ANGLES IN QUADRANTS (Signs) ①

class: IX, Mathematics FOUNDATION, SOLUTIONS

TEACHING TASK

01. $\sin\alpha + \operatorname{cosec}\alpha = 2$

$\Rightarrow \alpha = 90^\circ$

$\therefore \sin^n\alpha + \operatorname{cosec}^n\alpha = \sin^n 90^\circ + \operatorname{cosec}^n 90^\circ$

$= (1)^n + (1)^n$

$= 2$

Ans: C

02 $\tan 855^\circ = \tan(2\times 360^\circ + 135^\circ)$

$= \tan 135^\circ$

$= \tan(180^\circ - 45^\circ)$

$= -\tan 45^\circ$

$= -1$

Ans: B

03. $\alpha = n\pi - \theta$

$\Rightarrow \sin\alpha = \sin(n\pi - \theta)$

$= -\sin\theta$ if n is even

$= \sin\theta$ if n is odd

$\therefore \sin\alpha = (-1)^{n+1}\sin\theta$

Ans: A

04 $\cos\left((n+1)\frac{\pi}{2} - \theta\right) = -\sin\theta$ if n is odd

$= \sin\theta$ if n is even

$\therefore = (-1)^n \sin\theta$

Ans: B

05 $\alpha = \theta - \frac{13\pi}{2}$

$\cot\alpha = \cot\left(\theta - \frac{13\pi}{2}\right)$

$= -\cot\left(\frac{13\pi}{2} - \theta\right)$

$= -\cot\left[8\pi - \left(\frac{3\pi}{2} + \theta\right)\right]$

$= \cot\left(\frac{3\pi}{2} + \theta\right)$

$= -\tan\theta$

Ans: C

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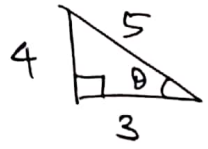
06 $\sin\theta = \frac{4}{5}$

$\Rightarrow \sin\theta$ is +ve

$\Rightarrow \theta \in Q_1$ or Q_2

$\Rightarrow \theta \in Q_2$ since given $\theta \notin Q_1$

$\therefore \cos\theta = -\frac{3}{5}$



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07 Complement of $\theta \rightarrow \frac{\pi}{2} - \theta$

Suplement of $\theta \rightarrow \pi - \theta$

Sum $= \frac{\pi}{2} - \theta + \pi - \theta = \frac{3\pi}{2} - 2\theta$

Ans: D

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08 $\sin\left(-\frac{11\pi}{3}\right) = -\sin\frac{11\pi}{3} = -\sin\left(\frac{12\pi}{3}\right) = -\sin\left(4\pi - \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3} = \sqrt{3}/2$

$\tan\left(\frac{35\pi}{6}\right) = \tan\left(6\pi - \frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

$\sec\left(-\frac{7\pi}{3}\right) = \sec\frac{7\pi}{3} = \sec\left(2\pi + \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

$\cos\left(\frac{5\pi}{4}\right) = \cos\left(\pi + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

$$\operatorname{cosec}\left(\frac{7\pi}{4}\right) = \operatorname{cosec}\left(2\pi - \frac{\pi}{4}\right) = -\operatorname{cosec}\frac{\pi}{4} = -\sqrt{2} \quad (3)$$

$$\cos\left(\frac{17\pi}{6}\right) = \cos\left(2\pi + \frac{5\pi}{6}\right) = \cos\frac{5\pi}{6} = \cos\left(\pi - \frac{\pi}{6}\right)$$

$$= -\cos\frac{\pi}{6}$$

$$= -\frac{\sqrt{3}}{2}$$

$$\therefore \text{Given expression} = \frac{\left(\frac{\sqrt{3}}{2}\right)\cdot\left(\frac{-1}{\sqrt{3}}\right)\cdot(2)}{\left(\frac{-1}{\sqrt{2}}\right)\cdot\left(-\sqrt{2}\right)\left(\frac{-\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

Ans: A

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09 $a\cos\theta - b\sin\theta = c \rightarrow (1)$

$a\sin\theta + b\cos\theta = x \rightarrow (2)$

$(1)^2 + (2)^2 \Rightarrow a^2 + b^2 = c^2 + x^2$

$\Rightarrow x = \pm\sqrt{a^2 + b^2 - c^2}$

Ans. D

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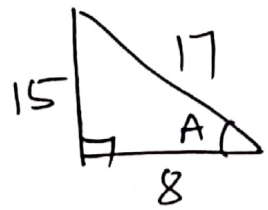
10. & $\tan A = -15$

$\Rightarrow \tan A = \frac{-15}{8}$

$\tan A$ is Negative

$A \in Q_2$ or Q_4

$A \in Q_2$ since $A \notin Q_4$



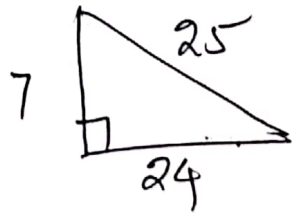
$25\sin B = -7$

$\Rightarrow \sin B = \frac{-7}{25}$

$\Rightarrow \sin B$ is negative

$\Rightarrow B \in Q_3$ or Q_4

$\Rightarrow B \in Q_3$ since $B \notin Q_4$

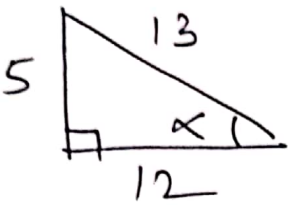


$\sin A\cos B + \cos A\sin B$

$= \left(\frac{15}{17}\right)\left(\frac{-24}{25}\right) + \left(\frac{-8}{17}\right)\left(\frac{-7}{25}\right) = \frac{-304}{425}$

Ans: C

11. $\sin\alpha = \frac{5}{13}$, $\alpha \in Q_2$



$$\frac{\sec\alpha + \tan\alpha}{\operatorname{cosec}\alpha - \cot\alpha} = \frac{\left(\frac{-13}{12}\right) + \left(\frac{-5}{12}\right)}{\frac{13}{5} + \frac{12}{5}}$$

$$= \frac{-3}{10} = -0.3$$

Ans: C

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12. $\frac{D}{90^\circ} = \frac{G}{100} = \frac{2C}{\pi}$

$$\frac{D}{90} = \frac{2C}{\pi} \Rightarrow D = \frac{2C}{\pi} \times 90$$

$$\Rightarrow \frac{G}{100} = \frac{2C}{\pi} \Rightarrow G = \frac{2C}{\pi} \times 100$$

$$G - D = \frac{2C}{\pi} \times 100 - \frac{2C}{\pi} \times 90 = \frac{2C}{\pi} \times 10 = \frac{20C}{\pi}$$

Ans: A

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13. $\sin(90^\circ + \theta) = \cos\theta$

$\sin(90^\circ - \theta) = \cos\theta$

Ans: A, D

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14. A) $\frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta} = \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

$$= \frac{\sin(1+\cos\theta)}{\sin^2\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta} \text{ (True)}$$

B) $\frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta} = \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$

$$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta} \text{ (True)}$$

Ans: A, B

15. Statement I: Conceptual (True)

Statement II: $l = 15\text{cm}$

$$\theta = 30^\circ = \frac{\pi}{6}^c$$

$$\therefore l = r\theta \Rightarrow r = \frac{l}{\theta} = \frac{15}{(\pi/6)} = \frac{15\times 6}{\pi}$$

$$= 90\times\frac{7}{22}$$

$$= \frac{630}{22} = \frac{315}{11}$$

$$= 28\frac{7}{11}$$

Ans: A

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16 Statement I: Conceptual (True)

Statement II: $\frac{D}{90} = \frac{G}{100} = \frac{2C}{\pi}$

$$\Rightarrow \frac{45}{90} = \frac{2C}{\pi}$$

$$\Rightarrow C = \frac{\pi}{4}$$

$$\Rightarrow C = \frac{11}{14} \text{ (False)}$$

Ans: C

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17 Statement I: $A+B = 90^\circ$

$$\Rightarrow \cot(A+B) = \cot 90^\circ$$

$$\Rightarrow \frac{\cot A\cdot\cot B - 1}{\cot A + \cot B} = 0$$

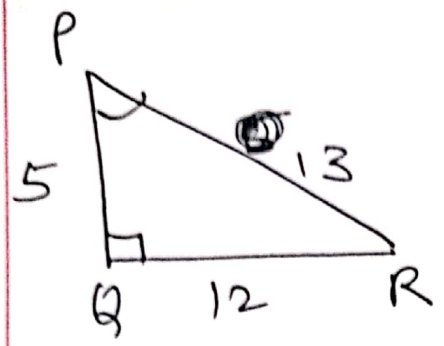
$\cot A\cdot\cot B = 1$ (False)

Statement II: $\cot\frac{\pi}{20}\cdot\cot\frac{5\pi}{20}\cdot\cot\left(\frac{\pi}{2}-\frac{3\pi}{20}\right)\cdot\cot\left(\frac{\pi}{2}-\frac{\pi}{20}\right)$

$= \cot\frac{\pi}{20}\cdot\cot\frac{5\pi}{20}\cdot\tan\frac{3\pi}{20}\cdot\tan\frac{\pi}{20} = \tan\frac{3\pi}{20}$ (False)

Ans: B

18



$PR + QR = 25$

By Trial & Error method

$PR = 13, \quad QR = 12$

$QR = 12\text{cm}$

(6)

Ans: B

19 $\sin P = \frac{12}{13}$

Ans: B

20. $\cos R = \frac{12}{13}$

Ans: D

21. Let $A = B = C = 60^\circ$

$= \tan A + \tan B + \tan C - \tan A \cdot \tan B \cdot \tan C$

$= \tan 60^\circ + \tan 60^\circ + \tan 60^\circ - \tan 60^\circ \cdot \tan 60^\circ \cdot \tan 60^\circ$

$= 3(\sqrt{3}) - (\sqrt{3})^3$

$= 0$

Ans: 0

22 a) $\dfrac{\tan A \cdot (-\cot A) \cdot (-\sin A)}{(-\cos A) \cdot (-\tan A) \cdot (\cos A)} = \dfrac{1}{\cos A}$

b) $\dfrac{-\sin A \cdot (-\cos A)}{\sin A} = \cos A$

c) $\sin^2\theta + \cos^2\theta = 1 \quad \forall \theta \in R$

d) $\sin\left(-\frac{11\pi}{3}\right) = -\sin\frac{11\pi}{3} = -\sin 660^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\sec\left(-\frac{7\pi}{3}\right) = \sec\frac{7\pi}{3} = \sec 420^\circ = \sec 60^\circ = 2$

$\cot\left(\frac{5\pi}{4}\right) = \cot 45^\circ = 1$

$\operatorname{cosec}\left(\frac{7\pi}{4}\right) = \operatorname{cosec} 315^\circ = \operatorname{cosec}(270^\circ + 45^\circ) = -\sec 45^\circ = -\sqrt{2}$

$\therefore G.E = \frac{\left(\frac{\sqrt{3}}{2}\right).(2)}{(1)(-\sqrt{2})} = -\sqrt{\frac{3}{2}}$

Ans: S, t, Q, Y

LEARNER'S TASK

CUQ'S

01. $135^\circ = 135 \times 1^\circ = 135 \times \frac{\pi}{180} = \frac{3\pi}{4}$

Ans: C

02 1 grade = 100 minutes

Ans: B

03 $\cos 7A = \sin(A - 6^\circ)$

$\Rightarrow 7A + A - 6 = 90^\circ$

$\Rightarrow 8A = 96$

$\Rightarrow A = 12^\circ$

Ans: A

04 $\cot(360^\circ - \theta) = -\cot\theta$

Ans: B

05 $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$

Ans: B

06. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \cot^2\theta - \operatorname{cosec}^2\theta = -1$

07. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

$\Rightarrow (K)(\operatorname{cosec}\theta - \cot\theta) = 1$

$\cot\theta - \operatorname{cosec}\theta = -\frac{1}{K}$

Ans: B

08 $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \operatorname{cosec}\theta = \pm\sqrt{1+\cot^2\theta}$

Ans: C

09. $\sin\theta \cdot \left(\frac{\cos\theta}{\sin\theta}\right) + \cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \cos\theta + \sin\theta$

Ans: A

10. $\sin(30^\circ + 60^\circ) = \sin 90^\circ = 1$

Ans: B

JEE MAINS LEVEL

01. $\cot\theta = \frac{7}{8}$

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \frac{49}{64}$

Ans A

02 $\operatorname{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow (\sqrt{10})^2 - \cot^2 A = 1$

$\Rightarrow \cot A = \pm 3$

Ans: D

03. $\sec\theta - \tan\theta = 3 \rightarrow (1)$

$\sec\theta + \tan\theta = \frac{1}{3} \rightarrow (2)$

$(1)+(2) \Rightarrow \sec\theta = \frac{5}{3}$ (positive)

$(1)-(2) \Rightarrow \tan\theta = -\frac{4}{3}$ (Negative)

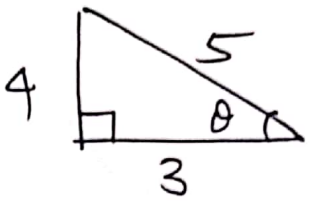
$\therefore \theta \in Q_4$

Ans: D

04. $\tan\theta = \frac{-4}{3}$ (Negative)

$\theta \in Q_2$ (or) Q_4

$\Rightarrow \theta \in Q_2$ since $\theta \notin Q_4$



$5\sin\theta + 10\cos\theta + 9\sec\theta$

$= 5\left(\frac{4}{5}\right) + 10\left(\frac{-3}{5}\right) + 9\left(\frac{-5}{3}\right)$

$= 4 - 6 - 15 = -17$

Ans: C

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05 $\sec\left(\frac{13\pi}{3}\right) = \sec\left(6\pi - \frac{5\pi}{3}\right) = \sec\frac{5\pi}{3}$

$= \sec 300^\circ = \sec(360^\circ - 60^\circ) = \sec 60^\circ = 2$

Ans. A

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06 $\left(\cot\frac{\pi}{16}\right)\cdot\cot\frac{2\pi}{16}\cdot\cot\frac{3\pi}{16}\cdot\ \ldots\ \cot\frac{7\pi}{16}$

$= \left(\cot\frac{\pi}{16}\cdot\cot\frac{7\pi}{16}\right)\left(\cot\frac{2\pi}{16}\cdot\cot\frac{6\pi}{16}\right)\left(\cot\frac{3\pi}{16}\cdot\cot\frac{5\pi}{16}\right)\cot\frac{4\pi}{16}$

$= \left[\cot\frac{\pi}{16}\cdot\cot\left(\frac{\pi}{2} - \frac{\pi}{16}\right)\right]\left[\cot\frac{2\pi}{16}\cdot\cot\left(\frac{\pi}{2} - \frac{2\pi}{16}\right)\right]$

$\left[\cot\frac{3\pi}{16}\cdot\cot\left(\frac{\pi}{2} - \frac{3\pi}{16}\right)\right]\cdot 1$

$= \left(\cot\frac{\pi}{16}\cdot\tan\frac{\pi}{16}\right)\left(\cot\frac{2\pi}{16}\cdot\tan\frac{2\pi}{16}\right)\left(\cot\frac{3\pi}{16}\cdot\tan\frac{3\pi}{16}\right)\cdot 1$

$= 1 \times 1 \times 1 \times 1$

$= 1$

Ans. A

(10)

07. $$= \frac{\cos(\pi - A)\cdot\cot\left(\frac{\pi}{2}+A\right)\cdot\cos(-A)}{\tan(\pi+A)\cdot\tan\left(\frac{3\pi}{2}+A\right)\cdot\sin(2\pi-A)}$$

$$= \frac{(-\cos A)\cdot(-\tan A)\cdot\cos A}{(\tan A)(-\cot A)(-\sin A)} = \cos A$$

Ans: C

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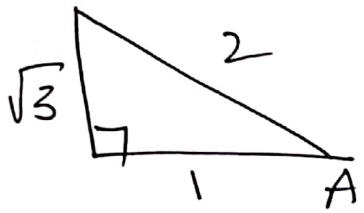
08 $\cos A = -\frac{1}{2}$ (Negative)

$A \in Q_2$ or Q_3

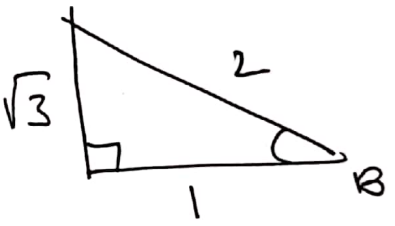
$\Rightarrow A \in Q_3$ since $A \notin Q_2$



$\cos B = -\frac{1}{2}$ (Negative)

$B \in Q_2$ or Q_3

$\Rightarrow B \in Q_2$ since $B \notin Q_3$



$$\frac{4\sin B - 3\tan A}{\tan B + \sin A} = \frac{4\left(\frac{\sqrt{3}}{2}\right) - 3\left(\sqrt{3}\right)}{\left(-\frac{\sqrt{3}}{1}\right) + \left(-\frac{\sqrt{3}}{2}\right)} = \frac{2}{3}$$

Ans: B

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09. $3\sin\theta + 4\cos\theta = 5 \longrightarrow (1)$

$4\sin\theta - 3\cos\theta = x \longrightarrow (2)$

$(1)^2 + (2)^2 \Rightarrow 9 + 16 = 25 + x^2 \Rightarrow x = 0$

Ans: C

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10. $(m^2 - n^2)\sin^2 B = (m+n)(m-n)\cdot\sin^2 B$

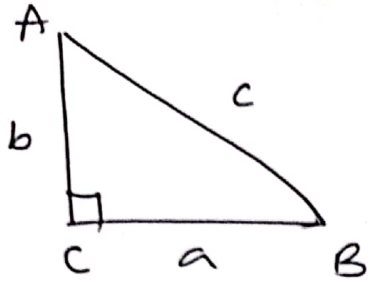
$$= \frac{\sin(A+B)}{\cos B\cdot\sin B}\cdot\frac{\sin(A-B)}{\cos B\cdot\sin B}\cdot\sin^2 B$$

$$= \frac{(\sin^2 A - \sin^2 B)}{\cos^2 B} = \frac{\cos^2 B - \cos^2 A}{\cos^2 B}$$

$$= 1 - n^2$$

Ans: A

11.



$\tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2+b^2}{ab}$

$= \frac{c^2}{ab}$

(11)

Ans. B

12 One minute → 360 revolutions

⇒ 60 sec → 360 revolutions

⇒ 1 sec → 6 rev.

1 rev → 2π radians

6 rev → 12π radians

Ans: D

13. $\frac{1-\cos\theta+\sin\theta}{1+\sin\theta} \times \frac{(1+\sin\theta)+\cos\theta}{(1+\sin\theta)+\cos\theta}$

$= \frac{(1+\sin\theta)^2 - \cos^2\theta}{(1+\sin\theta)(1+\cos\theta+\sin\theta)}$

$= \frac{1+\sin^2\theta+2\sin\theta-\cos^2\theta}{(1+\sin\theta)(1+\cos\theta+\sin\theta)}$

$= \frac{2(1+\sin\theta)\sin\theta}{(1+\sin\theta)(1+\cos\theta+\sin\theta)} = \frac{2\sin\theta}{1+\cos\theta+\sin\theta} = x$

14 $\cos^4\alpha + 2\cos^2\alpha(\sec^2\alpha - 1)$

$= \cos^4\alpha + 2\cos^2\alpha\left(\frac{1-\cos^2\alpha}{\cos^2\alpha}\right)$

$= \cos^4\alpha + 2\cos^2\alpha - 2\cos^4\alpha$

$= \cos^4\alpha + 2 - 2\cos^2\alpha$

$= 2\cos^2\alpha - \cos^4\alpha$

$= \cos^4\alpha + 2\sin^2\alpha$

$= (1-\sin^2\alpha)^2 + 2\sin^2\alpha$

$= 1 + \sin^4\alpha$

Ans. A

15. $$\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \cdot \tan 110^\circ} = \frac{\tan(180^\circ - 20^\circ) - \tan(90^\circ + 20^\circ)}{1 + \tan(180^\circ - 20^\circ) \cdot \tan(90^\circ + 20^\circ)}$$

$$= \frac{-\tan 20^\circ + \cot 20^\circ}{1 + (-\tan 20^\circ)(-\cot 20^\circ)}$$

$$= \frac{-n + \frac{1}{n}}{2}$$

$$= \frac{1 - n^2}{2n}$$

Ans: C

16. $m^2 - n^2 = (m+n)(m-n)$

$= 2\tan\theta \cdot 2\sin\theta$

$= 4 \cdot \tan\theta \cdot \sin\theta$

$4\sqrt{mn} = 4\sqrt{\tan^2\theta - \sin^2\theta}$

$= 4\sqrt{\sin^2\theta\left(\frac{1}{\cos^2\theta} - 1\right)}$

$= 4\sqrt{\sin^2\theta \times \frac{\sin^2\theta}{\cos^2\theta}}$

$= 4\sqrt{\tan^2\theta \times \sin^2\theta}$

$= 4\tan\theta \cdot \sin\theta$

$\therefore m^2 - n^2 = 4\sqrt{mn}$

Ans: C

17. $1 + \tan^2\theta = \sec^2\theta$

All the options are conceptually true.

18. $\cos(3\pi+\theta) = -\cos\theta$

Since $\cos(n\pi+\theta) = (-1)^n \cdot \cos\theta$

here $n = 3$ (odd)

Ans: B

19. $\tan(11\pi-\theta) = -\tan\theta$

Since $\tan(n\pi-\theta) = -\tan\theta$

Ans: C

20. $\cos(4\pi+\theta) \cdot \tan(4\pi-\theta)$

$= \cos\theta(-\tan\theta) = -\sin\theta$.

Ans: A

21. $\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)$

$= (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$

$= \sec^2\theta - \tan^2\theta$

$= 1$

Ans: 1

22 $\frac{\sin A + \cos A}{\sin A - \cos A} = \frac{1+\cot A}{1-\cot A} = \frac{1+\frac{3}{4}}{1-\frac{3}{4}} = 7$

Ans: 7

23 a) $\sin\theta = \sqrt{1-\cos^2\theta} = \frac{\sqrt{\sec^2\theta - 1}}{\sec\theta}$

b) $\cos\theta = \frac{1}{\sec A}$

c) $\tan\theta = \sqrt{\sec^2\theta - 1}$

d) $\operatorname{cosec}\theta = \frac{\sec\theta}{\sqrt{\sec^2\theta - 1}}$

Ans: r, s, p, r

ADDITIONAL PRACTICE QUESTIONS

01. $\sec A + \tan A = 3$

$\sec A - \tan A = \frac{1}{3}$

$\Rightarrow 2\sec A = 3 + \frac{1}{3}$

$\sec A = \frac{5}{3}$

Ans: B

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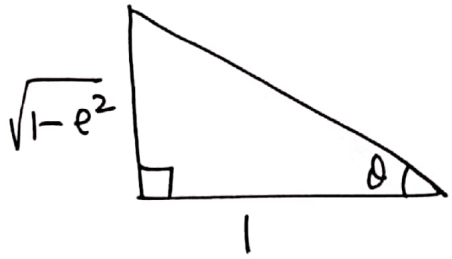
02. $\tan^2\theta = 1 - e^2$

$\Rightarrow \tan\theta = \frac{\sqrt{1-e^2}}{1}$



$\sec^2\theta - 1 = 1 - e^2$

$\Rightarrow \sec^2\theta = 2 - e^2 \Rightarrow \sec\theta = \sqrt{2-e^2}$

Now. $\sec\theta + \tan^3\theta \cdot \operatorname{cosec}\theta$

$= \frac{1}{\cos\theta} + \frac{\sin^3\theta}{\cos^3\theta} \times \frac{1}{\sin\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin^2\theta}{\cos^3\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^3\theta} = \sec^3\theta$

$= (\sqrt{2-e^2})^3 = (2-e^2)^{3/2}$

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03. $\tan\theta = \frac{-4}{3} \Rightarrow \sin\theta = \frac{4}{5}$ or $-\frac{4}{5}$

depending upon the quadrant in which θ lies

Ans. D

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04. $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ + \sin^2 90^\circ)$

$= 4\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right) - 3\left(\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right)$

$= 2$

$\Rightarrow$ THE END

Ans: 2