# MOTION IN A PLANE

***LEARNING OBJECTIVES***

- What is Projectile motion
- Path followed by the projectile
- How velocity changes during projectile motion
- Time of ascent, time of decent, time of flight of projectile
- Maximum height, horizontal range
- Energy of projectile.
- Horizontal projectile from a top of tower, velocity with which it hits the ground

**Real life applications of Kinematics:**

Φ Usefull in understanding the two dimentional motion in a simple way

Φ Usefull in fixing the target of missile and to track its path.

Φ Usefull in understanding the path follwed by the ball in foot ball, cricket, golf etc games.

Φ Usefull in understanding motion of a bomb relesed from an areoplane in level flight, a bullet fired from a gun, an arrow released from a bow, a javelin thrown by an athlet.



❖ **Important Formulae:**

1) $u_x = u\cos\theta$ = constant, $u_y = u\sin\theta$ (varies with time)

2) u= $\sqrt{u_x^2 + u_y^2}$ , v= $\sqrt{v_x^2 + v_y^2}$ ,

3) Angle made with horizonal $\tan\alpha = (v_y / v_x)$

4) General equation y= Ax - Bx$^2$, where A= $\tan\theta$, B = $\dfrac{g}{2u^2\cos^2\theta}$

5) $t_a = t_d = \dfrac{u\sin\theta}{g},\ T = t_a + t_d,\ H = \dfrac{u^2\sin^2\theta}{2g},\ R = \dfrac{u^2\sin 2\theta}{g}$

6) The horinzontal projectile from to top of tower , Range $R = u\sqrt{\dfrac{2h}{g}}$ , $t = \sqrt{\dfrac{2h}{g}}$

7) velocity with which it hits the ground $V = \sqrt{u^2 + 2gh} = \sqrt{u^2 + g^2t^2}$ ,

8) angle at which is strikes the ground $\theta = \tan^{-1}\left(\dfrac{gt}{u}\right) = \tan^{-1}\left(\dfrac{\sqrt{2gh}}{u}\right)$

## §§ PROJECTILE

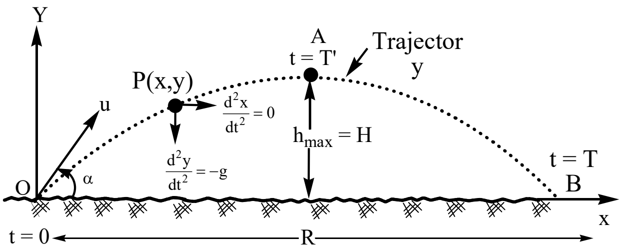
Projectile is any body projected into the air at an angle othe than 90° with the horizontal near the surface of the earth. The path followed by a projectile is called its trajectory.

The following assumptions are made in the projectile motion

1. The acceleration due to gravity 'g' is constant over the range of motion.
2. The air resistance is negligible.
3. The ground on which the projectile thrown is refernce level.

## §§ EQUATION OF THE PATH (TRAJECTORY)

Consider the motion of an object projected from the origin O of the co-ordinate system, with the initial velocity "u" inclined at an angle $\alpha$ with the horizontal or the *X*-axis as shown in figure.




The motion of the object P can be studied by resolving its velocity "u" in the horizontal and the vertical directions as shown.

Horizontal component of the velocity is , $v_x = u\cos\alpha$

Vertical component of the velocity is $v_y = u \sin \alpha$

The horizontal comp onent of velocity is $v_x$. It shall remain constant as no acceleration is acting in the horizontal direction (a = 0)

The initial velocity in the vertical direction $v_y$ shall go on decreasing because of the constant deceleration due to gravity (a = – g).

The object, therefore, is having horizontal and vertical motions simultaneously.The resultant motion would be the vector sum of these two motions and the path followed would be curvilinear.

Let P be the position of the object after a time *"t"* then distance travelled in the horizontal direction in time *t,* is x = (u cos a)t ......... (4.1)

The distance travelled in the vertical direction in time *t* is

$y = (u \sin a)\, t - \frac{1}{2}gt^2$ ......... (4.2)

The equations (4.1) and (4.2) are the time displacement relations for the projectile.

Eliminating the time *t, we* can obtain a relationship between *x* and *y* or the equation of the path of the projectile.

From equation (4.1) we can write $t = \frac{x}{u\cos\alpha}$

On substituting the value of "t" in equation (4.2) we can write

$$y = (u \sin a)\left(\frac{x}{u\cos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$$

$$y = (\tan a)\, x - \left(\frac{g}{2u^2\cos^2\alpha}\right)x^2 \quad \text{.......... (4.3)}$$

The above equation is of the form $y = Ax + Bx^2$ and represents a parabola. Thus the path of a projectile is a parabola.

**a) Time of ascent :-** Time Taken to Reach Maximum Height. We let this time be denoted $t_a$. At the maximum height the vertical velocity is zero i.e.,$V_y$=0

$$u\sin\theta - gt_a = 0 \quad \therefore t_a = \frac{u\sin\theta}{g}$$

**b) Time of Flight : -** The total time spent by a projectile in air during the time of motion is called time of flight.

On substituting y = 0 and t = T in equation (4.3) we can write

$$0 = u\sin\theta T - \frac{1}{2}gT^2 \quad \Rightarrow T = \frac{2u\sin\theta}{g} \text{ ..........(4.4)}$$

**c) Maximum Height : -** As the projectile ascends (moves up), the vertical component of its velocity decreases. At maximum height of the vertical component of velocity becomes zero. That means the projectile moves horizontally at its highest position.

From the equatrion of motion $v^2 - u^2 = 2as$, the maximum height attained by a projectile can be written as

$$(0)^2 - (u\sin\theta)^2 = 2 \times -g \times H_m \qquad \Rightarrow -u^2\sin^2\theta = -2gH_m$$

$$\Rightarrow H_m = \frac{u^2\sin^2\theta}{2g} \quad ................(4.5)$$

**Note:** When $\theta = 90^0, H_{max} = \frac{u^2}{2g}$. This is equal to the maximum height reached by a body projected vertically upwards.

**d) Horizontal Range : -** The horizontal distance travelled by the projectile while it touches the point on the same level of the point of projection is called horizontal range.

In the horizontal direction, velocity remains constant.

$$\therefore \text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$u\cos\alpha = \frac{R}{T} \qquad R = u\cos\alpha.T$$

$$R = (u\cos\alpha)\left(\frac{2u\sin\alpha}{g}\right) \text{ (or) } R = \frac{u^2\sin 2\alpha}{g}$$

**e) Angle of Projection for the Maximum Range of the Projectile**

Range, R = $\frac{u^2 \sin 2\alpha}{g}$

The range R for a given velocity **u** is maximum

if $\sin 2\alpha = 1$ (or) $2\alpha = \frac{\pi}{2}$ (or) $\alpha = \frac{\pi}{4} = 45^o$

Corresponding to this value of a, the maximum range is $R_{maximum} = \frac{u^2}{g}$ .......... (4.7)

**f) Relation between range and maximum height**

$R = \frac{v^2\sin 2\theta}{g}$ $\quad H = \frac{v^2\sin 2\theta}{2g}$

$$\frac{H}{R} = \frac{\left[\frac{v^2\sin^2\theta}{2g}\right]}{\left[\frac{v^2\sin 2\theta}{g}\right]} = \frac{1}{4}\tan\theta \text{ (or) } \boxed{\text{Tan}\theta = \frac{4H}{R}}$$

**Sub case :** Condition for range of projectile to be equal to maximum height attained by it

Tan $\theta$ = 4, $\quad \tan\alpha = 4 \quad \alpha = \tan^{-1}4 = 76^0$

**g) There are two angles of projection for same range**

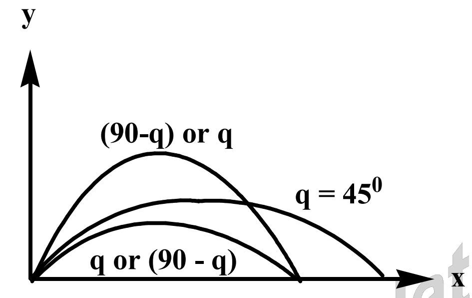
Replacing $\theta$ by $(90^0 - \theta)$ in the formula of range we get

$$R = \frac{u^2 \sin 2(90^0 - \theta)}{g} = \frac{u^2 \sin(180 - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g} = R$$

Thus, for a given velocity of projection, a projectile has the same range for angle of projection $\theta$ and $(90^0 - \theta)$

**Note :** In the above case range of two projections is same but time of flights are different.

**h) Relation between times of flights and range in case of projectiles having same range.**






Time of flight for angle of projection $\theta$.

$T_1 = \frac{2u\sin\theta}{g}$ and time of the flight for angleof projetion $(90^0 - \theta)$,

$$T_2 = \frac{2u\sin(90^0 - \theta)}{g} = \frac{2u\cos\theta}{g}$$

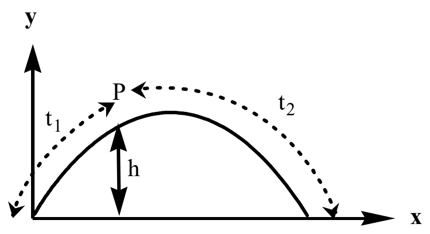
Multiplying $T_1$ and $T_2$ we get

$$T_1 T_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g}$$

or $T_1T_2 = \frac{2}{g}\left(\frac{u^2 \sin 2\theta}{g}\right)$ (or) $T_1T_2 = \frac{2R}{g}$ $\qquad \frac{T_1}{T_2} = \text{Tan}\theta$

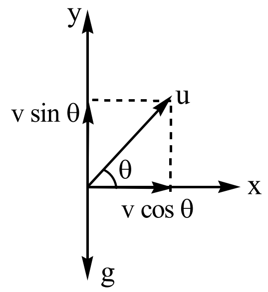
**(i) If $t_1$ is the time taken by projectile to reach a point P at height h and $t_2$ is the time taken from point P to ground level, then**

$t_1 + t_2 = T = \frac{2u\sin\theta}{g}$ or $u\sin\theta = \frac{g(t_1 + t_2)}{2}$




The height of point P, $h = u\sin\theta t_1 - \frac{1}{2}gt_1^2 = \frac{g(t_1 + t_2)}{2}t_1 - \frac{1}{2}gt_1^2$ or $h = \frac{1}{2}gt_1t_2$

## §§ VECTOR NOTATIONS INCASE OF PROJECTILE MOTION



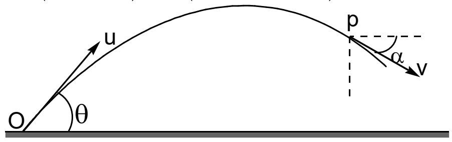

**a) Velocity after some time :** Initital velocity the projectile

$\vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$

$\vec{a} = -g\hat{j}$

Velocity of the projectile after some time $\vec{v} = \vec{u} + \vec{a}t$

$\vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j} = v_x\hat{i} + v_y\hat{j}$




$v = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

angle made by the velocity with horizontal. $\operatorname{Tan}\alpha = \dfrac{v_y}{v_x} = \dfrac{U\sin\theta - gt}{U\cos\theta}$

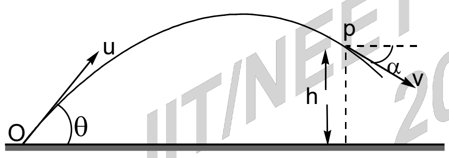
**b) Velocity after some hegiht**

$\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta)\hat{j}$

$\vec{a} = -g\hat{j}$

$\vec{s} = h\hat{j}$

$\vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{a}.\vec{s}$ and $v^2 = (u\cos\theta)^2 + (u\sin\theta)^2 - 2gh$



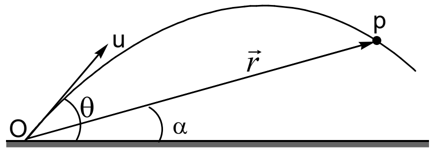



$v = \sqrt{(u\cos\theta)^2 + \left[(u\sin\theta)^2 - 2gh\right]}$

In vector form we can write it as $\vec{v} = (u\cos\theta)\hat{i} + \left(\sqrt{(u\sin\theta)^2 - 2gh}\right)\hat{j} = v_x\hat{i} + v_y\hat{j}$

angle made by the velocity with horizontal. $\operatorname{Tan}\alpha = \dfrac{v_y}{v_x} = \dfrac{\sqrt{(U\sin\theta)^2 - 2gh}}{U\cos\theta}$

**c) Position vector of the particle**




$\vec{U} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$

$\vec{a} = -g\hat{j}$

$\vec{s} = \vec{u}t + \dfrac{1}{2}\vec{a}t^2$

$\vec{r} = \left[(u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}\right]t + \dfrac{1}{2}(-g\hat{j})t^2$

$\vec{r} = (u\cos\theta\, t)\hat{i} + \left(u\sin\theta\, t - \dfrac{1}{2}gt^2\right)\hat{j}$

$\vec{r} = x\,\hat{i} + y\hat{j}$ $\quad x = (u\cos\theta)t$

$y = (u\sin\theta)t - \dfrac{1}{2}gt^2$

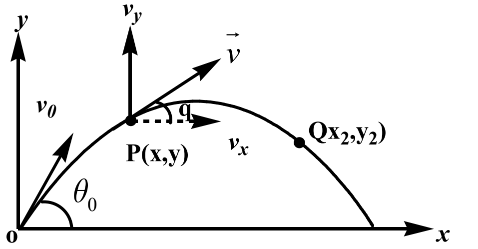
Magnitude of displacement of projecticle S = $\sqrt{x^2 + y^2}$

angle made by the position vector with horizontal.

$$\text{Tan}\,\alpha = \frac{(U\sin\theta)t - \frac{1}{2}gt^2}{(U\cos\theta)t} \qquad \text{Tan}\,\alpha = \frac{U\sin\theta - \frac{1}{2}gt}{U\cos\theta}$$

**(d) Position, Time and speed at any angular elevation $\theta$**

Let a particle be projected with $v_0$ at an angle $\theta_0$ .After a time t it moves with a velocity *v* at an angle $\theta$ with horizontal as shown in the figure. Since the horizontal component of velocity of a projectile remains constant i.e.,




$$v_x = v_0\cos\theta_0 = v\cos\theta \Rightarrow v = \frac{v_0\cos\theta_0}{\cos\theta}$$

As we know that $v^2 = v_0^2 - 2gy$, substituting the obtained value of V in this equation we obtain

$$y = \frac{v_0^2 - v^2}{2g} \qquad y = \frac{1}{2g}\left[v_0^{\,2} - \left(\frac{v_0\cos\theta_0}{\cos\theta}\right)^2\right] \Rightarrow y = \frac{v_0^{\,2}}{2g}\left(1 - \frac{\cos^2\theta_0}{\cos^2\theta}\right)$$

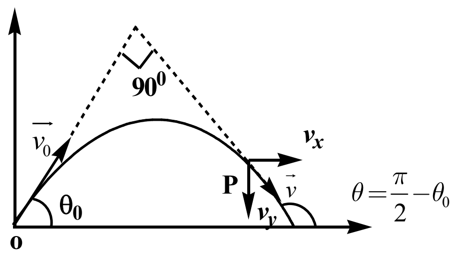
We know that $v_y = (v_y)_0 - gt \Rightarrow v\sin\theta = v_0\sin\theta_0 - gt$

substituting $v = \frac{v_0\cos\theta_0}{\cos\theta}$ we obtain $\left(\frac{v_0\cos\theta_0}{\cos\theta}\right)\sin\theta = v_0\sin\theta_0 - gt$

$$\Rightarrow t = \frac{v_0}{g}\left[\sin\theta_0 - \frac{\cos\theta_0 - \sin\theta}{\cos\theta}\right] \Rightarrow t = \frac{v_0\sin(\theta_0 - \theta)}{g\cos\theta}$$

The horizontal distance *x* covered during the time t is given as

$$x = (v_0\cos\theta_0)t \quad \Rightarrow x = (v_0\cos\theta_0)\frac{v_0\sin(\theta_0-\theta)}{g\cos\theta} \Rightarrow x = \frac{v_0^2\sin(\theta_0-\theta)\cos\theta_0}{g\cos\theta}$$




Referring to the adjoining figure when the velocity vector $\vec{v}$ becomes perpendicular to the initial velocity vector $\overrightarrow{v_0}$

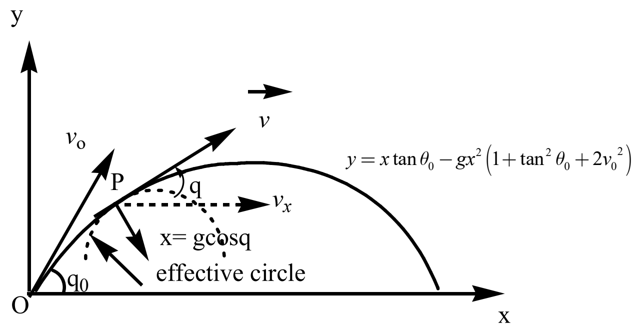
$v.\overrightarrow{v_0} = 0,$ where $v_0 = v_0\cos\theta_0\hat{i} + v_0\sin\theta_0\hat{j}$

and $v = (v_0\cos\theta_0)\hat{i} + (gt - v_0\sin\theta_0)(-\hat{j})$

$$\Rightarrow \left((v_0\cos\theta_0)\hat{i} + (v_0\sin\theta_0 - gt)\hat{j}\right).\ (v_0\cos\theta_0)^2 + (v_0\sin\theta_0 - gt)v_0\sin\theta_0 = 0$$

$$\Rightarrow v_0(\sin^2\theta_0 + \cos^2\theta_0) = gt\sin\theta_0 \Rightarrow t = \frac{v_0}{g\sin\theta_0}$$

## §§ RADIUS OF CURVATURE AT ANY POINT ON THE PATH OF APROJECTILE




Let at any time t the velocity vector $\vec{v}$ be inclined at an angle $\theta$ with horizontal at a point say, P as shown in figure. Since gravitational acceleration g is always acting vertically downwards the component of g perpendicular to the velocity vector $\vec{v}$ can be treated as a radial acceleration towards, the centre of a circular path of radius (let r) as shown in figure. r is known as radius of curvature of the parabola at p

$$\vec{v} \perp \vec{a} \text{ and } a_r = \frac{v^2}{r}, \text{when } a_r = g\cos\theta$$

$$\because a_r = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{a_r}, \text{ where } a_r = g\cos\theta \qquad \Rightarrow r = \frac{v^2}{g\cos\theta}$$

**EXAMPLE**

√ ***W.E - 1 :*** *A bullet fired at an angle of $30^0$ with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away? Assume the muzzle speed to be fixed, and neglect air resistance.*

**Sol** . We are given that angle of projection with the horizontal, $\theta = 30^0$, horizontal range R = 3km.

$$R = \frac{u_0^2 \sin 2\theta}{g}$$

$$3 = \frac{u_0^2 \sin 60^0}{g} = \frac{u_0^2}{g} \times \frac{\sqrt{3}}{2}$$

$$\frac{u_0^2}{g} = 2\sqrt{3} km$$

Since the muzzle speed $(u_0)$ is fixed

$$R_{\max} = \frac{u_0^2}{g} = 2\sqrt{3} = 2 \times 1.732 = 3.464 km$$

so, it is not possible to hit the target 5km away.

√ ***W.E -2:*** *A cannon and a target are 5.10 km apart and located at the same level. How soon will the shell launched with the initial velocity 240 m/s reach the target in the absence of air drag ?*

**Sol .** Here, $u_0$=240 $ms^{-1}$, R =5.10 km =5100m,

$g = 9.8ms^{-2}, \alpha = ?\quad R = \dfrac{u_0^2 \sin 2\alpha}{g}$

$\sin 2\alpha = \dfrac{Rg}{u_0^2} \qquad \Rightarrow \alpha = 30^0 \ \ or \ \ 60^0$

using, $T = \dfrac{2u_0 \sin\alpha}{g} \qquad When \ \ \alpha = 30^0, T_1 = \dfrac{2\times 240\times 0.5}{9.8} = 24.5s$

$When \ \ \alpha = 60^0, T_2 = \dfrac{2\times 240\times 0.867}{9.8} = 42.46s$

√ ***W.E -3:*** *The ceiling of a long hall is 20 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 $ms^{-1}$ can go without hitting the ceiling of the hall $\left(g = 10ms^{-2}\right)$?*

**Sol.** : Here, H =20 m, $u = 40ms^{-1}$.

Suppose the ball is thrown at an angle $\theta$ with the horizontal.

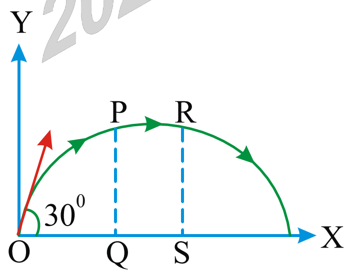
$Now \ \ H = \dfrac{u^2 \sin^2\theta}{2g} \Rightarrow 20 = \dfrac{(40)^2 \sin^2\theta}{2\times 10} \qquad = \dfrac{(40)^2 \times 0.866}{10} = 138.56m$

$or, \ \ \sin\theta = 0.5 \Rightarrow \ \theta = 30^0$

$Now \ \ R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{(40)^2 \times \sin 60^0}{10}$



√ ***W.E -4:*** *A ball projected with a velocity of 10m/s at angle of 30⁰ with horizontal just clears two vertical poles each of height 1m. Find separation between the poles.*




**Sol.** $h = u_y t + \dfrac{1}{2}gt^2 = \left(10\sin 30^0\right)t + \dfrac{1}{2}(-10)t^2$

$1 = 5t - 5t^2 \Rightarrow t = 0.72s, \ 2.76s$

are the instants at which projectile crosses the poles.

$\therefore$ separation between poles = OS - OQ

$= u\cos\theta\left(t_2 - t_1\right) = 10\cos 30^0\left(2.76 - 0.72\right) = 17.7m$

√ ***W.E -5 :*** *A body is projected with velocity u at an angle of projection $\theta$ with the horizontal.The body makes $30^0$ with horizontal at t = 2 second and then after 1 second it reaches the maximum height. Then find*

*a) angle of projection b) speed of projection.*

**Sol.** During the projectile motion, angle at any instant t is such that

$\tan\alpha = \dfrac{u\sin\theta - gt}{u\cos\theta}$

For t = 2 seconds, $\alpha = 30^0$

$\frac{1}{\sqrt{3}} = \frac{u\sin\theta - 2g}{u\cos\theta}$ ---------1

For t = 3 seconds, at the highest point $\alpha = 0^o$

$0 = \frac{u\sin\theta - 3g}{u\cos\theta}$ $\quad u\sin\theta = 3g$ ------------(2)

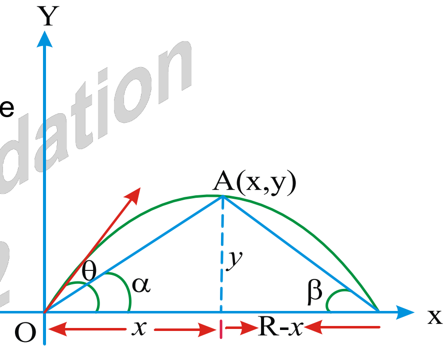
using eq. (1) and eq. (2) $\quad u\cos\theta = \sqrt{3}g$.....................(3)

Eq. (2) $\div$ eq.(3) give $\theta = 60^0$ squaring and adding equation (2) and (3)

$u = 20\sqrt{3}\ m/s.$

√ **W.E-6:** *A particle is thrown over a triangle from one end of horizontal base and grazing the vertex falls on the other end of the base.If $\alpha$ and $\beta$ are the base angles and $\theta$ be the angle of projection, prove that*

$\tan\theta = \tan\alpha + \tan\beta.$




**Sol.**: The situation is shown in figure.From figure,we have

$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x}$

$\tan\alpha + \tan\beta = \frac{yR}{x(R-x)}$ ________(1)

But equation of trajectory is $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

$\tan\theta = \left[\frac{yR}{x(R-x)}\right]$ ———(ii)

From Eqs. (i) and (ii), $\tan\theta = \tan\alpha + \tan\beta$

√ ***W.E -7:*** *The velocity of a projectile at its greatest height is $\sqrt{\frac{2}{5}}$ times its velocity, at half of its greatest height, find the angle of projection.*

**Sol.:** $u\cos\theta = \sqrt{\frac{2}{5}} \times u\sqrt{\frac{1+\cos^2\theta}{2}}$

Squaring on both sides $u^2\cos^2\theta = \frac{2}{5}u^2\left(\frac{1+\cos^2\theta}{2}\right)$

$10\cos^2\theta = 2 + 2\cos^2\theta \quad \Rightarrow 8\cos^2\theta = 2 \Rightarrow \cos^2\theta = \frac{1}{4} \Rightarrow \theta = 60^0$

√ ***W.E -8:*** *A foot ball is kicked off with an initial speed of 19.6 m/s to have maximum range. Goal keeper standing on the goal line 67.4 m away in the direction of the kick starts running opposite to the direction of kick to meet the ball at that instant. What must his speed be if he is to catch the ball before it hits the ground?*

**Sol.:** $R = \frac{u^2\sin 2\theta}{g} = \frac{(19.6)^2 \times \sin 90}{9.8}$ or R= 39.2 metre.

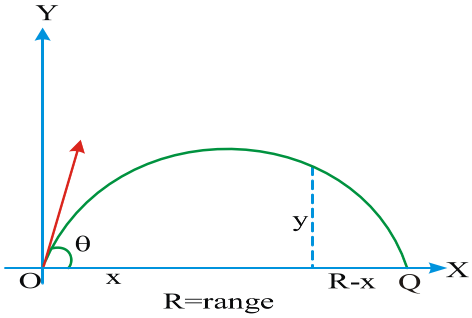
Man must run 67.4 m -39.2m=28.2m

in the time taken by the ball to come to ground Time taken by the ball.

$t = \frac{2u\sin\theta}{g} = \frac{2 \times 19.6 \times \sin 45^0}{9.8} = \frac{4}{\sqrt{2}}$

$t = 2\sqrt{2} = 2\times 1.41 = 2.82\sec.$ Velocity of man $= \frac{28.2m}{2.82\sec} = 10m/\sec.$

√ ***W.E -9:*** *A body projected from a point `0' at an angle $\theta$ , just crosses a wall `y' m high at a distance `x' m from the point of projection and strikes the ground at `Q' beyond the wall as shown, then find height of the wall*




**Sol .** We know that the equation of the trajectory is $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ can be written as

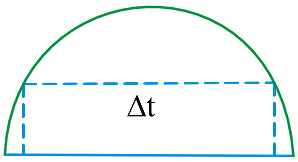
$$y = x\tan\theta - \left(\frac{gx^2}{2u^2\cos^2\theta}\right)\frac{\sin\theta}{\sin\theta}$$

$$y = x\tan\theta - \frac{gx^2\tan\theta}{u^2(2\sin\theta\cos\theta)} \Rightarrow y = x\tan\theta - \frac{x^2\tan\theta}{\frac{u^2\sin 2\theta}{g}}$$

$$\Rightarrow y = x\tan\theta\left[1-\frac{x}{R}\right] \quad [\because\ R = \frac{u^2\sin 2\theta}{g}]$$

√ ***W.E - 10:*** *A particle is projected with a velocity of $10\sqrt{2}$ m/s at an angle of $45^0$ with the horizontal . Find the interval between the moments when speed is $\sqrt{125}$ m/s $\left(g = 10m/s^2\right)$*

**Sol.** $v = \sqrt{125}\ m/s$




$u_x = 10\sqrt{2}\cos 45^0 = 10m/s,\ \ u_y = 10\sqrt{2}\sin 45^0 = 10m/s$

$v^2 = v_x^2 + v_y^2$

$125 = 100 + v_y^2 \Rightarrow v_y = 5m/s \quad (\because v_x = u_x)$

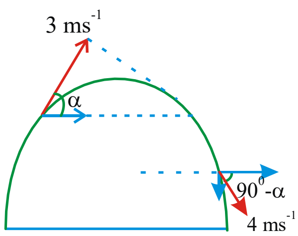
The required time interval is $\Delta t = \frac{2v_y}{g} = \frac{2\times 5}{10} = 1\ s$

√ ***W.E - 11:*** *A projectile of 2 kg has velocities 3 m/s and 4 m/s at two points during its flight in the uniform gravitational field of the earth. If these two velocities are $\perp$ to each other then the minimum KE of the particle during its flight is*

**Sol.** $V_1\cos\alpha = V_2\cos(90-\alpha)$

$3\cos\alpha = 4\sin\alpha$

$\tan\alpha = \frac{3}{4}$




$KE_{\min} = \frac{1}{2}mv_1^2\cos^2\alpha$

$= \frac{1}{2}\times 2\times 3^2\times\left(\frac{4}{5}\right)^2 = \frac{9\times 16}{25} = 5.76J$

√ ***W.E-12:*** *In the absence of wind the range and maximum height of a projectile were R and H. If wind imparts a horizontal acceleration a =g/4 to the projectile then find the maximum range and maximum height.*

**Sol** : $H^1 = H$ ($\therefore$ u sin$\theta$ remains same ) $T^1 = T$

$R^1 = u_x T + \frac{1}{2} a T^2 = R + \frac{1}{2}\frac{g}{4}T^2$

$= R + \frac{1}{8} g T^2 \quad = R + H$ $\boxed{R^1 = R + H}$ $\boxed{H^1 = H}$

➢ If a body is projected with a velocity

$\vec{u} = \vec{ai} + \vec{bj} + \vec{ck}$

($\vec{i} - east \quad \vec{j} - north \quad \vec{k} - \quad vertical$) then

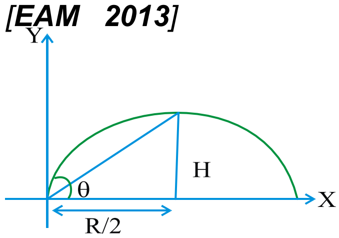
$u_x = \sqrt{a^2 + b^2}$ ; $u_y = c$

$T = \frac{2c}{g}; H = \frac{c^2}{2g}, R = \frac{2\left(\sqrt{a^2+b^2}\right)c}{g}$

√ ***W.E-13:*** *A particle is projected from the ground with an initial speed v at an angle $\theta$ with horizontal.The average velocity of the particle between its point of projection and highest point of trajectory is* ***[EAM 2013]***

**Sol:** $\vec{v}_{avg} = \frac{\vec{v} + \vec{u}}{2} = \frac{u\cos\theta\hat{i} + (u\cos\theta\hat{i} + u\sin\theta\hat{j})}{2}$

$v_{av} = \frac{v}{2}\sqrt{1 + 3\cos^2\theta}$




√ ***W.E-14 :*** *The speed with which a bullet can be fired is 150 ms$^{-1}$. Calculate the greatest distance to which it can be projected and also the maximum height to which it would rise.*

**Sol:**The greatest horizontal range is achieved at an angle of projection of 45°

Components of initial velocity = 150 cos 45° = 106.06 ms$^{-1}$.

Now, if T is the time of flight, then considering the vertical motion of the bullet,

u = 106.06 ms$^{-1}$; a = –9.8 ms$^{-1}$; s = 0; t = T

Using, s = ut + $\frac{1}{2}$ at$^2$

we get 0 = 106.06 T – $\frac{1}{2}$ 9.8 T$^2$

$\Rightarrow$ T = $\frac{106.06 \times 2}{9.8}$ = 21.64 sec.

Maximum Horizontal range = horizontal component of velocity ´ total time of flight.

= 106.06 × 21.64

= 2295.14 m

Again, if $H_{max}$ be the maximum height to which the bullet rises, then

u = 106.06 ms$^{-1}$; a = – 9.8 ms$^{-1}$;

$v = 0; s = H_{max} = \frac{u^2}{2g} \Rightarrow H_{max} = \frac{(106.06)^2}{19.6} = 573.91m$

√ ***W.E-15 :*** *The horizontal range of a projectile is $2\sqrt{3}$ times its maximum height. Find the angle of projection.*

**Sol:** If u and $\alpha$ be the initial velocity of projection and angle of projection respectively, then

the maximum height attained, $H_m = \frac{u^2 \sin^2 \alpha}{2g}$

and horizontal range , $R = \frac{2u^2 \sin\alpha \cos \alpha}{g}$

According to the problem, we can write

$$\frac{2u^2 \sin \alpha \cos \alpha}{g} = 2\sqrt{3}\left(\frac{u^2 \sin^2 \alpha}{2g}\right)$$

$\Rightarrow \tan \alpha = \left(\frac{2}{\sqrt{3}}\right) \Rightarrow \alpha = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

√ ***W.E-16 :*** *A stone is to be thrown so as to cover a horizontal distance of 3 m. If the velocity of the projectile is 7 $ms^{-1}$, find (i) the angle at which it must be thrown,*

*(ii) the largest horizontal displacement that is possible with the projection speed of 7 $ms^{-1}$.*

**Sol:**(a) Given that **,** u = 7 $\mathbf{ms^{-1}}$, range (R) = 3m

$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow 3 = \frac{(7)^2 \sin 2\theta}{9.8}$

$\Rightarrow$ sin 2q = 3/5 $\Rightarrow$ 2q = 37° or 180° – 2q = 37°.

$\Rightarrow$ 2q = 37° or 180° – 2q = 37°.

$\Rightarrow$ q = 18° 30' or q = 71° 30'

Hence a range of 3m is possible with two angles of projections.

(b) Range is maximum, if sin 2q is maximum.

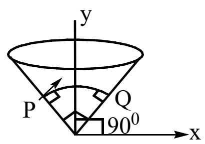
i.e., for sin 2q = 1

$\Rightarrow$ 2q = 90° or q = 45°

Hence for maximum range with a given velocity, the angle of projection, q = 45°

$$R_{max} = \frac{(7)^2 \sin 2(45^0)}{9.8} = 5m$$

√ ***W.E-17:*** *A particle is projected from point P with velocity $5\sqrt{2}ms^{-1}$ perpendicular to the surface of a hollow right angle cone whose axis is vertical. It collides at Q normally. The time of the flight of the particle is*

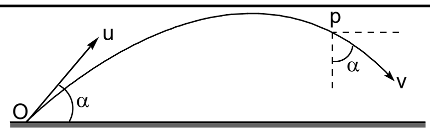



**Sol:** $t = \frac{u}{g \sin \theta} = \frac{5\sqrt{2} \times \sqrt{2}}{10} = 1 sec$

√ ***W.E-18 :*** *If at point of projection, the velocity of a particle is "u" and is directed at an angle "$\alpha$" to the horizontal, then show that it will be moving at right angles to its initial direction after a time* $\frac{(u \operatorname{cosec} \alpha)}{g}$

**Sol:** Let "t" be the time after which velocity becomes perpendicular to its initial direction.

As u and v are perpendicular, the angle between v and vertical will be a.




Initial velocity 'u' $= (u\cos\alpha\ \hat{i} + u\sin\alpha\ \hat{j})$

After *t* sec, velocity 'v' $= \left\{u\cos\alpha\ \hat{i} + (u\sin\alpha - gt)\hat{j}\right\}$

$\because$ These are perpendicular their dot product is zero.

$\therefore (u\cos\alpha\, i + u\sin j)\cdot\left\{u\cos\alpha\, i + (u\sin\alpha - gt)j\right\} = 0$

and $t = \dfrac{u\,\text{cosec}\,\alpha}{g}$

√ ***W.E-19 :*** *If y = x –* $\mathbf{\frac{1}{2}}$ *$x^2$ is the equation of a trajectory, find the time of flight.*

**Sol:** We have $y = x - \frac{1}{2}x^2 = x\,(1 - x/2)$

If y = 0, then either x = 0 or x = 2.

Hence the range of the motion is 2.

For half the range, x = 1, then y = 1/2

Hence maximum height attained = 1/2.

Time to reach maximum height, $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{1}{g}}$

Time of flight, $T = 2t = 2\sqrt{\frac{1}{g}}$ i.e. $T = \frac{2}{\sqrt{g}}$

√ ***W.E-20 :*** *The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 $ms^{-1}$ can go without hitting the ceiling of the hall?*

**Sol:** Here, H = 25 m, u = 40 $ms^{-1}$.

Suppose the ball is thrown at an angle q with the horizontal.

Now, $H = \dfrac{u^2\sin^2\theta}{2g} \Rightarrow 25 = \dfrac{(40)^2\sin^2\theta}{2g}$

or sin q = 0.5534 or q = 33.6°

Now $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{2u^2\sin\theta\cos\theta}{g}$

$= \dfrac{2\times(40)^2\times 0.554\times 0.833}{9.8}$ or R = 150.17 m

√ ***W.E-21 :*** *If R is the horizontal range for $\theta$ inclination and h is the maximum height reached by the projectile, show tht the maximum range is given by $\frac{R^2}{8h} + 2h$.*

**Sol:** We know that horizontal range,

$R = \dfrac{u^2\sin 2\theta}{g}$ and maximum height, $h = \dfrac{u^2\sin^2\theta}{2g}$

$$\therefore \frac{R^2}{8h} + 2h = \frac{\left[\frac{u^2\sin 2\theta}{g}\right]^2}{8\left[\frac{u^2\sin^2\theta}{2g}\right]} + 2\left[\frac{u^2\sin^2\theta}{2g}\right] = \frac{u^4\left(2\sin\theta\cos\theta\right)^2}{g^2\times 8\frac{u^2\sin^2\theta}{2g}} + \frac{u^2\sin^2\theta}{g}$$

$= \frac{u^2}{g}\left(\cos^2\theta + \sin^2\theta\right) \quad = \frac{u^2}{g} = R_{max}$

√ ***W.E-22 :*** *A cannon and a target are 5.10 km apart and located at the same level. How soon will the shell launched with the initial velocity 240 m/s reach the target in the absence of air drag?*

**Sol:** Here, $v_0$ = 240 ms$^{-1}$, R = 5.10 km = 5100 m,

g = 9.8 ms$^{-2}$, a = ?

From formula, R = $\frac{v_0^2 \sin 2\alpha}{g}$ We have, sin 2a = $\frac{Rg}{{v_0}^2}$

Putting values, we get, sin 2a = $\frac{5100 \times 9.8}{240 \times 240} = 0.8677 = \frac{\sqrt{3}}{2}$ = sin 60° or sin 120°

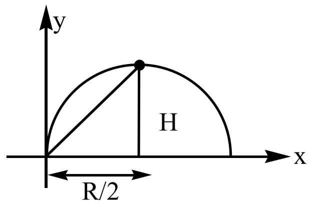
a = 30° or 60°

From formula, T = $\frac{2v_0 \sin\alpha}{g}$ When, a = 30°, $T_1$ = $\frac{2 \times 240 \times 0.5}{9.8}$ = 24.5 sec.

When, a = 60°, $T_2$ = $\frac{2 \times 240 \times 0.867}{9.8}$ = 42.41 sec.

√ ***W.E-23 :*** *A particle is projected from the ground with an initial speed of v at an angle q with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is*

**Sol:** Average velocity $= \frac{\text{displacememnt}}{\text{time}}$




$V_{av} = \frac{\sqrt{H^2 + \frac{R^2}{4}}}{\frac{T}{2}}$ …(i)

Here H = maximum height $= \frac{v^2 \sin^2\theta}{2g}$

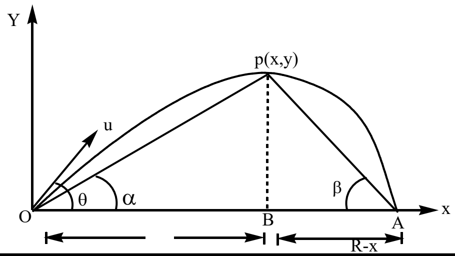
R = range $= \frac{v^2 \sin 2\theta}{g}$

and T = time of flight $= \frac{2v\sin\theta}{g}$

Substituting in (i), we get $v_{av} = \frac{v}{2}\sqrt{1 + 3\cos^2\theta}$

√ ***W.E-24 :*** *A particle is projected over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If a and b be the base angles and q the angle of projection then show that tanq = tan a + tan b.*

**Sol:** The situation concerning the problem is described in figure .



Now, the equation of trajectory is

$$y = x \tan q - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

or $\quad y = x \tan q \left[1 - \frac{gx}{2u^2 \cos^2 \theta \tan \theta}\right]$

or $\quad y = x \tan q \left[1 - \frac{gx}{(2/g)\, u^2 \sin\theta \cos\theta}\right]$ or

$y = x \tan q \left[1 - \frac{x}{u^2 \sin 2\theta / g}\right]$

or $\quad y = x \tan q \left[1 - \frac{x}{R}\right]$ or $\tan q = \frac{y}{x} \times \frac{R}{(R-x)}$

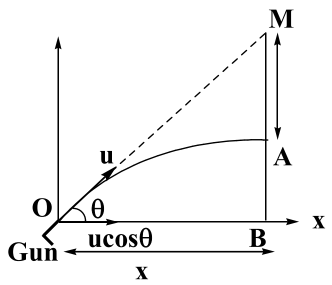
$$= \frac{y}{x}\left[\frac{R-x+x}{R-x}\right] = \frac{y}{x} + \frac{y}{R-x} = Tan\,\alpha + Tan\,\beta$$

tan q = tan a + tan b.

√ ***W.E-25 :*** *A hunter aims his gun and fires a bullet directly at a monkey on a tree. At the instant the bullet leaves the barrel of the gun, the monkey drops. Will the bullet hit the monkey?*

**Sol:** Suppose the gun situated at O directed towards the monkey at position M. Let bullet leaves the barrel of the gun with velocity *u* at an angle $\theta$ with the horizontal. Let bullet crosses the vertical line MB at A after time t. Horizontal distance travelled

$OB = x = u\cos\theta\, t$ or $t = \frac{x}{u\cos\theta}$




For motion of bullet from O to B, the vertical height

$AB = u\sin\theta\, t - \frac{1}{2}gt^2 = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}gt^2$

$= x\tan\theta - \frac{gt^2}{2}$ ......(i)

Also $\quad MB = x\tan\theta$

Now $\quad y = MA = MB - AB$

$= x\tan\theta - \left(x\tan\theta - \frac{gt^2}{2}\right) = \frac{1}{2}gt^2$ Thus, in time t the bullet passes through A a vertical distance $\frac{1}{2}gt^2$ below M.

The vertical distance through which the monkey fall in time t. $s = \frac{1}{2}gt^2$

Thus, the bullet and the monkey will always reach at point A at the same time.

## TEACHING TASK

**Single Answer Type:**

**1.** A particle is projected from ground with some initial velocity making an angle of $45^0$ with the horizontal. It reaches a height of 7.5 m above the ground while it travels a horizontal distance of 10m from the point of projection.The initial speed of the projection is

1) 5 m/s 2) 10 m/s 3) 20 m/s 4) 40 m/s

**2.** A particle is projected from ground at an angle $45°$ with initial velocity $20\sqrt{2}\,ms^{-1}$. The magnitude of average velocity in a time interval from t = 0 to t = 3 s in $ms^{-1}$ is

1) 20.62 2) 10.31 3) 41.14 4) 5.15

**3.** A ball is thrown with a velocity of u making an angle $\theta$ with the horizontal. Its velocity vector normal to initial vector (u) after a time interval of

1) $\frac{u\sin\theta}{g}$ 2) $\frac{u}{g\cos\theta}$ 3) $\frac{u}{g\sin\theta}$ 4) $\frac{u\cos\theta}{g}$

**4.** A stone is projected with a velocity 20 $\sqrt{2}$ m/s at an angle of $45^0$ to the horizontal. The average velocity of stone during its motion from starting point to its maximum height is

1) 10$\sqrt{5}$ m/s 2) 20$\sqrt{5}$ m/s 3) 5$\sqrt{5}$ m/s 4) 20 m/s

**5.** A player kicks a foot ball obliquely at a speed of 20 m/s so that its range is maximum. Another player at a distance of 24m away in the direction of kick starts running at that instant to catch the ball. Before the ball hits the ground to catch it, the speed with which the second player has to run is (g=10 $ms^{-2}$)

1) 4 $m/s^{-1}$ 2) 4$\sqrt{2}$ $m/s^{-1}$ 3) 8$\sqrt{2}$ $m/s^{-1}$ 4) 8 $m/s^{-1}$

**6.** A particle is fired with velocity u making angle $\theta$ with the horizontal . What is the change in velocity when it is at the highest point ?

1) $u\cos\theta$ 2) u 3) $u\sin\theta$ 4) $(u\cos\theta - u)$

**7.** Two projectiles A and B are thrown from the same point with velocities *v* and $\frac{v}{2}$ respectively. If B is thrown at an angle 45° with horizontal. What is the inclination of A. when their ranges are the same?

1) $\sin^{-1}\left(\frac{1}{4}\right)$ 2) $\frac{1}{2}\sin^{-1}\left(\frac{1}{4}\right)$ 3) $2\sin^{-1}\left(\frac{1}{4}\right)$ 4) $\frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$

**8.** A particle is projected with a velocity v such that its range on the horizontal plane is twice the greatest height attained by it, The range of the projectile is (when it is acceleration due to gravity is 'g')

1) $\frac{4v^2}{5g}$ 2) $\frac{4g}{5v^2}$ 3) $\frac{v^2}{g}$ 4) $\frac{4v^2}{\sqrt{5g}}$

**9.** A number of bullets are fired in all possible directions with the same initial velocity u. The maximum area of ground covered by bullets is

1) $\pi\left(\frac{u^2}{g}\right)^2$ 2) $\pi\left(\frac{u^2}{2g}\right)$ 3) $\pi\left(\frac{u}{g}\right)^2$ 4) $\pi\left(\frac{u}{2g}\right)^2$

**10.** A ball is projected from the ground with a velocity 'u' such that its range is maximum. Then

1) Its velocity at half the maximum height is $\frac{\sqrt{3}}{2}u$
2) Its velocity at the maximum height is 'u'.
3) Change in its velocity when it returns to the ground is 'u'.
4) all the above are true.

**Assertion and Reason type:**

- *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.*

*1) Both A & R are true and R is correct explanation of A*
*2) Both A & R are true and R is not correct explanation of A*
*3) A is true but R is false.* *4) Both A & R are false.*

**11.** **Assertion(A):** In projectile motion, the angle between the instantaneous velocity and acceleration at the height point is $180^0$.
**Reason(R):** At the highest point, velocity of projectile will be vertically upward.

**12.** **Assertion :-** Two particles of different mass, projected with same velocity at same angles. The maximum height attained by both the particle will be same.
**Reason :-** The maximum height of projectile is independent of particle mass.

**Matrix Match Type:**

- *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*
*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*



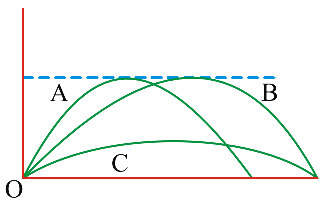
**13.** Match List I with List II for a projectile

| **List - I** | **List - II** |
|---|---|
| a) For two angles $\theta$ and $(90-\theta)$ with same magnitude of velocity of projection | e) $\frac{P\vec{i}.P\vec{i}}{g}$ |
| b) Equation of parabola of a projectile $y = Px - Qx^2$ | f) Maximum height = 25% of $\frac{P^2}{Q}$ |
| c) Radius of curvature of path of a body projected with velocity $(P\vec{i}+Q\vec{j})$ $ms^{-1}$ at highest point | g) Range = Maximum height |
| d) Angle of projection $\theta = \tan^{-1}(4)$ | h) Range is same |

1) $a \to f; b \to h; c \to g; d \to e$ 2) $a \to h; b \to f; c \to e; d \to g$
3) $a \to e; b \to g; c \to f; d \to h$ 4) $a \to e; b \to g; c \to h; d \to f$

**14.** Trajectories are shown in figure for three kicked footballs. Initial vertical and horizontal velocity components are $u_y$ *and* $u_x$ respectively. Ignoring air resistance, choose the correct statement from column-2 for the value of variable in column-1.

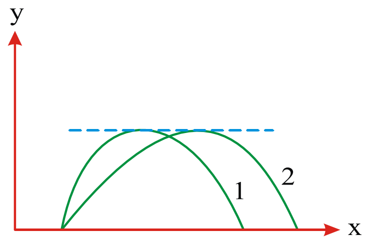
| Column-1 | Column-2 |
|---|---|
| A) Time of flight | P) greatest for A only |
| B) $u_y / u_x$ | Q) greatest for C only |
| C) $u_x$ | R) equal for A and B |
| D) $u_x u_y$ | S) equal for B and C |



## Multi Answer Type:

- *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options.*

**15.** Trajectories of two projectiles are shown in figure. Let $T_1$ and $T_2$ be the time periods and $u_1$ and $u_2$ their speeds of projection. Then




(a) $T_2 > T_1$ (b) $T_1 = T_2$

(c) $u_1 > u_2$ (d) $u_1 < u_2$

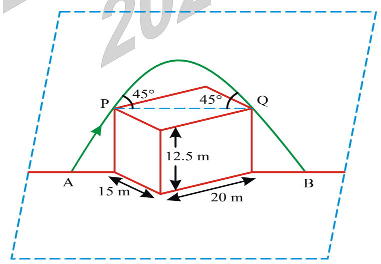
**16.** Two particles are projected from ground with same intial velocities at angles $60^0$ and $30^0$ (with horizontal). Let $R_1$ and $R_2$ be their horizontal ranges, $H_1$ and $H_2$ their maximum heights and $T_1$ and $T_2$ are the time of flights. Then

(a) $\frac{H_1}{R_1} > \frac{H_2}{R_2}$ (b) $\frac{H_1}{R_1} < \frac{H_2}{R_2}$ (c) $\frac{H_1}{T_1} > \frac{H_2}{T_2}$ (d) $\frac{H_1}{T_1} < \frac{H_2}{T_2}$

## Comprehsion Type:

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C ) and (D) out of which **ONLY ONE** is correct. Choose the correct option.*

A particle is fired from 'A' in the diagonal plane of a building of dimension 20m (length) x 15m(breadth) x 12.5m (height), just clears the roof diagonally & falls on the other side of the building at B. It is observed that the particle is traveling at an angle $45^0$ with the horizontal when it clears the edges P and Q of the diagonal. Take g=10m/s$^2$.




**17.** The speed of the particle at point P will be :

a) $5\sqrt{10}m/s$ b) $10\sqrt{5}m/s$ c) $5\sqrt{15}m/s$ d) $5\sqrt{5}m/s$

**18.** The speed of projection of the particle at A will be :

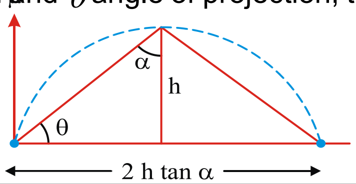
a) $5\sqrt{10}m/s$ b) $10\sqrt{5}m/s$ c) $5\sqrt{15}m/s$ d) $5\sqrt{5}m/s$

**19.** The range that is AB will be :

a) $5\sqrt{10}m$ b) $25\sqrt{3}m$ c) $5\sqrt{15}m$ d) $25\sqrt{5}m$

## Integer Type:

**20.** A projectile is fired from the base of cone-shaped hill. The projectile grazes the vertex and strikes the hill again at the base. If $\alpha$ be the half - angle of the cone, h its height, u the initial velocity of projection and $\theta$ angle of projection, then $\tan\theta\tan\alpha$ is



## KEY

**ΦΦ TEACHING TASK :**

1) 3, 2) 1, 3) 3, 4) 1, 5) 2, 6) 3, 7) 2, 8) 1, 9) 1, 10) 1, 11) 4,
12) 1, 13) 2, 14) a-r b-p c-q d-s, 15) b,d, 16) a,c, 17) a,
18) b, 19) b, 20) 2

## LEARNER'S TASK

### BEGINNERS ( Level - I )



**Single Answer Type:**

**1.** Keeping the speed of projection constant, the angle of projection is increased from 0º to 90º. Then the horizontal range of the projectile

1) goes on increasing up to $90^0$

2) decreases up to 90º

3) increases up to $45^0$ and decreases afterwards

4) decreases up to $45^0$ and increases afterwards

**2.** Keeping the speed of projection constant, the angle of projection is increased from 0° to 90º. Then the maximum height of the projectile

1) goes on increasing upto $90^0$

2) decreases upto $90^0$

3) increases upto $45^0$ and decreases beyond it

4) decreases upto $45^0$ and increases beyond it

**3.** The path of one projectile as seen from another projectile is a ( if horizontal components of velocities are equal )

1) straight line 2) parabola 3) hyperbola 4) circle

**4.** Two particles are projected with same speed but at angles of projection $\left(45^0-\theta\right)$ and $\left(45^0+\theta\right)$. Then their horizontal ranges are in the ratio of

1) 1 : 2 2) 2 : 1 3) 1 : 1 4) none of the above

**5.** The acceleration of a projectile relative to another projectile is

1) -g 2) g 3) 2g 4)0

**6.** A particle is projected in xy plane with y-axis along vertical, the point of projection is origin. The equation of the path is $y=\sqrt{3}x-\frac{g}{2}x^2$. where y and x are in m. Then the speed of projection in $ms^{-1}$ is

1) 2 2) $\sqrt{3}$ 3) 4 4) $\sqrt{3}$/2

**7.** If a body is thrown with a speed of 19.6m/s making an angle of $30^0$ with the horizontal, then the time of flight is

1) 1 s 2) 2 s 3) $2\sqrt{3}$ s 4) 5 s

**8.** A particle is projected with an initial velocity of 200 m/s in a direction making an angle of $30^0$ with the vertical. The horizontal distance covered by the particle in 3s is

1) 300 m 2) 150 m 3) 175 m 4) 125 m

**9.** A body is projected with an initial velocity 20 m/s at $60^0$ to the horizontal. Its initial velocity vector is ____(g=10 $m/s^2$)

1) $10\hat{i}-20\hat{j}$ 2) $10\sqrt{3}\,\hat{i}+10\hat{j}$ 3) $10\hat{i}+10\sqrt{3}\,\hat{j}$ 4) $5\hat{i}+5\sqrt{3}\,\hat{j}$

**10.** A body is projected at an angle of 30° with the horizontal with momentum P. At its highest point the magnitude of the momentum is:

1) $\frac{\sqrt{3}}{2}P$ 2) $\frac{2}{\sqrt{3}}P$ 3) P 4) $\frac{P}{2}$

**11.** The potential energy of a projectile at its maximum height is equal to its kinetic energy there. If the velocity of projection is 20 $ms^{-1}$, its time of flight is (g=10 $ms^{-2}$)

1) 2s 2) $2\sqrt{2}\,s$ 3) $\frac{1}{2}s$ 4) $\frac{1}{\sqrt{2}}s$

**12**. From a point on the ground a particle is projected with initial velocity $u$, such that its horizontal range is maximum. The magnitude of average velocity during its ascent.

1) $\frac{\sqrt{5}u}{2\sqrt{2}}$ 2) $\frac{5u}{4}$ 3) $\frac{\sqrt{3}}{2\sqrt{2}}$ 4) none

**13.** The horizontal and vertical displacements of a projectile are given as $x = at$ & $y = bt - ct^2$. Then velocity of projection is

1) $\sqrt{a^2+b^2}$ 2) $\sqrt{b^2+c^2}$ 3) $\sqrt{a^2+c^2}$ 4) $\sqrt{b^2-c^2}$

**14.** Two bodies are thrown from the same point with the same velocity of 50ms$^{-1}$. If their angles of projection are complimentary to each other and the difference of maximum heights is 30m, the minimum and maximum heights are(g=10 m/s$^2$)

1) 50 m & 80 m 2) 47.5 m & 77.5 m 3) 30 m & 60 m 4) 25 m & 55 m

**15.** A missile is fired for maximum range with an initial velocity of $20\,m\,s^{-1}$, the range of the missile is $\left(g=10\,m/s^2\right)$

1) 50m 2) 60 m 3) 20m 4) 40 m

**16.** If $\vec{u}=a\hat{i}+b\hat{j}+c\hat{k}$ with $\hat{i},\hat{j},\hat{k}$ are in east, north and vertical directions, horizontal component of velocity of projectile is

1) a 2) b 3) $\sqrt{a^2+b^2}$ 4) $\sqrt{b^2+c^2}$

**17.** If the time of flight of a projectile is doubled, what happens to the maximum height attained?

1) halved 2) remains unchanged 3) doubled 4) become four times

**18.** If $\vec{u}=a\hat{i}+b\hat{j}+c\hat{k}$ with $\hat{i},\hat{j},\hat{k}$ are in east, north and vertical directions, the maximum height of the projectile is

1) $\frac{a^2}{2g}$ 2) $\frac{b^2}{2g}$ 3) $\frac{c^2}{2g}$ 4) $\frac{b^2c^2}{2g}$

**19.** The parabolic path of a projectile is represented by $y=\frac{x}{\sqrt{3}}-\frac{x^2}{60}$ in MKS units : Its angle of projection is (g = 10ms$^{-2}$)

1) $30^0$ 2) $45^0$ 3) $60^0$ 4) $90^0$

**20.** A body is projected at angle $30^0$ to horizontal with a velocity 50 $ms^{-1}$. Its time of flight is

1) 4 s 2) 5 s 3) 6 s 4) 7 s

**21**. A body is projected with velocity $60\,m/s$ at $30^0$ to the horizontal. The velocity of the body after 3 seconds is

1) $20\hat{i}+20\sqrt{3}\hat{j}$ 2) $30\hat{i}$ 3) $10\sqrt{3}\hat{j}$ 4) $30\sqrt{3}\hat{i}$

**22.** A body is projected with velocity u such that in horizontal range and maximum vertical heights are same.The maximum height is

1) $\frac{u^2}{2g}$ 2) $\frac{3u^2}{4g}$ 3) $\frac{16u^2}{17g}$ 4) $\frac{8u^2}{17g}$

**23.** A cricket ball is hit for a six leaving the bat at an angle of $60^0$ to the horizontal with kinetic energy 'k'. At the top, K.E. of the ball is

1) Zero 2) k 3) $\frac{k}{4}$ 4) $\frac{k}{\sqrt{2}}$

**24.** A bomb at rest is exploded and the pieces are scattered in all directions with a maximum velocity of $20ms^{-1}$. Dangerous distance from that spot is (g = 10 $m/s^2$)

1) 10 m 2) 20 m 3) 30 m 4) 40 m

**25.** A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be

1) $20\sqrt{2}$ m 2) 10 m 3) $10\sqrt{2}$ m 4) 20 m

**26**. A grass hopper can jump a maximum horizontal distance of 0.3 m. If it spends negligible time on the ground, its horizontal component of velocity is (g=10 $m/s^2$)

1) 3/2 m/s 2) $\sqrt{\frac{3}{2}}$ m/s 3) 1/2 m/s 4) $\sqrt{\frac{2}{3}}$ m/s

**27.** A stone is thrown with a velocity $v$ at an angle $\theta$ with the horizontal. Its speed when it makes an angle $\beta$ with the horizontal is

1) $v\cos\theta$ 2) $\frac{v}{\cos\beta}$ 3) $v\cos\theta\cos\beta$ 4) $\frac{v\cos\theta}{\cos\beta}$

**28.** A body is projected with a certain speed at angles of projection of $\theta$ and $90-\theta$. The maximum heights attained in the two cases are 20 m and 10 m respectively. The maximum possible range is

1) 60 m 2) 30 m 3) 20 m 4) 80 m

**29.** The launching speed of a certain projectile is five times the speed it has at its maximum height. Its angle of projection is

1) $\theta = \cos^{-1}(0.2)$ 2) $\theta = \sin^{-1}(0.2)$ 3) $\theta = \tan^{-1}(0.2)$ 4) $\theta = 0^0$

**30.** A person throws a bottle into a dustbin at the same height as he is 2m away at an angle of $45^0$. The velocity of thrown is

1) g 2) $\sqrt{g}$ 3) 2g 4) $\sqrt{2g}$

**31.** A particle projected from the level ground just clears in its ascent a wall 30 m high and 120$\sqrt{3}$ away measured horizontally. The time since projection to clear the wall is two second. It will strike the ground in the same horizontal plane from the wall on the other side at a distance of (in metres)

1)150$\sqrt{3}$ 2)180$\sqrt{3}$ 3) 120$\sqrt{3}$ 4) 210$\sqrt{3}$

**32.** A stone is projected with a velocity 20$\sqrt{2}$ m/s at an angle of $45^0$ to the horizontal. The average velocity of stone during its motion from starting point to its maximum height is (g=10 $m/s^2$)

1)10$\sqrt{5}$ m/s 2)20$\sqrt{5}$ m/s 3)5$\sqrt{5}$ m/s 4)20m/s

**33.** A ball is thrown with velocity $8\ ms^{-1}$ making an angle 60° with the horizontal. Its velocity will be perpendicular to the direction of initial velocity of projection after a time of

1) $\frac{1.6}{\sqrt{3}}$ s 2) $\frac{4}{\sqrt{3}}$ s 3) 0.6 s 4) $1.6\sqrt{3}$ s

**34.** The range of a projectile, when launched at an angle of $15^0$ with the horizontal is 1.5m. The additional horizontal distance the projectile would cover when projected with same velocity at $45^0$ is

1) 3 km 2) 4.5 km 3) 1.5 km 4) 2.5 km

**35.** A body is projected obliquely from the ground such that its horizontal range is maximum.If the change in its linear momentum, as it moves from half the maximum height to maximum height, is P, the change in its linear momentum as it travels from the point of projection to the landing point on the ground will be

1) P 2) $\sqrt{2}$ P 3) 2 P 4) $2\sqrt{2}$ P

**36.** A projectile is thrown at angle $\beta$ with vertical. It reaches a maximum height H. The time taken to reach the highest point of its path is

1) $\sqrt{\frac{H}{g}}$ 2) $\sqrt{\frac{2H}{g}}$ 3) $\sqrt{\frac{H}{2g}}$ 4) $\sqrt{\frac{2H}{g\cos\beta}}$

**37.** The maximum height attained by a projectile is increased by 5%.Keeping the angle of projection constant, What is the percentage increase in horizontal range?

1)5% 2)10% 3)15% 4)20%

**38.** A gardener wants to wet the garden without moving from his place with a water jet whose velocity is 20 $\text{m s}^{-1}$ the maximum area that he can wet $\left(g = 10 \text{ m s}^{-2}\right)$ ( in metre$^2$ )

1) $1600\pi$ 2) $40\pi$ 3) $400\pi$ 4) $200\pi$

**39.** A particle is projected with speed u at angle $\theta$ to the horizontal. find the radius of curvature at highest point of its trajectory

1) $\frac{u^2\cos^2\theta}{2g}$ 2) $\frac{\sqrt{3}u^2\cos^2\theta}{2g}$ 3) $\frac{u^2\cos^2\theta}{g}$ 4) $\frac{\sqrt{3}u^2\cos^2\theta}{g}$

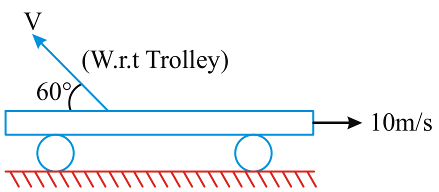


## ACHIEVERS ( Level - II )

**Solve the following:**

**1.** A golfer standing on level ground hits a ball with a velocity of $u = 50ms^{-1}$ at an angle $\alpha$ above the horizontal. If $\tan\alpha = \frac{5}{12}$, then the time for which the ball is at least 15 m above the ground will be (take g $g = 10ms^{-2}$ )

**2.** A particle is projected from a stationary trolley. After projection, the trolly moves with velocity $2\sqrt{15}m/s$. For an observer on the trolley, the direction of the particle is as shown in the figure while for the observer on the ground, the ball rises vertically. The maximum height reached by the ball from the trolley is $h$ metre. The value of $h$ will be



## EXPLORERS ( Level - III )

**Assertion and Reason type:**

♦ *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.*

*1) Both A & R are true and R is correct explanation of A*

*2) Both A & R are true and R is not correct explanation of A*

*3) A is true but R is false.* *4) Both A & R are false.*

**1.** **A :** The horizontal displacement of a projectile varies linearly with time.
**R :** Projectile motion is uniform motion along horizontal direction.

**2.** **A :** The path followed by one projectile as observed by another projectile is a straight line in uniform gravitation field.
**R :** The relative velocity between two projectiles at a given place doesnot change with time. Because their relative acceleration is zero.

**3.** **A :** A ball is projected with $60ms^{-1}$ at $60^0$ with the horizontal simultaneously a toy car starts moving with $30ms^{-1}$ from the same point and in the same horizontal direction as the ball moves. The ball always lies above the toy car.
**R :** Bodies moving with same uniform velocity cover equal displacements in equal intervals of time.

**4.** **A :** Only vertical component of velocity of a projectile is known, time of flight can be calculated but horizontal range cannot be calculated.
**R :** Time of flight depends on horizontal component and range depends on vertical component of velocity projection.

**5.** **A :** In case of projectile the angle between velocity and acceleration changes from point to point.
**R :** Because its horizontal component of velocity remains constant while vertical component of velocity changes from point to point due to acceleration due to gravity.

**6.** **A :** In projectile motion, the angle between the instantaneous velocity and acceleration at the height point is $180^0$.
**R :** At the highest point, velocity of projectile will be in horizontal direction only.

**7.** **A :** When range of a projectile is maximum, its angle of projection may be $45^0$ or $135^0$.
**R :** Horizontal range = $\frac{u^2 \sin 2\theta}{g}$. When q = $45^0$ or $135^0$ the range is same.

**8.** **A :** When a body is projected at an angle $45^0$, its maximum height is half than that of horizontal range.
**R :** Horizontal range = $\frac{u^2 \sin 2\theta}{g}$ and maximum height = $\frac{u^2 \sin^2 \theta}{2g}$

**Matrix Match Type:**

♦ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*
*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

**9.** Angle between velocity and acceleration vectors in the following cases
a) Vertically projected body e) $90^0$

b) For freely falling body — f) changes from point to point
c) For projectile — g) Zero
d) In uniform circular motion — h) $180^0$

1) a – h ; b – g ; c – f ; d – e   2) a – f ; b – g ; c – h ; d – e
3) a – e ; b – f ; c – h ; d – g   4) a – g ; b – h ; c – e ; d – f

**10.** For a projectile 'R' is range and 'H' is maximum height

a) R = H — e) Angle of projection $\tan^{-1}(1)$
b) R = 2H — f) Angle of projection $\tan^{-1}(4)$
c) R = 3H — g) Angle of projection $\tan^{-1}(2)$
d) R = 4H — h) Angle of projection $\tan^{-1}(4/3)$

1) a – g ; b – h ; c – e ; d – f   2) a – h ; b – g ; c – e ; d – f
3) a – f ; b – g ; c – h ; d – e   4) a – e ; b – g ; c – f ; d – h

**11.** For a projectile relation between Range, velocity of projection and angle of projection ($g=10 m/s^2$)

a) u = 20 m/s q = $30^0$ — e) $45\sqrt{3}$m
b) u = 30 m/s q = $60^0$ — f) 40m
c) u = 30 m/s q = $45^0$ — g) $20\sqrt{3}$m
d) u = 20 m/s q = $45^0$ — h) 90m

1) a – e ; b – g ; c – h ; d – f   2) a – g ; b – h ; c – g ; d – f
3) a – f ; b – h ; c – g ; d – e   4) a – g ; b – e ; c – h ; d – f



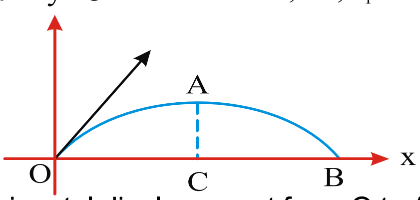
**Multi Answer Type:**

◆ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options.*

**12.** A particle is projected from the ground with velocity u at angle $\theta$ with horizontal. The horizontal range, maximum height and time of flight are R, H and T respectively. Now keeping u as fixed, $\theta$ is varied from $30^\circ$ to $60^\circ$. Then

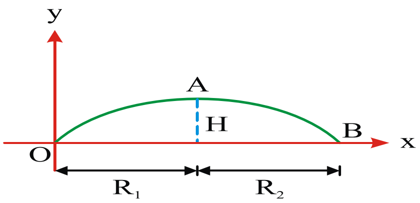
(a) R will first increase. H will increase and T will decrease
(b) R will first increase then dcrease while H and T both will increase
(c) R will decrease while H and T will increase
(d) R will increase while H and T will decrease

**13.** Suppose in the abscence of air resistance, $R = OB,\ H = AC,\ t_1 = t_{OA}$ and $t_2 = t_{AB}$. If air resisitance is taken into consideration and the corresponding values are $R', H', t_1'$ and $t_2'$ then




(a) $R' < R, H' < H, t_1' > t_1$ and $t_2' > t_2$
(b) $R' < R, H' < H, t_1' > t_1$ and $t_2' < t_2$
(c) $R' < R, H' > H, t_1' > t_1$ and $t_2' < t_2$
(d) $R' < R, H' < H, t_1' < t_1$ and $t_2' > t_2$

**14.** In a projectile motion let $t_{OA} = t_1$ and $t_{AB} = t_2$. The horizontal displacement from O to A is $R_1$ and from A to B is $R_2$. Maximum height isH and time of flight is T. If air drag is to be considered, then choose the correct alternative(s).




(a) $t_1$ will decrease while $t_2$ will increase
(b) H will increase
(c) $R_1$ will decrease while $R_2$ will increase
(d) T may increase or decrease

**15.** A projectile is projected from the ground making an angle of $30^\circ$ with the horizontal. Air exerts a drag which is proportional to the velocity of the projectile

(a) at highest point velocity will be horizontal
(b) the time of ascent will be equal to the time of descent
(c) the time of descent will be greater than the time of ascent
(d) the time of ascent will be greater than the time of descent

**16.** A particle is projected from ground with velocity $40\sqrt{2}\,m/s$ at $45^{o}$. At time t = 2 s
(a) displacement of particle is 100 m
(b) vertical component of velocity is 20 m/s
(c) velocity makes an angle of $\tan^{-1}(2)$ with vertical
d) particle is at height of 60 m from ground

**Comprehsion Type:**

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C ) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

**Passage-I**

A body is projected with a velocity 60 $ms^{-1}$ at $30^0$ to horizontal.

**17.** Its initial velocity vector is
1) $10\hat{i} + 10\sqrt{3}\hat{j}$ 2) $30\hat{i} + 30\sqrt{3}\hat{j}$ 3) $30\sqrt{3}\hat{i} + 30\hat{j}$ 4) $30\sqrt{3}\hat{i}$

**18.** Velocity after 3 seconds is __
1) $20\hat{i} + 20\sqrt{3}\hat{j}$ 2) $30\hat{i}$ 3) $10\sqrt{3}\hat{j}$ 4) $30\sqrt{3}\hat{i}$

**19.** The displacement after 2 s is __
1) $30\sqrt{3}\hat{i} + 30\hat{j}$ 2) $60\sqrt{3}\hat{i} + 40\hat{j}$ 3) $10\sqrt{3}\hat{i} + 10\hat{j}$ 4) $40\sqrt{3}\hat{j} + 40\hat{i}$

**Passage-II**

A ball is projected at an angle of $30^0$ and $60^0$ to the horizontal with the same initial velocity in each case.

**20.** Ratio of their times of flight is
1) 1:1 2) 1:3 3) 1: $\sqrt{3}$ 4) 2: $\sqrt{3}$

**21.** Ratio of maximum height is
1) 1:1 2) 1:3 3) 1: $\sqrt{3}$ 4) 2: $\sqrt{3}$

**22.** Ratio of ranges is
1) 1:1 2) 1:3 3) 1: $\sqrt{3}$ 4) 2: $\sqrt{3}$



## KEY

**ΦΦ LEARNER'STASK :**

☐ **BEGINNERS :**

1) 3 2) 1 3) 1 4) 3 5) 4 6) 1 7) 2 8) 1 9) 3 10) 1 11) 2
12) 1 13) 1 14) 2 15) 4 16) 3 17) 4 18) 3 19) 1 20) 2 21) 4 22) 4
23) 3 24) 4 25) 4 26) 2 27) 4 28) 1 29) 1 30) 4 31) 2 32) 1 33) 1
34) 3 35) 4 36) 2 37) 1 38) 1 39) 3

☐ **ACHIEVERS :** 1) 9 2) 3

☐ **EXPLORERS :**

1) 1 2) 1 3) 2 4) 3 5) 1 6) 4 7) 1 8) 1 9) 1 10) 3 11) 4
12) b 13) b 14) a,d 15) a,d 16) a,b,c,d 17) 3 18) 4 19) 2 20) 3
21) 2 22) 3

## §§ BODY PROJECTED FROM TOP OF TOWER.

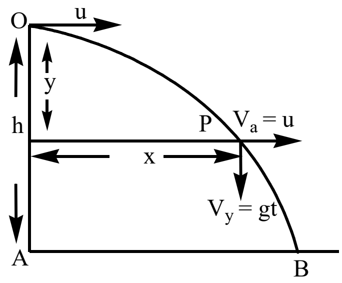
Suppose body is projected with an initial velocity u making an angle $\theta$ above horizontal from top of a tower t = 0.

In this case horizontal acceleration is zero so horizontal velocity remains constant but in vertical direction there is an acccelerations 'g'.

### ¶¶ Case (i) Body projected horizontally from top of tower (If $\theta = 0$ ).

**Equation for Path (Trajectory):**

Suppose a body is projected horizontally with an initial velocity u from the top of a tower at time t=0.




Let the height of the tower be ‘h’. As the body is horizontally projected the angle of projection is zero ( $\theta$ =0). Because no horizontal acceleration is present the horizontal velocity remains constant throught the motion. Hence after time t the velocity in horizontal direction will be $v_x$=u.

The initial vertical velocity at the time of projection is zero i.e., $u_y$=0 . There is an acceleration ‘g’ along the -ve y-direction. The vertical component of velocity goes on increasing.

Let the body reach a point ‘P’ in time t, and x and y are the coordinates of the body.

We can write the equation for the y-coordinate after time t and it is

$y = \frac{1}{2}gt^2$ ............(i)

and for x-coordinate as $x=ut$ .....(ii)

$\Rightarrow t = x/u$

From Eqs.(i) and (ii) we get

$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2 = \left(\frac{g}{2u^2}\right)x^2$ ............(iii)

g and u being constants, $\left(\frac{g}{2u^2}\right)$ is a constant. Let $\frac{g}{2u_2}$ =k.

Then Eq (iii) can be writien as y=k$x^2$. .........(iv)

This equation represents the equation of a parabola.

### (a) Time of Descent

It is the time the body takens to the ground after it is projected from the height ‘h’.

From Eq(i) and putting y = h and t = $t_d$ we get

$h = \frac{1}{2}gt_d^2 \Rightarrow t_d = \sqrt{\frac{2h}{g}}$ ......(v)

We can see that the time of descent is independentof initial velocity with which the body is projected and depends only on the height from which it is projected. $t_d$ is the time of flight in the case .

**(b) Range**

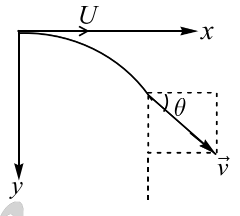
The maximum horizontal distance travelled by the body while it touches the ground is the range (R). It is AB in the fig.

As the horizontal velocity is constant

Range R = (horizontal velocity) time of descent =(u)$t_d$.

From Eq. (v), $R=(u)\sqrt{\frac{2h}{g}}$

**(c) Velocity after some time**




$\vec{U}=U\hat{i}$ $\quad\vec{a}=g\,\hat{j}$

$\vec{v}=\vec{U}+\vec{a}t$ $\quad\vec{v}=U\hat{i}+gt\,\hat{j}$ $\quad\vec{v}=v_x\hat{i}+v_y\hat{j}$

$\tan\theta=\frac{v_y}{v_x}\Rightarrow\theta=\tan^{-1}\left(\frac{gt}{U}\right)$

This is the angle made by the velocity vector with horizontal.

**d) Velocity vector after travelling some vertical displacement.**

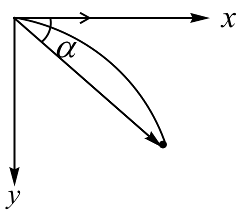
$\vec{U}=U\hat{i};a=g\,\hat{j}$ $\quad\vec{s}=y\hat{j};\vec{v}.\vec{v}=\vec{u}.\vec{u}+2\vec{a}.\vec{s}$

$v^2=u^2+2ay;v=\sqrt{u^2+2ay}$

in vector form $\vec{v}=u\hat{i}+\sqrt{2ay}\,\hat{j}$ .

Angle made by velocity vector with horizontal. $Tan\,\alpha=\sqrt{\frac{2gy}{U}}$ ;

**e) Position vector after some time**




$\vec{U}=U\hat{i}$ $\quad\vec{a}=g\,\hat{j}$ $\quad\vec{s}=\vec{U}t+\frac{1}{2}\vec{a}t^2$ ;

$\vec{s}=Ut\hat{i}+\frac{1}{2}gt^2\hat{j}$ $\quad\vec{s}=x\hat{i}+yj$

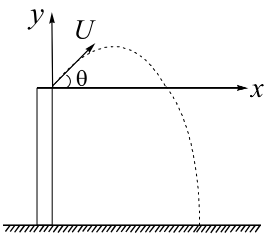
$x=Ut$ $\quad y=\frac{1}{2}gt^2$

angle made by position vector with horizontal $\tan\alpha=\frac{y}{x}=\frac{\frac{1}{2}gt^2}{Ut}$; $\tan\alpha=\frac{gt}{2U}$

¶¶ **Case (ii)** <u>When body is projected at an angle above horizontal from the top of tower(If $\theta>0$)</u>

$\vec{U}=U\cos\theta\hat{i}+u\sin\theta\hat{j}$

$\vec{a}=-g\hat{j}$




**a) Velocity vector after some time.**

$\vec{V} = \vec{U} + \vec{a},t$

$\vec{V} = U\cos\theta\hat{i} + (U\sin\theta - gt)\hat{j}$

$\vec{V} = v_x\hat{i} + v_y\hat{j}$ **i.e.,** $V_x = U\cos\theta$ $V_y = U\sin\theta - gt$

**b) Time of ascent**

at maximum height Vy = 0 $t_a = \dfrac{U\sin\theta}{g.}$

Time of ascent is same as projectle from horizontal surface.

**c) Posiiton vector after some time**

$\vec{S} = \vec{U}t + \dfrac{1}{2}\vec{a}t^2$

$\vec{r} = \left[(U\cos)\hat{i} + U\sin\theta\hat{j}\right]t + \dfrac{1}{2}(-g\hat{j})t^2$ **or** $\vec{r} = (U\cos\theta)t\ \hat{i} + \left[(U\sin\theta)t - \dfrac{1}{2}gt^2\right]\hat{j}$

$= x\hat{i} + y\hat{j}$

from the above equation we can write $x = (U\cos\theta)t$ , $y = (U\sin\theta)t - \dfrac{1}{2}gt^2$



**d) Time of flight :**

$y = (U\sin\theta)t - \dfrac{1}{2}gt^2$ (or) $-h = (U\sin\theta)T - \dfrac{1}{2}gT^2$

$\dfrac{1}{2}gT^2 - (U\sin\theta)T - h = 0$

$T = \dfrac{U\sin\theta \pm \sqrt{(U\sin\theta)^2 + 4\left(\frac{1}{2}g\right)h}}{2\left(\frac{1}{2}g\right)}$ (or) $T = \dfrac{U\sin + \sqrt{(U\sin\theta)^2 + 2gh}}{g}$

**e) Range of projectile :-** $R = (U\cos\theta)T$

**¶¶ Case (iii)** Body projected below the horizontal level.(If $\theta < 0$ )

Suppose body is projected with an initial velocity u making an angle $\theta$ below the horizontal from top of a tower t = 0.

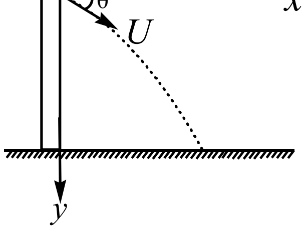
In this case horizontal acceleration is zero so horizontal velocity remains constant but in vertical direction there is an acccelerations 'g'.

$\vec{U} = U\cos\theta\hat{i} + U\sin\theta\hat{j}$ , $\vec{a} = g\hat{j}$ ,

$\vec{V} = \vec{U} + \vec{a}t = U\cos\theta\hat{i} + U\sin\theta\hat{j} + gt\hat{j}$

$\vec{V} = U\cos\theta\hat{i} + (U\sin\theta + gt)\hat{j}$

$\vec{V} = v_x\hat{i} + v_y\hat{j}$ **i.e.,** $V_x = U\cos\theta$ $V_y = U\sin\theta - gt$



**a) Position vector after some time**

$$\vec{S}=\vec{U}t+\frac{1}{2}\vec{a}t^2 \textbf{ (or) } \vec{r}=\left[(U\cos\theta)\hat{i}+U\sin\theta\hat{j}\right]t+\frac{1}{2}(g\,\hat{j})t^2$$

$$\vec{r}=(U\cos\theta)t\ \hat{i}+\left[(U\sin\theta)t+\frac{1}{2}gt^2\right]\hat{j} \qquad =x\hat{i}+y\hat{j}$$

from the above equation we can write $x=(U\cos\theta)t \quad y=(U\sin\theta)t+\frac{1}{2}gt^2$

**d) Time of flight**

$$y=(U\sin\theta)t-\frac{1}{2}gt^2 \text{ (or) } h=(U\sin\theta)T+\frac{1}{2}gT^2 \text{ (or) } \frac{1}{2}gT^2+(U\sin\theta)T-h=0$$

$$T=\frac{-U\sin\theta\pm\sqrt{(U\sin\theta)^2+4\left(\frac{1}{2}g\right)h}}{2\left(\frac{1}{2}g\right)} \text{ (or) } T=\frac{-U\sin\theta+\sqrt{(U\sin\theta)^2+2gh}}{g}$$

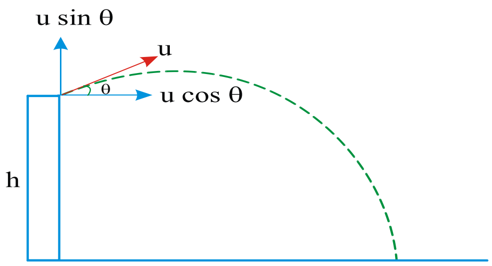
**d) Range of projectile :** $R=(U\cos\theta)T$



**EXAMPLE**

√ ***W.E -1:*** *A ball is thrown from the top of a tower of 61 m high with a velocity 24.4 $ms^{-1}$ at an elevation of $30^0$ above the horizontal . What is the distance from the foot of the tower to the point where the ball hits the ground?*

**Sol :**



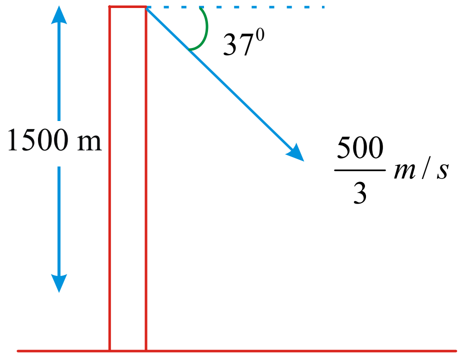

$$h=\frac{1}{2}gt^2-(u\sin\theta)t \quad\Rightarrow t=5\sec onds$$

Also, d $=\left(u\cos\theta\right)t=105.65m$

√ ***W.E -2:*** *A particle is projected from a tower as shown in figure, then find the distance from the foot of the tower where it will strike the ground.* $\left(g=10m/s^2\right)$

**Sol:** $u_y=u\sin\theta=\frac{500}{3}\sin 37^0$

$$s=ut+\frac{1}{2}at^2 \text{ (or) } 1500=\left(\frac{500}{3}\sin 37\right)t+\frac{1}{2}10t^2$$




$1500 = \frac{500}{3}\left(\frac{3}{5}\right)t + 5t^2$ (or) $300 = 20t + t^2 \Rightarrow t = 20s$

$\therefore$ horizontal distance = ( u cos $\theta$) t

$= \frac{500}{3}\left(\frac{4}{5}\right)10 = \frac{4000}{3}m$

√ ***W.E - 3:*** *A golfer standing on the ground hits a ball with a velocity of 52 m/s at an angle $\theta$ above the horizontal if $\tan\theta = \frac{5}{12}$ find the time for which the ball is at least 15m above the ground?* $\left(g = 10m/s^2\right)$

**Sol:** $v_y = \sqrt{u_y^2 - 2gy}$ , $u_y = u\sin\theta$

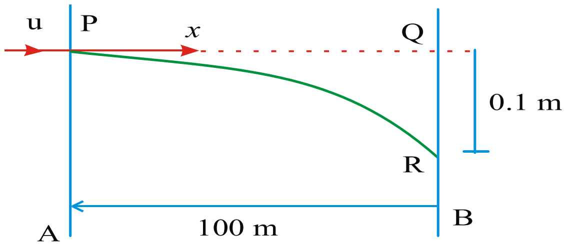
$v_y = \sqrt{52\times52\times\frac{5\times5}{13\times13} - 2\times10\times15} = \sqrt{16\times25 - 300} = 10$

$\Delta t = \frac{2v_y}{10} = \frac{2\times10}{10} = 2s$



√ ***W.E-4:*** *Two paper screens A and B are separated by a distance of 100m. A bullet pierces A and B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. Neglect resistance of paper and air.*

**Sol:** The situation is shown in Fig.




$d = u\sqrt{\frac{2(h_1 - h_2)}{g}} \Rightarrow 100 = u\sqrt{\frac{2\times0.1}{9.8}}$

$\Rightarrow u = 700m/s.$

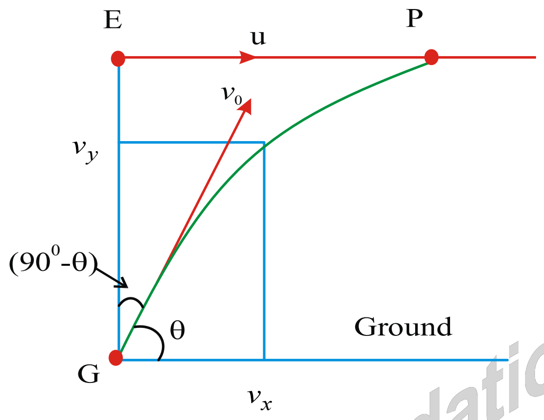
√ ***W.E -5:*** *A boy aims a gun at a bird from a point, at a horizontal distance of 100m. If the gun can impart a velocity of 500m/sec to the bullet, at what height above the bird must he aim his gun in order to hit it?*

**Sol :** $x = vt$ or $100 = 500\times t$ ; $t = 0.2\sec.$

Now h $= 0 + \frac{1}{2}\times10\times(0.2)^2$ = 0.20m = 20cm.

√ ***W.E - 6:*** *An enemy plane is flying horizontally at an altitude of 2 km with a speed of 300 $ms^{-1}$. An army man with an anti - aircraft gun on the ground sights enemy plane when it is directly overhead and fires a shell with a muzzle speed of $600ms^{-1}$. At what angle with the vertical should the gun be fired so as to hit the plane?*

**Sol:** Let G be the position of the gun and E that of the enemy plane flying horizontally with speed.




u = 300$ms^{-1}$, when the shell is fired with a speed $v_0$, $v_x = v_0\cos\theta$

The shell will hit the plane, if the horizontal distance EP travelled by the plane in time t = the distance travelled by the shell in the horizontal direction in the same time, i.e.

$u \times t = v_x \times t$ or $u = v_x \Rightarrow u = v_0\cos\theta$

or $\cos\theta = \frac{u}{v_0} = \frac{300}{600} = 0.5$ or $\theta = 60^0$

Therefore, angle with the vertical $= 90^0 - \theta = 30^0$.



√ ***W.E -7:*** *From the top of a tower, two balls are thrown horizontally with velocities $u_1$ and $u_2$ in opposite directions. If their velocities are perpendicular to each other just before they strike the ground, find the height of tower.*

**Sol:** Time taken to reach ground $t = \sqrt{\frac{2h}{g}}$

at time of reaching ground respective velocities are $\vec{v_1} = u_1\hat{i} + gt\hat{j}$, $\vec{v_2} = -u_2\hat{i} + gt\hat{j}$

Given $\vec{v_1}.\vec{v_2} = 0$, $t = \frac{\sqrt{u_1u_2}}{g}$

$\therefore \sqrt{\frac{2h}{g}} = \frac{\sqrt{u_1u_2}}{g} \Rightarrow h = \frac{u_1u_2}{2g}$ is the height of the tower.

√ ***W.E -8:*** *A block of ice starts sliding down from the top of an inclined roof of a house along a line of the greatest slope. The inclination of the roof with the horizontal is 30º. The heights of the highest and lowest points of the roof are 8.1 m and 5.6 m respectively. At what horizontal distance from the lowest point will the block hit the ground? Neglect any friction. [g = 9.8 m/s²]*

**Sol:** Acceleration of the block along the greatest slope is equal to a = g sin 30º

Distance travelled by the block along the greatest slope is equal to

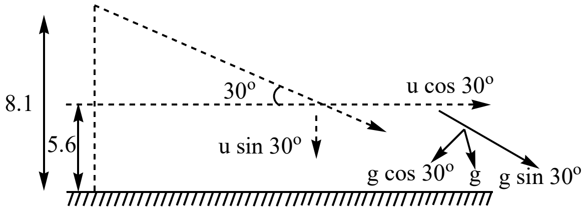
$S = \frac{(8.1 - 5.6)}{\sin 30^0} = 5m$.

If u be the speed of the block when it is just about to leave the roof then

$u^2 = 0 + 2g \sin 30^o \times 5 \Rightarrow u = 7$ m/s

If t be the time taken to hit the ground then

$$5.6 = u \sin 30^O t + \frac{1}{2} g t^2 = \frac{7}{2} t + \frac{1}{2} \times 9.8 t^2$$




$\Rightarrow 7t^2 + 5t - 8 = 0$

$$t = \frac{-5 \pm \sqrt{25-(4)(7)\times(-8)}}{2\times 7}$$

$\Rightarrow t = \frac{-5 \pm 15.78}{14}$ s , -ve value is to be rejected.

i.e., $t = \frac{-5+15.78}{14} = 0.77$ sec.

Horizontal distance travelled is equal to x= u cos 30°t = $\frac{7\sqrt{3}}{2} \times \frac{10.78}{14} m \Rightarrow$ x = 4.67 m 8.

√ ***W.E -9:*** *Two paper screens A and B are separated by a distance of 100 m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. Neglect the resistance of paper and air.*

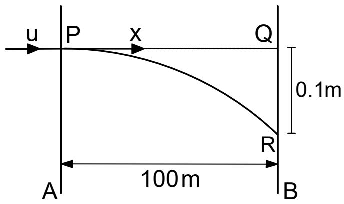
**Sol:** The situation is shown in Fig.

Let the bullet be travelling with a velocity u when it strikes the screen A at P. It is a problem of horizontal projectile.

Let the bullet take time t in reaching from P to R.

The vertical distance fallen, y = 0.1 m

From formula, $y = \frac{1}{2} g t^2$ We have, $t = \sqrt{\frac{2y}{g}}$




Putting values, we get $t = \sqrt{\frac{2\times 0.1}{9.8}} = \sqrt{\frac{1}{49}} = \frac{1}{7}$ sec.

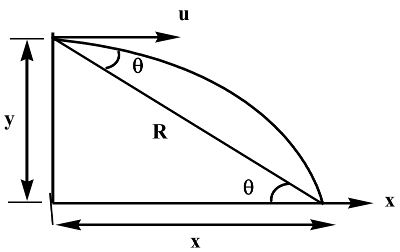
For horizontal motion, x = 100 m, t = 1/7 sec

From formula, x = ut We have, u = x/t

Putting values, we get, $u = \frac{100}{1/7}$ = 700 or u = 700 $ms^{-1}$

√ ***W.E -10:*** *A particle is projected horizontally with a speed " u" from the top of plane inclined at an angle "$\theta$ " with the horizontal. How far from the point of projection will the particle strike the plane?*

**Sol:** Suppose particle is projected from height y, it strikes the ground at a horizontal distance x, then distance $R = \sqrt{x^2 + y^2}$ ................(i)




If "t" is the time of motion, then x = ut .............(ii)

and $y = \frac{1}{2} g t^2$ From equation (i) , we get $t = \frac{x}{u}$

$\therefore y = \frac{1}{2} g \left(\frac{x}{u}\right)^2 = \frac{gx^2}{2u^2}$ ...........(iii)

From figure, $\frac{y}{x} = \tan\theta$ or $y = x \tan\theta$ ..................(iv)

From equatin (iii) and (iv), we have $x \tan\theta = \frac{gx^2}{2u^2}$ or $x\left[\frac{gx}{2u^2} - \tan\theta\right] = 0$

As x = 0 is not possible, So, $\left[\frac{gx}{2u^2} - \tan\theta\right] = 0$

or $x = \frac{2u^2 \tan\theta}{g}$ ........(v)

Now from equation (i), (iv) and (v), we get

$R = \sqrt{x^2 + y^2} = \sqrt{x^2 + (x\tan\theta)^2} = x\sqrt{1+\tan^2\theta} = x\sec\theta$ or $= \frac{2u^2}{g}\tan\theta\sec\theta$

√ ***W.E -11:*** *An aeroplane is flying in a horizontal direction with a velocity of 600 km/hour at a height of 1960 m. When it is vertically above a point A on the ground, a body is dropped from it. The body strikes the ground at a point B. Calculate the distance AB.*

**Sol:** Here, $v_0$ = 600 km hr$^{-1}$ = $\frac{600\times1000}{3600} = \frac{500}{3}$ ms$^{-1}$

For vertically downward motion of the body under gravity

u = 0, g = 9.8 ms$^{-2}$, S = h = 1960 m, t = ?

From formula, $S = ut + \frac{1}{2}gt^2$

Putting values, we get, $1960 = 0 + \frac{1}{2}\times 9.8\times t^2 = 4.9\,t^2$ i.e., $t^2 = \frac{1960}{4.9} = 400$ or t = 20 sec.

Horizontal distance covered by the body in the above time is given by, $R = v_0 t$

Putting values, we get, $R = \frac{500}{3}\times 20 = \frac{10^4}{3}$ m $R = \frac{10}{3}$ km = 3.33 km.

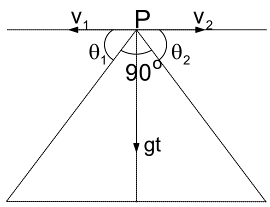


√ ***W.E -12:*** *Two particles move in a uniform gravitational field with an acceleration "g". At the initial moment the particles were located at one point and moved with velocities $u_1$ = 3.0 ms$^{-1}$ and $u_2$ = 4.0 ms$^{-1}$ horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular.*

**Sol:** The situation is shown Fig. Let the velocity vectors become perpendicular after time "t" when both particles has fallen same vertical distance $\frac{1}{2}gt^2$ and have acquired same vertical velocities gt.

Let their resultant velocities make angles " $q_1$ " and " $q_2$" with horizontal.

Then $\tan q_1 = \frac{gt}{v_1}$ and $\tan q_2 = \frac{gt}{v_2}$




Since, velocity vectors are perpendicular

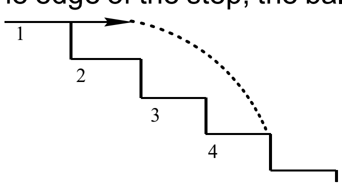
$q_1 + q_2 = 90°$, hence $\tan q_2 = \cot q_1$

It makes $\tan q_1 = \frac{gt}{v_1}$ and $\cot q_1 = \frac{gt}{v_2}$

Multiplying we get.

√ ***W.E -13:*** *A ball rolls off the top of a stairway with a horizontal velocity of magnitude 150 cm/sec. The steps are 20 cm high and 20 cm wide. Which step will the ball hit first?*

**Sol:** Let **x** be the horizontal distance measured from the foot of the edge of the step, the ball leaves the top of stairway to the point it hits the n$^{th}$ step.




$x = v_0 t = 1.5\,t$ ....... (i)

where $v_0$ is the initial horizontal speed.

The height of the ball drops through is

y = nH = 0.2 n ...... (ii)

where n is number of steps and H = 20 cm is the height of each step. As the vertical component of initial velocity is zero, the vertical distance traversed in the same time **t** is $y = \frac{1}{2}gt^2$ ...... (iii)

and x = nw = 0.2 n ....... (iv)

x = y = 0.2n .... (iv)

$V_0t = \frac{1}{2}gt^2$ $t = \frac{2V_0}{g} = \frac{2\times150\times10^{-2}}{10} \Rightarrow t = 0.3$ s

0.2n = 1.5 (0.3) $\Rightarrow$ n = 2.25 m $\Rightarrow$ n = 3

## TEACHING TASK



**Single Answer Type:**

**1.** A stair case contains ten steps each 10 cm high and 20 cm wide. The minimum horizontal velocity with which the ball has to be rolled off the upper most step, so as to hit directly the edge of the lowest step is (approximately)

1) 42ms$^{-1}$ 2)4.2ms$^{-1}$ 3) 24ms$^{-1}$ 4)2.4ms$^{-1}$

**2.** From certain height 'h' two bodies are projected horizontally each with velocity v. One body is projected towards North and the other body is projected towards east. Their separation on reaching the ground

1) $\sqrt{\frac{2v^2h}{g}}$ 2) $\sqrt{\frac{4v^2h}{g}}$ 3) $\sqrt{\frac{v^2h}{g}}$ 4) $\sqrt{\frac{8v^2h}{g}}$

**3.** An object is projected horizontally from a top of the tower of height h. The line joining the point of projection and point of striking on the ground makes an angle $45^0$ with ground,Then with what velocity the object strikes the ground

1) $\sqrt{\frac{11gh}{2}}$ 2) $\sqrt{\frac{9gh}{2}}$ 3) $\sqrt{\frac{7gh}{2}}$ 4) $\sqrt{\frac{5gh}{2}}$

**4.** A ball is thrown horizontally from a cliff such that it strikes the ground after 5s. The line of sight makes an angle $37^\circ$ with the horizontal.The initial velocity of projection in ms$^{-1}$ is

1) 50 2) $\frac{100}{\sqrt{3}}$ 3) $\frac{100}{\sqrt{2}}$ 4) $\frac{100}{3}$

**5.** An object is launched from a cliff 20 m above the ground at an angle of $30^0$ above the horizontal with an initial speed of 30 m/s. How far does the object travel before landing on the ground? (in metre)

1) 20 2) $20\sqrt{3}$ 3) 60 4) $60\sqrt{3}$

**6.** A bomber flying upward at an angle of $53^\circ$ with the vertical releases a bomb at an altitude of 800 m. The bomb strikes the ground 20 s after its release. If $g = 10\,m\,s^{-2}$, the velocity at the time of release of the bomb in ms$^{-1}$ is

1) 400 2) 800 3) 100 4) 200

**7.** Two particles move in a uniform gravitational field with an acceleration g. At the initial moment the particles were located at same point and moved with velocities $u_1 = 9\,m\,s^{-1}$ and $u_2 = 4\,m\,s^{-1}$ horizontally in opposite directions. The time between the particles at the moment when their velocity vectors are mutually perpendicular in s is (take $g = 10\,m\,s^{-2}$)

1) 0.36 2) 3.6 3) 0.6 4) 6

**8.** An aeroplane is flying horizontally at a height of 980 m with velocity 100 $ms^{-1}$ drops a food packet. A person on the ground is 414 m ahead horizontally from the dropping point. At what velocity should he move so that he can catch the food packet.

1) $50\sqrt{2}ms^{-1}$ 2) $\frac{50}{\sqrt{2}}ms^{-1}$ 3) $100ms^{-1}$ 4) $200ms^{-1}$

**Assertion and Reason type:**

- *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.*

*1) Both A & R are true and R is correct explanation of A*

*2) Both A & R are true and R is not correct explanation of A*

*3) A is true but R is false.* *4) Both A & R are false.*



**9.** **A:** For a body projected horizontally from the top of a tower, the velocity on reaching the ground depends both on velocity of projection and height of the tower.

**R :** For a projectile velocity varies both in horizontal and vertical directions.

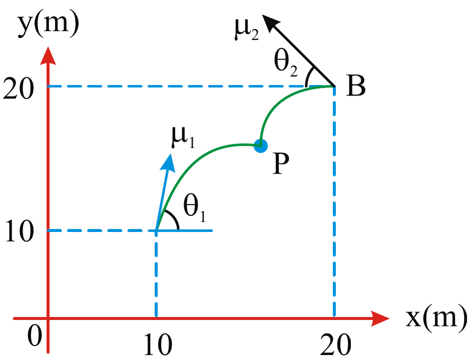
**10.** **A :** If a bomb is dropped from an aeroplane moving horizontally with constant velocity then the bomb appears to move along a vertical straight line for the pilot of the plane.

**R :** Horizontal component of velocity of the bomb remains const and same as thevelocity of the plane during the motion under gravity.

**Multi Answer Type:**

- *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options*

**11.** Two projectiles A and B are fired simultaneously as shown in figure. They collide in air at point at time t. Then




(a) $t(u_1\cos\theta_1 - u_2\cos\theta_2) = 20$

(b) $t(u_1\sin\theta_1 - u_2\sin\theta_2) = 10$

(c) Both (a) and (b) are correct

(d) Both (a) and (b) are wrong

**12.** An aeroplane at a constant speed releases a bomb. As the bomb drops away from the aeroplane,

a) It will always be vertically below the aeroplane

b) It will always be below the aeroplane only if the aeroplane was flying horizontallly.

c) It will always be vertically below the aeroplane only if the aeroplane was flying at an angle of $45^0$ to the horizontal

d) It will gradually fall behind the aeroplane if the aeroplane was flying horizontally.

**Matrix Match Type:**

- *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*
  *If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

**13.** When a body is projected horizontally with velocity 'u' 'from certain height, it reaches the ground with the velocigty 4u, then

| | |
|---|---|
| 1) height of projection | a) 4u |
| 2) Range | b) $\frac{u}{g}\sqrt{15}$ |
| 3) Time of flight | c) $\frac{u^2}{g}\sqrt{15}$ |
| 4) Velocity to reches the ground | d) $\frac{15u^2}{2g}$ |

A) 1-d,2-c,3-b,4-a B) 1-d,2-b,3-c,4-a
C) 1-a,2-c,3-b,4-d D) 1-b,2-c,3-d,4-a



**Comprehsion Type:**

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C ) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

**Passage: 1**

Two projectiles are projected simultaneously from the top and bottom of a vertical tower of height h at angles $45^0$ and $60^0$ above horizontal respectively. Body strike at the same point on ground at distance 20m from the foot of the tower after same time.

**14.** The speed of projectile projected from the bottom is

a) 40 m/s b) $\frac{20}{\sqrt{3}}m/s$ c) $40\sqrt{3}m/s$ d) $\frac{20}{\sqrt{\sqrt{3}}}m/s$

**15.** The ratio of the speed of the projectile projected from the top and the speed of the projectile projected from the bottom of tower is

a) $1:\sqrt{2}$ b) $1:\sqrt{3}$ c) $\sqrt{5}:1$ d) $\sqrt{7}:1$

**16.** The time of flight of projectiles is

a) $(3)^{\frac{1}{4}}$ b) $2(3)^{\frac{1}{4}}$ c) $3(3)^{\frac{1}{4}}$ d) $4(3)^{\frac{1}{4}}$

**Passage:2** **(IIT JEE 1996)**

Two guns situated on top of a hill of height 10m fire one shot each with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontally and the other fires upwards at an angle of $60^0$ with the horizontal. The shots collide in air at a point P. Find

**17.** The time interval between the firings and

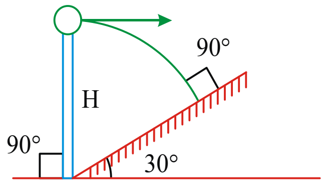
a) 1 s  b) 2s  c) 3 s  d) 4s

**18.** the coordinates of point P. Take the origin of coordinate system at the foot of the hill right below the muzzle and trajectories in the xy-plane.

a) $(5m, 5m)$ b) $(5\sqrt{3}m, 5\sqrt{3}m)$ c) $(5\sqrt{3}m, 5m)$ d) $(5m, 5\sqrt{3}m)$

**Integer Type:**

**19.** In the given figure, the angle of inclination of the inclined plane is $30^0$. A particle is projected with horizontal velocity $V_0$ from height H. Find the horizontal velocity $V_0$(in m/s) so that the particle hits the inclined plane perpendicularly. Given, H = 4m, g =10 m/s$^2$




## KEY

**ΦΦ TEACHING TASK :**

1) 2, 2) 2, 3) 4, 4) 4, 5) 4, 6) 3, 7) 3, 8) 1, 9) 3, 10) 1,
11) b, 12) a, 13) A, 14) d, 15) a, 16) b, 17) a, 18)c, 19) 4

## LEARNER'S TASK

### BEGINNERS ( Level - I )

**Single Answer Type:**

**1.** A stone is just dropped from the window of a train moving along a horizontal straight track with uniform speed. The path of the stone is

1) a parabola for an observer standing by the side of the track

2) a horizontal straight line for an observer inside the train

3) both (1)&(2) are true

4) (1) is true but (2) is false

**2.** A bomb is dropped from an aeroplane flying horizontally with uniform speed. The path of the bomb is

1) a vertical straight line for a stationary observer on the ground

2) a parabola for the pilot of the aeroplane

3) a vertical straight line for the pilot and parabola for a stationary observer on the ground

4) a horizontal straight line for the pilot and parabola for a stationary observer on the ground

**3.** A and B are two trains moving parallel to each other. If a ball is thrown vertically up from the train A, the path of the ball is

1) parabola for an observer standing on the ground

2) vertical straight line for an observer in B when B is moving with the same speed in the same direction of A

3) a parabola for an observer in B when B is moving with same speed but in opposite direction

4) all the above are true

**4.** A ball is thrown from rear end to the front end of a compartment of a train which is moving at constant horizontal velocity. An observer sitting in the compartment and another observer standing on the ground draw the trajectory of the ball. They will have

1) equal horizontal and equal vertical ranges

2) equal vertical ranges but different horizontal ranges

3) different vertical ranges but equal horizontal ranges

4) different vertical and different horizontal ranges

**5.** For body thrown horizontally from the top of a tower,

1) the time of flight depends both on h and v

2) the horizontal Range depends only on v but not on h

3) the time of flight and horizontal Range depend on h but not on v

4) the horizontal Range depends on both v and h

**6.** A body projected horizontally with a velocity 'v' from a height 'h' has a range 'R'. With what velocity a body is to be projected horizontally from a height h/2 to have the same range ?

1) $\sqrt{2}$ v 2) 2v 3) 6v 4) 8v

**7.** A stone is thrown horizontally with velocity g $ms^{-1}$ from the top of a tower of height g metre. The velocity with which it hits the ground is (in $ms^{-1}$)

1) g 2) 2g 3) $\sqrt{3}g$ 4) 4g

**8.** A body is thrown horizontally from the top of a tower. It reaches the ground after 4s at an angle $45^0$ to the ground. The velocity of projection is

1)9.8$ms^{-1}$ 2)19.6$ms^{-1}$ 3)29.4$ms^{-1}$ 4) 39.2$ms^{-1}$

**9.** Two cliff of heights 120 m and 100.4 m are separated by a horizontal distance of 16 m if a car has to reach from the first cliff to the second the horizontal velocity of car should be

1) 16 m/s 2) 4 m/s 3) 2 m/s 4) 8 m/s

**10.** A body projected horizontally from the top of a tower follows $y = 20x^2$ parabola equation where x , y are in m $\left(g = 10 \ ms^{-2}\right)$.Then the velocity of the projectile is ($ms^{-1}$)

1) 0.2 2) 0.3 3) 0.4 4) 0.5

**11.** A bomb is dropped from an aeroplane flying horizontally with a velocity of 720 kmph at an altitude of 980m. Time taken by the bomb to hit the ground is

1) 1 s 2) 7.2 s 3) 14.14 s 4) 0.15 s

**12.** A body is projected horizontally from a height of 78.4 m with a velocity 10 $ms^{-1}$. Its velocity after 3 seconds is (g=10 $ms^{-2}$)(Take direction of projection as i and vertically upward direction as j)

1) $10\hat{i}-30\hat{j}$ 2) $10\hat{i}+30\hat{j}$ 3) $20\hat{i}-30\hat{j}$ 4) $10\hat{i}+10\sqrt{3}\hat{j}$

**13.** Two thin wood screens A and B are separated by 200 m a bullet travelling horizontally at speed of 600 m/s hits the screen A penetrates through it and finally emerges out from B making holes in A and B the resistance of air and wood are negligible the difference of heights of the holes in A and B is.

1) 5 m 2) $\frac{49}{90}m$ 3) $\frac{7}{\sqrt{90}}m$ 4)zero

**14.** A body is projected horizontally from the top of a tower with a velocity 9.8 m/s. The time elapse before the vertical component of is twice of the horizontal velocity is.

1) 1s 2) 2s 3)0.5s 4) 1.5s

**15.** A particle is projected horizontally from the table to touches the ground at distance 'd' from the edge, then the velocity of projection is

1)$h\sqrt{\frac{2g}{d}}$ 2)$d\sqrt{\frac{2g}{h}}$ 3)$h\sqrt{\frac{g}{2d}}$ 4)$d\sqrt{\frac{g}{2h}}$

**16.** Two stones A and B are thrown with velocity 10 m/s and 20 m/s from the to of a tower horizontally if they reach the ground after times $t_1$ and $t_2$ respectively, then

1) $t_1=2\,t_2$ 2) $t_2=2\,t_1$ 3) $t_1=t_2$ 4) $t^2{}_1=2t^2{}_2$

**17.** A body is projected horizontally from certain height. After 2s its direction of motion makes $30^0$ to the horizontal the its initial velocity is

1)$\frac{2g}{\sqrt{3}}$ 2)$2\sqrt{3}g$ 3) 3)$\frac{\sqrt{3}g}{2}$ 4) none

**18.** A stone is thrown horizontally with velocity 30 m/s from the top of a tower height 0f 60m. The velocity with which it hits ground is (in m/s)

1) 0 2) 20 3) $10\sqrt{21}$ 4) $20\sqrt{3}$

**19.** A ody is projected horizontally with 98 m/s from the top a tower reaches the ground in 10s. Then itr strikes the ground at an angle

1) $0^0$ 2) $45^0$ 3) $30^0$ 4) $60^0$

**20.** A body is projected horizontally with a velocity 'u' from the top of a tower strikes the ground with a velocity '2u'. the height of the tower is

1) $u^2/2g$ 2) $u^2/g$ 3) $3u^2/g$ 4) $3u^2/4g$

**21.** A body is projected horizontally with 10 m/s from the top of a tower of height 20m on reaching the ground its displacement is (g=10m/s)

1) 40m 2) 20m 3) $20\sqrt{2}$ m 4) $40\sqrt{2}$ m

**22.** Two stones are proje andcted horizontally from the top of a tower of height 78.4m with velocities 25 m/s and 45 m/s. The seperation between them on reaching the ground if oth are projected in same direction is

1) 40m 2) 80m 3) 120m 4) 160m

**23.** A bomb is dropped from an aeroplane flying horizontally with a velocity of 720kmph at an altitude of 980m. Time taken by the bomb to hit the ground is

1) 1s 2) 7.2s 3) 14.14s 4) 0.15s

**24.** From the top of a tower of height 78.4 m two stones are projected horizontally with 10 m/s and 20 m/s in opposite directions. On reaching the ground, their separation is

1) 120 m 2) 100 m 3) 200 m 4) 150 m

**25.** A body is projected vertically upwards. At its highest point it explodes into two pieces of masses in the ratio of 2:3 and the lighter piece flies horizontally with a velocity of 6 $ms^{-1}$. The time after which the lines joining the point of explosion to the position of particles are perpendicular to each other is

1) $\sqrt{\frac{6}{25}}$ s 2) $\sqrt{\frac{12}{15}}$ s 3) $\sqrt{\frac{24}{25}}$ s 4) 2 s.

**26.** From the top of a building 80 m high, a ball is thrown horizontally which hits the ground at a distance. The line joining the top of the building to the point where it hits the ground makes an angle of $45^0$ with the ground. Initial velocity of projection of the ball is

($g = 10\ m/s^2$)

1) 10 m/s   2) 15 m/s   3) 20 m/s   4) 30 m/s

**27.** A stone is thrown from the top of a tower of height 50 m with a velocity of $30\,ms^{-1}$ at an angle of $30^0$ above the horizontal . Find the time during which the stone will be in air

1) 2 sec   2) 3 sec   3) 4 sec   4) 5 sec

**28**. From the top of a tower 40 m high a ball is projected upwards with a speed of 20 m/s at an angle 30° with the horizontal. The ratio of the total time of flight to hit the ground to the time taken by it to come back to the same initial elevation is ($g = 10\,m\,s^{-2}$)

1) 2:1   2) 3:1   3) 3:2   4) 4 :1

**29.** A body is thrown horizontally with a velocity u from the top of a tower.The displacement of the stone when the horizontal and vertical velocities are equal is

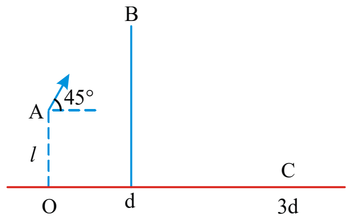
1) $\frac{u^2}{g}$   2) $\frac{u^2}{2g}$   3) $\sqrt{5}\left(\frac{u^2}{2g}\right)$   4) $\frac{2u^2}{g}$

## ACHIEVERS ( Level - II )

***Solve the following questions:***

**1.** A balll is thrown horizontally from a height of 20 m. If hits the ground with a velocity of '3' times the velocity of projection. The velocity of projection is 3.5*x* m/s, then *x* is

**2.** A projectile is launched at time t = 0 from point A which is at height 1m above the floor with speed v m/sec and at an angle $\theta = 45^0$ with the floor. It passes through a hoop at $B$ which is 1 m above $A$ and $B$ is the highest point of the trajectory. The horizontal distance between $A$ and $B$ is d metres. The projectile then falls into a basket, hitting the floor at C a horizontal distance 3d metres from A. Find *l* (in m).




## EXPLORERS ( Level - III )

**Assertion and Reason type:**

- *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.*

*1) Both A & R are true and R is correct explanation of A*

*2) Both A & R are true and R is not correct explanation of A*

*3) A is true but R is false.*   *4) Both A & R are false.*

**1.** **A) :** If a bomb is dropped from an aeroplane moving horizontally with constant velocity then the bomb appears to move along a vertical straight line for the pilot of the plane.

**R) :** Horizontal component of velocity of the bomb remains const and same as the velocity of the plane during the motion under gravity.

**2.** **A) :** Time taken by the bomb to reach the ground from a moving aeroplane depends on height of aeroplane only

**R) :** Horizontal component of velocity of bomb remains constant and vertical component of velocity of bomb changes due to gravity

**3.** **A) :** For a body projected horizontally from the top of a tower, the velocity on reaching the ground depends both on velocity of projection and height of the tower.

**R :** For a projectile velocity varies both in horizontal and vertical directions.

**Multi Answer Type:**

♦ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which* ***ONE or MORE*** *is correct. Choose the correct options*

**4.** A bomber flying horizontally with constant speed releases a bomb from an aeroplane.

a) The path of bomb as seen by the observer on the ground is parabola

b) The path of the bomb as seen by a pilot is a straight line.

c) The path of the aeroplane with respect to bomb is a straight line

d) The path of the bomb as seen by pilot observed as parabola.

1) a is correct 2) a and b are correct

3) a,b and c are correct 4) only d is correct

**5.** An aeroplane moving horizontally from west to east with some velocity and with an acceleration $5 m/s^2$ drops a food packet at some instant. Then :

a) The path of the packet is parabolic with respect to ground

b) A person sitting on the aeroplane shall see the packet is always vertically below the plane.

c) With respect to plane, the packet travels in a stright line making an angle $\tan^{-1}(1/2)$ west of vertical

d) With respect to plane, the packet travels in a stright line making an angle $\tan^{-1}(2)$ east of vertical



**Matrix Match Type:**

♦ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in* ***Column–I*** *have to be matched with statements (p, q, r, s) in* ***Column–II****. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*
*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

**6.** A ball is projected horizontally with a speed of 9.8 m/s from the top a tower of height 4.9 m. Then match the following two columns.

| **Column-I** | **Column-II** |
|---|---|
| A) Time of flight in seconds | p) 4.9 |
| B) Magnitude of velocity in m/s on reaching the ground | q) 9.8 |
| C) Horizontal range in meters | r) $9.8\sqrt{2}$ |
| D) Magnitude of change in vertical velocity in $\frac{1}{2}$s in m/s | s) 1 |

**7.** a) Equation of trajectory — e) $\sqrt{u^2+2gh}$

b) Range — f) $Tan^{-1}(gt/u)$

c) velocity aqt ground — g) $y=(g/2u^2)x^2$

d) The angle at which projectile strikes the ground — h) $u\sqrt{\frac{2h}{g}}$

1) a –g ; b – d ; c – a ; d – b    2) a – g ; b – h ; c – g ; d – f
3) a – f ; b – h ; c – g ; d – e    4) a – g ; b – e ; c – h ; d – f

**Comprehsion Type:**

♦ *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C ) and (D) out of which* ***ONLY ONE i****s correct. Choose the correct option.*

**Passage-I**

From a tower of height 19.6m two bodies are simultaneously projected horizontal in opposite directions, with velocites of 10m/s and 4 m/s respectively

**8.** The time taken for the velocites vectors of two bodies to become perpendicular to each othe is

1) 0.1s    2) 0.2s    3) 0.4s    4) 0.8s

**9.** The horizontal distance between the two bodies when their velocity vectors are perpendicular to each other is

1) 1m    2) 0.5m    3) 2m    4) 4m

**10.** The time taaken for the displacement vectors of the two bodies to become perpendicular to each other is

1) 0.1s    2) 0.2s    3) 0.4s    4) 0.8s

**Passage-II**

When a stone is projected horizontally from top of the tower height is 45m, with the velocityof 30m/s then,

**11.** Time of descent

1) 3s    2) 2s    3) 1s    4) 4s

**12.** The velocity with which it hits the ground

1) 20m/s    2) 30 m/s    3) $30\sqrt{2}m/s$    4) $20\sqrt{2}m/s$

**13.** The horizontal displacement or range

1) 45m    2) 90m    3) 25m    4) 30m

## RESEARCHERS ( Level - IV )

**I) Choose the correct option:**

**1.** A boat is moving with a velocity (3i+4j) with respect to ground. The water in the river is moving with a velocity (-3i-4j) with respect to ground. The relative velocity of boat with respect to water is **(2011E)**

1) 6i+8j    2) zero    3) 6i    4) 8j

**2.** Two persons A and B are located in x -y plane at the points (0, 0) and (o, 10) respectively (the distances are measured in mks units). At time t = 0, they start moving simultaneously with velocities $\vec{v}_A = 2\hat{j}$ m/s and $\vec{v}_B = 2\hat{i}$ m/s. The time after which A and B are at their closest distance **(2009M )**

1) 2.5 s 2) 4 s 3) 1 s 4) 10/ s

**3.** Two bodies are projected simultaneously in the same verticle plane from the same point with velocities $v_1$ and $v_2$ with angles $\theta_1$ and $\theta_2$ respectively with the horizantal. If $v_1 \cos\theta_1 = v_2 \cos\theta_2$, the path of one ball from the position of other ball is **(2010M)**

1) Parabola 2) Horizantal straight line

3) Vertical straight line 4) straight line making $45^0$ with the vertical

**4.** The maximum height reached by projectile is 4 metres. The horizontal range is 12m. Velocity of projection in m/s is (g - acceleration due to gravity) **(2004M)**

1) $5\sqrt{\frac{g}{2}}$ 2) $3\sqrt{\frac{g}{2}}$ 3) $\frac{1}{3}\sqrt{\frac{g}{2}}$ 4) $\frac{1}{5}\sqrt{\frac{g}{2}}$

**5.** Two stones are projected with the same speed but making different angles with the horizontal. Their horizontal ranges are equal. The angle of projection of one is p/3 and the maximum height reached by it is 102m. Then the maximum height reached by the other in metres is **(2003M)**

1) 336 2) 224 3) 56 4) 34

**6.** A projectile has initially the same horizontal velocity as it would acquire if it had moved from rest with uniform acceleration of $3m/s^2$ for 0.5 minute. If the maximum height reached by it is 80m, then the angle of projection is ($g=10m/s^2$) **(2002M)**

1) $\tan^{-1}(3)$ 2) $\tan^{-1}(3/2)$ 3) $\tan^{-1}(4/9)$ 4) $\sin^{-1}(4/9)$

**7.** The horizontal and vertical displacements of a projectile at a time 't' are x = 36t, y = 48t - $4.9t^2$ respectively. Initial velocity of the projectile is (in m/s) **(2012M)**

1) 15 2) 30 3) 45 4) 60

**8.** It is possible to project a particle with a given speed in two possible ways so that it has the same horizontal range R. The product of the times taken by it in the two possible ways is **(2014M)**

1) $\frac{R}{g}$ 2) $\frac{2R}{g}$ 3) $\frac{3R}{g}$ 4) $\frac{4R}{g}$

**9.** A body of mass $m_1$, projected vertically upwards with an initial velocity 'u' reaches a maximum height 'h'. Another body of mass $m_2$ is projected along an inclined plane making an angle $30^0$ with the horizontal with a velocity 'u'. The maximum distance travelled along the incline is **(2018M)**

1) 2h 2) h 3) h/2 4) h/4

**10.** A boy playing on the roof of a 10m high building throws a ball with a speed of 10 m/s at an angle of $30^0$ with the horizontal. How far from the throwing point will the ball be at the height of 10m from the ground? **[JEE-2016]**

$(g = 10\ m/s^2, \sin 30^0 = 1/2, \cos 30^0 = \sqrt{3}/2)$

a) 8.66 m b) 5.20 m c) 4.33 m d) 2.60 m

**11.** A projectile can have the same range R for two angles of projection. If $T_1$ and $T_2$ be the time of flights in the two cases, then the product of the two time of flights is directly proportional to [**JEE-2014**]

a) $1/R^2$ b) 1/R c) R d) $R^2$

**12.** The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is [**AIEEE-2007**]

a) $-2av^3$ b) $2av^2$ c) $-2abv^2$ d) $2bv^3$

**13.** A projectile can have the same range 'R' for two angles of projection. If '$t_1$' and '$t_2$' be the times of flights in the two cases, then the product of the two times of flight is proportional to [**AIEEE-2005**]

a) 1 / R b) R c) $R^2$ d) 1 / $R^2$

**14.** A particle is projected at $60^0$ to the horizontal witha kinetic energy K. The kinetic energy at the highest point is [**AIEEE-2007**]

a) K/2 b) K c) zero d) K/4

**15.** A projectile is projected under gravity with velocity $\sqrt{2ag}$ from a point at a height 'h' above the level plane at an anle '$\theta$' to it. The maximum range R on the ground is [**AIEEE2007**]

1) $\sqrt{(a^2+1)h}$ 2) $\sqrt{a^2h}$ 3) $\sqrt{a\,h}$ 4) $2\sqrt{a(a+h)}$

**16.** A ball is thrown vertically upwards with a speed of 10m/s from the top of the tower 200m high and another is thrown vertically downwards with the same speed simultaneously. The time difference between them in reaching the ground is (g=10m/s) (**1994E**)

1) 12s 2) 6s 3) 2s 4) 1s

**17.** A ball iis released from the top of a tower of height h meter. it takes T seconds to reach the ground. what is the position of the ball in T/3 seconds. (**2014A**)

1) h/9 meters from the ground 2) 7h/9 meters from the ground

3) 8h/9 meters from the ground 4) 17h/18 meters from the ground

**18.** A wooden block of mass 10g is dropped from the top of a cliff 1`00m high. simultameously a bullet of mass 10g is fired from the foot of the cliff upward with velocity 100m/s. The bullet and the wooden block will meet each after a time (**2011M**)

1) 10s 2) 0.5s 3) 7s 4) 1s

**II)** **Additional Worksheet for practice:**

**1.** A ball is projected at an angle of $30^0$ and $60^0$ to the horizontal with the same initial velocity in each case. Ratio of their times of flight is

1) 1:1 2) 1:3 3) 1:$\sqrt{3}$ 4) 2:$\sqrt{3}$

**2.** Two stones are projected with the same speed but making different angles with the horizontal. Their horizontal ranges are equal. The angle of projection of one is $\pi/3$ and the maximum height reached by it is 102m. Then the maximum height reached by the other in metres is

1) 336 2) 224 3) 56 4) 34

**3.** In the above problem, ratio of maximum height is

1) 1:1 2) 1:3 3) 1: $\sqrt{3}$ 4) 2: $\sqrt{3}$

**4.** For a projectile, the ratio of maximum height reached to the square of flight time is ($g=10ms^{-2}$)

1) 5:4 2) 5:2 3) 5:1 4) 10:1

**5.** In the above problem, ratio of ranges is

1) 1:1 2) 1:3 3) 1: $\sqrt{3}$ 4) 2: $\sqrt{3}$

**6.** The horizontal range of a projectile is $4\sqrt{3}$ times themaximum height achieved by it, then the angle of projection is

1) $30^0$ 2) $45^0$ 3) $60^0$ 4) $90^0$

**7.** If the range and maximum height of a projectile are respectively 'R' and 'H', the maximum range that could be obtained with the same velocity of projection is

1) 4H 2) 2R 3) $2H+\frac{R^2}{8H}$ 4) $2R+\frac{H^2}{8R}$

**8.** A body is projected horizontally from a height of 78.4 m with a velocity10 $ms^{-1}$ . Its velocity after 3 seconds is [ g = 10 $ms^{-2}$]

1) $10\hat{i}+30\hat{j}$ 2) $10\hat{i}+10\hat{j}$ 3) $20\hat{i}+30\hat{j}$ 4) $10\hat{i}+10\sqrt{3}\hat{j}$

**9.** A bomb is dropped from an aeroplane flying horizontally with a velocity of 720 kmph at an altitude of 980m. Time taken by the bomb to hit the ground is

1) 1 s 2) 7.2 s 3) 14.14 s 4) 0.15 s

**10.** A stone is thrown horizontally with velocity g $ms^{-1}$ from the top of a tower of height g metre. The velocity with which it hits the ground is (in $ms^{-1}$)

1) g 2) 2g 3) $\sqrt{3g}$ 4) 4g

**11.** Two particles A and B are prejected simultaneously freom the two towers of height 10m and 20m respectively. Particle A is projected with an initail speed of 10 $\sqrt{2}$ m/s at angle $45^0$ with horizontal, while particle B is projected horizontally with speed 10m/s. If they collide in aiar what is the distance 'd' between the towers

1) 10m 2)5m 3)2m 4) 15m

**12.** A ball is thrown horizontally from the top a tower with a speed 12.5m/s strikes the ground 75m away from the tower , if $g=10m/s^2$ , height of the tower is

1) 45m 2) 90m 3) 150 4) 180m

## KEY

**ΦΦ LEARNER'STASK :**

- **BEGINNERS :** 1) 3 2) 3 3) 4 4) 2 5) 4 6) 1 7) 3 8) 4 9) 4
  10) 4 11) 3 12) 1 13) 2 14) 2 15) 4 16) 1 17) 2 18) 3
  19) 2 20) 3 21) 3 22) 2 23) 3 24) 1 25) 3 26) 3 27) 4
  28) 1 29) 3
- **ACHIEVERS :** 1) 4 2) 3
- **EXPLORERS :** 1) 1 2) 2 3) 3 4) 3 5) a,c 6) A–s; B–r; C–q; D–p
  7) 1 8) 3 9) 1 10) 2 11) 1 12) 3 13) 2
- **RESEARCHERS:**

  **I)** 1) a 2) c 3) a 4) b 5) d 6) 1 7) 1 8) 3 9) 1 10) 4 11) 3
  12) 4 13) 2 14) 1 15) 4 16) 3 17) 3 18) 4

  **II)** 1) 3 2) 4 3) 2 4) 1 5) 1 6) 1 7) 3 **8**) 1 9) 3 10) 3 11)3
  12) 4