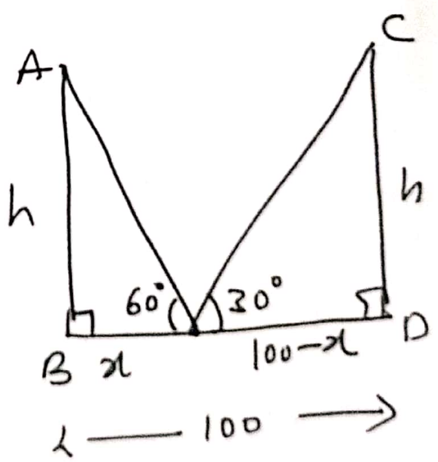
$\tan 30^\circ = \frac{h}{200+x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{200+h} \Rightarrow h = 100(\sqrt{3}+1) = 273.2$ mt

Ans. B

---

09 AB, CD $\rightarrow$ two pillars of equal height



BD = 100 mt

Now, $\tan 60^\circ = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow x = \frac{h}{\sqrt{3}} \rightarrow (i)$

$\tan 30^\circ = \frac{h}{100-x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100-x}$

$\Rightarrow 100 - x = \sqrt{3}h$

$\Rightarrow 100 - \frac{h}{\sqrt{3}} = \sqrt{3}h$ (from (i))

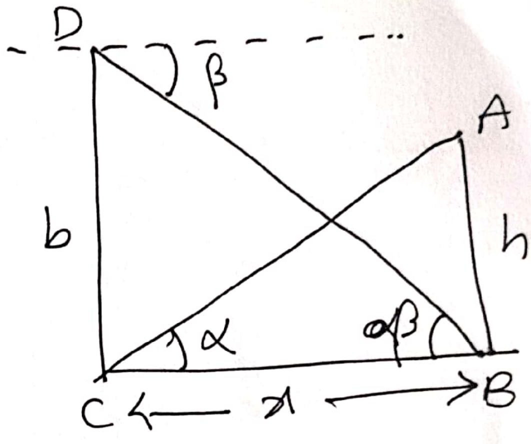
9. $\Rightarrow 100\sqrt{3} - h = 3h$

$\Rightarrow h = 25\sqrt{3}$

$\Rightarrow h = 43.3$ mt

Ans. A

10 AB → Tower



$\tan\alpha = \frac{h}{x}$

$\Rightarrow x = h\cot\alpha \rightarrow (i)$

$\Rightarrow \tan\beta = \frac{b}{x}$

$\Rightarrow x = b\cot\beta \rightarrow (ii)$

∴ From (i) & (ii)

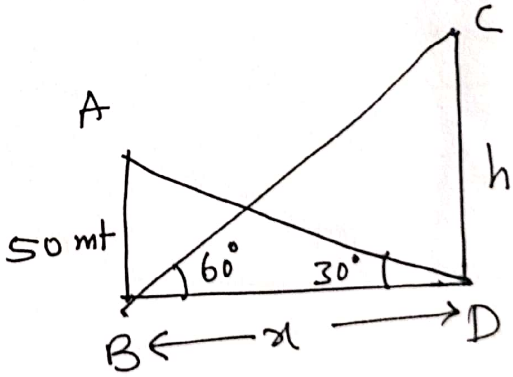
∴ $h\cot\alpha = b\cot\beta$

$\Rightarrow h = b\cot\beta\tan\alpha$

Ans. A

11 AB → Tower

CD → hill



$\tan 30^\circ = \frac{50}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{x}$

$\Rightarrow x = 50\sqrt{3} \rightarrow (i)$

$\tan 60^\circ = \frac{h}{x}$

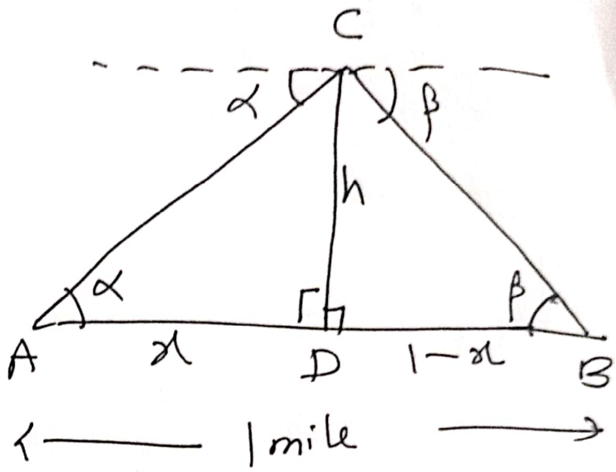
$\Rightarrow \sqrt{3} = \frac{h}{50\sqrt{3}}$ (∵ from i)

$\Rightarrow h = 150$ mt

Ans: C

JEE ADVANCED LEVEL QUESTIONS

01 From the figure



$$\tan\alpha = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\tan\alpha} \longrightarrow \text{(i)}$$

Again, $\tan\beta = \dfrac{h}{1-x}$

$$\Rightarrow 1 - x = \frac{h}{\tan\beta}$$

$$\Rightarrow 1 - \frac{h}{\tan\alpha} = \frac{h}{\tan\beta}$$

$$\Rightarrow \frac{h}{\tan\alpha} + \frac{h}{\tan\beta} = 1$$

$$\Rightarrow h\left(\frac{\tan\alpha + \tan\beta}{\tan\alpha \cdot \tan\beta}\right) = 1$$

$$h = \frac{\tan\alpha \cdot \tan\beta}{\tan\alpha + \tan\beta}$$

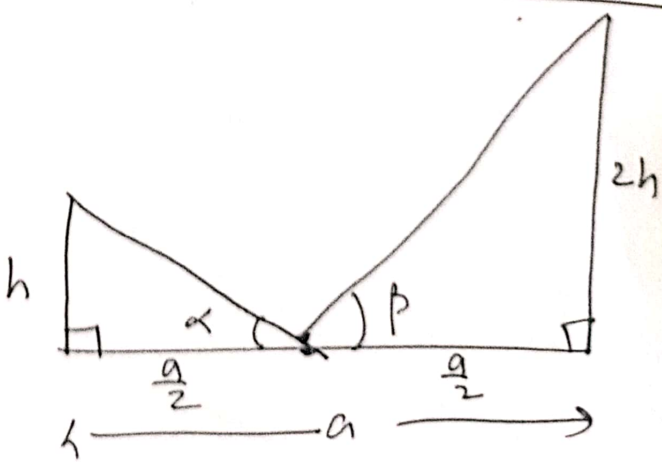
(or)

$$h = \frac{\left(\frac{\sin\alpha}{\cos\alpha}\right) \cdot \left(\frac{\sin\beta}{\cos\beta}\right)}{\left(\frac{\sin\alpha}{\cos\alpha}\right) + \left(\frac{\sin\beta}{\cos\beta}\right)}$$

$$\therefore h = \frac{\sin\alpha \cdot \sin\beta}{\sin\alpha\cos\beta + \cos\alpha \cdot \sin\beta} = \frac{\sin\alpha \cdot \sin\beta}{\sin(\alpha+\beta)}$$

Ans: B, D

02 Given $\alpha + \beta = 90°$



$$\Rightarrow \tan\alpha \cdot \tan\beta = 1$$

$$\Rightarrow \frac{h}{\left(\frac{a}{2}\right)} \cdot \frac{2h}{\left(\frac{a}{2}\right)} = 1$$

$$\Rightarrow 8h^2 = a^2$$

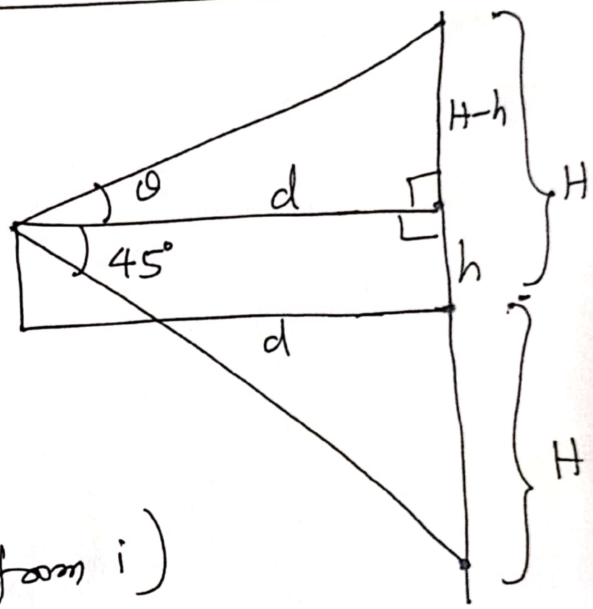
$$\Rightarrow h = \frac{a}{2\sqrt{2}}$$

$\therefore$ The height of the Smaller pole $= \frac{a}{2\sqrt{2}}$ mt

The height of the Longer pole $= 2h = \frac{a}{\sqrt{2}}$ mt

Ans. B, C

03



$\tan 45^\circ = \frac{H+h}{d}$

$\Rightarrow 1 = \frac{H+h}{d}$

$\Rightarrow d = H+h \rightarrow$ (i)

Again, $\tan\theta = \frac{H-h}{d}$

$\Rightarrow \tan\theta = \frac{H-h}{H+h}$ (from i)

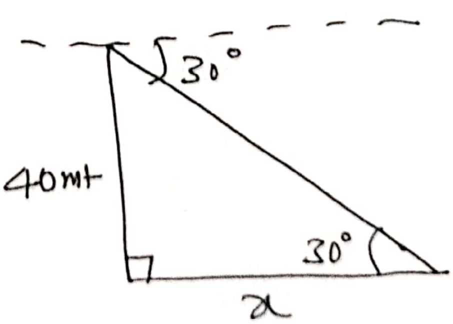
$\Rightarrow H = h\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

$\Rightarrow \; = h\left(\frac{\cot\theta+1}{\cot\theta-1}\right) = h\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$

$\therefore H = h\cot(45^\circ-\theta)$

Ans. B, D

04. Statement I:



$\tan 30^\circ = \frac{40}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{x}$

$\Rightarrow x = 40\sqrt{3}$ (True)

Statement II: Conceptual (True)

Ans.. A

05 Statement I: Conceptual (False)

Statement II: Conceptual (False)

Ans. B

06 Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans. A

07 Assertion: Conceptual (False)

Reason: Conceptual (True)

Ans: C

08 We have $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

Given $\alpha + \beta = 105^\circ$

let $\alpha = 60^\circ$ and $\beta = 45^\circ$

$$\therefore \frac{a}{b} = \frac{\cos 60^\circ - \cos 45^\circ}{\sin 45^\circ - \sin 60^\circ} = \frac{\left(\frac{1}{2}\right) - \frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right) - \frac{\sqrt{3}}{2}}$$

$$\Rightarrow \frac{a}{b} = \frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{2}}$$

Ans: A

09 Given $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha} = \frac{-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\beta+\alpha}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)}$

$$= \tan\left(\frac{\alpha+\beta}{2}\right)$$

$$= \tan\left(\frac{30^\circ}{2}\right)$$

$$= \tan 15^\circ$$

$$\therefore \frac{a}{b} = 2-\sqrt{3}$$

$$\Rightarrow \frac{b}{a} = \frac{1}{2-\sqrt{3}} = 2+\sqrt{3}$$

$$\Rightarrow \frac{b}{a} + 1 = 2 + \sqrt{3} + 1$$

$$\Rightarrow a + b = a(3 + \sqrt{3})$$

Ans : B

10. $$\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$$

$$\Rightarrow \frac{a}{b} = \tan\left(\frac{\alpha+\beta}{2}\right) = \tan\left(\frac{0^\circ}{2}\right) = 0$$

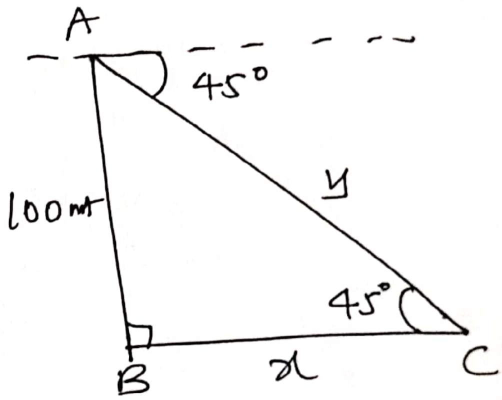
$$\Rightarrow a = 0$$

Ans: C

11 $$\tan 45^\circ = \frac{100}{x}$$

$$\Rightarrow 1 = \frac{100}{x}$$

$$\Rightarrow x = 100 \text{ mt}$$



Ans: B

12 $$\sin 45^\circ = \frac{100}{y}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{100}{y}$$

$$\Rightarrow y = 100\sqrt{2}$$
$$= 100\,(1.414)$$
$$= 141.4 \text{ mt}$$

Ans: B

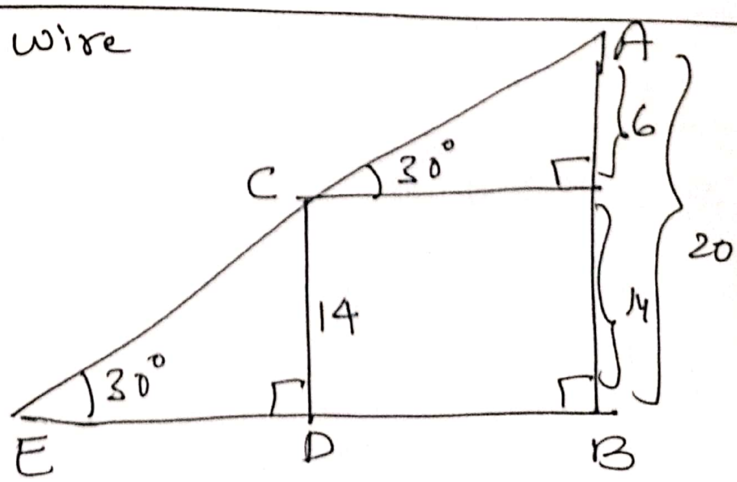
13 AC → Length of the wire

AB, CD → Poles

$$\sin 30^\circ = \frac{6}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{6}{AC}$$

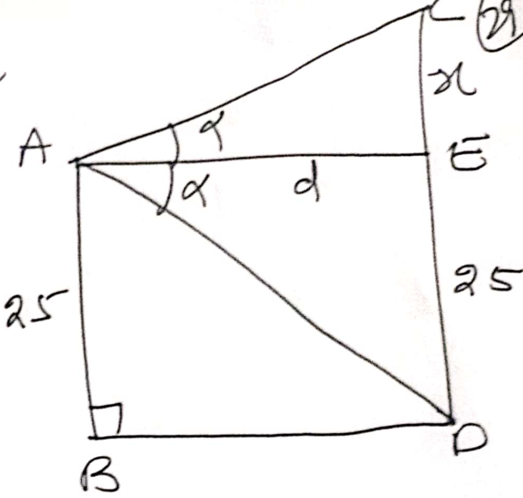
$$\Rightarrow AC = 12$$



Ans: 12

14 AB → Height of the cliff

CD → Height of the tower



$$\tan\alpha = \frac{25}{d} = \frac{x}{d}$$

$$\Rightarrow x = 25$$

Height of the tower

$$= 25 + 25$$

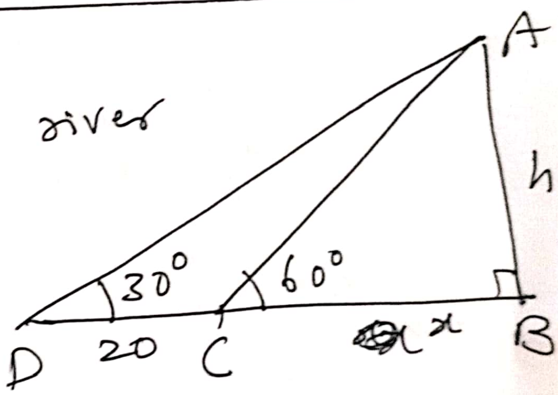
$$= 50 \text{ mt}$$

Ans: 50

---

15 a) AB → tree

BC → width of the river



$$\tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \cancel{\boxed{x = \frac{h}{\sqrt{3}}}} \quad h = \sqrt{3}x$$

Also, $\tan 30^\circ = \dfrac{h}{20+x}$

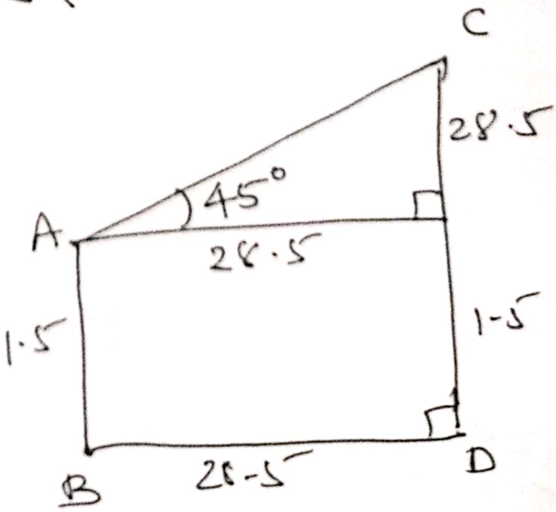
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$$

$$\Rightarrow 20 + x = \sqrt{3}h = \sqrt{3}(\sqrt{3}x) = 3x$$

$$\Rightarrow x = 10 \ (1)$$

b) AB → observer

CD → Tower



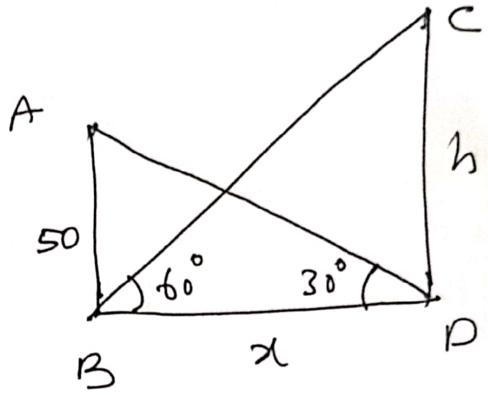
$$\tan 45^\circ = \frac{CE}{AE}$$

$$\Rightarrow 1 = \frac{CE}{28.5}$$

$CE = 28.5$

$\therefore$ The height of the observer $= 28.5 + 1.5$
$= 30$ mt (P)

c) $AB \rightarrow$ Tower
$CD \rightarrow$ hill



$\tan 30° = \frac{50}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{x}$

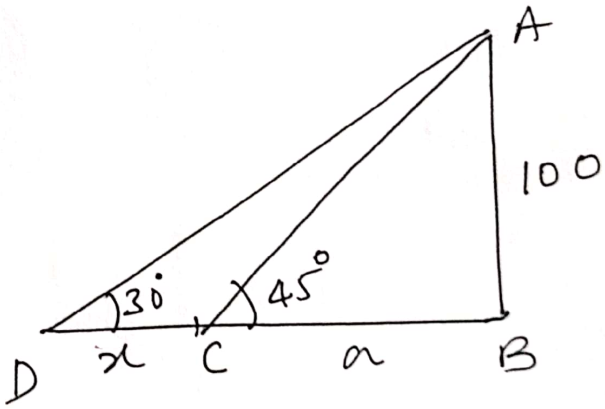
$\Rightarrow x = 50\sqrt{3}$

Again, $\tan 60° = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{50\sqrt{3}}$

$\Rightarrow h = 150$ mt (r)

d) $AB \rightarrow$ tower



$\tan 45° = \frac{100}{a}$

$\Rightarrow a = 100$

$\tan 30° = \frac{100}{x+a}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x+100}$

$x + 100 = 100\sqrt{3}$

$\Rightarrow x = 100(\sqrt{3}-1)$ mt ( )

Ans: s, p, r, —

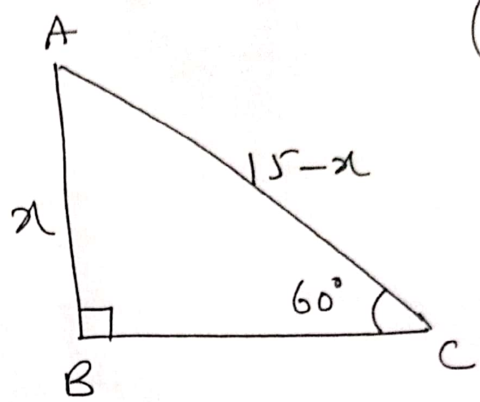
16 a) Conceptual (r)
b) Conceptual (p)
c) Conceptual (q)
d) Conceptual (r, s)

Ans. r, p, q, (r, s)

ADDITIONAL PRACTICE QUESTIONS

a.

Tan



$$\sin 60^\circ = \frac{x}{15-x}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{x}{15-x}$$

$$\Rightarrow 15\sqrt{3} - \sqrt{3}x = 2x$$

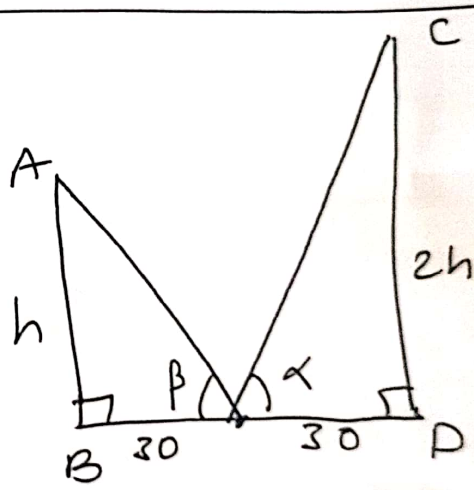
$$\Rightarrow x(2+\sqrt{3}) = 15\sqrt{3}$$

$$\Rightarrow x = \frac{15\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 15\sqrt{3}(2-\sqrt{3})$$

Ans. B

---

02 Given $\alpha + \beta = 90^\circ$



$$\Rightarrow \tan\alpha \cdot \tan\beta = 1$$

$$\Rightarrow \frac{2h}{30} \cdot \frac{h}{30} = 1$$

$$\Rightarrow h^2 = 450$$

$$\Rightarrow h = \sqrt{450}$$

$$\Rightarrow h = 21.21$$

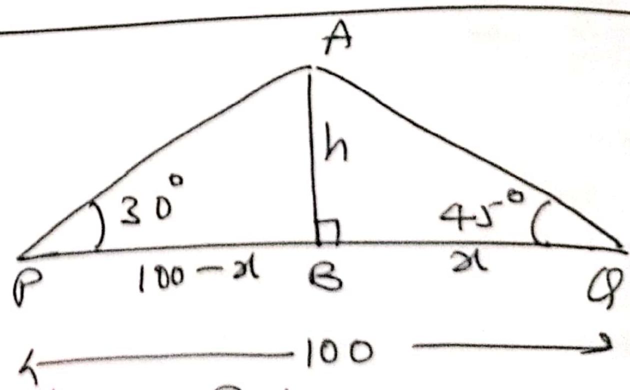
$$\Rightarrow 2h = 42.42$$

$\boxed{\text{Ans. A}}$

---

03 PQ = width of the river

AB → Tower



$$\tan 45^\circ = \frac{h}{x} \Rightarrow h = x$$

$$\tan 30^\circ = \frac{h}{100-x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100-h}$$

$$\Rightarrow 100 - h = \sqrt{3}h$$

$$\Rightarrow h(\sqrt{3}+1) = 100$$

$$\Rightarrow h = \frac{100}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$h = 50(\sqrt{3}-1) = 50 \times (0.732)$$

$$= 36.5 \text{ mt}$$

Ans D

⇒ THE END ⇐