

9th CLASS
MATHEMATICS
IIT FOUNDATION
STUDY MATERIAL

8. DECIMAL REPRESENTATION OF RATIONAL NUMBERS.

TEACHING TASK

JEE MAIN LEVEL QUESTIONS

① Let $x = 1.\overline{27} = 1.272727\ldots$

product 100 on both sides

$$100x = 127.272727$$

Subtract $100x = 127.2727\ldots$

$$- x = 1.2727\ldots$$

$$\hline 99x = 126$$

$$x = \frac{126}{99} = \frac{14}{11}$$

Ans: A

② let $x = 0.\overline{47} - \text{①}$

multiplying equation by 100

$$100x = 100 \times 0.47\overline{7} \ldots$$

$$100x = 47.777\ldots - \text{②}$$

Subtract ① from ②

$$100x = 47.777$$

$$10x = 4.777$$

$$\hline 90x = 43.000$$

Multiplying with 10

$$10x = 4.777\ldots - \text{①}$$

$$x = \frac{43}{90}$$

Ans: C

③ $x = 0.82, y = 0.17, A = 0.7, B = 0.2$

$$= (x+y+A+B)^{(x+y)(A+B)}$$

$$\Rightarrow (0.82+0.17+0.7+0.2)^{(0.82+0.17)+(0.7+0.2)}$$

$$\Rightarrow (1.89)^{1.89}, \text{ here } 1.89 \approx 2$$

$$= (2)^2 = 4. \quad \text{Ans: A}$$

④ Let x be the number to be added.

$$2.34 + x = 4.53$$

$$x = 4.53 - 2.34$$

Ans: D

$$x = 2.19.$$

The closest option to 2.19 is 2.18

⑤ Let y be the real number

$$5.36 - y = 2.75$$

$$\Rightarrow 5.36 - 2.75 = y \quad \text{Ans: B}$$

$$\Rightarrow 2.61 = y.$$

⑥
★

$$a \times 3.27 = 7.83$$

$$a = \frac{7.83}{3.27} = \frac{783}{327} = \frac{261}{109} \quad \text{Ans: } \star$$

⑦

y be the number that divides 8.46

$$8.46 \div y = 3.43$$

$$y = 2.466$$

$$2.466 \approx \frac{2466}{1000} = \frac{1233}{500}$$

The closest option to $\frac{1233}{500}$ is $\frac{245}{103}$

Ans: A

⑧

$$1.4\overline{34} \times 100$$

$$\Rightarrow \frac{14\overline{34} \times 100}{100}$$

$$\Rightarrow \frac{1434}{10 \times 99}$$

Ans: C

$$\Rightarrow \frac{1434}{990} \approx \frac{1420}{990}$$

9) let $2.\overline{37} + 3.\overline{04} = 5.41$

Now, $\frac{x}{2} = 5.41$

$$x = 5.41 \times 2 = 10.8\overline{2}$$

$$\therefore 10.8\overline{2} \approx 10.8\overline{4} \quad \text{Ans: A}$$

10) $(8 + 0.\overline{6}) \frac{1}{2}$

$$\Rightarrow (8 + 0.\overline{6}) \frac{1}{2} = 8.\overline{6} \frac{1}{2} = 4.3 \Rightarrow \frac{43}{10}$$

Comparing with the Options:

$$A: 4\frac{9}{3} = \frac{12+9}{3} = \frac{21}{3} = 7$$

$$B: 3\frac{3}{9} = \frac{27+3}{9} = \frac{30}{9} = 3.\overline{3} \quad \text{Ans: C}$$

$$C: 4\frac{8}{9} = 4.3$$

11) Through Options

$$A: \frac{1}{6} = 0.166$$

$$B: \frac{1}{5} = 0.2$$

$$C: \frac{1}{7} = 0.142$$

$$D: \frac{1}{8} = 0.125$$

Ans: A

$\therefore \frac{1}{6}$ is closest Rational.

⑫ The product of $0.666\dots$ and $0.777\dots$
 $\Rightarrow 0.666 \times 0.777\dots$
 $= 0.5185\dots$ Ans: B

⑬ The decimal representation of $\frac{1}{7}$ is
 $\frac{1}{7} = 0.142857.$

The repeating block is 142857.

$\therefore$ The period length of this repeating block is 6.

Ans: C

⑭ The irrational number among the options is $\sqrt{2}$. The other options are rational.

Ans: C

⑮ $x = 0.123123\dots$ $\frac{1000x}{9} = \frac{1000 \times (0.123123\dots)}{9}$
 $= \frac{123.123123\dots}{9} = 13.68 \pm 13.67$

Ans: A

(16)

$$0.777\ldots = 0.\overline{7}$$

$$x = 0.\overline{7}$$

Multiply by 10 to shift the decimal point-

$$10x = 7.777\ldots$$

$$10x - x = 7.777\ldots - 0.777\ldots$$

$$9x = 7$$

$$\therefore x = 7/9$$

Ans: A

(17)

The decimal $0.142857142857\ldots$ is a Repeating decimal.

Through options:

Ans: A

$$A: \frac{1}{7} = 0.1428571429\ldots$$

ADVANCED LEVEL QUESTIONS
MULTIPLE CORRECT ANSWERS TYPE:

① Through options

A: $\frac{7}{12}$; this will change into a terminating decimal because $12 = 2^2 \times 3$

B: $\frac{5}{64}$, this will change into a terminating decimal because $64 = 2^6$

C: $\frac{13}{125}$ this will change into a terminating decimal because $125 = 5^3$

Ans: A, B and C.

② Through options

A: $\frac{15}{99} = 0.151515\ldots$ its a non-terminating.

D: $\frac{23}{60} = 0.38333\ldots$ its a non-terminating

Ans: A and D.

③

★ the rational number $\frac{3}{7} = 0.42857142\ldots$

Ans: $\frac{3}{7}$

(option wrong)

④ converting $\frac{7}{15}$ into decimal

$$\frac{7}{15} = 0.46666\ldots \quad \text{Ans: A and D.}$$

REASON AND ASSERTION TYPE

⑤ The assertion is true because any terminating decimal can be written as a fraction with a finite numerator and denominator.

For example 0.75 can be written as $\frac{75}{100}$ or simplified to $\frac{3}{4}$.

Ans: A

⑥ The assertion is true because any rational number can be represented as a terminating or repeating decimal.

The reason is also true because rational numbers are defined as numbers that can be expressed as a fraction of two integers.

Ans: A.

⑦ There any terminating decimals can be expressed as $a/10^n$. Repeating decimals cannot be expressed in this form.

→ The decimal system is indeed based on powers of 10, and any decimal number can be expressed in the way.

Ans: C

STATEMENT TYPE:

⑧ Statement-I: Not all rational numbers have a terminating decimal representation, some have repeating decimal representations.

Statement-II: Irrational numbers have non-terminating, non-repeating decimal representations.

Ans: D

⑨ Statement-I: While some rational numbers have non-terminating decimal representation, this is not true for all rational numbers.

Some have terminating decimal representation.

Statement-II: Irrational numbers always have non-terminating, non-repeating decimal representations.

Ans: C

(10) Statement-1: The decimal representation of $\frac{137}{25}$ is terminating, $\frac{137}{25} = 5.48$.

Statement-11: For a rational number to have a terminating decimal, its denominator, in simplest form must be of the form

$$2^m \times 5^n.$$

Ans: A

(11) Statement-1: When $a=5$, $b=5$.

$$\therefore \frac{8a}{12b} = \frac{8 \times 5}{12 \times 5} = \frac{40}{60} = \frac{2}{3} = 0.6666 \dots$$

This is incorrect as, $\frac{2}{3}$ has a repeating decimal not terminating.

Statement-11: For a rational number to have a terminating decimal, its denominator in simplest form must be of the form $2^m \times 5^n$.

Ans: D

COMPREHENSION TYPE

Comprehension - 1

(10) A number that can be expressed as a fraction with integer in the numerator and denominator. Ans: C

(11) Divide the numerator by the denominator. Ans: C

(12) Decimal representation of $\frac{5}{8} = 0.625$ Ans: A

(14) $0.333 \dots$ is the example for the repeating decimal. Ans: C

(15) It has a repeating pattern of digits after the decimal point. Ans: B

Comprehension- II

⑩ Through options

$$A: \frac{7}{12} = 0.5833\ldots$$

$$B: \frac{5}{44} = 0.113636\ldots$$

$$C: \frac{13}{125} = 0.104.$$

Ans: C

⑪ Through options

$$A: \frac{15}{99} = 0.151515\ldots \quad \text{Ans: A}$$

$$B: \frac{-9}{75} = 0.12$$

$$C: \frac{7}{20} = 0.35$$

$$D: \frac{23}{100} = 0.23$$

INTEGER TYPE:

⑫ The periodicity of $\frac{5}{13}$ is 6.

The decimal representation is 0.384615, where 384615 repeats indefinitely.

(19) The periodicity of $5.232323\dots$ is 2.
The digits 23 repeat indefinitely.

(20) Prime factorization of $1000 = 10^3 = (2 \times 5)^3 = 2^3 \times 5^3$
Prime factorization of the denominator in the form of $2^m \times 5^n$, $m=3, n=3$.

(21) The number of digits after the decimal point in the decimal representation of $\frac{25}{1000} = 0.025$ is 3.

Matrix matching type:

(20)
(a) $\frac{1}{3} = 0.333\dots$ Ans: Y

(b) $\frac{1}{16} = 0.0625$ Ans: Q

(c) $\frac{1}{7} = 0.142857\dots$ Ans: S

(d) $\frac{5}{2} = 2.5$ Ans: P

(21)

(a) $0.1616\ldots$ Ans: p

(b) $0.\overline{2}$ Ans: r

(c) $3.142857\ldots$ Ans: s

(d) $0.101001000\ldots$ Ans: q

LEARNERS TASK

Conceptual Understanding Question (CUP's)

- ① If the decimal representation of a number is non-terminating and non-repeating, then the number is "an irrational number".

Ans: D

②

$$\frac{3}{11} = 0.272727 \dots$$

$\therefore$ The period of $\frac{3}{11}$ is 27 Ans: B

③

$$\frac{1}{6} = 0.1\overline{666} \dots$$

Ans: A

$\therefore$ The periodicity of $\frac{1}{6}$ is 1.

④

$$1.\overline{79} = \frac{179}{99}$$

Ans: C

⑤

$$\frac{9}{11} = 0.818181 \dots$$

$$= 0.\overline{81}$$

Ans: A

$$\textcircled{6} \quad \frac{5}{13} = 0.384615\ldots$$

$$= 0.\overline{384615}$$

Ans: B

$$\textcircled{7} \quad 0.\overline{6} = \frac{2}{3}$$

Ans: C

$$\therefore 2.\overline{6} = 2 + \frac{2}{3}$$

$\textcircled{8}$ Through options

$$A: \frac{39}{24} = 1.625$$

$$B: \frac{3}{13} = 0.23076$$

Ans: C

$$C: \frac{3}{11} = 0.2727\ldots$$

$$D: \frac{137}{25} = 5.48$$

$\textcircled{9}$ The value of π is $\frac{22}{7}$

Ans: A

$\textcircled{10}$ Through options

$$A: \frac{1}{8} = 0.125$$

$$B: \frac{1}{7} = 0.14285\ldots$$

Ans: B

$$C: \frac{1}{2} = 0.5$$

⑫ $2.\bar{4} = \frac{22}{9}$

$\therefore 2.\bar{4} \times 3 = \frac{22}{\cancel{9}_3} \times \cancel{3}^1 = \frac{22}{3} = 7\frac{1}{3}$ Ans: B

⑪ Through options terminating decimal is

A: $\frac{17}{200} = 0.085$ Ans: A

⑬ $0.654 = \frac{654}{1000}$ Ans: B

JEE MAINS LEVEL QUESTIONS

① Through options

A: $\sqrt{\frac{4}{9}} = \frac{2}{3}$ it is a rational number

B: $\frac{4}{5}$ it is a rational number

C: $\sqrt{7}$ it is an irrational number

D: $\sqrt{81} = 9$ it is also a rational number.

Ans: C

② For the decimal $5.232323\dots$

Period: The repeating part is 32

Periodicity: The length of the repeating part, which is 2. Ans: D

③ $1.272727\dots = \frac{14}{11}$ Ans: B

④ $0.\overline{123} = \frac{123}{990}$ Ans:
★

⑤ $3.\overline{3} = 3\frac{1}{3}$, $7 - 3.\overline{3}$ is
 $7 - 3.\overline{3} = 7 - 3\frac{1}{3} \Rightarrow 7 - \frac{10}{3}$
 $\Rightarrow \frac{21-10}{3} = \frac{11}{3} = 3\frac{2}{3}$ Ans: D

⑥ $0.\overline{7} = \frac{7}{9}$, $0.\overline{8} = \frac{8}{9}$
 $\therefore 0.\overline{9} = \frac{9}{9} = 1$ Ans: D

⑦ $\frac{263}{125} = 2.104$

Ans: B

⑧ Through options

A: $\frac{7}{12} = 0.58\overline{3}$

B: $\frac{5}{44} = 0.11\overline{36}$

C: $\frac{13}{125} = 0.104$

D: $\frac{2}{9} = 0.22\overline{2}$

Ans: C

⑨ Through options

A: $\frac{3}{5} = 0.6$

B: $\frac{-9}{75} = 0.12$

C: $\frac{7}{20} = 0.35$

D: $\frac{23}{60} = 0.38\overline{33}$

Ans: D

⑩ $8.9\overline{34} \times 100 = x$

multiply by 1000 $\Rightarrow 8934.934934$

multiply by 10 $\Rightarrow 89.34349$

subtract $= 8934.93 \Rightarrow 1000x$

$- 89.34 = 10x$

$8845.5855 = 990x$

$x = \frac{8845.5855}{990} \times 10$

$= \frac{88450}{990}$

Ans: A

11

$$4.53 - 2.34 = 2.19$$

Ans: D

12

$$5.36 - 2.75 = 2.61$$

Ans: A

13

$$\frac{8.46}{x} = 3.43$$

$$x = \frac{8.46}{3.43} \Rightarrow \frac{\frac{846}{100}}{\frac{343}{100}} = \frac{846}{343} = \frac{254}{103}$$

$$x = \frac{254}{103}$$

Ans: D

14

$$A: \frac{7}{12} = 0.58\overline{33}$$

$$B: \frac{5}{44} = 0.11\overline{36}$$

Ans: C

$$C: \frac{13}{125} = 0.104$$

$$D: \frac{2}{9} = 0.2\overline{2}$$

ADVANCED LEVEL QUESTIONS

MULTIPLE CORRECT ANSWERS TYPE:

① $0.6 = \frac{6}{10}$ or $\frac{\cancel{3}6}{\cancel{10}5} = \frac{3}{5}$ Ans: A and D

② $0.\overline{16} = \frac{5}{30} = \frac{1}{6}$

B: $\frac{15}{90} = \frac{1}{6}$ Ans: B and C.

③ $\left| -\frac{3}{5} \right| - \left| \frac{1}{4} \right|$

= $\frac{3}{5} - \frac{1}{4} = \frac{12-5}{20} = \frac{7}{20}$ (or) 0.35 Ans: A and B

④ $0.\overline{3} - 0.\overline{18}$

= $0.333... - 0.1818... = 0.1515...$

$\therefore 0.1515 = \frac{15}{99} = \frac{5}{33}$

Ans: C

through options

A: $\frac{13}{99} = 0.1313...$

C: $0.14 = 0.1414...$

B: $\frac{13}{90} = 0.144...$

D: $\frac{13}{990} = 0.01313...$

⑤ $(7 + 0.\overline{26}) \frac{1}{2}$

let $x = 0.\overline{26}$

$$100x = 26.\overline{26}$$

Sub: $100x - x = 26$

$$99x = 26 \Rightarrow x = \frac{26}{99}$$

$$\Rightarrow (7 + 0.\overline{26}) = \left(7 + \frac{26}{99}\right) \Rightarrow \frac{693 + 26}{99} = \left(\frac{719}{99}\right) \times \frac{1}{2}$$

$$\Rightarrow \frac{719}{198} = 3.63$$

Ans: D

REASON AND ASSERTION TYPE:

- ⑥ Assertion: Any decimal number can be converted into a rational number of the form $\frac{p}{q}$ where p and q are integers $q \neq 0$.

Reason: The decimal representation of a number is a finite or repeating pattern of digits, which can indeed be expressed as a fraction.

Ans: A.

⑦ Assertion: Every rational number can be expressed as a terminating or repeating decimal.

Reason: The decimal representation of a rational number is determined by the prime factorization of its denominator.

Ans: A

⑧ Assertion: Every rational number can be expressed as a decimal, either terminating or repeating.

Reason: Rational numbers are those that can be represented as $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Ans: A

STATEMENT TYPE

⑨ Statement-1: $\frac{7}{16} = 0.4375$, true, as this is the correct decimal representation.

Statement-11: To convert a rational number into a decimal, we divide the numerator by the denominator. True, and it explains how to derive the decimal.

Ans: A

⑩ Statement-1: The correct approximation of π is 3.14159..., which is an irrational number and does not have a repeating decimal expansion.

Statement-11: In the case of non-terminating, repeating decimal expansion, the decimal representation has an infinite number of digits.

Thus, Statement-1 is false, while Statement-11 is true.

Ans: D

COMPREHENSION TYPE:

Comprehension-I:

- ⑪ $\sqrt{2}$ is a non-terminating, non-repeating decimal, approximately 1.414213....

Ans: A

Comprehension-II:

⑫ $\frac{1}{2} (2.\overline{37} + 3.\overline{04}) =$

$$\frac{1}{2} (2.\overline{37} + 3.\overline{04}) \Rightarrow \frac{5.41}{2} = 2.705$$

Rounding to two decimal places gives

2.71. Ans: A

⑬ $\frac{1}{2} (3.\overline{04} - 2.\overline{37})$

$$\Rightarrow \frac{1}{2} (0.67) = 0.335 \approx 0.\overline{3}$$

Ans: B

INTEGER TYPE :

(14) $\frac{3}{125}$; denominator $125 = 5^3$

Express in the required form: $125 = 2^0 \times 5^3$

$\therefore$ The power of 2 is 0.

(15) factor: $1000 = 10^3 = (2 \times 5)^3 = 2^3 \times 5^3$

$$2^m \times 5^n = 2^3 \times 5^3$$

$$\therefore m=3, n=3.$$

(16) $\frac{25}{1000} = 0.025$

calculate the digits after the decimal point.

There are 2 digits after the decimal point.

Matrix matching type:

(17)

(a) $\frac{1352}{90} = 15.02\overline{2} \dots = 15.0\overline{2}$ Ans: γ

(b) $\frac{4}{3} = 1.333\dots = 1.\overline{3}$ Ans: p

(c) $\frac{1}{4} = 0.25$ Ans: q

(d) $\frac{169}{450} = 0.37555\dots = 0.37\overline{5}$ Ans: t

(18)

(a) $\frac{2}{100} + \frac{3}{100} = \frac{5}{100} = 0.05$ Ans: q

(b) $10 \frac{1}{10} = \frac{100+1}{10} = \frac{101}{10} = 10.1$ Ans: γ

(c) $3 \frac{1}{4} = \frac{12+1}{4} = \frac{13}{4} = 3.25$ Ans: s

(d) $\frac{6}{25} = 0.24$ Ans: p

(19)

(a) $\frac{7}{8} = 0.875$ Ans: S

(b) $\frac{5}{3} = 1.666... = 1.\overline{6}$ Ans: P

(c) $\frac{66}{25} = 2.64$ Ans: Q

(d) $\frac{100}{9} = 11.1111... = 11.\overline{1}$ Ans: R